# HOMODYNE DETECTION

## Cavity Quantum Optomechanics

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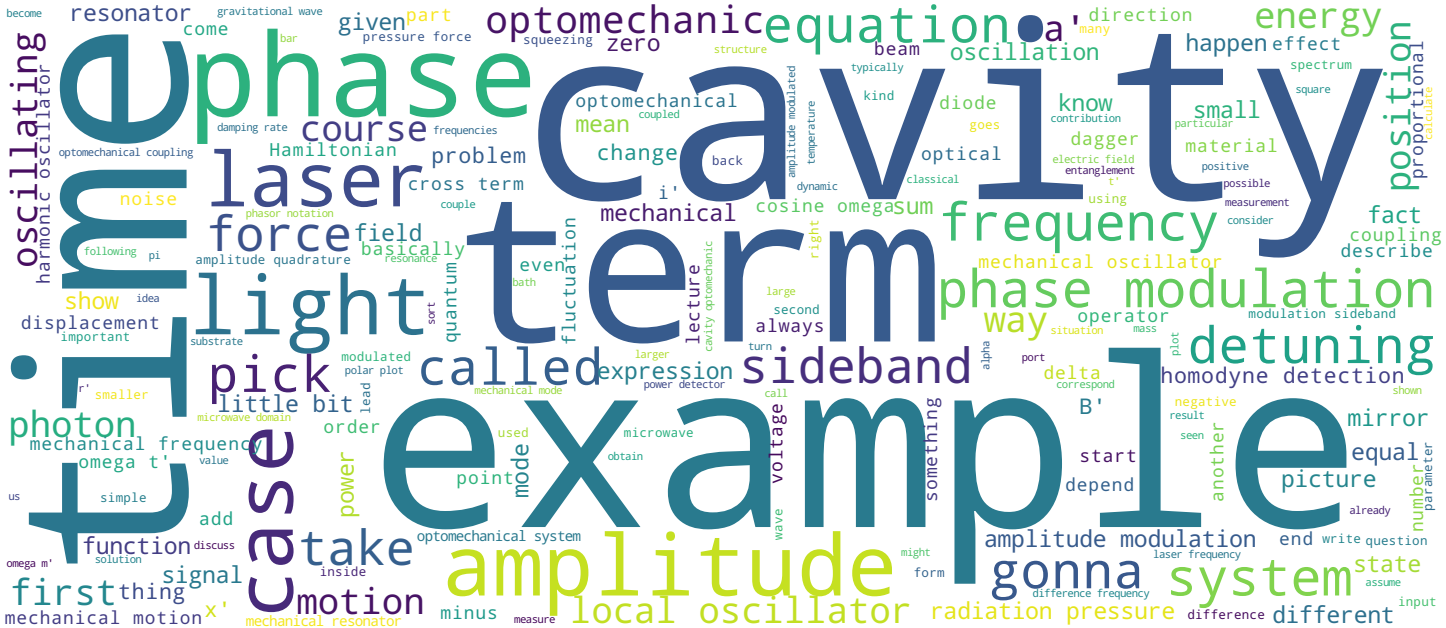


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# WELCOME & OUTLINE

## In this lesson, you will learn about homodyne detection in optomechanics

### The topics we will cover will include:

- **Review of concept of cavity modulation by mechanical motion**
- **Modulation sidebands in optomechanics**
- **Why diode rectification isn't enough**
- **How homodyne detection fixes the problem**
- **Optimizing the "homodyne angle"**
- **Homodyne with mixers in the microwave domain**

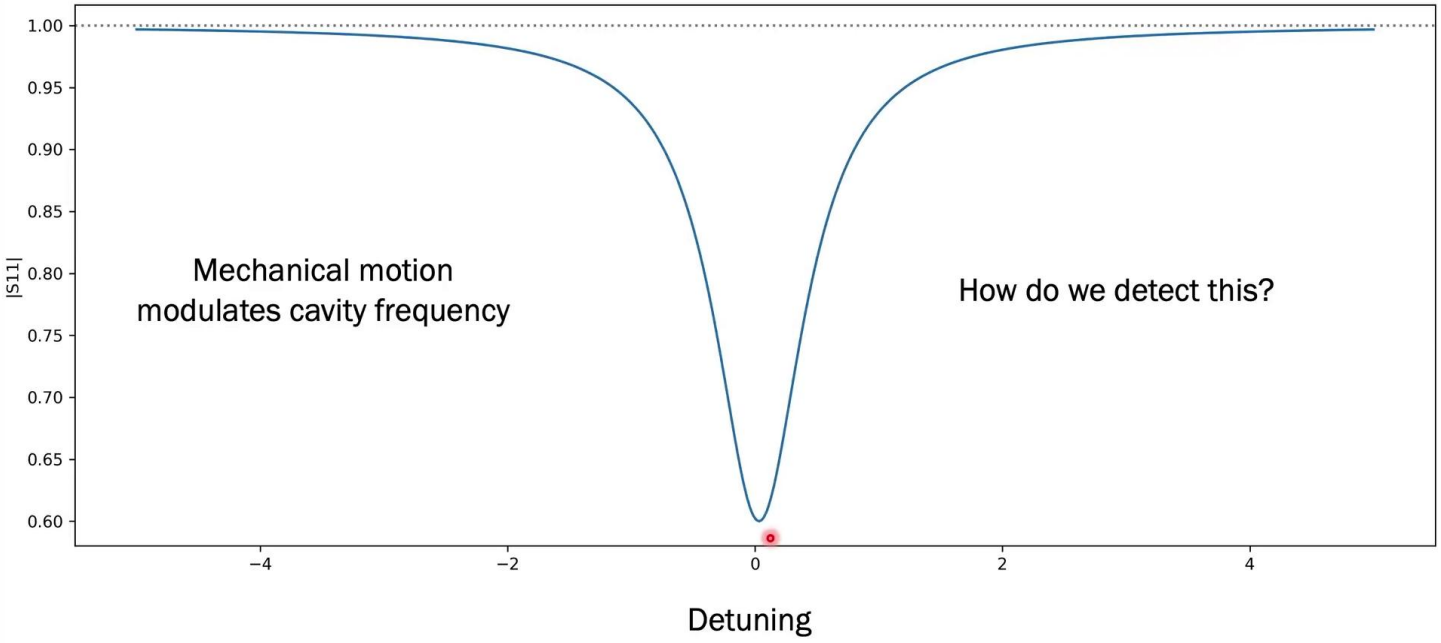
Hello. My name is Gary Steele. I'm a professor at the Delft University of Technology in the Netherlands and I'm going to tell you today about homodyne detection in optomechanics as part of the cavity optomechanics mooc. So in this lesson I'll teach you about homodyne detection in optomechanics and the topics we will cover will include a review of the concept of cavity modulation by mechanical motion. I'll tell you about modulation sidebands in optomechanics. I will tell you why rectification and power by power and meters or photodiodes or regular diodes just isn't enough. I'll tell you about how homodyne detection fixes this problem. I'll show you a little straight to you the idea of the homodyne angle and I'll tell you also a little bit about homodyne with mixers in the microwave domain.

