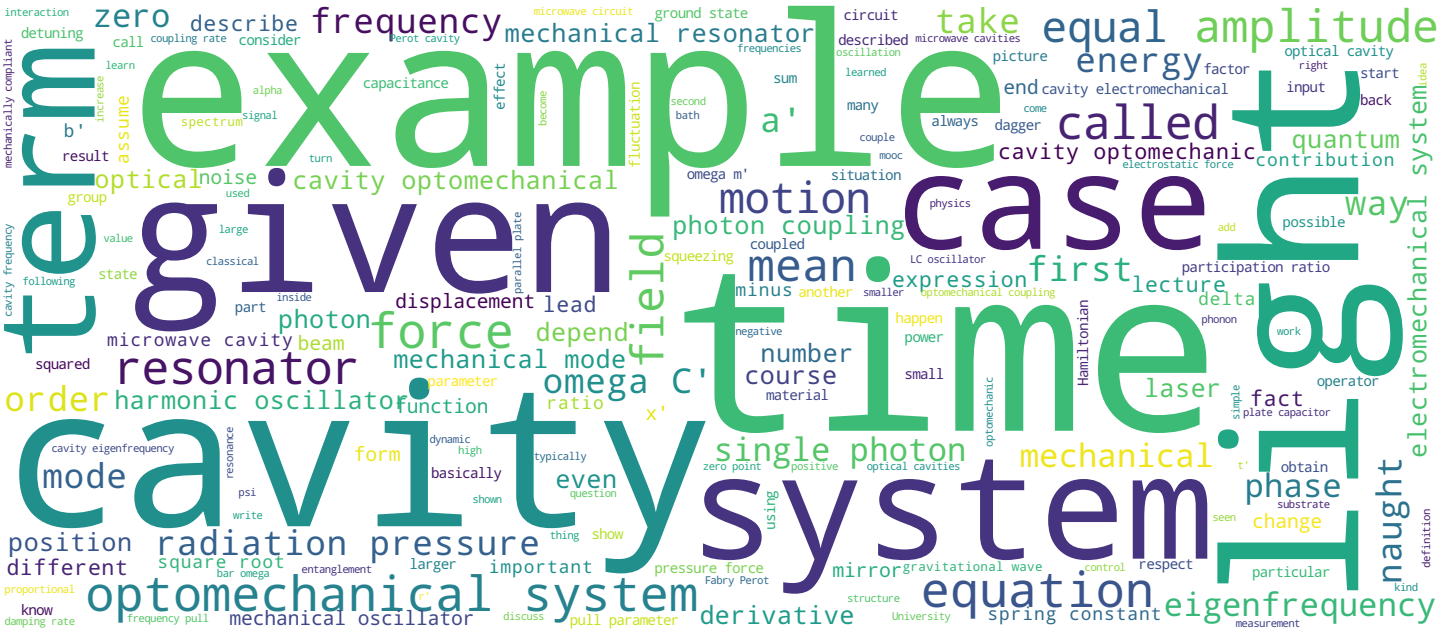
OPTOMECHANICAL FORCES IN CIRCUITS

Cavity Quantum Optomechanics

Eva Weig
University of Konstanz

Universität Konstanz



EPFL

OUTLINE

In this lesson, you will learn how to translate the concepts of optomechanical forces introduced in the previous lecture to circuits:

- **Cavity optomechanical systems can be implemented in microwave circuits**
- **Cavity opto- and cavity electromechanical systems are described by the same theoretical framework.**
- **Both implementations have their advantages and disadvantages.**
- **Cavity electromechanical systems offer additional, electrostatic controls**

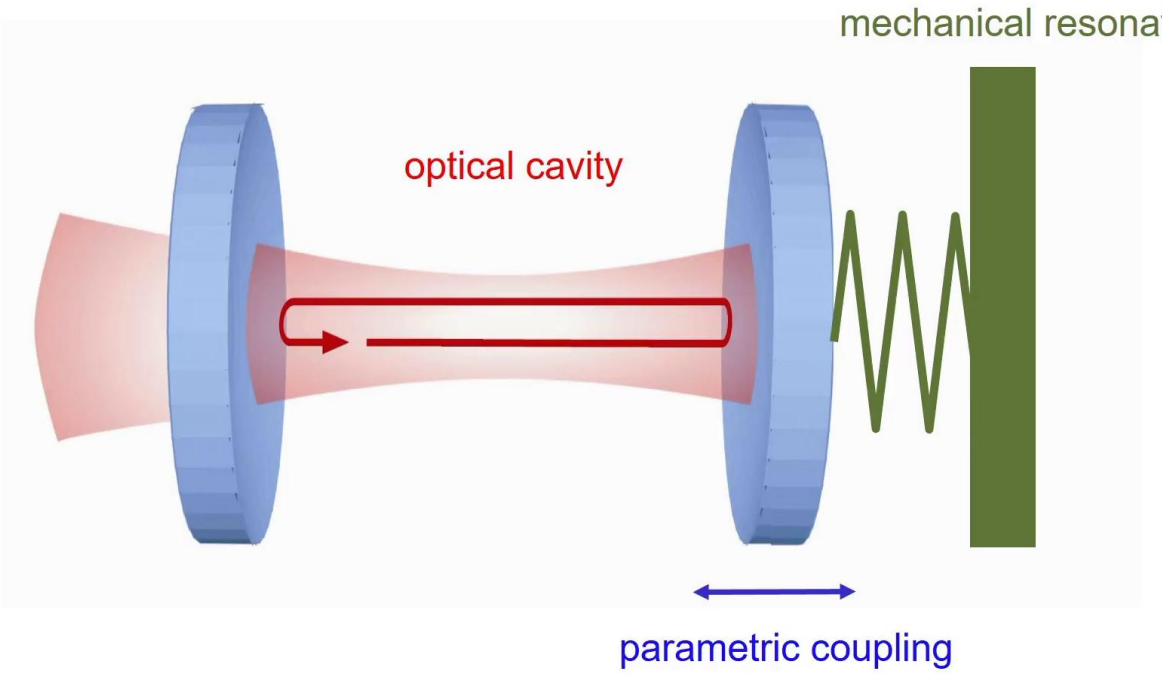
Welcome. My name is Eva Weig and I'm a professor at the University of Konstanz in Germany. In the previous lecture, you learned about optical forces. In this lecture, we will see that optomechanical forces also exist in circuits. This is lecture five of the mooc on cavity optomechanics. Here is a brief outline. In this lecture, you will learn how to translate the concepts of optomechanical forces, which have been introduced in the previous lectures, to circuits. In particular, we will see that cavity optomechanical systems can be implemented in microwave circuits and that cavity optomechanical systems and the resulting cavity electromechanical systems are described by the same theoretical framework. We will discuss the advantages and disadvantages of both implementations and we will finally see that cavity electromechanical systems offer additional electrostatic means to control the state of the resonator.

