THE STANDARD QUANTUM LIMIT (SQL)

Cavity Quantum Optomechanics

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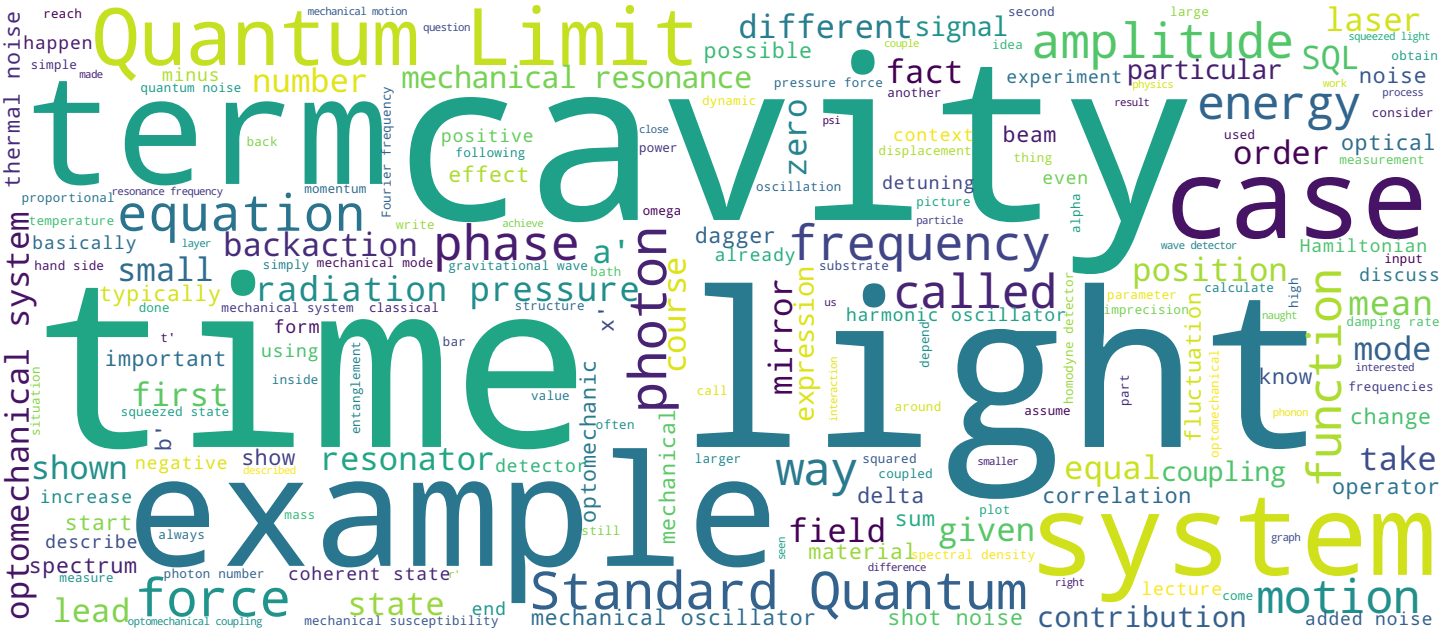

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OUTLINE

In this lesson, you will learn about the standard quantum limit (SQL) in optomechanical displacement measurements

- Measurement imprecision and backaction
- The Standard Quantum Limit (SQL)
- Frequency dependence of the SQL
- Reaching the SQL in practice
- Beating the SQL with quantum non-demolition (QND) measurement

