NONLINEAR DYNAMICS

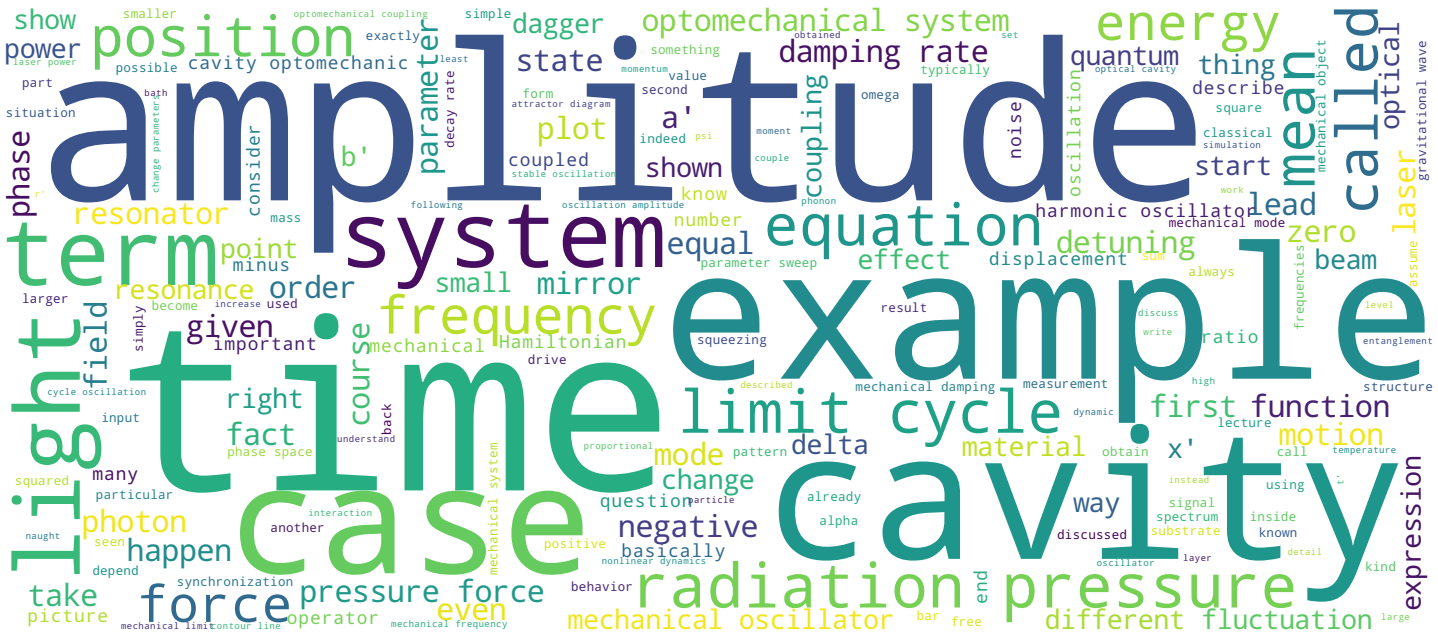
Cavity Quantum Optomechanics

Florian Marquardt

Max Planck Institute for the Science of Light &
Friedrich-Alexander-Universität Erlangen-Nürnberg

MAX PLANCK INSTITUTE
FOR THE SCIENCE OF LIGHT

FAU FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG


amplitude
time
example
cavity
case
system
light
term
limit cycle
radiation pressure
mechanical oscillator
equation
frequency
position
energy
optomechanical system
damping rate
state
dagger
parameter
coupling
mean
called
optical
laser
resonator
point
resonance
order
small
mirror
given
important
right
fact
force
pressure force
photon
happen
negative
change
mode
material
motion
first function
harmonic oscillator
detuning
displacement
effect
zero
beam
lead
start
shown
plot
coupled
quantum
thing
describe
noise
oscillation
expression
different fluctuation
mechanical damping
measurement
Hamiltonian
squeezing
delta
pattern
obtain
x'
way
discussed
substrate
known
inside
signal
spectrum
using
take
picture
operator
even
synchronization
basically
alpha
already
question
behavior
end
bar
free
large
kind
structure
entanglement
ratio
lecture
temperature
call
obtain
dynamic
high
instead
layer
oscillator
detail
work
level
assume
gravitational wave
mechanical object
optical cavity
simulation
square
classical
mechanical mode
always
result
discuss
write
frequencies
parameter sweep
stem
displacement
number
equal
mass
consider
phase
field
light
course
back
input
cycle oscillation
phase space
understand
proportional
mechanical system
nonlinear dynamics
drive
mechanical
used
increase
become
simply
larger
minus
point
resonator
show
power
laser power
possible
cavity optomechanic
exactly
smaller
optomechanical coupling
simple
obtained
set
momentum
value
second
something
typically
omega
form
attractor diagram
decay rate
parameter
b'
a'
indeed
psi
moment
couple
phonon
oscillation amplitude
stable oscillation
following
know
part
bath
situation
change parameters
many
squared
particular
seen
naught
depend
synchronization
interaction
another
particle
positive
mechanical limit
contour line
mechanical frequency
operator
force
pressure force
mechanical oscillator
bar
free
end
behavior
nonlinear dynamics
oscillator
layer

OUTLINE

In this lesson, you will learn about the nonlinear classical dynamics of an optomechanical system.

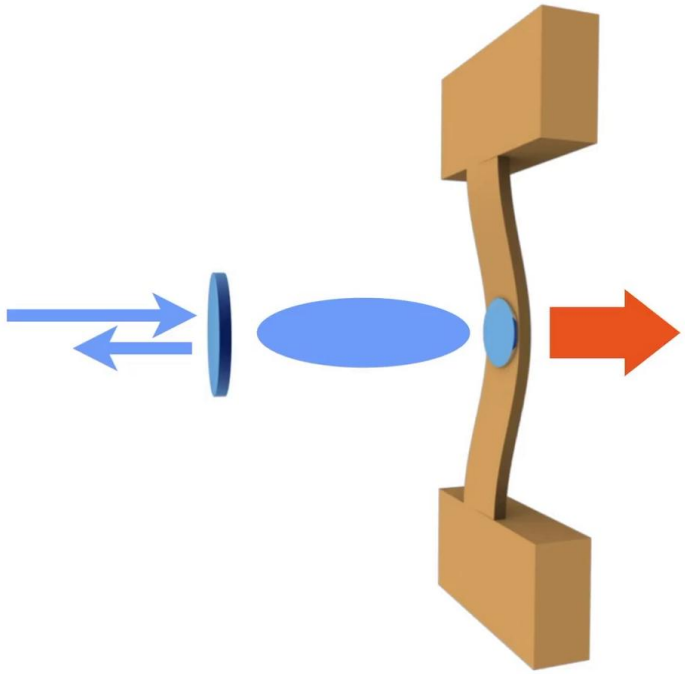
- **The dynamical instability and mechanical limit cycles**
- **The optomechanical attractor diagram**
- **Synchronization and chaos**
- **…explore for yourself using the Cavity Optomechanics Simulator on the Web**

Nonlinear dynamics of optomechanical systems. So if you consider the classical dynamics of an optomechanical system and you drive it in just the right way, you will see that there can be an instability and this instability we will learn leads to mechanical limit cycles. If you analyze these so-called limit cycles more closely, then you'll arrive at something that is called the optomechanical attractor diagram and from there it's only a small step to more advanced topics like synchronization and chaos. All of these things are a little bit hard to explore purely analytically and so it's great to do numerics and for that purpose we have supplied you with an simulator that can be accessed on the web.