Notes

Summary

0m 00s

THE CANONICAL CAVITY OPTOMECHANICAL SYSTEM



Here you can see the canonical cavity optomechanical system which has been discussed in the previous lectures of this mooc. It consists of an optical cavity formed by two opposing mirrors and a mechanical element, a spring which is parametrically coupled to the cavity. In this case this is realized by having one of the mirrors attached to the spring such that the motion of the mechanically compliant mirror modulates the cavity eigenfrequency. These are the key ingredients of every optomechanical system.

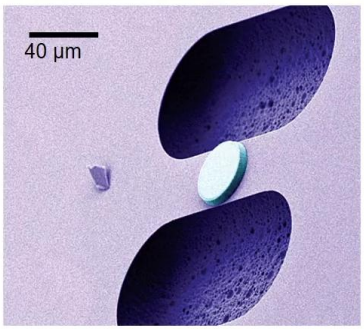
Notes

Summary

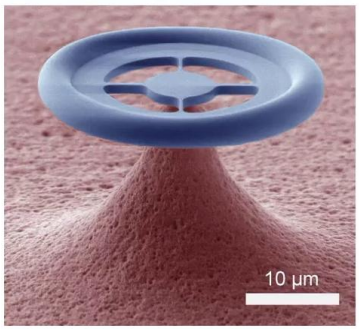
1m 09s

CAVITY OPTOMECHANICS

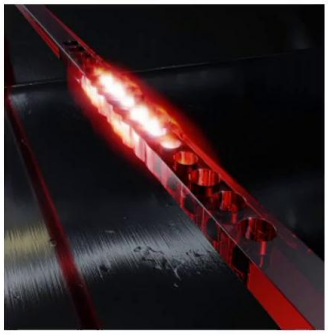
- So far, cavity optomechanics has been described in the language of optics.



Aspelmeyer group
University of Vienna



Kippenberg group
EPFL



Seidler group
IBM Research

So far cavity optomechanics has been described in the language of optics and here you can see a few prominent implementations which are based on optical cavities. A one-on-one realization of a mechanically compliant mirror from the group of Markus Aspelmeyer in Vienna. A microtoroid cavity in which light is circulating around its circumference to form a whispering-gallery mode while a mechanical bending type vibration of the structure is changing the length of the path of the light and thus induces parametric coupling. This is from the group of Tobias Kippenberg in Lausanne. A photonic nanostructure featuring a periodic arrangement of holes that is designed in a way that both light and sound waves are confined in a small defect in the center. Here a mechanical deformation changes the effective electrodynamic properties such as the refractive index which again leads to the desired parametric coupling. All these examples rely on cavities pumped by lasers. But is that the only way?

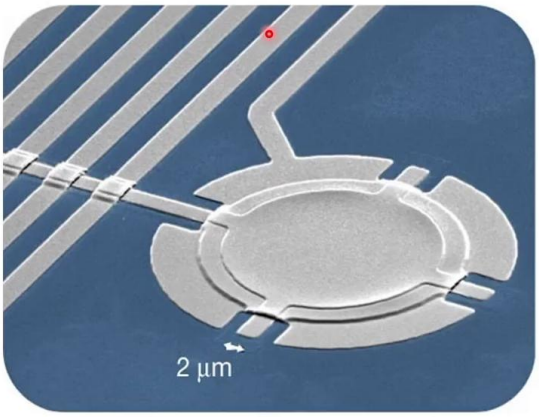
Notes

Summary

1m 49s

CAVITY ELECTROMECHANICS

- Is this a cavity optomechanical system?



John Teufel, NIST

Here you can see something different. A circular drum made from a superconducting material which can certainly vibrate and which is part of some circuit. Is that a cavity optomechanical system too?

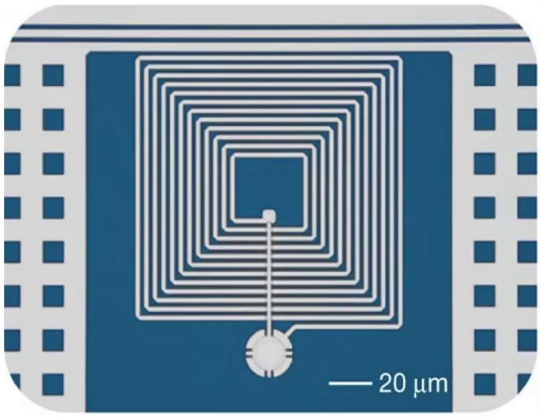
Notes

Summary

3m 09s

CAVITY ELECTROMECHANICS

- Is this a cavity optomechanical system?
- It is an electrical circuit made from superconducting metal, featuring an LC oscillator coupled to a mechanically compliant drum
- This type of optomechanical system is called a cavity *electro*mechanical system (or: *circuit electro*mechanical system)



John Teufel, NIST

If we inspect this a little more carefully, we see that the drum is actually one plate of a parallel plate capacitor. And that the capacitor is connected to a spiral inductor. The two form a lumped element LC oscillator and as the motion of the drum modulates the capacitance of the capacitor and thus the eigenfrequency of the LC circuit, we can recover the parametric coupling. So yes. This is a cavity optomechanical system. Optomechanical systems implemented in circuits are often denoted cavity electromechanical systems or even circuit electromechanical systems. They have the benefit of being compatible with a large toolbox available in superconducting circuitry including superconducting qubits. This is particularly useful for the generation of non-Gaussian states even though this aspect will not be addressed in this lecture.