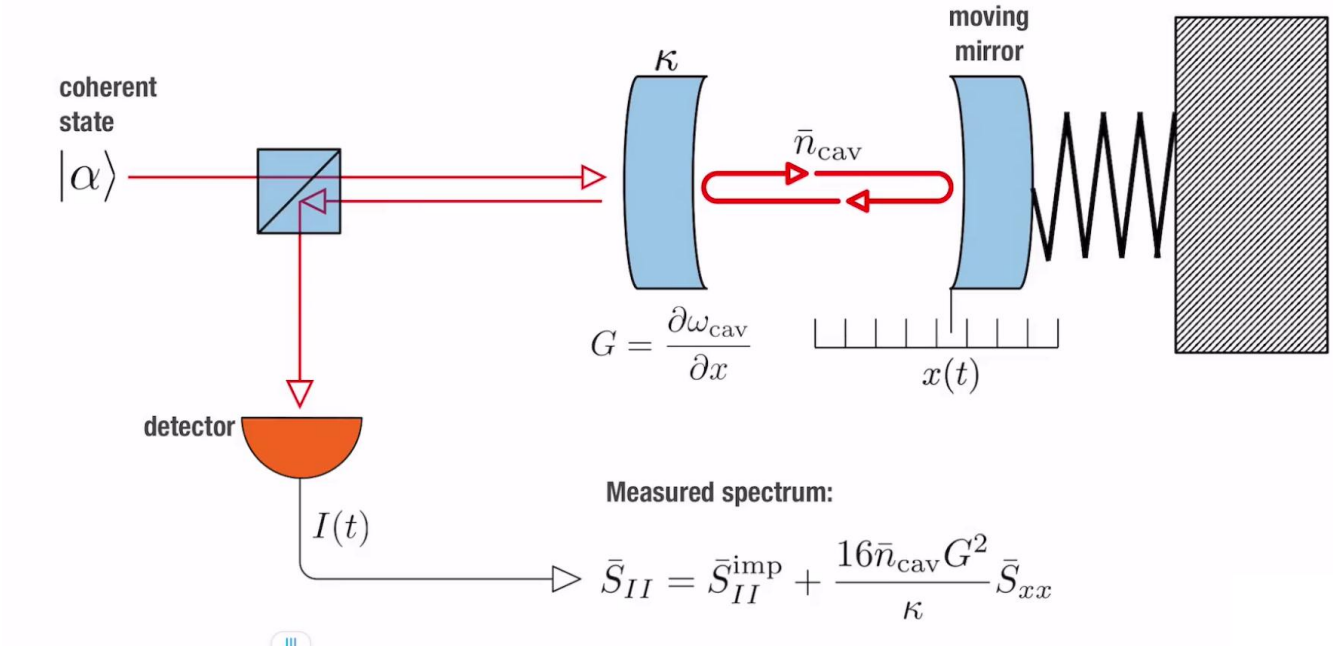
Welcome to this lecture on the Standard Quantum Limit. My name is Albert Schliesser. I'm at the Niels Bohr Institute of Copenhagen University. In this lesson, we will look at the Standard Quantum Limit in the specific context of optomechanical displacement measurements. And we will start by considering the two sources of quantum noise in such a measurement namely, measurement imprecision and measurement backaction. We will see how these two give rise to the Standard Quantum Limit, the SQL. We will discuss the frequency dependence of the SQL. We will see what it takes to reach the SQL in practice. And finally, we will briefly touch upon ways to achieve a sensitivity better than the SQL using quantum non-demolition measurements.