Notes

Summary

0m 01s

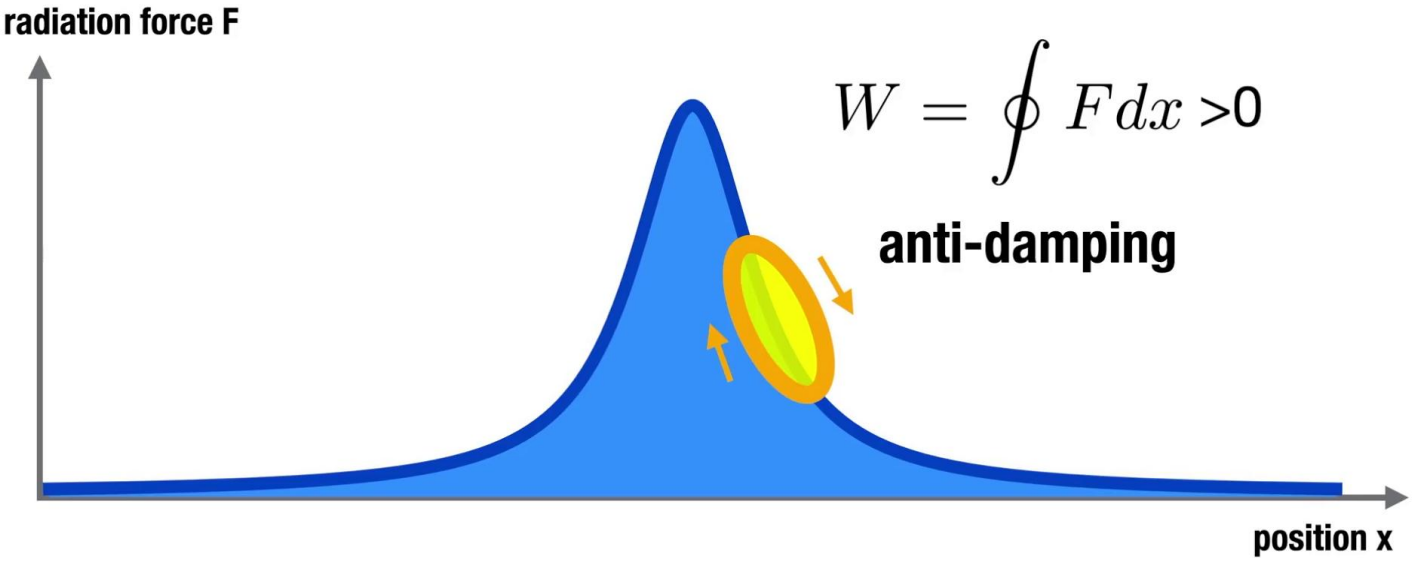


But let's start simple. Here's the standard optomechanical system and optical cavity driven by a laser, the radiation pressure force pushing some mechanical object.

Notes

Summary

0m 50s



Now, we already learned in an earlier lecture that if you plot the radiation pressure force versus the position of the mechanical object, then what you essentially do is you're sweeping through the resonance with the laser and so the force and intensity, they go up and down. But this is only a picture of the static force as a function of position. We also learned that a very important aspect is that the force is time delayed because the light needs some time to fill up the cavity and to leak out of the cavity. This time delayed force can lead to things such as friction but also if you position the mechanical object just right or position the laser frequency just right, it can lead to anti-damping.

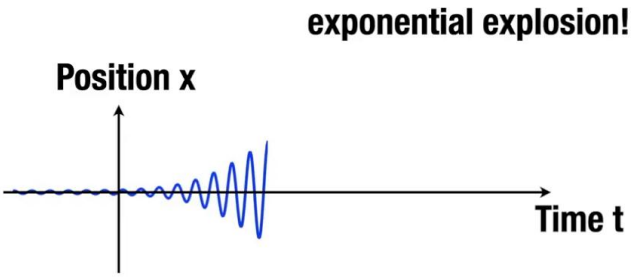
Notes

Summary

1m 03s

INSTABILITY

$$\Gamma_{\text{total}} = \Gamma + \Gamma_{\text{opt}} \quad < 0 \Rightarrow ?$$

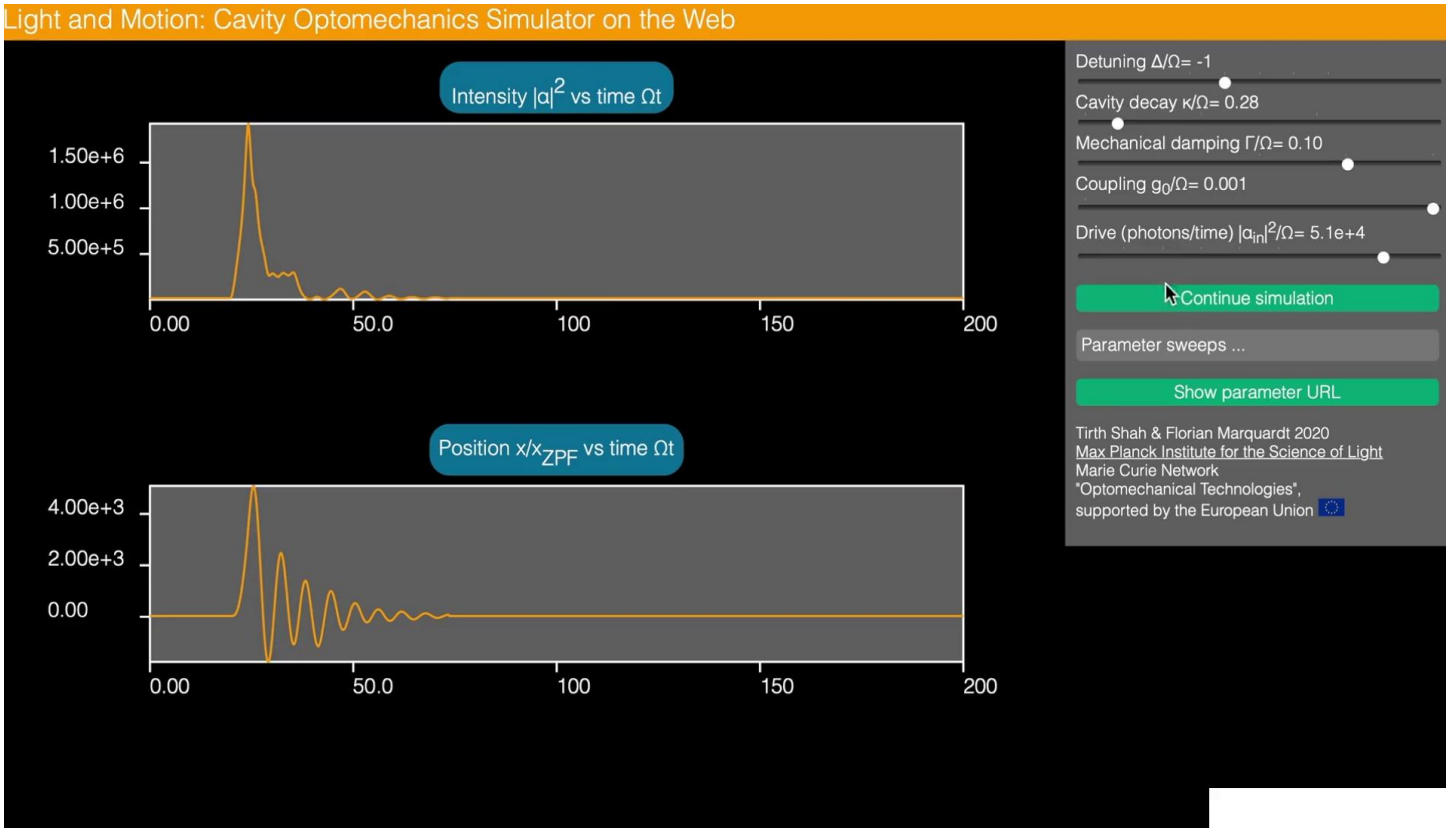


So what does that mean? In that case the total damping constant of the mechanical system consists of two parts, the intrinsic damping rate gamma and the damping rate contribution supplied by the optomechanical dynamics here called gamma-opt. And the case we're considering here, this second contribution is actually negative and so the question arises what happens if not only the second contribution is negative but the whole sum that is the total damping rate of the mechanical system turns out to be negative? What then? This is a very unconventional situation. And indeed if you take it serious, then what this means is if you start out from any small amplitude oscillations as time progresses, the oscillations will grow an amplitude. They will actually grow exponentially with a rate determined by this total gamma that is now negative. So there's an exponential explosion and the question is what happens next?