Notes

Summary

0m 00s

# CAVITY MODULATION (REVIEW)



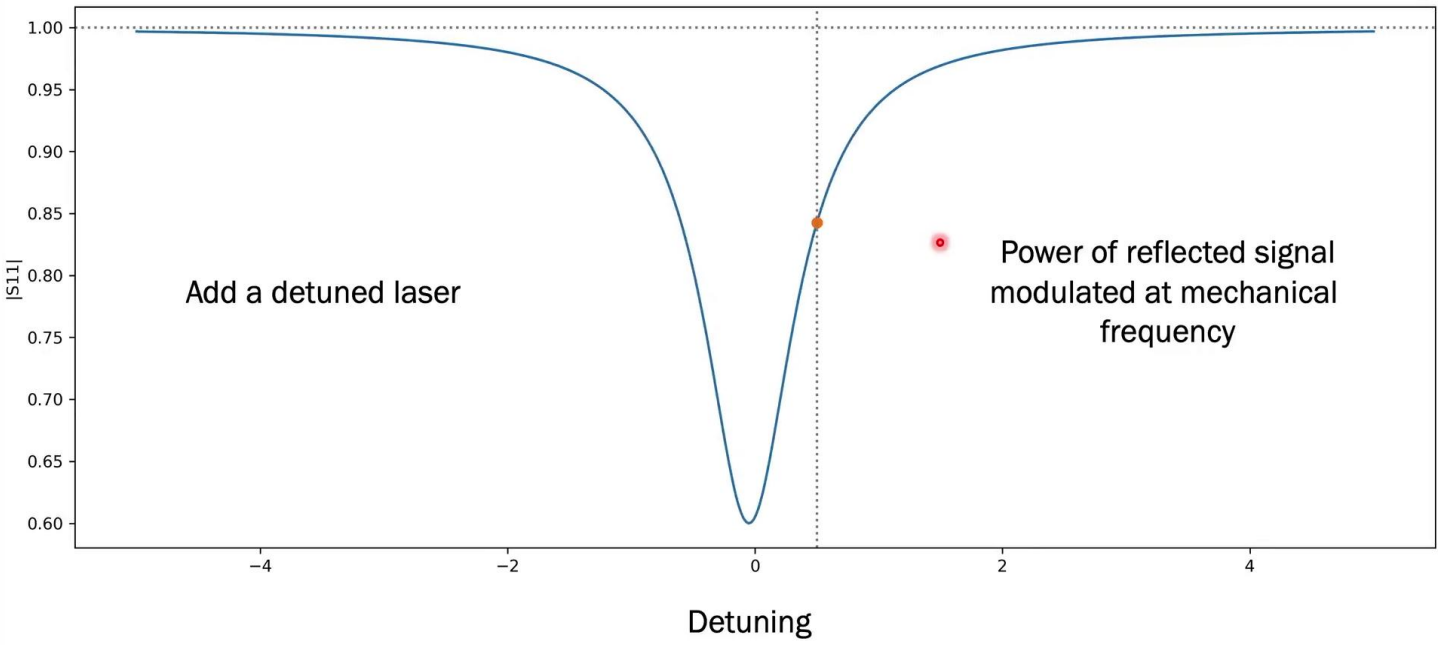

So this here is an illustration of the basic idea of optomechanics. So I have this optomechanical cavity. My mechanical resonator is oscillating and the result of the optomechanical coupling is that the frequency of my optical or microwave cavity is then also oscillating as a function of time at the mechanical frequency. And our goal in optomechanics is to use this oscillating frequency of the cavity to detect the motion. But how do we do this?

Notes

Summary

0m 54s

# ADD LASER



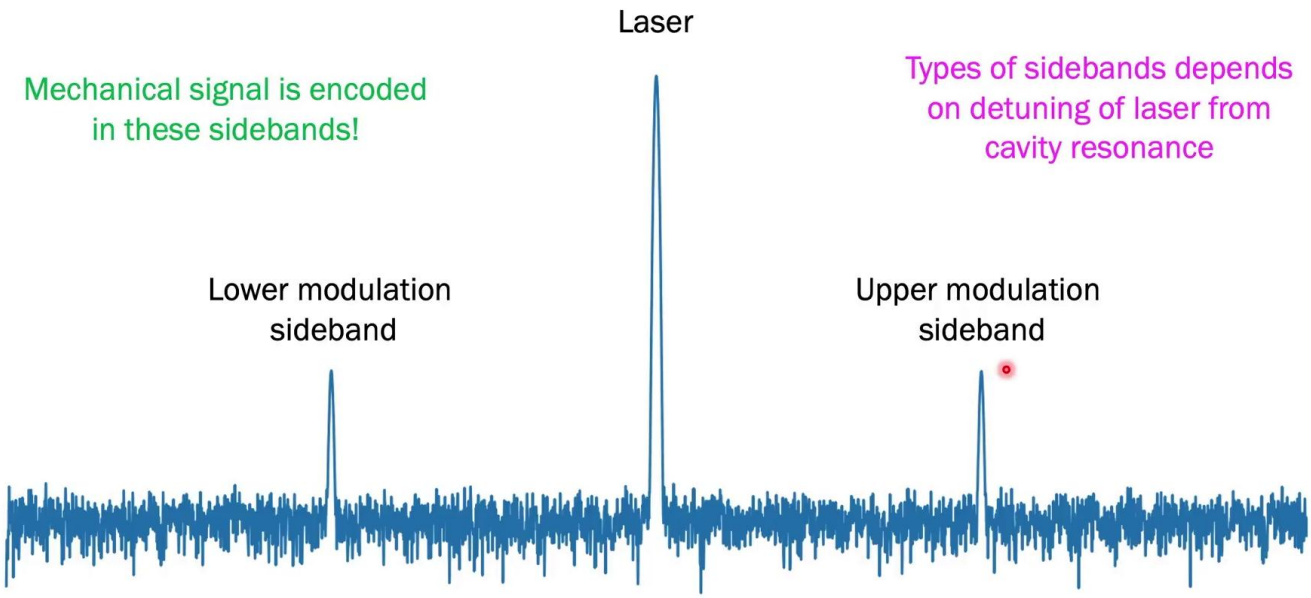

Well, one way to do this is to shine a laser at our cavity. And what this reflection coefficient tells us this laser is at a fixed frequency, say right here and this reflection coefficient tells us for example, what is the amplitude of the reflected light. And now we can see that if my cavity frequency is oscillating due to the mechanical motion of a mechanical resonator, that, for example, where I've put laser here, I will get an oscillating power coming out of my optomechanical cavity and this oscillating power for this choice of position will be oscillating at the mechanical frequency.

