



# WELCOME & OUTLINE

In this lesson, we will learn about the coupling, at a classical level, between optics / mechanics and the dynamical effects due to retarded nature of the radiation-pressure force :

- Theoretical description of the coupling
- Optical spring effect
- Optomechanical damping rate : Amplification & Cooling

Hello everyone. I am Rémy Braive from Université de Paris and Center of Nanoscience and Nanotechnology. Today I'm going to explain you Dynamical Backaction in the frame of optomechanics. In this lesson we will learn about the coupling at a classical level between optics and mechanics and the dynamical effect due to retarded nature of the radiation-pressure force. In order to look at this, we will start with a theoretical description of the coupling between optics and mechanics and we will look at the induced effect of this coupling; namely, the optical spring effect and the change in the optomechanical damping rate going towards amplification and cooling of the mechanical resonators.

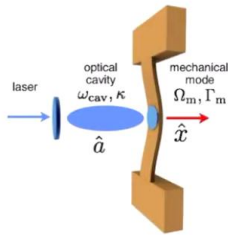
Notes

Summary



# COUPLED EQUATION OF MOTION

## Classical Optomechanical equation of motion



Light amplitude

$$\dot{\alpha} = -\frac{\kappa}{2}\alpha + i(\Delta + Gx)\alpha + \sqrt{\kappa_{ex}}\alpha_{in}$$

Mechanical displacement

$$m_{eff}\ddot{x} = -m_{eff}\Omega_m^2x - m_{eff}\Gamma_m\dot{x} + \hbar G|\alpha|^2$$

Linearized version :  $\alpha = \bar{\alpha} + \delta\alpha$

$$\left\{ \begin{array}{l} \delta\dot{\alpha} = \left(i\Delta - \frac{\kappa}{2}\right)\delta\alpha + iG\bar{\alpha}x \\ m_{eff}\ddot{x} = -m_{eff}\Omega_m^2x - m_{eff}\Gamma_m\dot{x} + \hbar G(\bar{\alpha}^*\delta\alpha + \bar{\alpha}\delta\alpha^*) \end{array} \right.$$

In this frame we look at a standard classical optomechanical device; namely, an optical cavity and coupled to a mechanical mode. So let's start with the optical cavity. The light evolution amplitude inside this cavity is given by this first equation here with the linewidth of the cavity and the power injected inside the cavity. Of course, you have the coupling with the optomechanical oscillator which is shown with this capital 'G' term here. In the case of the mechanical displacement of the mechanical mode, you have this equation here. With the acceleration of the mechanical mode which is vibrating at the mechanical frequency omega 'm' with a mechanical damping gamma 'm' and again the coupling term for the optomechanical interaction which is given by this term 'G' alpha squared. If we try to linearize this equation taking into account a mean value of photons and some fluctuation around this mean value, we can rewrite the previous expression with this set of two expression here and we have the mean value of the number of photon in the cavity with some fluctuation.

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0m 45s

# COUPLED EQUATION OF MOTION

In the frequency space :  $\delta\alpha = \delta\alpha[\omega]e^{-i\omega t}$   $x = x[\omega]e^{-i\omega t}$

$$\begin{cases} -i\omega\delta\alpha[\omega] = \left(i\Delta - \frac{\kappa}{2}\right)\delta\alpha[\omega] + iG\bar{\alpha}x[\omega] \\ -m_{eff}\omega^2x[\omega] = -m_{eff}\Omega_m^2x[\omega] + i\omega m_{eff}\Gamma_m x[\omega] + \hbar G(\bar{\alpha}^*\delta\alpha[\omega] + \bar{\alpha}(\delta\alpha^*)[\omega]) \end{cases}$$

$$(\delta\alpha^*)[\omega] = \delta\alpha[-\omega]^*$$

Solving the linearized  
coupled equation of motion  
for the light and mechanics



Dynamical effects due to  
retarded nature of radiation  
pressure

If we want to try now to look in the frequency space for this equation, we inject this expression alpha, delta alpha and 'x' depending on the frequency omega inside this expression and we get this new set of equation with a frequency dependent displacement and a fluctuation of photons inside the cavity. In this case, we can solve the linear couple equation of motion for the light and mechanics and it's this induced effect, dynamical effect due to the retarded nature of radiation pressure. This is what we're going to see in the next few slides.