Notes

Summary

1m 56s

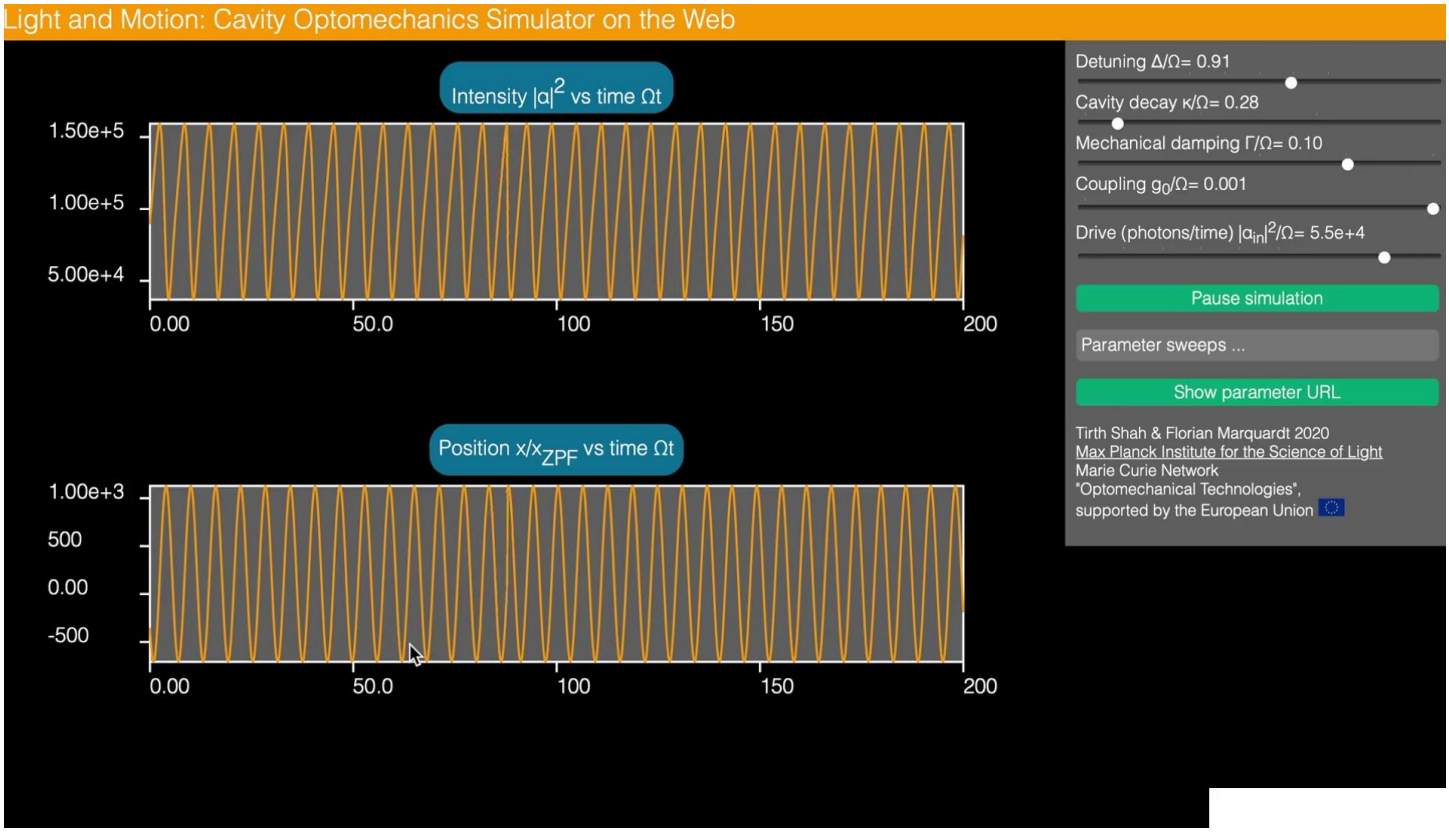


In order to answer this question, it's not enough to only look at the linearized dynamics of the system, you really have to delve into the full nonlinear dynamics. There's no better way to do that than by a numerical simulations. And so what we have provided for you is actually a full-scale optomechanic simulator that you can access from any web browser. And in this lecture, we will show you how to use that simulator to understand the nonlinear dynamics of an optomechanical system. Now, what you see here is the cavity optomechanic simulator. You see two plots and a control panel. In the plots what is shown right now is the light intensity in the cavity versus time and in the lower plot, the mechanical position versus time. At the moment the simulation has stopped. In the control panel on the other hand, you can control things like the detuning between laser and cavity, the cavity decay rate, mechanical damping, coupling constant between mechanics and light field and the laser drive. Now let's just continue the simulation for now and what I will do in a moment is I'll just apply a very strong radiation pressure force by ramping up the light intensity temporarily.

Notes

Summary

3m 10s



And so what you can see is while I took back the light intensity, the sudden kick in the radiation pressure force gave rise to an oscillatory motion in the mechanical oscillator that then is slowly damped away. In this case, let's pause the simulation, in this case, we had set the detuning to be negative, that is, we are in the red detuned regime and so we expect that actually the optomechanics contributes some extra term to the damping rate so the damping is stronger than intrinsically expected. What we can do now is we can switch to the opposite regime, that is, we can make sure that we have a positive detuning and then we can see what happens. So here now I'm going to suddenly switch the detuning to a positive value and watch what happens with the oscillation. It really rings up. The amplitude grows and now it has settled down to a stationary pattern. So the amplitude has saturated and we have an oscillation that will go on forever despite the presence of intrinsic mechanical damping. So that is what happens when you run into the instability. At first the amplitude rises and then you settle into a pattern that will go on forever until you switch off the laser drive. This is what we call in more mathematical terms a mechanical limit cycle oscillation.

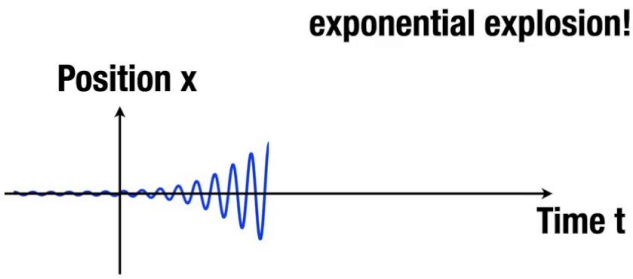
Notes

Summary

4m 42s

INSTABILITY

$$\Gamma_{\text{total}} = \Gamma + \Gamma_{\text{opt}} \quad < 0 \Rightarrow ?$$



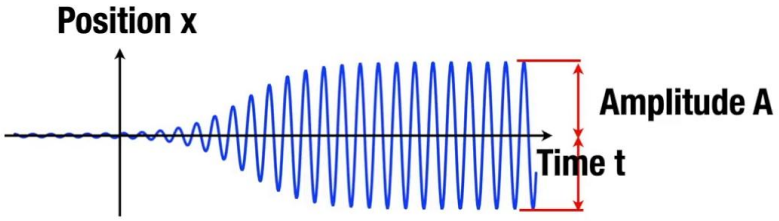
Started out with a question. What happens if the overall damping rate of an optomechanical system turns out to be negative and indeed, in the numerical simulations we saw an exponential explosion, but after some time the system settles to a stable oscillation amplitude.

Notes

Summary

6m 23s

INSTABILITY: SATURATION AT FINITE AMPLITUDE



experiments:

Carmon, Rokhsari, Yang, Kippenberg, Vahala, PRL 2005

Höhberger, Karrai, IEEE proceedings 2004

Metzger et al., PRL 2008

...