Notes

Summary

1m 29s

# MODULATION SIDEBANDS



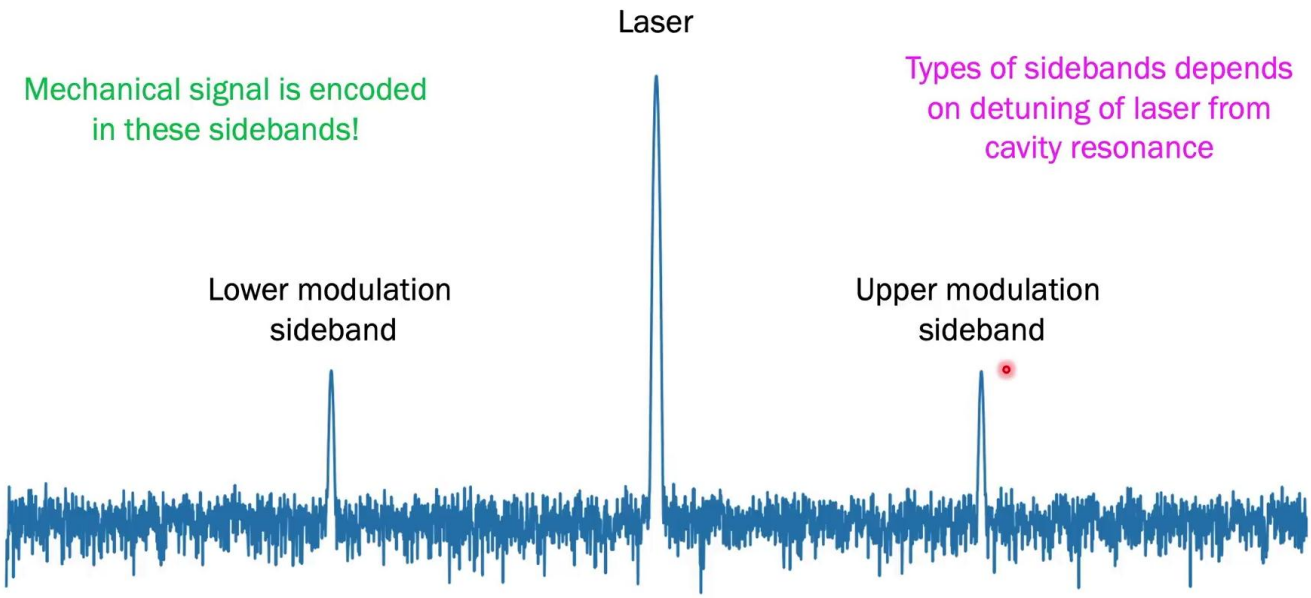

Now if I had any power that is any microwave source laser, any kind of wave that is being modulated in amplitude then we know that if we look at the Fourier transform of this we will get something that is called modulation sideband. So this is my simulated plot of the power spectral density of a laser that is being modulated. And so we see here the laser frequency and then at a detuning from laser frequency that is equal to the modulation frequency, I will have both an upper modulation sideband and a lower modulation sideband and this is something we will see later mathematically. So in optomechanics, the mechanical signal is encoded in these sidebands. Our goal is to try to get information out of these sidebands. And actually the type of sideband we get depends on actually the detuning of the laser from the cavity resonance.

Notes

Summary

2m 14s

# MODULATION SIDEBANDS




And so I want to show you first with pictures and then later with math what are the types of sidebands that you can get in optomechanics.

Notes

Summary

3m 19s

# Modulation Sidebands: The Pictures

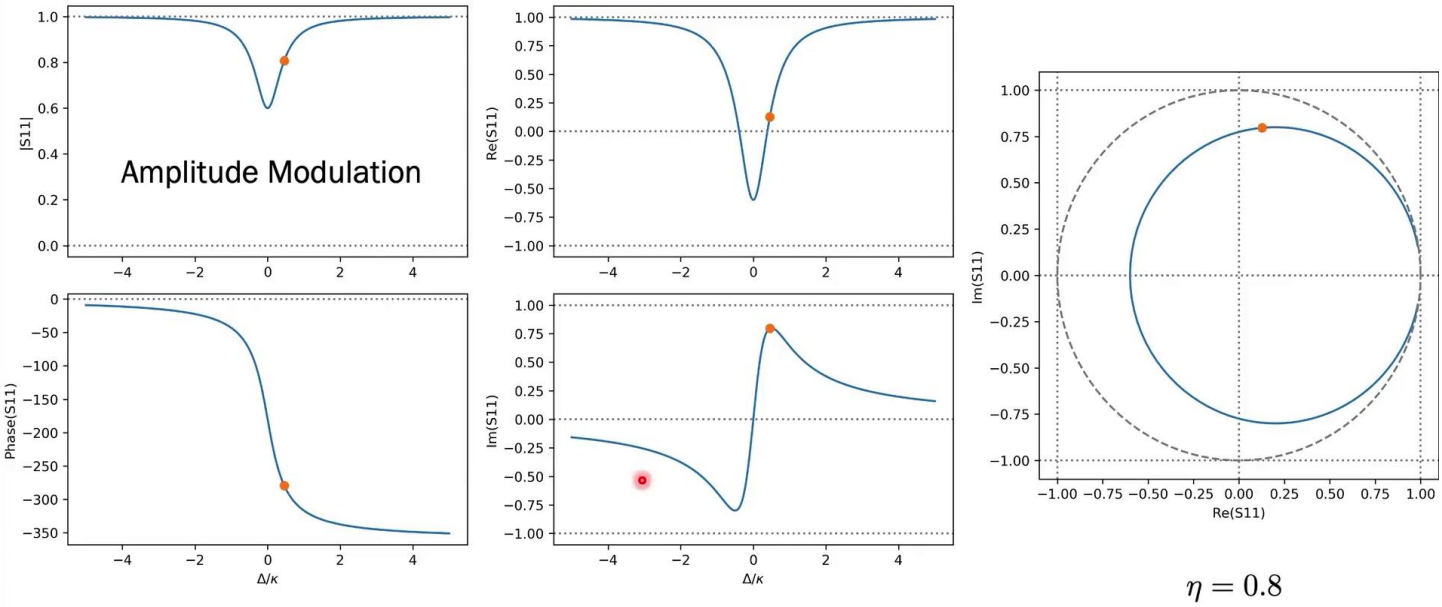
So first pictures.