Notes

Summary



2m 30s

# FREQUENCY DEPENDENT MECHANICS

Without optomechanical coupling :

The mechanical susceptibility  $\chi_m^{-1}(\omega) = m_{eff}[(\Omega_m^2 - \omega^2) - i\omega\Gamma_m]$

Assume a weak test force on the mechanical oscillator in the presence of optomechanical interaction

$$\chi_{m,eff}^{-1}(\omega) = \chi_m^{-1}(\omega) + \Sigma(\omega)$$

So without optomechanical coupling, the mechanical susceptibility can be written as function of the mechanical frequency and the mechanical damping with this imaginary susceptibility. If this it's true, if there is no coupling, optomechanical coupling between the two element. If now we assume that there is a weak test force on the mechanical oscillator in the presence of the optomechanical interaction, the mechanical susceptibility, the effective mechanical susceptibility can be written as the mechanical susceptibility without optomechanical coupling plus an extra term which called sigma and we want to try to express this sigma.

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3m 17s

# FREQUENCY DEPENDENT MECHANICS

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Assume a weak test force on the mechanical oscillator in the presence of optomechanical interaction

$$\chi_{m,eff}^{-1}(\omega) = \chi_m^{-1}(\omega) + \Sigma(\omega)$$

from light amplitude equation:

$$\delta\alpha[\omega] = -\frac{G\bar{\alpha}}{(\Delta - \omega) + i\frac{\kappa}{2}}\alpha[\omega]$$

Re-injected it in term of coupling in the mechanical equation :

$$\Sigma(\omega) = 2m_{eff}\Omega_m g^2 \left\{ \frac{1}{(\Delta + \omega) + i\frac{\kappa}{2}} + \frac{1}{(\Delta - \omega) - i\frac{\kappa}{2}} \right\}$$

$$\hbar G^2 |\alpha|^2 = 2m_{eff}\Omega_m g^2$$

From the light amplitude equation, we can have an relationship between the evolution of the field inside the cavity and the displacement. And if we re-inject this expression here, we can re-inject it in term of coupling in the mechanical equation and we get the expression for sigma, the extra term in the mechanical susceptibility. And we have two terms here which can be expressed and we will discuss this in the following.

Notes

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4m 15s

# FREQUENCY DEPENDENT MECHANICS

Separate real and imaginary parts :  $\Sigma(\omega) \equiv m_{eff}\omega[2\delta\Omega_m(\omega) - i\Gamma_{opt}(\omega)]$

$\delta\Omega_m(\omega)$  Frequency dependent mechanical frequency shift

$\Gamma_{opt}(\omega)$  Optomechanical damping rate

$$\delta\Omega_m(\omega) = g^2 \frac{\Omega_m}{\omega} \left[ \frac{(\Delta + \omega)}{(\Delta + \omega)^2 + \frac{\kappa^2}{4}} + \frac{(\Delta - \omega)}{(\Delta - \omega)^2 + \frac{\kappa^2}{4}} \right]$$

$$\Gamma_{opt}(\omega) = g^2 \frac{\Omega_m}{\omega} \left[ \frac{\kappa}{(\Delta + \omega)^2 + \frac{\kappa^2}{4}} - \frac{\kappa}{(\Delta - \omega)^2 + \frac{\kappa^2}{4}} \right]$$

At sufficiently weak laser drive, it is permissible to evaluate  $\delta\Omega_m(\omega)$  and  $\Gamma_{opt}(\omega)$  at the original, unperturbed frequency  $\omega = \Omega_m$

So we can separate the real and imaginary part of this extra susceptibility in term of a frequency dependent mechanical frequency shift which is the real part and plus optomechanical damping rate 'gamma-opt' as function of frequency. And from the previous equation we can rewrite this term and this term with different expressions which are given by this where we see that it's frequency dependent and we will discuss this two term in the following. So let's start with the frequency dependent mechanical frequency shift. At sufficiently weak laser drive, it is permissible to evaluate this frequency dependent mechanical frequency shift and this optomechanical damping rate at the original unperturbed frequency; namely, when the frequency omega is equal to the mechanical frequency omega 'm'.

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Summary



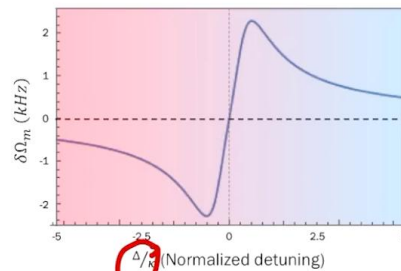
4m 53s

# OPTICAL SPRING EFFECT

$$\delta\Omega_m(\omega = \Omega_m) = g^2 \left[ \frac{(\Delta + \Omega_m)}{(\Delta + \Omega_m)^2 + \frac{\kappa^2}{4}} + \frac{(\Delta - \Omega_m)}{(\Delta - \Omega_m)^2 + \frac{\kappa^2}{4}} \right]$$