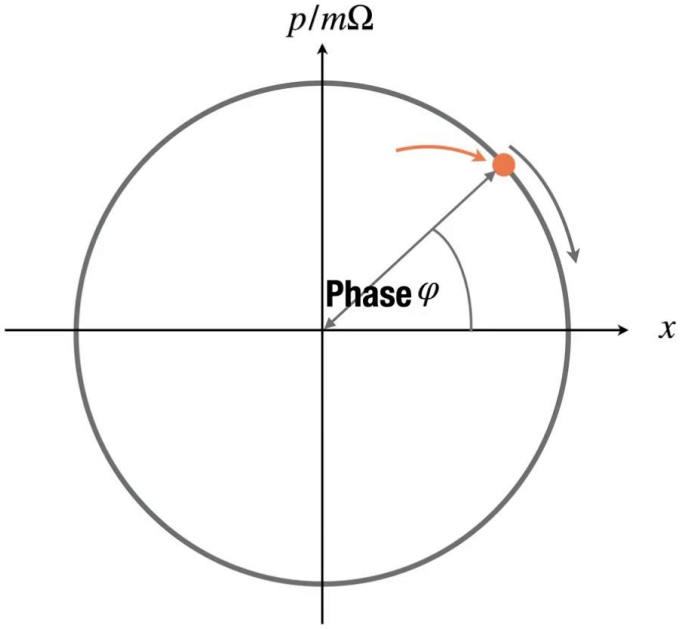
This is the optomechanical dynamical instability leading to these stable oscillations which are called limit cycle oscillations. Such a behavior has seen in a multitude of different optomechanical experiments and it is best understood in a phase space picture.

Notes

Summary

6m 39s

LIMIT CYCLE: PHASE SPACE PICTURE



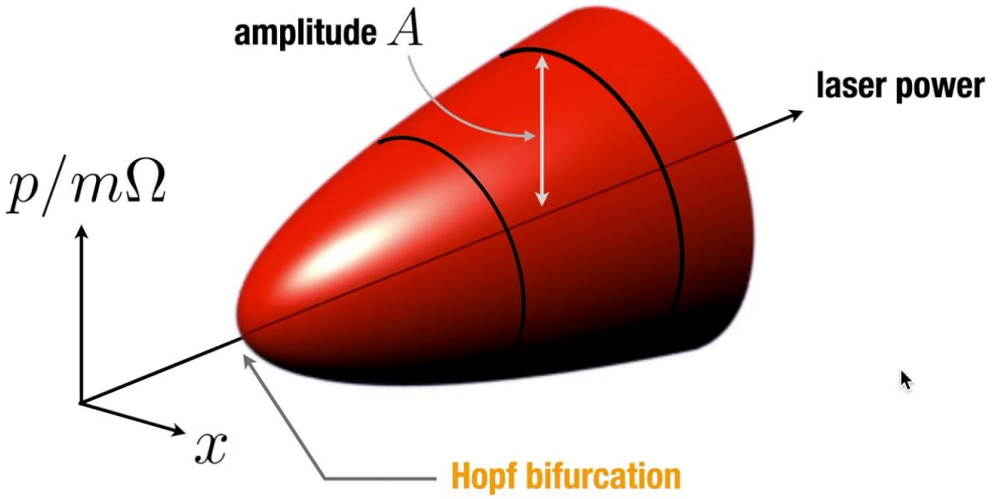
So here we plot the position to the right and the momentum to the vertical so that at every point in time the system will be represented by a point in phase space as shown here and the amplitude of the oscillations is simply the distance from the origin. This point in phase space will revolve around the origin in a clockwise manner at a steady rate corresponding to the frequency. Now if you go towards another amplitude for a short moment, for example, by a perturbation, what you will find is that the system relaxes back to the same old amplitude and the same is true if you start with a lower amplitude again that makes the amplitude increase towards the limit cycle amplitude so the limit cycle is really stable attractor of the system. Besides the amplitude, of course, there's also the phase of the mechanical oscillation and that in itself is free to take on whatever value is prescribed, for example, by the initial conditions. In addition, It can be easily modified by external noise coming into the system and we will come back to it later. Now from a mathematicians point of view, it is very interesting trick inquire how this limit cycle behaves when you change parameters? And you can imagine that if the parameters are not right at all then there is not even any such limit cycle and instead the system would be at rest in the center of the phase space coordinate system.

Notes

Summary

6m 57s

LIMIT CYCLE: HOPF BIFURCATION



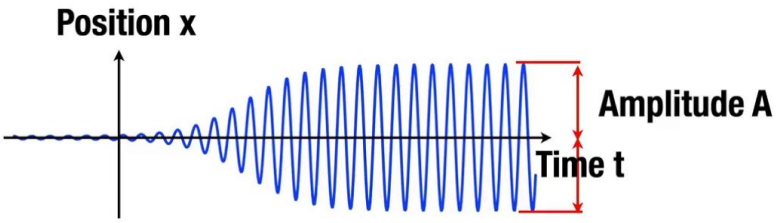
This can be depicted in another type of diagram where you're not only show phase space ‘x’ and ‘p’, but also plot some other parameter like, for example in our case, the laser power. As you increase the laser power, the effects of the radiation pressure force become more and more prevalent until finally we reach the point where indeed the overall damping rate becomes at first zero and afterwards negative and at this point the limit cycle starts to grow at first with a small amplitude and then with ever-increasing amplitudes. This point is called a Hopf bifurcation and it is a typical occurrence in nonlinear dynamical systems that are driven that have some energy input and, therefore, can develop these kinds of limit cycles.

Notes

Summary

8m 36s

POWER BALANCE CONDITION



Find amplitude A via the "power balance condition"

$$P_{\mathrm{rad}}/P_{\mathrm{fric}} = \langle F\dot{x}\rangle / m\Gamma \left\langle \dot{x}^2 \right\rangle$$

Power fed into system by radiation pressure

Power dissipated by friction

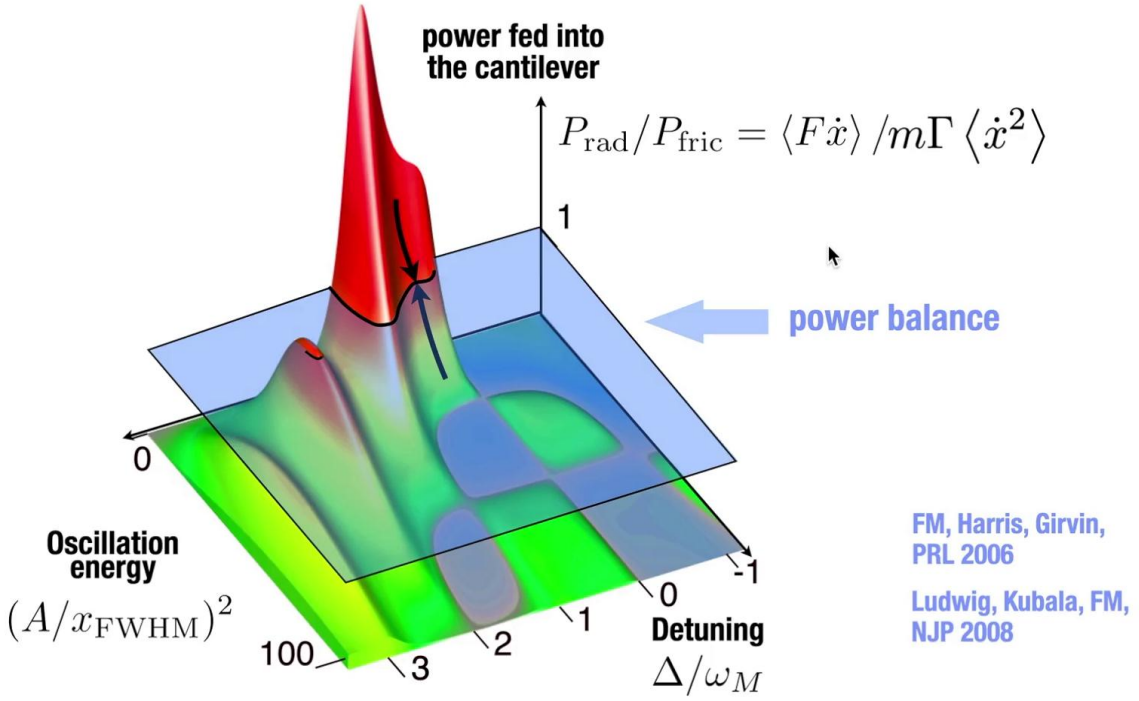
Now moving on if we want to understand what determines exactly the amplitude of the oscillations, it turns out there is a very easy way to find it and that is simply to realize that the amplitude is determined by a balance, a balance between two terms, balancing the power lost by mechanical friction that would usually reduce the amplitude down to zero against the power being fed into the system by the radiation pressure force 'F'. The power, of course, is the force times the velocity. You will have to average this over an oscillation period and then you want to look at the ratio between this power input and the power lost by friction. When this ratio is exactly one, the balance is established and the amplitude will not change and so that is the condition for the amplitude. In order to work out the details, you will have to figure out how exactly does the radiation pressure force depend on time given that you have a mechanical limit cycle oscillation which you assume a sinusoidal. There is actually some analytical calculation to be able to do this but we will not go into the details here.