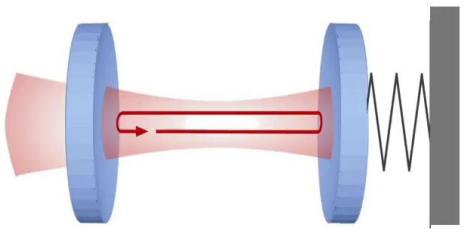
Notes

Summary

3m 27s

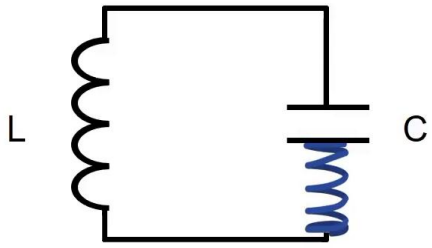
THE CIRCUIT WORLD OF CAVITY OPTOMECHANICS

- The standard description of cavity optomechanics uses the language of optics.



This cavity is populated with optical photons.

- Here we will develop an equivalent language parametrizing the cavity as a circuit.



This cavity is populated with microwave photons.

The standard description of cavity optomechanics uses the language of optics. Now we will develop a different language which parameterizes the cavity as an electrical circuit but what you should keep in mind is that a cavity optomechanical system is a harmonic oscillator which is parametrically coupled to another mechanical harmonic oscillator. The optical cavity is populated with optical photons. The LC cavity is populated with microwave photons. Both are bosons and both live inside a harmonic oscillator which forms the cavity. So both languages will be correctly describing the physics of our system.

Notes

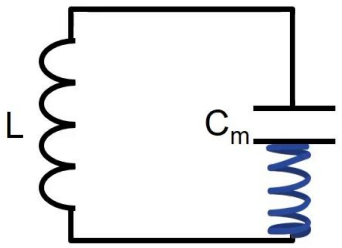
Summary

4m 39s

OPTOMECHANICAL COUPLING IN AN LC OSCILLATOR

Simple model:

- Eigenfrequency: $\omega_c(x) = \frac{1}{\sqrt{L \cdot C_m(x)}}$



- Frequency pull parameter:

$$G = \frac{\partial \omega_c(x)}{\partial x} = -\frac{\omega_c}{2C_m(x)} \cdot \frac{\partial C_m(x)}{\partial x}$$

For a parallel plate capacitor with $C_m = \frac{\varepsilon A}{x}$:

$$G = -\frac{\omega_c}{2C_m} \cdot \left(-\frac{\varepsilon A}{x^2}\right) = \frac{\omega_c}{2x}$$

So let's take a closer look. In the most simple model of an LC oscillator, we have the capacitor shunted with the inductor. The eigenfrequency of this cavity omega 'C' is given by one over the square root of 'L' times 'C' and if the capacitor has a mechanically compliant plate as indicated by the little spring, ‘C’ will depend on the plate position 'x'. Now we can compute the frequency pull parameter which tells us how much the cavity frequency changes upon a displacement of the mechanical resonator. It is given by the derivative of the cavity eigenfrequency with respect to the displacement 'x'. And if we differentiate the cavity frequency with respect to 'x', we obtain 'G' as minus omega 'C' over two times the capacitance 'Cm' times the derivative of the capacitance with respect to a displacement. Let's assume we have a parallel plate capacitor the capacitance of which is given by the ratio of the plate area over the plate separation 'A' over 'x' times the dielectric constant epsilon. We can easily compute the derivative which leads us to 'G' being equal to omega 'C' times two times 'x'.

Notes

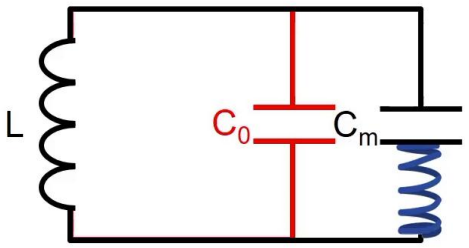
Summary

5m 31s

OPTOMECHANICAL COUPLING IN AN LC OSCILLATOR

Model incorporating the parasitic capacitance C_0 of the circuit:

- Eigenfrequency: $\omega_c(x) = \dfrac{1}{\sqrt{L \cdot (C_0 + C_m(x))}}$



- Frequency pull parameter:

$$G = \frac{\partial \omega_c(x)}{\partial x} = -\frac{\omega_c}{2C_0} \cdot \frac{\partial C_m(x)}{\partial x}$$

For a parallel plate capacitor with $C_m = \dfrac{\varepsilon A}{x}$:

$$G = -\frac{\omega_c}{2C_0} \cdot \left(-\frac{\varepsilon A}{x^2}\right) = \frac{\omega_c}{2x} \cdot \frac{C_m}{C_0 + C_m}$$ with participation ratio $\dfrac{C_m}{C_0 + C_m} < 1$

- Single photon coupling rate: $g_0 = G \cdot x_{zpf} = \dfrac{\omega_c}{2x} \cdot \dfrac{C_m}{C_0 + C_m} \cdot x_{zpf}$

And once we know 'G', we can easily compute one of the most important figures of merit of a cavity optomechanical system, the single photon coupling rate 'g-naught' by simply multiplying capital 'G' with the amplitude of the zero-point motion of the resonator 'xzpf'. That already gives us a basic understanding of our cavity optomechanical system. But this model is maybe a bit too simplistic. What we have neglected completely is that every circuit has a parasitic capacitance which adds to the capacitance of the moving drum. Let's call this extra capacitance C-naught and add it to our circuit by putting it in parallel with the mechanical capacitance 'Cm'. Then the cavity eigenfrequency is the sum of the two capacitances; C-naught plus 'Cm' rather than just 'Cm' and we can repeat our discussion which will now lead to the appearance of C-naught in all equations. All modifications to the cavity eigenfrequency, the frequency pull parameter and the single photon coupling rate are highlighted in red. For the parallel plate capacitor in the end we find the same result for capital 'G' and for g-naught times an additional factor which describes the ratio of the mechanically compliant capacitance 'Cm' over the total capacitance C-naught plus 'Cm'.