Notes

Summary

3m 31s

# Δ = 0.5, AMPLITUDE MODULATION




$\eta = 0.8$

*No "phase quadrature" modulation*

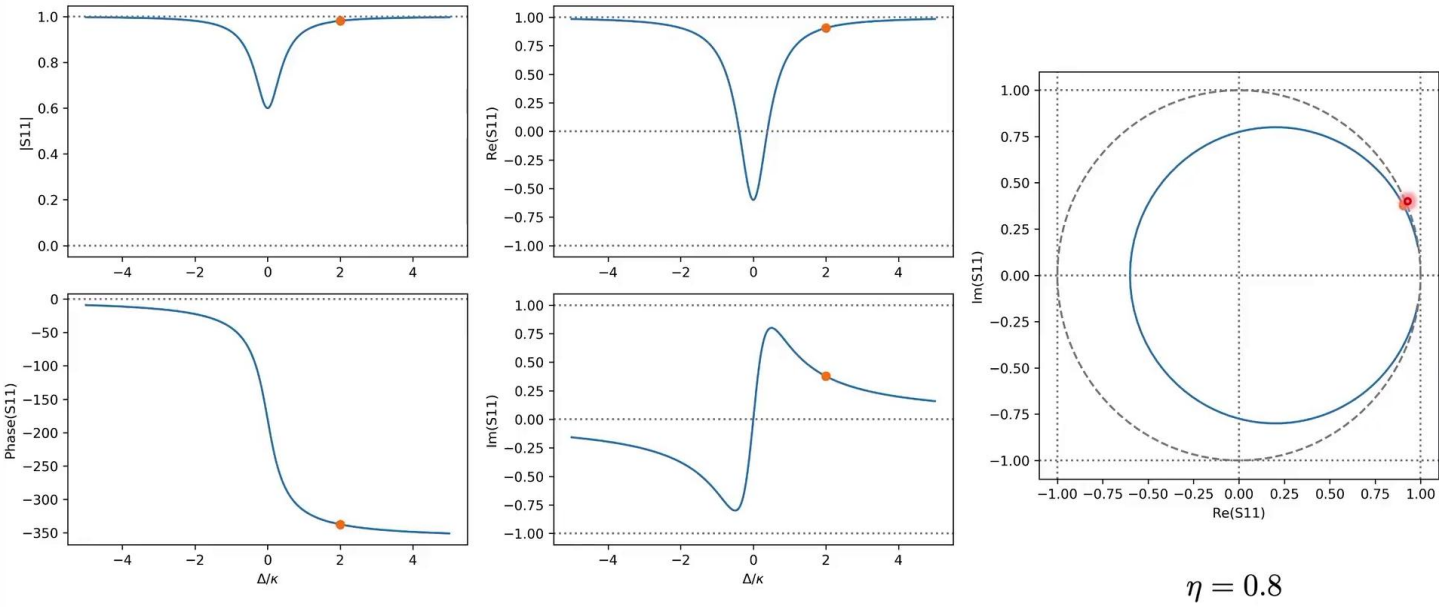
So this is what I showed you on the first slide the idea of a oscillating, well, in the first slide this cavity frequency was oscillating. This is now plotted versus detuning and it doesn't really matter whether I detune my laser or the or the cavity so and for the simplicity of all these pictures and making these animations, I'm gonna modulate this little dot back and forth. It's completely equivalent. So if I pick, for example, a detuning between my laser and my cavity of 0.5 then what I will see is I will get, for example, here I will get some amplitude modulation. Clearly the amplitude of my laser will be modulated. There will actually be a little tiny bit of phase modulation. If we look at my favorite plots in terms of quadratures, what I will see is that actually my amplitude here will be oscillating quite large but my phase my absolute quadrature but my phase quadrature is actually the first order not oscillating so this picture this point actually in this case of detuning of 0.5 is the case where I have only amplitude quadrature modulation.

Notes

Summary

3m 32s

# OTHER Δ: A MIX




$\eta = 0.8$

*Both amplitude and phase modulation*

You can also see this in this picture of this polar plot that I introduced earlier and actually for this detuning, we pick this point where this thing is oscillating back and forth along this line purely in this one direction this one quadrature direction which in this is the real part and the amplitude what I call sometimes the amplitude quadrature. So we can also do same thing for detuning a zero. We can see that if we pick zero detuning, the first order we have no modulation of the amplitude of the reflected light. Also not of the amplitude quadrature of the reflected light. You can see clearly we're gonna have the maximum of the phase modulation because of the fact that the phase has a higher slope at zero detuning and we can also see that that's gonna result in a large oscillation of the phase quadrature. And that is then seen over here as this oscillation in the polar plot so in the polar plot this circle is going up and down along this direction there. So we can also pick actually a different place and a different place where, for example, there's a detuning of about two. We have a little bit of amplitude modulation, a little bit of phase modulation, a little bit of amplitude quadrature modulation, a little bit of amplitude phase modulation and you can see what we're doing is we're actually going back and forth on this point of this polar plot up here.

Notes

Summary

4m 40s

# Modulation Sidebands: The Math

So that is the that is the the sort of touchy feely pictures of what this modulation looks like.

Notes

Summary

6m 10s

# AMPLITUDE MODULATION

$V = A e^{i\omega t}$ ← Phasor notation

$A = A_0 (1 + m \cos \omega_m t)$ ← $m \ll 1$; modulate amplitude

Use identity:

$2\cos\theta = e^{i\theta} + e^{-i\theta}$

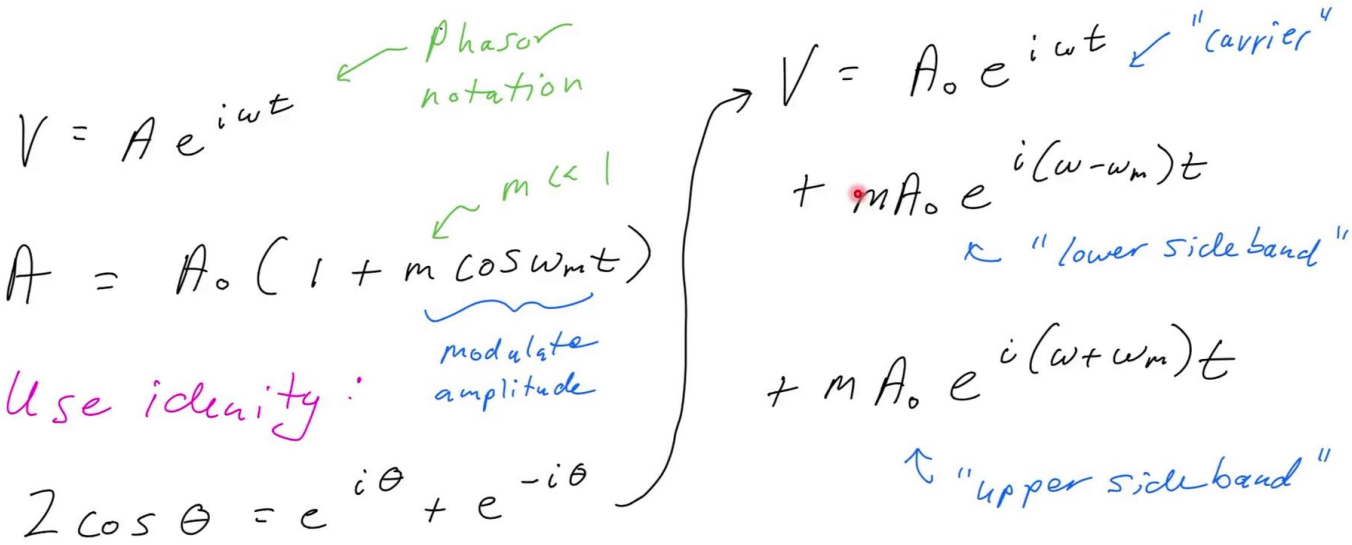
Let's look a little bit of math. So I promise that I will show you, in the previous plot we could not see anything about the sidebands light. We could just see how the light was being modulated. We could see there could be phase or amplitude modulated. So I want to show now two examples of how mathematically this amplitude of phase modulation results in sidebands and actually importantly how they're different. So let's we're gonna take an oscillating, we're gonna consider an oscillating voltage here which is given in phasor notation by 'A' times 'e' to the 'i omega t'. Now we're going, amplitude modulation means that this amplitude is not constant in time but being modulated at a modulation frequency which is also the mechanical frequency and the important thing is that this modulation is small so we're not turning the amplitude the wave all the way off up and down. It's not turning the amplitude to zero but we're just wiggling it a little bit like I showed in the previous plots. So now, what is math we're going to do? We're going to use a trigonometric identity that we can write a cosine as the sum of these two complex exponentials.

Notes

Summary

6m 22s

# AMPLITUDE MODULATION




We're just gonna plug this guy in and you can see that if you write this out, we're gonna get three terms. In the voltage, we're gonna get an amplitude oscillating at the original frequency. This is what people often call the carrier frequency. But we're gonna get also two sidebands which correspond to the sum and the differences. So this one of these terms will give a plus 'i omega mt' and that will be the upper sideband. The other one will give a minus 'i omega mt' and that will give us the lower sideband. And then amplitude modulation these are actually important later but the coefficients of these are real numbers not imaginary numbers. They're real-valued and they have this coefficient the amplitude of these modulation sidebands are smaller by a factor 'm'.

Notes

Summary

7m 29s

# PHASE MODULATON

$V = A_0 e^{i\omega t} e^{i\phi}$

$\phi = m \cos \omega_m t$

modulate phase

Taylor expand:

$e^x \approx 1 + x$

$V = A_0 (1 + i m \cos \omega_m t) e^{i\omega t}$

this is the difference to amplitude modulation

$= A_0 e^{i\omega t} + i m A_0 e^{i(\omega - \omega_m)t} + i m A_0 e^{i(\omega + \omega_m)t}$

→ Same sidebands as amplitude modulation but with different phase!

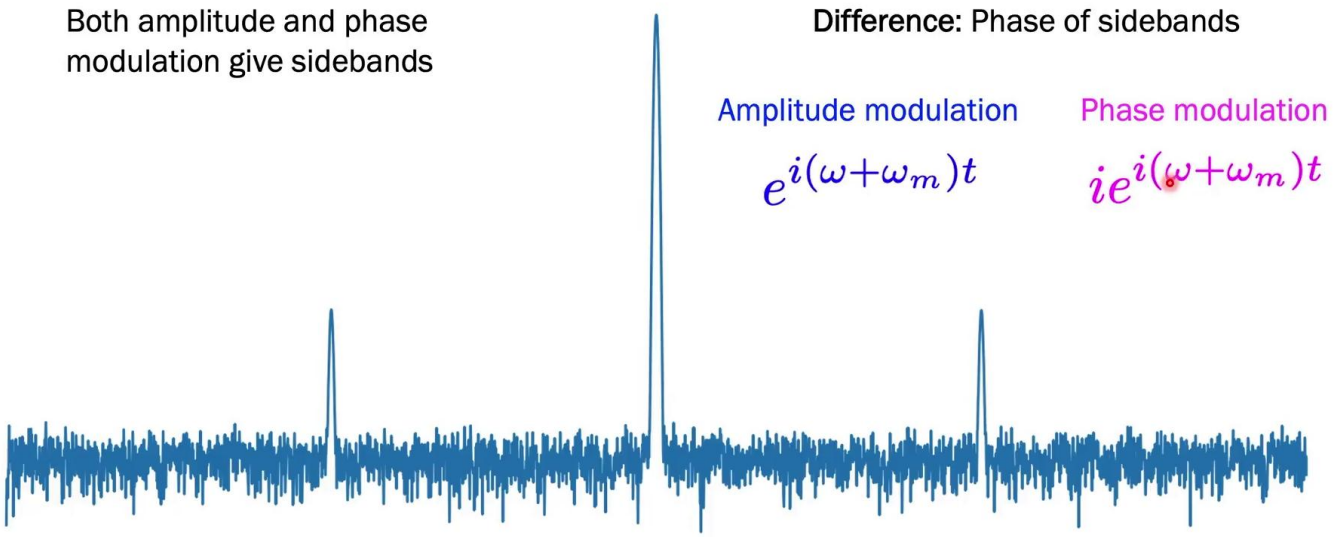
Let's now take a look at a different case of phase modulation. So in phase modulation what we do is we don't, we have a constant amplitude but then we allow the phase to oscillate so now we're gonna write it like this. We have our wave and there's gonna be a second term 'e' to the 'i phi' and phi is then going to be oscillating 'm cosine omega mt'. Now, the trick here is we're going to tailor expand this. We're gonna approximate 'e' to the 'ix' as one plus 'x' for small 'x' and that we're allowed to do because 'm' is always a small number. And then what we're going to get is we're gonna get this expression over here. This looks awful lot like our expression for amplitude modulation except for this pesky little 'i' up here and that will actually become quite important later. So we can run it out. We will get the same sum and difference frequencies and now with this the same coefficients actually as I parametrized it here but only with a different phase with this imaginary 'i' in front of this coefficient of our phasor.

Notes

Summary

8m 22s

# RECAP: SIDEBANDS




So to summarize to recap all this both phase modulation and amplitude modulation will give you these two sidebands of your carrier in optomechanics and they will both have sideband power and the only actually the difference, the important difference is that these sidebands have have a real coefficient for amplitude modulation and imaginary coefficient for phase modulation.

Notes

Summary

9m 26s

# THE PROBLEM

We don’t have 100 THz voltmeters.

How do we detect the mechanical sidebands?

So now, of course, all the mechanical motion is encoded in these sidebands and so, you know, our job is then to just measure these sidebands and now the problem these sidebands, the problem, of course, is that we don't have 100 terahertz voltmeters so how are we going to detect these mechanical sidebands?