Notes

Summary

9m 27s

POWER BALANCE CONDITION



FM, Harris, Girvin, PRL 2006

Ludwig, Kubala, FM, NJP 2008

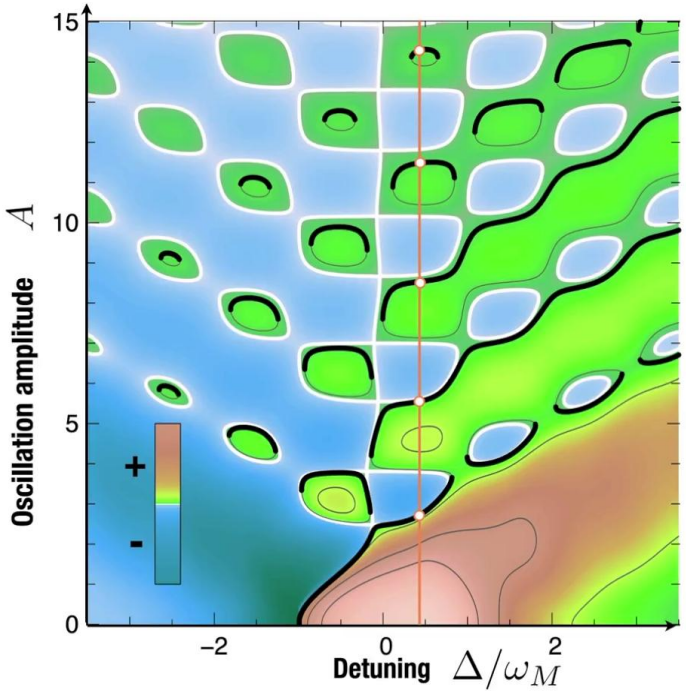
Now when you depict the outcome of such a calculation graphically, you can show the ratio that we just discussed between the power input and the power dissipated and you can plot this ratio as a function of parameters. In this case, we plotted it as a function on the one hand, of detuning which is one of the parameters you can control experimentally and on the other hand, as a function of amplitude of oscillations or more precisely here some normalized oscillation energy going like the square of the amplitude. If you plot this power ratio versus these parameters, you see a rather complicated landscape but the balance is obtained when the power ratio is one which determines a contour line and so your attractor sits on this contour line. Again as we discussed before if the amplitude is not right, for example, if it's larger than expected then it will decrease. If the amplitude is smaller and it should be then it will increase and it will always come back to the same value that determines the limit cycle amplitude.

Notes

Summary

10m 48s

OPTOMECHANICAL ATTRACTOR DIAGRAM



theory:

FM, Harris, Girvin, PRL 2006

observation of attractor diagram:

Krause et al., PRL 2015

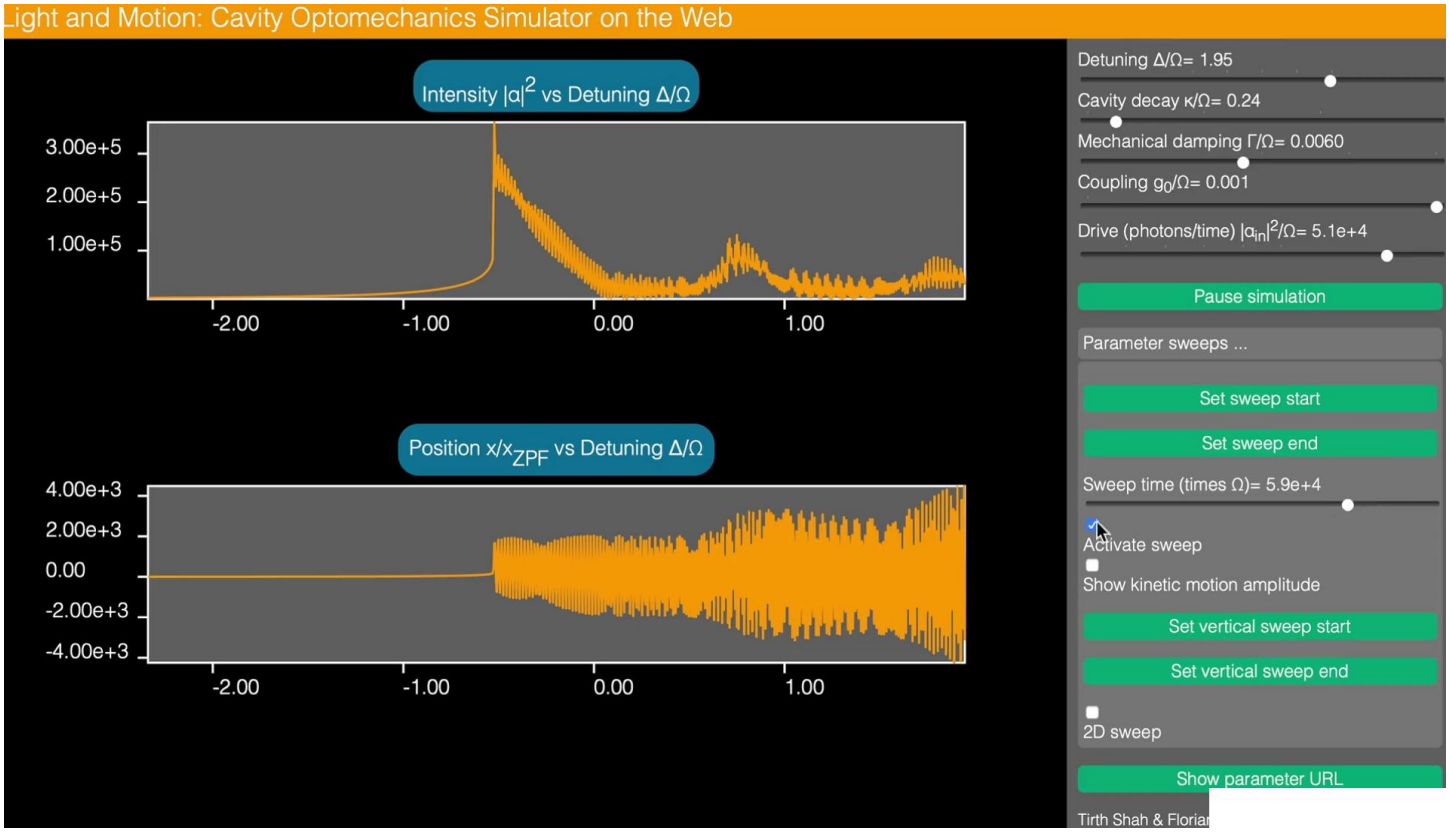
Multistability!

We can plot the same kind of graph in a slightly different way 2D just to see more of it. Here the colors represent the power input and if it's positive, it will be orange or green. So what that means is the power balance condition will be fulfilled on contour lines of this plot, which one you have to pick depends on the mechanical damping rate. For example, the thick black contour line corresponds to very low mechanical damping rate almost zero. What we see here is that if we fix some external parameter like the detuning, then there is not only one stable oscillation amplitude but in fact, there are several stable oscillation amplitudes at least if the mechanical damping rate is low enough. So this behavior is known as dynamical multistability and it's a very important feature of these optomechanical attractor diagrams. What this means in practice is also that you can get effects such as hysteresis so that would mean if you change external parameters like the detuning or the laser power, you will wonder around on these attractors and you might easily jump from one attractor to another and then if you change parameters back to their old values, you may not necessarily end up at the same behavior of the system so the typical behavior of a multistable system hysteresis is also observed in optomechanical systems.