Notes

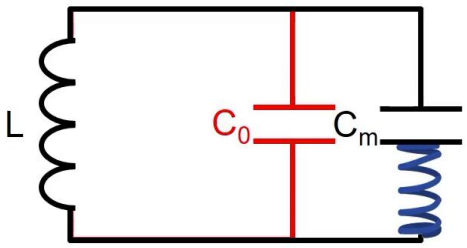
Summary

7m 15s

OPTOMECHANICAL COUPLING IN AN LC OSCILLATOR

Model incorporating the parasitic capacitance C_0 of the circuit:

- Eigenfrequency: $\omega_c(x) = \dfrac{1}{\sqrt{L \cdot (C_0 + C_m(x))}}$



- Frequency pull parameter:

$$G = \frac{\partial \omega_c(x)}{\partial x} = -\frac{\omega_c}{2C_0} \cdot \frac{\partial C_m(x)}{\partial x}$$

For a parallel plate capacitor with $C_m = \dfrac{\varepsilon A}{x}$:

$$G = -\frac{\omega_c}{2C_0} \cdot \left(-\frac{\varepsilon A}{x^2}\right) = \frac{\omega_c}{2x} \cdot \frac{C_m}{C_0 + C_m}$$ with participation ratio $\dfrac{C_m}{C_0 + C_m} < 1$

- Single photon coupling rate: $g_0 = G \cdot x_{zpf} = \dfrac{\omega_c}{2x} \cdot \dfrac{C_m}{C_0 + C_m} \cdot x_{zpf}$

This factor is called the participation ratio and describes how much of the circuit's capacitance is contributing to the mechanical coupling. Clearly we obtain the largest pull parameter in single photon coupling when the participation ratio is one. So in practical implementations the goal is always to keep the parasitic capacitance C-naught as small as possible to get the participation ratio approaching one.

Notes

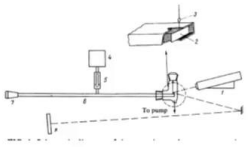
Summary

9m 10s

THE ZOO OF CAVITY ELECTROMECHANICAL SYSTEMS

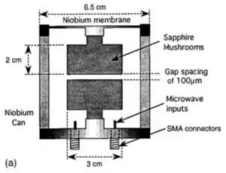
gravitational waves

Moscow State Univ.:



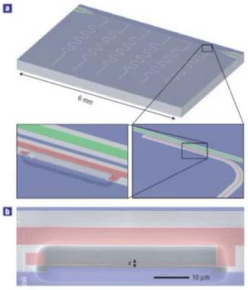
Braginskii et al., Sov. Phys. JETP 31, 829 (1970)

Univ. Western Australia:



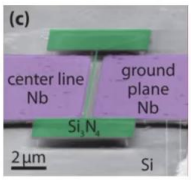
Cuthbertson et al., Rev. Sci. Instr. 67, 2435 (1996)

JILA:



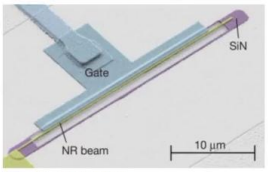
Regal et al., Nat. Phys. 4, 555 (2008)

WMI/LMU/Konstanz:



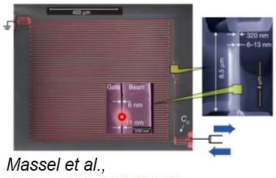
Pernpeintner et al., APL 105, 123106 (2014)

Cornell/Caltech:



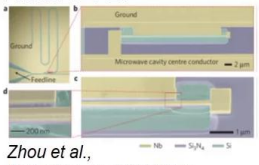
Rocheleau et al., Nature 463, 72 (2010)

Aalto:



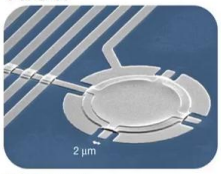
Massel et al., Nature 480, 351 (2011)

WMI/EPFL:



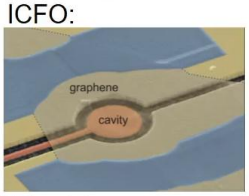
Zhou et al., Nat. Phys. 9, 179 (2013)

NIST:



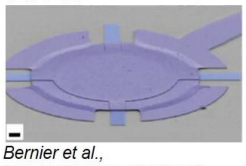
Teufel et al., Nature 471, 204 (2011)

ICFO:



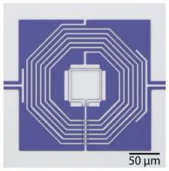
Weber et al., Nano Lett. 14, 2854 (2014)

EPFL:



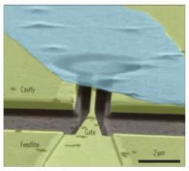
Bernier et al., Nat. Comms. 8, 604 (2017)

Caltech:



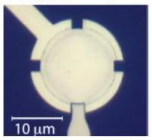
Wollman et al., Science 349, 952 (2015)

TU Delft:



Singh et al., Nat. Nano 9, 820 (2014)

Aalto:



Before we take a look at the specs of cavity electromechanical systems, I'd like to show you some examples of implementations to get you an idea of what I call the zoo of cavity electromechanical systems. Basically it turns out that the field is actually quite old and dates back to before the days the term cavity optomechanics was coined. The first papers have actually been published by the gravitational wave sensing community as early as in the 1970s, for example, at Moscow State University or at the University of Western Australia. But essentially the systems under investigation there are cavity electromechanical systems featuring pretty macroscopic microwave cavities and mechanical resonators. The field was sort of rediscovered in the early 2000s under the name of cavity optomechanics when on-chip nano or microscale implementations were on the rise and since then the field has been thriving. At the beginning, one-dimensional resonators, such as nanoscale strings, have been considered which were coupled to coplanar waveguide resonators such as in the landmark experiment from Konrad Lehnert's group at JILA.