Notes

Summary

9m 56s

# ONE SOLUTION: DIODE DETECTOR (PHOTODETECTOR)

Diode rectifier

$$V_{out} \propto (V_{in})^2$$

Photodiode

$$I \propto P_{opt}$$

So one solution that I'm going to present is to just take your laser and shine it into a photodiode and measure what comes out the other side. If you're working in microwaves, you might send it into an actual just diode and both of these things are what are called square law power detectors. So, for example, here the output voltage is proportional to the input voltage squared and it says here the power-out is proportional to the field amplitude squared.

Notes

Summary

10m 15s

# AMPLITUDE MODULATION + POWER DETECTION

$$V = A_0 (1 + m\cos\omega_m t) e^{i\omega t}$$

$$|V|^2 = V^* V$$

$$= A_0^2 (1 + m\cos\omega_m t)^2$$

$$= A_0^2 \left(1 + 2m\cos\omega_m t + m^2 \cos^2\omega_m t\right)$$

↑ "cross term"

↖ small oscillating term at $2\omega_m$

So let's take a look at what happens. So we're going to first take a look at amplitude detection, amplitude modulation and power detection so this is my amplitude modulated signal. To calculate the power, I actually need to take the magnitude squared which means I need to multiply 'V' by its complex conjugate. If I do that then I'm going to get here A-naught squared times one plus m-omega, this modulation term squared and when I square this term here, you can see that I'm gonna get this blue guy here. This is an important one. This is actually a sort of cross term that comes from the one multiplying by this 'm cosine omega t'. So you can see that actually what's gonna come out of my power diode or is going to be a constant voltage plus a voltage that is just oscillating at the mechanical frequency and this is actually the how we can use a diode to detect amplitude modulated sidebands in optomechanics.

Notes

Summary

10m 45s

# AMPLITUDE MODULATION + POWER DETECTION

$$V = A_0 (1 + m\cos\omega_m t) e^{i\omega t}$$

$$|V|^2 = V^* V$$

$$= A_0^2 (1 + m\cos\omega_m t)^2$$

$$= A_0^2 \left(1 + 2m\cos\omega_m t + m^2 \cos^2\omega_m t\right)$$

Detected power contains mechanical signal at $\omega_m$!

↑ "cross term"

↖ small oscillating term at $2\omega_m$

This detector diode will just directly contain the cosine omega 't' of my the encoding of my mechanical motion. You notice there's a second term up here which is then a small oscillating term at twice the mechanical frequency. But for a small modulation index this is very negligible.

Notes

Summary

11m 54s

# PHASE MODULATION + POWER DETECTION

$$V = A_0 (1 + i m \cos \omega_m t) e^{i \omega t}$$

$$V V^* = A_0^2 (1 + i m \cos \omega_m t)(1 - i m \cos \omega_m t)$$

$$= A_0^2 (1 + m^2 \cos^2 \omega_m t)$$

→ No cross term!

→ No (first order) mechanical signal!

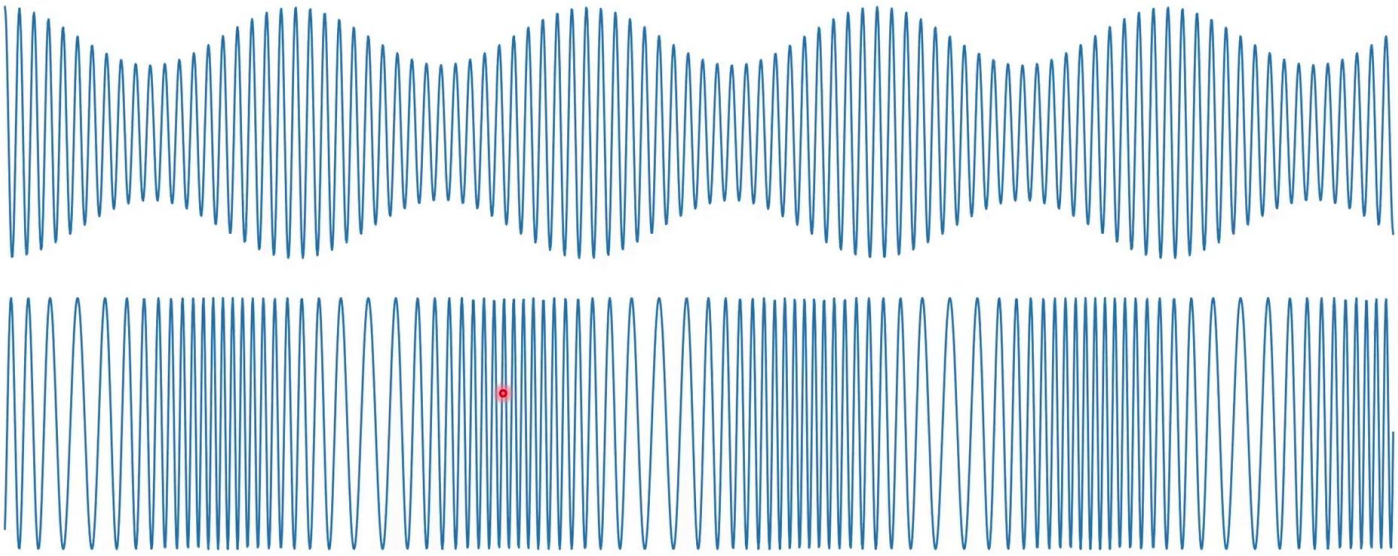
Now, let's take a look at what happens with phase modulation so phase modulation up here, we have an important difference is this 'i' and now what happens is when I multiply 'V' by its complex conjugate, I'm going to get one plus 'i' and one minus 'i' and now you can see because of this 'i', I have now completely lost my cross term. There is no cross term. And so for phase modulation there is no first order detectable mechanical signal if you just send your light or your microwaves into a diode. And that's a problem.

Notes

Summary

12m 14s

# INTUITION: ENVELOPE DETECTOR



Diode detector cannot detect phase modulation!

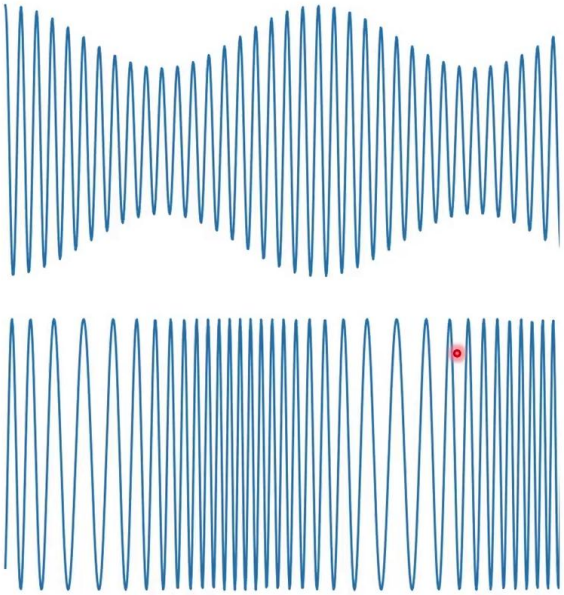
Well, let's see the intuitive, you can sort of get an intuitive understanding of this. You know a power diode is like an envelope detector and this is an example of an amplitude modulated signal where here the amplitude is being modulated so you can see obviously the envelope is being modulated but, of course, if you modulate the phase then in principle the envelope is not modulated at all.