Notes

Summary

11m 53s

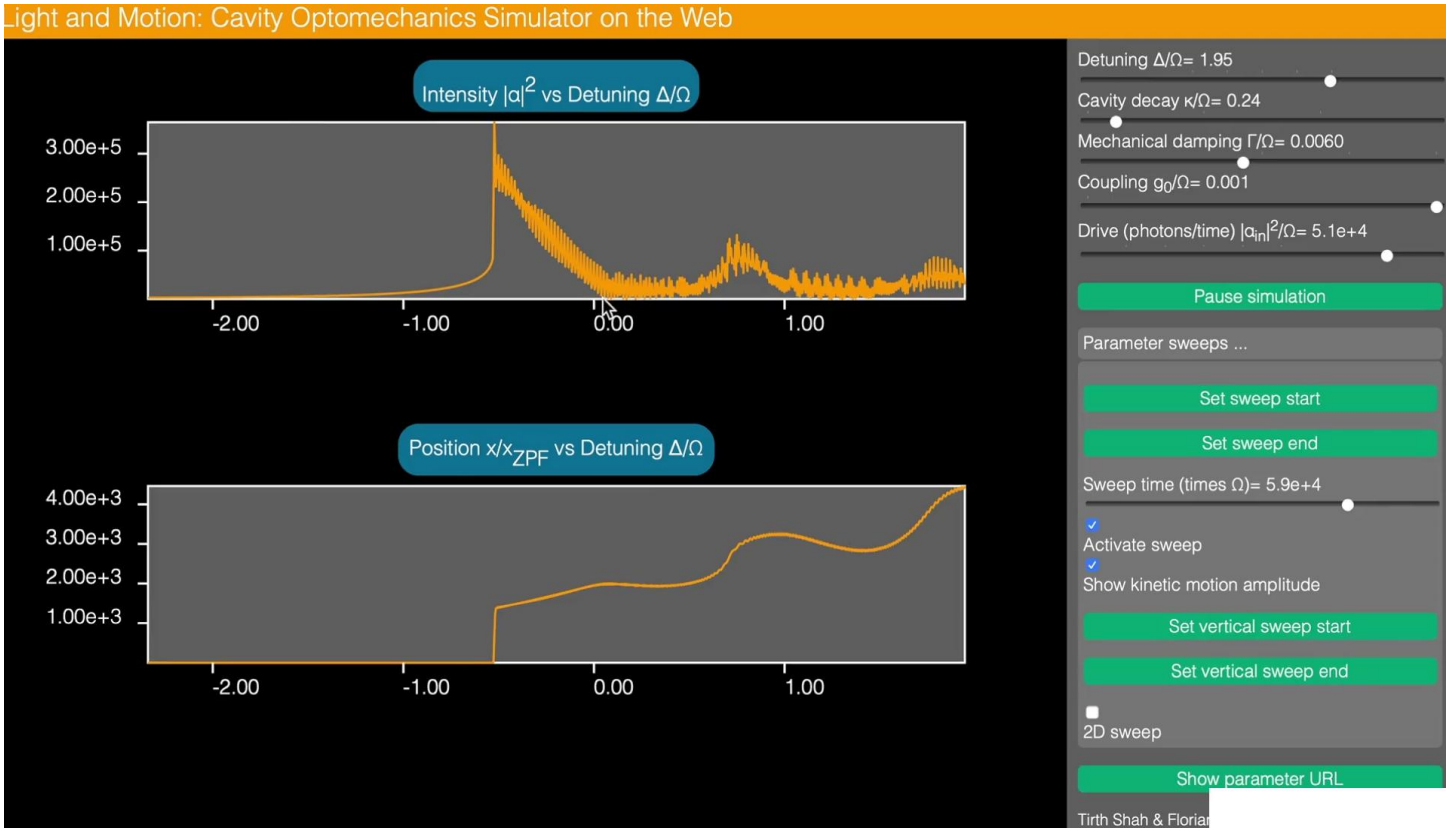


Now, all the complex behavior we have seen just now can, of course, be observed in the numerical simulations. Here you see a limit cycle oscillation where the light field pattern is already rather complex but more insights will be gained if we are able to change parameters and see at a glance what happens at different parameters. And that's what parameter sweeps are there for. So you can, for example, pick a small detuning, set this as the start of a parameter sweep then pick a large detuning, set this as the end of the sweep. And if you activate the sweep, instead of having a curve versus time, you will now find the intensity and the position plotted versus detuning. As in any real experiment, the details of the outcome of such a parameter sweep depend on how fast you sweep the parameter. So you can play around with a sweep time. Maybe you want to take it rather large in order to approximate an adiabatic parameter sweep. You also see the behavior looks quite complicated and the reason is, of course that on this scale you already don't resolve the individual oscillations anymore.