Notes

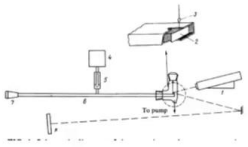
Summary

9m 45s

THE ZOO OF CAVITY ELECTROMECHANICAL SYSTEMS

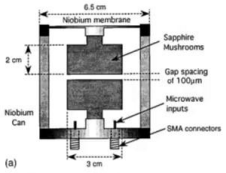
gravitational waves

Moscow State Univ.:



Braginskii et al., Sov. Phys. JETP 31, 829 (1970)

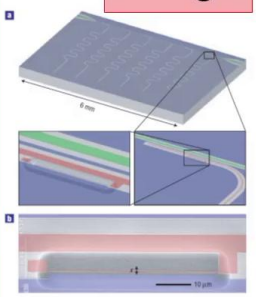
Univ. Western Australia:



Cuthbertson et al., Rev. Sci. Instr. 67, 2435 (1996)

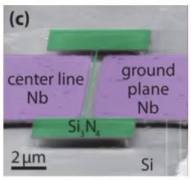
strings

JILA:



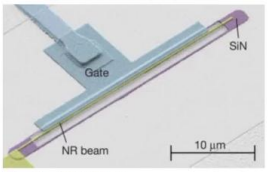
Regal et al., Nat. Phys. 4, 555 (2008)

WMI/LMU/Konstanz:



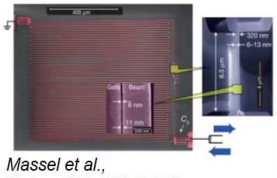
Pernpeintner et al., APL 105, 123106 (2014)

Cornell/Caltech:



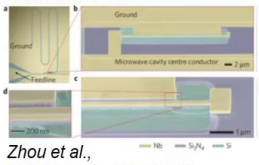
Rocheleau et al., Nature 463, 72 (2010)

Aalto:



Massel et al., Nature 480, 351 (2011)

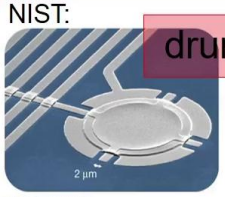
WMI/EPFL:



Zhou et al., Nat. Phys. 9, 179 (2013)

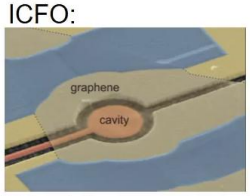
drums

NIST:



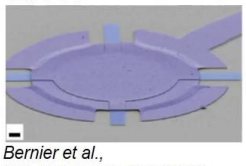
Teufel et al., Nature 471, 204 (2011)

ICFO:



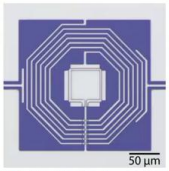
Weber et al., Nano Lett. 14, 2854 (2014)

EPFL:



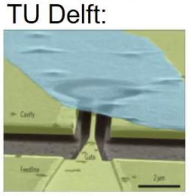
Bernier et al., Nat. Comms. 8, 604 (2017)

Caltech:



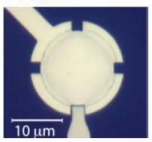
Wollman et al., Science 349, 952 (2015)

TU Delft:



Singh et al., Nat. Nano 9, 820 (2014)

Aalto:



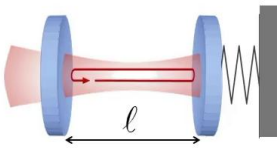
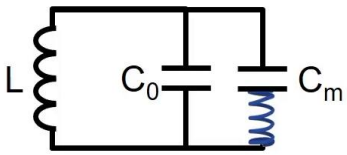
As you can see variations of that system were studied in many other groups, for example, at Cornell Caltech, Aalto, the WMI and the EPFL or the WMI, LMU and University of Konstanz. In recent years, however, the community largely switched to two-dimensional resonators, that is, micron scale drums forming parallel plate capacitors with a very small gap between the plates. This was pioneered by John Tefuel at NIST but rapidly picked up by others. For instance, at Caltech, the TU Delft, Alto, EPFL or ICFO. Why? Well, these drums have a very large participation ratio. In the drum capacitor, the electromagnetic field is concentrated in an extremely small volume which is much smaller than the wavelength of the cavity and the field is precisely where the mechanical mode is located so that the modal overlap is very large so all in all, the drums allow to achieve very large single photon couplings g-naught.

Notes

Summary

11m 18s

COUPLING & RADIATION PRESSURE FORCE

	 $\frac{\omega_c}{2\pi} = \frac{c}{\lambda}$ typ. 300 THz	 $\frac{\omega_c}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{L \cdot C}}$ typ. 10 GHz
Single photon coupling: $g_0 = \frac{\partial \omega_c}{\partial x} \cdot x_{zpf} = G \cdot x_{zpf}$	$g_0 = \frac{\omega_c}{\ell} \cdot x_{zpf}$	$g_0 = \frac{\omega_c}{2x} \cdot x_{zpf} \cdot \frac{C_m}{C_0 + C_m}$
Radiation pressure force: $F_{rad} = -\frac{\partial E}{\partial x} = -\frac{\partial (N\hbar\omega_c)}{\partial x} = N\hbar G$	$F_{rad} = \frac{N\hbar\omega_c}{\ell}$	$F_{rad} = \frac{N\hbar\omega_c}{x} \cdot \frac{C_m}{C_0 + C_m}$