In the limit of large cavity  
decay rate ( $\kappa \gg \Omega_m$ ) :  
(i.e., Doppler regime / Unresolved sideband regime)

$$\delta\Omega_m(\Delta) \Big|_{\kappa \gg \Omega_m} = g^2 \frac{2\Delta}{\Delta^2 + \frac{\kappa^2}{4}}$$



For  $\Delta < 0$  (red-detuned laser), **spring softening**      For  $\Delta > 0$  (blue-detuned laser), **spring hardening**

In this case we can inject omega and omega 'm' in the equation which simplify a little bit the expression and then we can discuss the evolution of this expression. In the limit of a large cavity decay rate; namely, when the linewidth of the optical cavity is much bigger than the mechanical mode, the mechanical frequency, it's also called a Doppler regime or unresolved sideband regime, this frequency shift this optical spring effect can be even simplified. Now if we look at this, the evolution of this expression as function of the normalized detuning, delta over kappa, we can see that depending on this value if it's negative or positive, we can even have either have spring softening here when delta is negative in the red-detuned laser or when it can be also hardening, we can have spring hardening when delta is positive.

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# OPTOMECHANICAL DAMPING RATE

At mechanical resonance :

$$\Gamma_{opt}(\omega = \Omega_m) = g^2 \left[ \frac{\kappa}{(\Delta + \Omega_m)^2 + \frac{\kappa^2}{4}} - \frac{\kappa}{(\Delta - \Omega_m)^2 + \frac{\kappa^2}{4}} \right]$$

Full effective mechanical damping rate:  $\Gamma_{eff} = \Gamma_m + \Gamma_{opt}(\omega = \Omega_m)$

For  $\Gamma_{opt} > 0$ ,

Increased damping



Cooling

Resolved sideband regime ( $\kappa \ll \Omega_m$ )

$$\Gamma_{opt}(\Delta = -\Omega_m) \Big|_{\kappa \ll \Omega_m} = \frac{4g^2}{\kappa}$$

Unresolved sideband regime ( $\kappa \gg \Omega_m$ )

$$\Gamma_{opt} \left( \Delta = -\frac{\kappa}{2} \right) \Big|_{\kappa \gg \Omega_m} = 8 \left( \frac{g}{\kappa} \right)^2 \Omega_m$$

For  $\Gamma_{opt} < 0$ ,

Extra antidamping



Amplification

Even instabilities if  $\Gamma_{eff} < 0$

Now let's shift to the optomechanical damping rate. At the mechanical resonance again when omega is equal to omega 'm', we have this expression where only omega 'm' appears here and here and the full effective mechanical damping rate can be written as follow. So this effective damping rate is equal to the natural mechanical damping rate of the oscillator of the mechanical mode that we are looking at plus this optomechanical damping rate at omega 'm' so in this case if the optomechanical damping rate is positive, it's gonna increase the damping and so leads to the cooling of the mechanical mode. In this case, resolved sideband regime, in this resolved sideband regime when kappa is much smaller than omega 'm', the optomechanical damping rate can reach can be equals to four times the optomechanical coupling squared divided by the linewidth of the cavity at when the detuning is equal to minus omega 'm'. In the case of the unresolved sideband regime, as the optimal optomechanical damping rate is reached when when the detuning is equal to minus kappa over two and this gives you optimum optomechanical damping rate given by this expression here. At the opposite when the optomechanical damping rate is negative you have an extra antidamping which can lead to amplification and even if even instabilities are possible if this effective damping rate, mechanical damping rate is negative.

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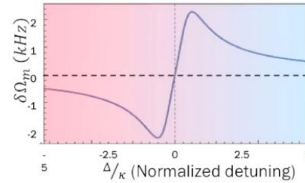
7m 09s

# DYNAMICAL BACKACTION

Retardated nature of radiation pressure induces modification of the mechanical behavior of the resonator

- Mechanical resonance : Optical spring effect

$$\delta\Omega_m(\omega = \Omega_m) = g^2 \left[ \frac{(\Delta + \Omega_m)}{(\Delta + \Omega_m)^2 + \frac{\kappa^2}{4}} + \frac{(\Delta - \Omega_m)}{(\Delta - \Omega_m)^2 + \frac{\kappa^2}{4}} \right]$$



- Mechanical damping : Cooling / Amplification

$$\Gamma_{opt}(\omega = \Omega_m) = g^2 \left[ \frac{\kappa}{(\Delta + \Omega_m)^2 + \frac{\kappa^2}{4}} - \frac{\kappa}{(\Delta - \Omega_m)^2 + \frac{\kappa^2}{4}} \right]$$

So in order to conclude, I've shown here the retardated nature of radiation pressure which induces modification of the mechanical resonator and you have two features; the optical spring effect and the cooling or the amplification of the mechanical damping.

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Summary