Notes

Summary

13m 31s

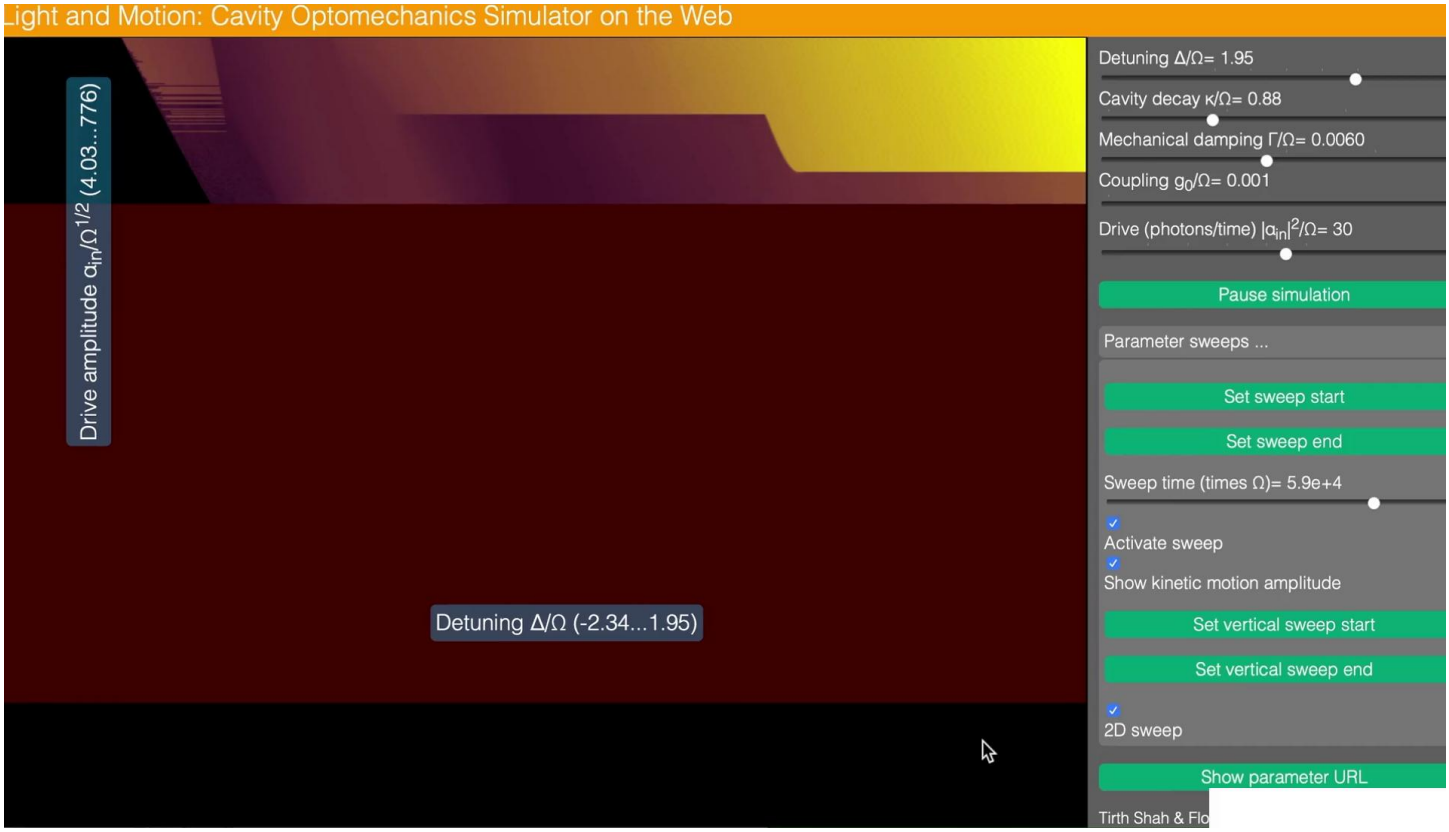


In order to ameliorate this a little bit, you can check out flag that says show only the kinetic motion amplitude which means you extract something like the amplitude that we just discussed and so then this is much easier to understand and what you'll see here is really something that has to do with the attractor diagram we just showed where as you sweep a parameter like the detuning, the oscillation amplitude will change. You also see the typical pattern of different kind of resonances that depend on how far away you are from zero detuning. So resonances are expected to occur when the detuning is an integer multiple of the mechanical frequency omega but there's one very important aspect to take into account here which is that all of this curve is anyway shifted simply because the radiation pressure force will expand the cavity and lower the cavity resonance and so that will shift everything.

Notes

Summary

14m 46s



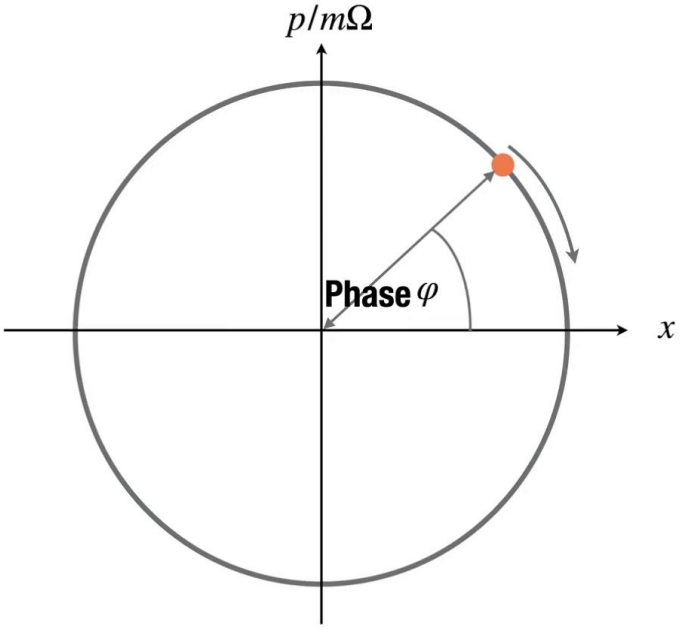
The cavity optomechanic simulator is also able to do 2D parameter sweeps. For example, the one shown here where you plot the results both against detuning and the drive amplitude and this works simply by setting a vertical sweep start and vertical sweep end and then selecting the 2D sweep. Once you have the 2D sweep, you can then in addition change parameters like for example, the cavity decay rate which will automatically lead to a recalculation of the whole sweep. So have fun exploring the wonderful dynamics of optomechanical systems.

Notes

Summary

16m 04s

SYNCHRONIZATION



Let us briefly explore another topic which is synchronization. Suppose you have such a limit cycle. We already discussed the amplitude is fixed but the phase is free to do what it wants. For example, it can be influenced by external noise but also it can be influenced by the forces of other oscillators that are coupled to the given oscillator.

Notes

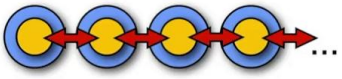
Summary

16m 44s

SYNCHRONIZATION

Collective dynamics in an array of coupled cells?

Phase-locking: synchronization!



And so that leads us to the general question. What about the collective nonlinear dynamics in an array of such coupled optomechanical cells consisting of a mechanical mode and an optical mode. Now, one of the most important phenomena that can occur in such a situation when you couple limit cycles to each other is synchronization. That means the phases can lock to each other and the frequencies can become the same even if initially they had not been identical. This behavior was already understood to some extent hundreds of years ago but it can also happen in optomechanics.

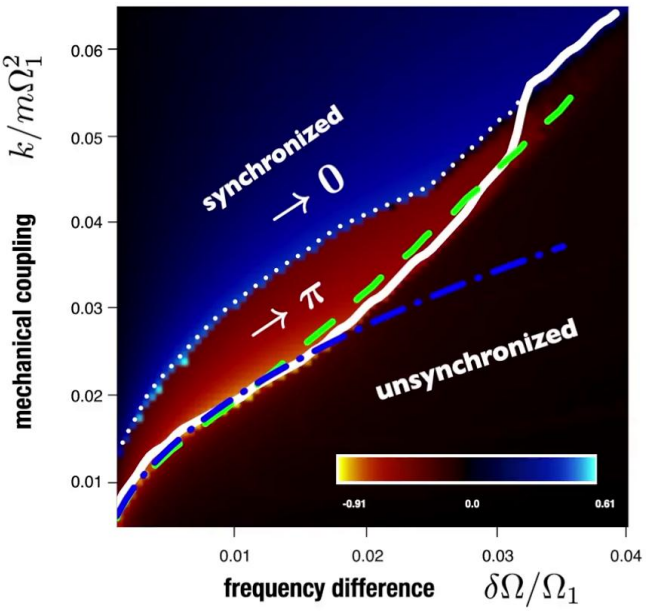
Notes

Summary

17m 08s

SYNCHRONIZATION

different intrisic frequencies



theoretical prediction: G. Heinrich et al., PRL 2011

experiments:

Zhang et al, PRL 2012 (Lipson lab)
Bagheri et al, PRL 2013 (Tang lab)
...

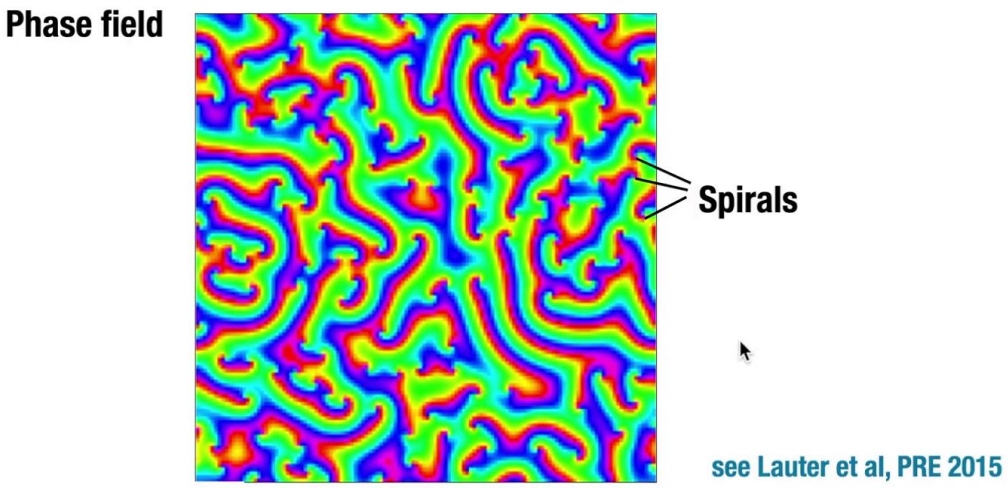
Here's an example. If you have just two optomechanical cells that are coupled to each other and you vary both the difference of the intrinsic mechanical frequencies and the mechanical coupling strength, you see a phase diagram like the one shown here. If the mechanical coupling is too weak and there is a some frequency difference then they will simply remain unsynchronized but if you cross some threshold in coupling then you observe synchronization and there can even be different types of synchronization where the phases tend to be opposite to each other so there's a pi difference and when the phases want to be close to each other so there's a zero phase difference. First theoretical predictions of optomechanical synchronization were obtained in 2011 and then very quickly first experiments followed. You can go on from two optomechanical oscillators to many.