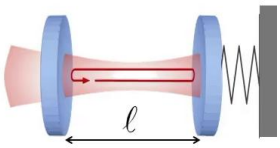
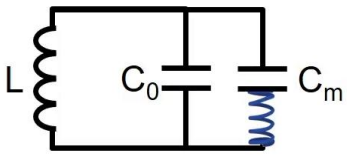
Now let's compare the standard Fabry-Perot cavity with our LC oscillator approach. Obviously, the two systems have very different eigenfrequencies that of the Fabry-Perot cavity is given by the ratio of the speed of light ‘C’ over the wavelength of the light which is of order of one micron which leads to an eigenfrequency of the cavity of typically around 300 terahertz. For the LC oscillator, the eigenfrequency is given by the square root of one over LC which typically ends up being a few or up to ten gigahertz. This is four orders of magnitude below that of the optical cavity. If we now consider the single photon coupling rate g-naught, we find that for the Fabry-Perot cavity, it depends on the ratio of the cavity eigenfrequency omega 'C' over the length of the cavity 'L' which can be millimeters or even centimeters times the zero-point motion. For the LC oscillator, it scales as omega 'C' over '2x' as we just discussed. The factor of two comes from the derivative of the square root so it's nothing to worry about. And, of course, we have to multiply with the zero-point motion and with the participation ratio. For now let's consider we have a drum and we can assume the participation ratio being close to one.

Notes

Summary

12m 40s

COUPLING & RADIATION PRESSURE FORCE

	 $\frac{\omega_c}{2\pi} = \frac{c}{\lambda}$ typ. 300 THz	 $\frac{\omega_c}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{L \cdot C}}$ typ. 10 GHz
Single photon coupling: $g_0 = \frac{\partial \omega_c}{\partial x} \cdot x_{zpf} = G \cdot x_{zpf}$	$g_0 = \frac{\omega_c}{\ell} \cdot x_{zpf}$	$g_0 = \frac{\omega_c}{2x} \cdot x_{zpf} \cdot \frac{C_m}{C_0 + C_m}$
Radiation pressure force: $F_{rad} = -\frac{\partial E}{\partial x} = -\frac{\partial (N\hbar\omega_c)}{\partial x} = N\hbar G$	$F_{rad} = \frac{N\hbar\omega_c}{\ell}$	$F_{rad} = \frac{N\hbar\omega_c}{x} \cdot \frac{C_m}{C_0 + C_m}$

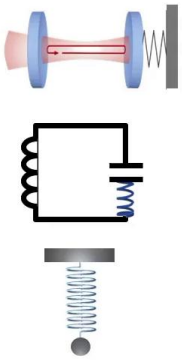
Then in the numerator we lose four orders of magnitude compared to the optical cavity which does not make it sound like a good idea to use microwave cavities. However, in the denominator we win because here the ‘x’ is the separation between the plates which can be engineered to get down to the order of a few tens of nanometers. So we can gain at least the same four orders of magnitude back or even more. This means we can get a very large single photon coupling in microwave circuits. The same argument can be made for the radiation pressure force. This force is obtained as the derivative of the electrostatic energy which is given by the number of photons in the cavity 'N' times the characteristic energy scale h-bar omega 'C'. So it depends on the number of photons times h-bar times the derivative of the cavity frequency which is the frequency pull parameter. For the Fabry-Perot cavity this again has omega 'C' over the cavity length 'l' whereas for the microwave cavity we find omega 'C' over plate separation 'x' and, of course, times the participation ratio.

Notes

Summary

14m 29s

FIGURES OF MERIT



$\omega_c/2\pi$	T	$\hbar\omega_c/k_BT$	$n_{c,th} = \frac{1}{e^{\hbar\omega_c/k_BT}-1}$	$P = \frac{1}{1+n_{c,th}}$	$g_0/2\pi = \frac{\omega_c \cdot C_m}{C_0+C_m}\frac{x_{zpf}}{4\pi x}$	n_d	$g/2\pi = \frac{g_0}{2\pi}\sqrt{n_d}$
300 THz	300 K	50	10^{-21}	1	10^2 Hz *10^7 Hz* (*)	10^3	10^3 Hz *10^8 Hz* (*)
10 GHz	50 mK	10	10^{-4}	0.9999	10^2 Hz	10^8	10^6 Hz
10 MHz	50 mK	0.01	10^{+2}	0.01			

(*) *photonic implementations of optomechanical systems, e.g. Leijssen et al., Nat. Comms. 8, 16024 (2017)*

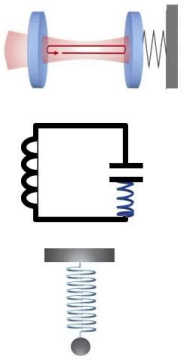
So let's look at some more specs to be able to compare the two kinds of optomechanical systems in more detail. We've seen that the optical cavities have eigenfrequencies around 300 terahertz so even when they are operated at room temperature 300 kelvin the ratio of their characteristic energy over the thermal energy h-bar omega 'C' over 'kBT' is very large of order 50 or so. This translates into a very small thermal population of the cavity mode. The thermal population in c-thermal is computed using the Bose–Einstein distribution and as you can see it's essentially zero here. So we can safely assume to have our cavity in the ground state and indeed if you work out the ground state population you find this to be one. And this is what we need as we want to use it to control the resonator. For the microwave cavity, the situation is a little bit different. Its eigenfrequency is around ten gigahertz. This means we need to cool it to about 50 millikelvin to have a reasonably small thermal occupation and a ground state probability approaching one. At room temperature the microwave cavity would be in a more complicated displaced thermal state.