Notes

Summary

12m 53s

# PROBLEM!



Part of, or even all of, the mechanical signal is phase modulation

But power detectors can measure phase modulation!

What are we going to do???!!?!

So really there's a problem and the problem is that part or even depending on our detuning, all of our mechanical signal is in phase modulation and power detectors cannot measure phase modulation so we're either throwing away part or throwing away all of our signal that we're after. So what are we going to do?

Notes

Summary

13m 18s

# THE SOLUTION

Homodyne!

And the solution, of course, is the topic of this lecture and that's homodyne detection.

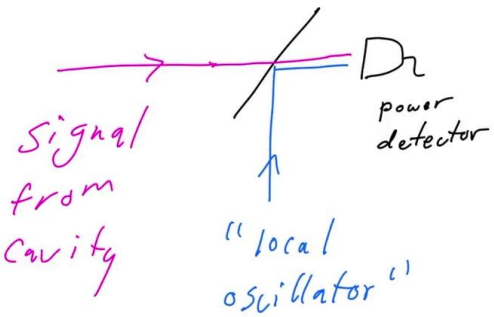
Notes

Summary

13m 42s

# HOMODYNE: MAKE OUR OWN CROSS TERM!

If we don't have a cross term, let's just add one!




$$V = V_{sig} + V_{LO}$$

$$= A_0 (1 - i m \cos\omega_m t) e^{i\omega t} +$$

phase modulation

So what is homodyne detection? Homodyne detection is basically if we don't have a cross term, let's just make our own cross term and the way that works works is that we have the signal coming up our cavity over here and then we shine an extra laser or we put an extra signal into our directional coupler if we're working in microwave domain and then we add, we have a mirror that adds a beam splitter that adds these and then that goes to our power detector. Now, of course, these two things will actually add in voltage or electric field so they will coherently interfere actually. And so what I'm gonna get in my voltage before it hits the power detector is gonna be the sum of the signal voltage and this what is called a local oscillator voltage. That's a funny old story. That's the term local oscillator actually comes with radio transmitters. They use a similar concept and the local oscillator was the oscillator you had in your house and then there was another oscillator on the radio tower kilometers away. But okay. So let's see what happens? So here this is I'm gonna look at the case of phase modulation which was the troublesome one before.

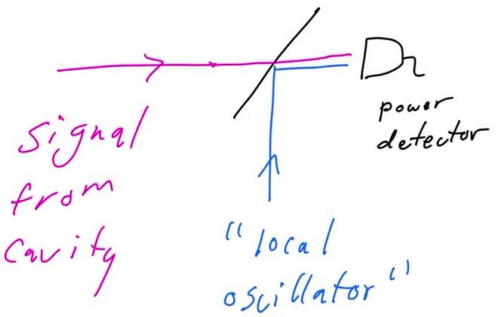
Notes

Summary

13m 45s

# HOMODYNE: MAKE OUR OWN CROSS TERM!

If we don't have a cross term, let's just add one!




$$V = V_{sig} + V_{LO}$$

phase modulation

$$= A_0 (1 - i m \cos\omega_m t) e^{i\omega t} + A_{LO} e^{i\omega t} e^{i\phi_{LO}}$$

To get the cross term we are after, we want $e^{i\phi_{LO}} = i$

→ Pick $\phi_{LO} = \pi/2$ (or $3\pi/2$)

So we're gonna have this 'A', this 'i' times the modulation index times cosine omega 't' and now we're gonna add our local oscillator. Now the key is that we can pick, you know, as long as we have some control over it, we can pick the phase of our local oscillator. You know, optical setup this might be by splitting part of your laser off before it goes to your device and changing the length of an arm. In a microwave setup there's a device called the fascia for you to turn the knob and it changes the phase and so now the key is that if we want to get this cross term that gave us our linear detection for amplitude modulation, all we have to do is pick the phase of this local oscillator to be 'i' because if this oscillator has phase 'i' then when I take the modulus squared like I did before then I will also get this cross term because it has the same phase as this term over here that I'm after. And so for this phase modulation case, I would pick the local oscillator phase of pi over two or I could also pick three pi over two if I want.