Notes

Summary

17m 53s

PATTERN FORMATION IN OSCILLATOR ARRAYS



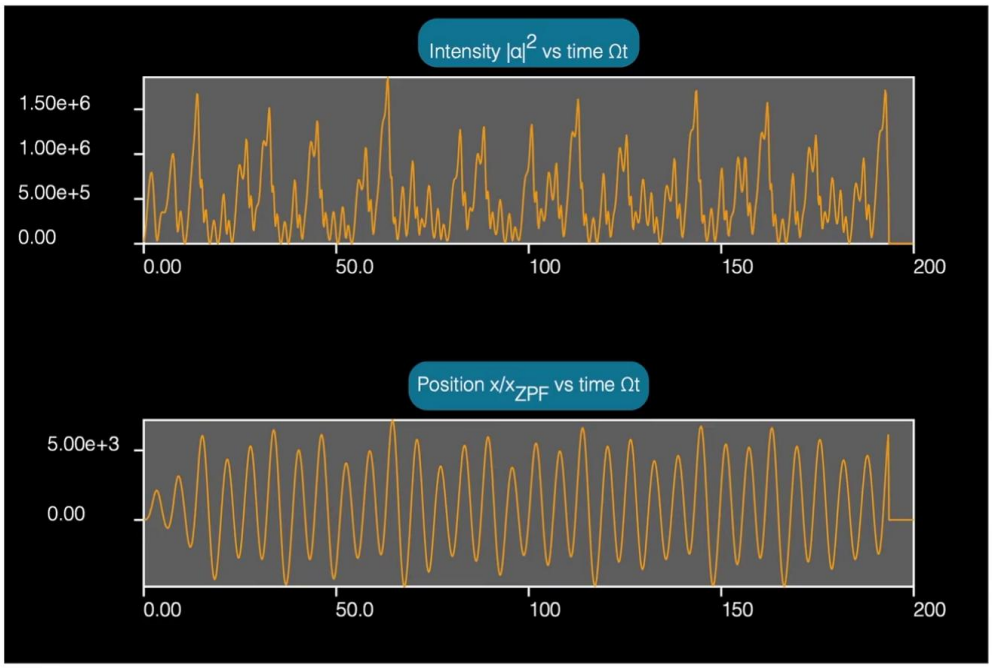
For example, you can consider at least on paper a whole array of oscillators. Here each pixel represents a single oscillator and color represents its phase and what you see here is the typical kinds of patterns that will evolve in the phase field of such an optomechanical oscillator array including topological defects that is spirals. At the moment this has not yet been observed experimentally because it's hard to build large arrays and drive them into a nonlinear regime but this is an interesting area for exploration. So that is synchronization.

Notes

Summary

18m 50s

CHAOS



first experiments: Carmon et al PRL 2007

And then finally, last but not least I want to mention another aspect. If your drive even harder then instead of just getting limit cycles, you see other phenomena like period doubling and eventually they will lead to chaos. Chaotic irregular motion is the one depicted here. First experiments also are relatively early already but the study of this is not yet very far advanced and there's certainly a lot to be explored in this direction.

Notes

Summary

19m 27s

CONCLUSION / SUMMARY

- **The dynamical instability and mechanical limit cycles**
- **The optomechanical attractor diagram**
- **Synchronization and chaos**
- **…explore for yourself using the**
  Cavity Optomechanics Simulator on the Web

for further pointers to the literature, see the review
Aspelmeyer, Kippenberg, Marquardt, Reviews of Modern Physics **86**, 1391 (2014)
also: Lectures Notes by Florian Marquardt for lectures delivered at the
Les Houches School "Quantum Machines", July 2011 (Oxford University Press)

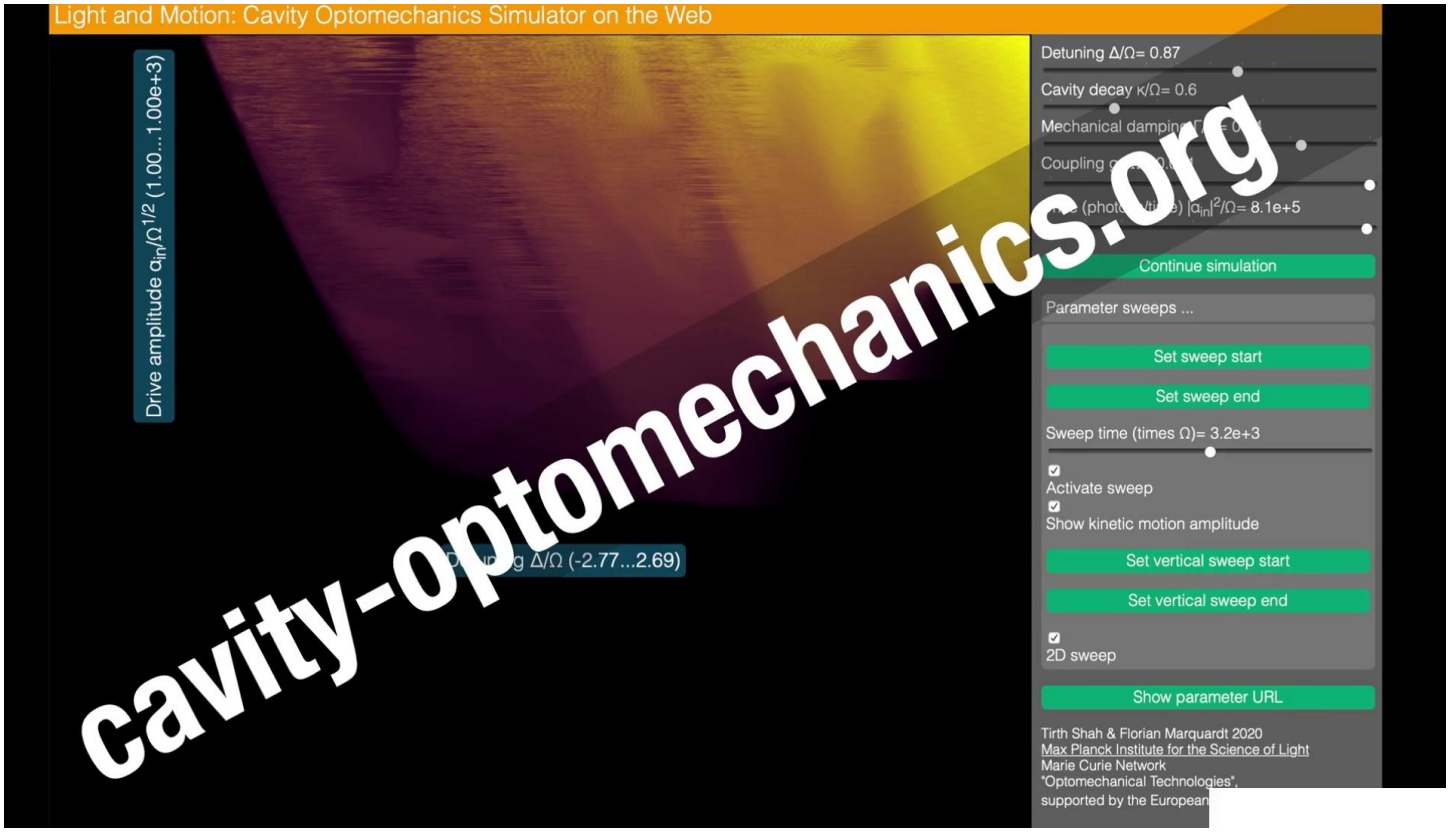
…and the further lectures in this series!

So let me conclude. What we saw in this little lecture is that first, optomechanical systems can display a dynamical instability where suddenly you have a growth of the amplitude of oscillations until they saturate to some value that leads to mechanical limit cycles. These can even have several different amplitudes so if you plot things as a function of parameters, you explore what is known as the optomechanical attractor diagram. And then finally, we also found that there are even further interesting phenomena like synchronization and chaos. You can explore many of these things yourself numerically in the cavity optomechanics simulator.

Notes

Summary

19m 59s



You can find the cavity optomechanics simulator at this web address.

Notes

Summary

20m 45s