Notes

Summary

15m 56s

FIGURES OF MERIT



$\omega_c/2\pi$	T	$\hbar\omega_c/k_BT$	$n_{c,th} = \frac{1}{e^{\hbar\omega_c/k_BT}-1}$	$P = \frac{1}{1+n_{c,th}}$	$g_0/2\pi = \frac{\omega_c \cdot C_m}{C_0+C_m}\frac{x_{zpf}}{4\pi x}$	n_d	$g/2\pi = \frac{g_0}{2\pi}\sqrt{n_d}$
300 THz	300 K	50	10^{-21}	1	10^2 Hz *10^7 Hz* (*)	10^3	10^3 Hz *10^8 Hz* (*)
10 GHz	50 mK	10	10^{-4}	0.9999	10^2 Hz	10^8	10^6 Hz
10 MHz	50 mK	0.01	10^{+2}	0.01			

(*) *photonic implementations of optomechanical systems, e.g. Leijssen et al., Nat. Comms. 8, 16024 (2017)*

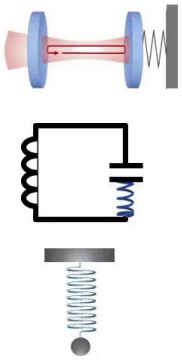
But temperatures in the tens of millikelvins are easily attainable in dilution refrigerators so this is not a big issue. Especially keeping in mind that the microwave circuits considered so far are based on superconducting materials which need to be cooled to a very similar temperature range to function properly anyway. Finally, for the sake of comparison let's also take a look at the mechanical resonator. Typical eigenfrequencies of mechanical resonators are of order ten megahertz. So here even at 50 millikelvin, we have a substantial thermal population. In order to get also the mechanical mode into the ground state by conventional cooling would require a temperature below a 100 microkelvins which is not feasible for most of these systems. That is why we use the optical or the microwave cavity to cool and to manipulate the mechanics. Now let's go back to the optical versus the microwave cavity. We have already discussed that despite the smaller eigenfrequency, the single photon coupling rate can be quite comparable for the optical and the microwave cavity.

Notes

Summary

17m 32s

FIGURES OF MERIT



$\omega_c/2\pi$	T	$\hbar\omega_c/k_BT$	$n_{c,th} = \frac{1}{e^{\hbar\omega_c/k_BT}-1}$	$P = \frac{1}{1+n_{c,th}}$	$g_0/2\pi = \frac{\omega_c \cdot C_m}{C_0+C_m}\frac{x_{zpf}}{4\pi x}$	n_d	$g/2\pi = \frac{g_0}{2\pi}\sqrt{n_d}$
300 THz	300 K	50	10^{-21}	1	10^2 Hz *10^7 Hz* (*)	10^3	10^3 Hz *10^8 Hz* (*)
10 GHz	50 mK	10	10^{-4}	0.9999	10^2 Hz	10^8	10^6 Hz
10 MHz	50 mK	0.01	10^{+2}	0.01			

(*) *photonic implementations of optomechanical systems, e.g. Leijssen et al., Nat. Comms. 8, 16024 (2017)*

As a result of the small separation between the two capacitor plates in the microwave circuit, a typical number would be roughly a 100 hertz but I should also mention that the same trick of concentrating the electromagnetic field near the mechanical mode can also be applied to the optical cavities. For example, in nanophotonic implementations much larger couplings of up to ten megahertz can be found as indicated as a footnote here. But in many cases the single photon coupling is small. In the so-called linearized regime of cavity optomechanics, the optomechanical coupling 'g' can be boosted by pumping photons into the cavity. You will learn more about this in other lectures of this mooc. And here's a huge advantage of microwave cavities whereas optical cavities can typically host a few thousand photons before unwanted effects occur. This number is orders of magnitude larger for microwave cavities which can hold up to almost a billion of photons. So the total coupling 'g' which amounts to g-naught times the square root of the number of cavity photons 'nd' can actually reach the megahertz regime which is larger than what is possible in many optical implementations. So to conclude, there is no clear better optomechanical system. The different implementations vary in their specifications and it really depends on the designated experiment to find out what is the best choice.

Notes

Summary

18m 55s

ELECTROSTATIC ACTUATION

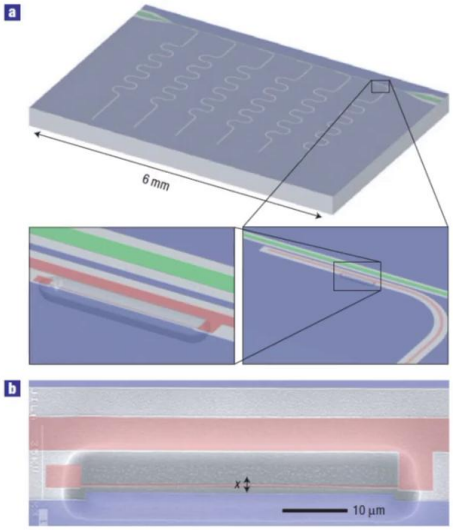
The circuit can be employed to directly actuate the mechanical resonator:

- Electrostatic energy: $E = -\frac{1}{2}CV^2$
- Force:

$$F = -\frac{\partial E}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}V^2 = \frac{1}{2}\frac{\partial C}{\partial x}\left(V_{DC} + V_{AC}\right)^2$$

$$= \underbrace{\frac{1}{2}\frac{\partial C}{\partial x}V_{DC}^{\,2}}_{DC} + \underbrace{\frac{1}{2}\frac{\partial C}{\partial x}V_{AC}^{\,2}}_{\sim 2\omega_m} + \underbrace{\frac{\partial C}{\partial x}V_{DC}V_{AC}}_{\sim \omega_m}$$

$$= \frac{\partial C}{\partial x}V_{DC}V_{AC}$$



Regal et al., Nat. Phys. 4, 555 (2008)

There are a few extra benefits of electromechanical systems. For example, the circuit can be employed to directly actuate the mechanical resonator electrostatically as it is conveniently done also in Microelectromechanical Systems, MEMS. We start with the electrostatic energy which is given by minus one-half times the capacitance 'C' times the voltage squared. From that we obtain the force by differentiating with respect to 'x' which is equal to one-half del-C over del-x times V-squared. Now we assume that we can apply both a DC voltage and an AC voltage which is approximately at resonance with the mechanical mode. In this example here from the Konrad Lehnert group, this is done through a small coupling of the mechanical resonator to the microwave cavity feed line. If we factor the contribution of the DC and the AC voltage out, we get one contribution to the force which does not depend on time and we neglect this constant first term. A second term goes with VAC-squared which means that it oscillates at two omega 'm' rather than at omega ‘m’. We also neglect this faster oscillating term. Finally, we end up with a resonant driving term del-C over del-x times VDC times VAC which drives the resonator at omega 'm' and that's our electrostatic drive term.