Notes

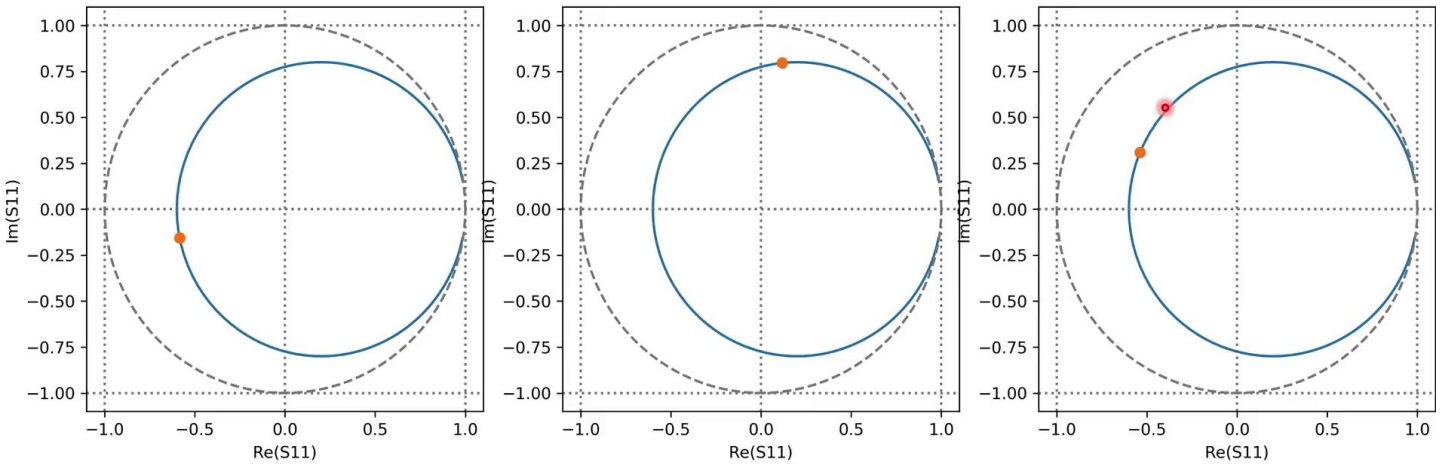
Summary

14m 57s

# HOMODYNE ANGLE

Optimal angle depends on detuning

Aim is to match the slope




90 degrees

0 degrees

~ 45 degrees

So that brings us to an illustration of this so the question is what phase do you want to pick for your local oscillator? Well, it turns out you can see from these plots actually very intuitively that the angle you want to pick for your local oscillator depends on the detuning. So this is the case of zero detuning and then then we want to pick actually a phase that is pointing up and down so either 90 degrees or minus 90 degrees so that our local oscillator is in the same direction as the oscillations of our of our that are induced by optomechanics. This one here is zero is actually when we have this pure amplitude quadrature modulation and then we actually want a local oscillator in this direction and actually the reason why amplitude modulation works with a diode is because the carrier signal is already oscillating in that direction so it's got a built-in local oscillator if you want. But actually if we pick an arbitrary detuning there might be any kind of to maximize the amount of signal to make sure we don't miss any of the signal from our mechanical resonator, we want to pick a phase of our local oscillator that is tangential to the slope at the position of the detuning that we're oscillating around.

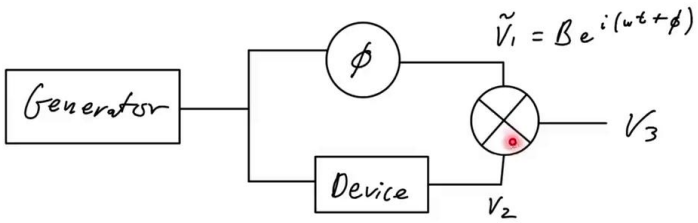
Notes

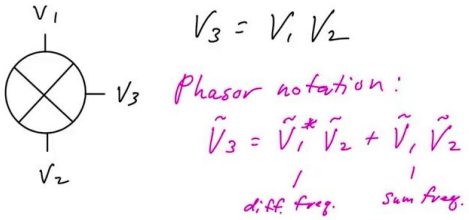
Summary

16m 03s

# MICROWAVE HOMODYNE

Image from Marki Microwave




So I want to end very quickly with a microwave because microwave is a little bit different. The difference is that in microwave domain we have different tools. One of them is called a mixer. It's actually made of diodes, a bunch of diodes together. But a mixer is a very special object that outputs basically you have two to three ports in it and you feed in two signals on one port on port one port two and then port three magically outputs the product of the two. Not the square, it just outputs their designed output 'V1' times 'V2'. In the phasor notation that's actually V1-star times 'V2' to get the sum plus or difference frequency and 'V1 V2' to get the sum frequency. So how does it look like in the setup? We'll take our generator, microwave generator lab, we'll split it into two, we'll send part of it to our device in our fridge, it'll come back out and then we have a second arm from the generator that we apply a phase shift and then we adjust this. We can pick we can adjust that phase shift to whatever we want. And we feed those into a mixer and we get a voltage out and the question is what is a voltage that comes out.

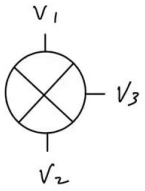
Notes

Summary

17m 30s

# MICROWAVE HOMODYNE

Image from Marki Microwave

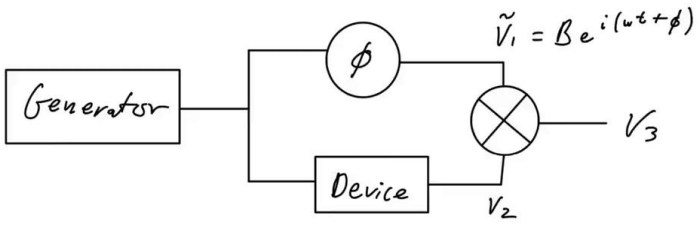



$V_3 = V_1 V_2$

Phasor notation:

$\tilde{V}_3 = \tilde{V}_1^* \tilde{V}_2 + \tilde{V}_1 \tilde{V}_2$

diff. freq. / sum freq.




Consider only difference frequency:

$$\tilde{V}_3 = A_0(1 + i m \cos\omega_m t) e^{i\omega t} B e^{-i\omega t} e^{-i\phi}$$

$$= A_0 B (1 + i m \cos\omega_m t) e^{-i\phi}$$

$V_3(t) = Re(\tilde{V}_3)$ → $\phi = 0$: $V_3(t)$ does not oscillate!

↳ max signal for $\phi = \pi/2$ to $3\pi/2$

So I'm gonna consider only the difference frequency because I'm gonna high pass filter the sum frequency or low pass filter the sum frequency away. So the difference frequency is then just this term here. So that's gonna be A-naught times 'i' omega 'i m cosine omega t'. This is a phase modulation term. Here's my 'i omega t' from the oscillation or frequency of my microwave generator. And then I'm gonna pick this guy actually has the same frequency but I've complex conjugated him to get a difference frequency and then here I have 'e' to the 'i phi'. And so these two guys here cancel and what I'm gonna get is gonna look a lot like homodyne except without the extra terms. It's going to have an 'A' and a 'B' and one plus 'i m cosine omega m t' 'e' to the 'i phi'. And now here is a little bit of a trick. We have to remember how phasor notation works. You would say yeah but this is always oscillating at omega 'm' but actually the oscillating signal we measured in lab is only the real part of this so if there's any oscillations in the imaginary part, that actually is something that we're not gonna measure just this is the mathematical construction of the phasor notation. And so actually if we pick in this case phi equals zero or pi, we will again get no oscillations and actually to get the optimal signal, we want to pick phi as pi over two or three pi over two.

Notes

Summary

18m 39s

# CONCLUSION / SUMMARY

- **Optomechanical detection generates sidebands of your laser that can correspond to amplitude modulation, phase modulation, or a mixture of both**
- **Diode / photo detectors can only detect amplitude modulation**
- **By adding a signal "in phase" with the one you want to detect before rectifying, you can recover all of your signal!**
- **This is called "homodyne detection"**
- **There are analogues of homodyne detection using mixers in the microwave domain**

*Code for plots and animations available at https://gitlab.tudelft.nl/gsteele/omt-mooc*

Okay, so in summary I've told you about how optomechanical detection generates sidebands of your laser that can be amplitude modulation, phase modulation or the mixture of whatever you wants pretty much depending on what detuning you choose. I've told you about how photo detectors and diodes can only detect amplitude modulation which is a real problem because sometimes all of your signals in phase or and actually always some of your signals in phase. I showed you how by adding an 'in phase', a signal 'in phase' with the one you want to detect, this can recover all of your signal back. This is something called homodyne detection. And I told you about an analog of homodyne detection commonly used in the microwave domain that uses mixers instead of diodes.

Notes

Summary

20m 16s