Notes

Summary

20m 45s

CAPACITIVE SOFTENING

The circuit further allows for eigenfrequency tuning of the mechanical resonator:

- DC component of electrostatic force:

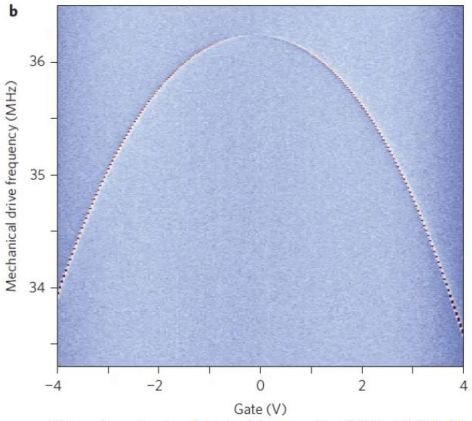
$$F = \frac{1}{2}\frac{\partial C}{\partial x}V_{DC}^{\ 2}$$

- Spring constant:

$$k = k_0 + k_{el} = k_0 - \frac{\partial F}{\partial x} = k_0 - \frac{1}{2}\frac{\partial^2 C}{\partial x^2}V_{DC}^{\ 2}$$

- Eigenfrequency:

$$\omega_m = \sqrt{\frac{k_0 + k_{el}}{m_{eff}}} \approx \sqrt{\frac{k_0}{m_{eff}}}\left(1 - \frac{1}{4k_0}\frac{\partial^2 C}{\partial x^2}V_{DC}^{\ 2} + \ldots\right)$$



Singh et al., Nat. Nano 9, 820 (2014)

The eigenfrequency of the mechanical resonator can also be tuned electrostatically. Here we focus on the DC component of the electrostatic force, the first term on the previous slide. It was not important for driving but as we will see shortly, it creates a contribution to the spring constant of the resonator. The spring constant of the resonator 'k' is given by the intrinsic spring constant k-naught arising from the elastic properties of the resonator plus the electrostatic correction k-el which can be expressed as the derivative of the electrostatic force with respect to the displacement del-F over del-x. That's the definition of a spring constant. Working out the derivative we find that 'k' is equal to k-naught minus one-half times the second derivative of the capacitance times the DC voltage squared. We can express the change of the spring constant as a change of the eigenfrequency by remembering that omega 'm' is the square root of spring constant over effective mass. Assuming that k-el will be small compared to k-naught, we can Taylor expand to find that omega 'm' tunes quadratically with VDC and again, we have a negative sign that is the eigenfrequency is tuning towards smaller frequencies as VDC is increased.

Notes

Summary

22m 47s

CAPACITIVE SOFTENING

The circuit further allows for eigenfrequency tuning of the mechanical resonator:

- DC component of electrostatic force:

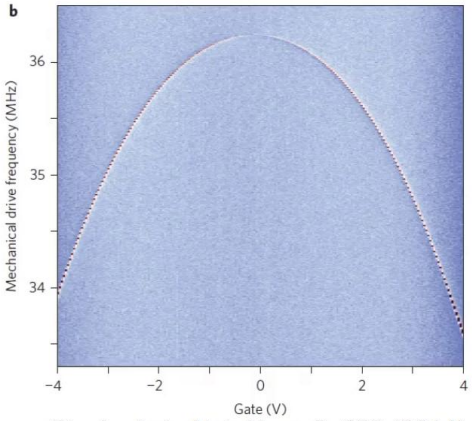
$$F = \frac{1}{2}\frac{\partial C}{\partial x}V_{DC}^{\ 2}$$

- Spring constant:

$$k = k_0 + k_{el} = k_0 - \frac{\partial F}{\partial x} = k_0 - \frac{1}{2}\frac{\partial^2 C}{\partial x^2}V_{DC}^{\ 2}$$

- Eigenfrequency:

$$\omega_m = \sqrt{\frac{k_0 + k_{el}}{m_{eff}}} \approx \sqrt{\frac{k_0}{m_{eff}}}\left(1 - \frac{1}{4k_0}\frac{\partial^2 C}{\partial x^2}V_{DC}^{\ 2} + \ldots\right)$$



Singh et al., Nat. Nano 9, 820 (2014)

You can see this in this example from Gary Steele's group at TU Delft. Tuning towards lower frequencies is referred to as a softening behavior of the resonator so here we see an example of capacitive softening induced by the electrostatic force. Note that there is also another frequency tuning mechanism elastic stiffening that can increase the eigenfrequency. This is observed when an increase of VDC changes the static deflection of the resonator so much that the tension increases significantly. In the example shown here this mechanism is negligible.

Notes

Summary

24m 35s

CONCLUSION / SUMMARY

In this lesson, you learned how to translate the concepts of optomechanical forces to circuits:

- **Cavity electromechanical systems are the circuit analogy of cavity optomechanical systems**
- **They can exhibit large single photon coupling and can host a large number of photons, but they need to be operated at cryogenic temperatures**
- **The mechanical mode can be additionally driven and tuned in frequency by means of electrostatic forces**

So let me conclude. In this lesson, you learned how to translate the concepts of optomechanical forces to circuits. In particular, you saw that cavity electromechanical systems are the circuit analogy of cavity optomechanical systems, that they can exhibit large single photon coupling and can host a large number of photons but that they need to be operated at cryogenic temperatures. And we saw that the mechanical mode can be additionally driven and tuned in eigenfrequency by means of the electrostatic forces at play.

Notes

Summary

25m 21s