Notes

Summary

0m 00s

MEASUREMENT IMPRECISION



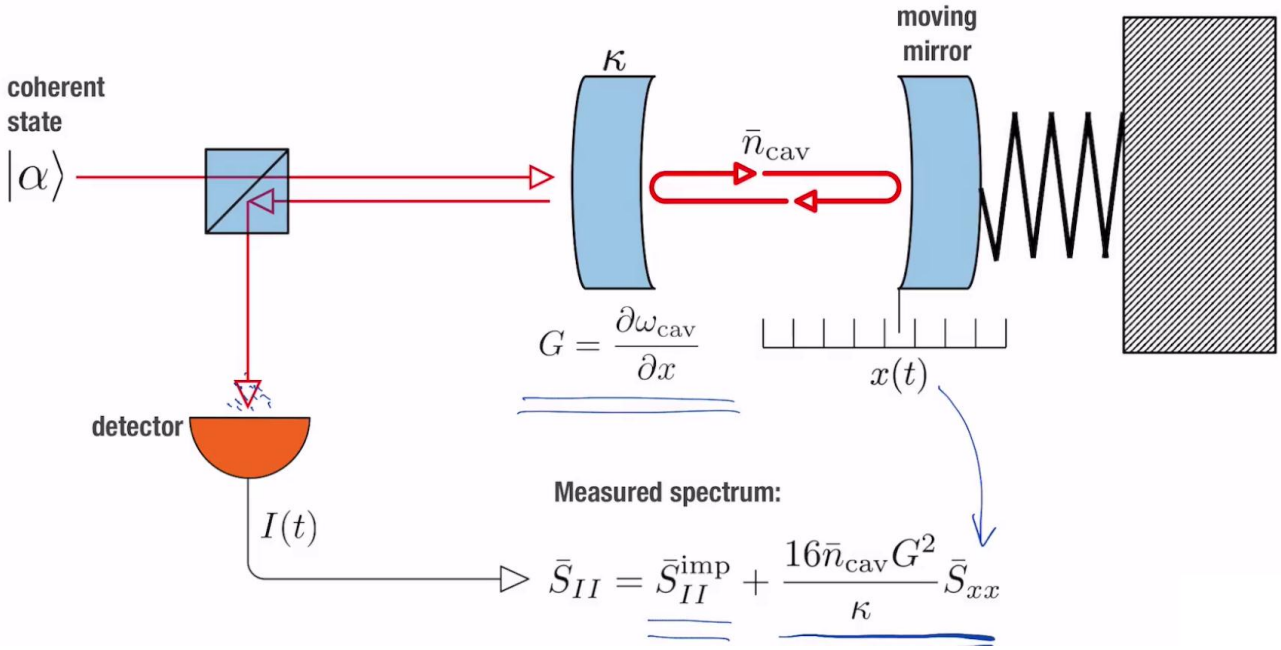
So suppose we want to measure the position of some object. We call this position 'x' and it can be a function of time and since we're dealing with optomechanics here, let's assume we're doing that with an optomechanical system. That means that I make this moving object reflective. It is a mirror and I'm combining it with a second mirror here which has a transmission rate for the light of kappa so that I can store a typically very large number of photons in this cavity n-bar cavity. I do this by basically shining a coherent state of light towards the system. It gets transmitted through the first mirror. The light makes many many round trips in the cavity. It comes back out and I'm sending it towards a detector. This will typically be a homodyne detector, I will measure the phase of the light that re-emerges and this homodyne detector outputs a current as a function of time. Now if I do analyze this current and in particular, I look at the spectrum that I get for the photocurrent then I would certainly expect to find a a signal that is just proportional to the displacement 'x' so I find the spectrum of the motion here multiplied with a transduction factor which involves the number of photons in the cavity but also the frequency pull parameter of this optomechanical system so by how much the resonance frequency of the cavity shifts for a given displacement 'x'.

Notes

Summary

0m 51s

MEASUREMENT IMPRECISION



But that's not the only contribution that I have in my photocurrent. There's also a second term which is typically just a wide spectrum and that's just a shot noise in my detection essentially due to the fact that I have random photon arrival times at my detector giving rise to shot noise. This noise term is referred to in the context of the Standard Quantum Limit as the measurement imprecision.

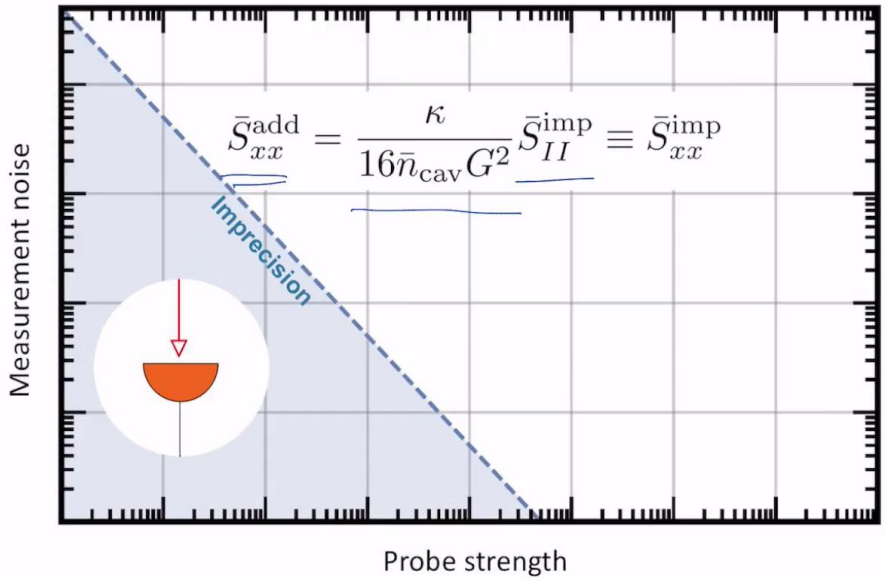
Notes

Summary

2m 52s

MEASUREMENT IMPRECISION

Increased probe strength lowers measurement imprecision



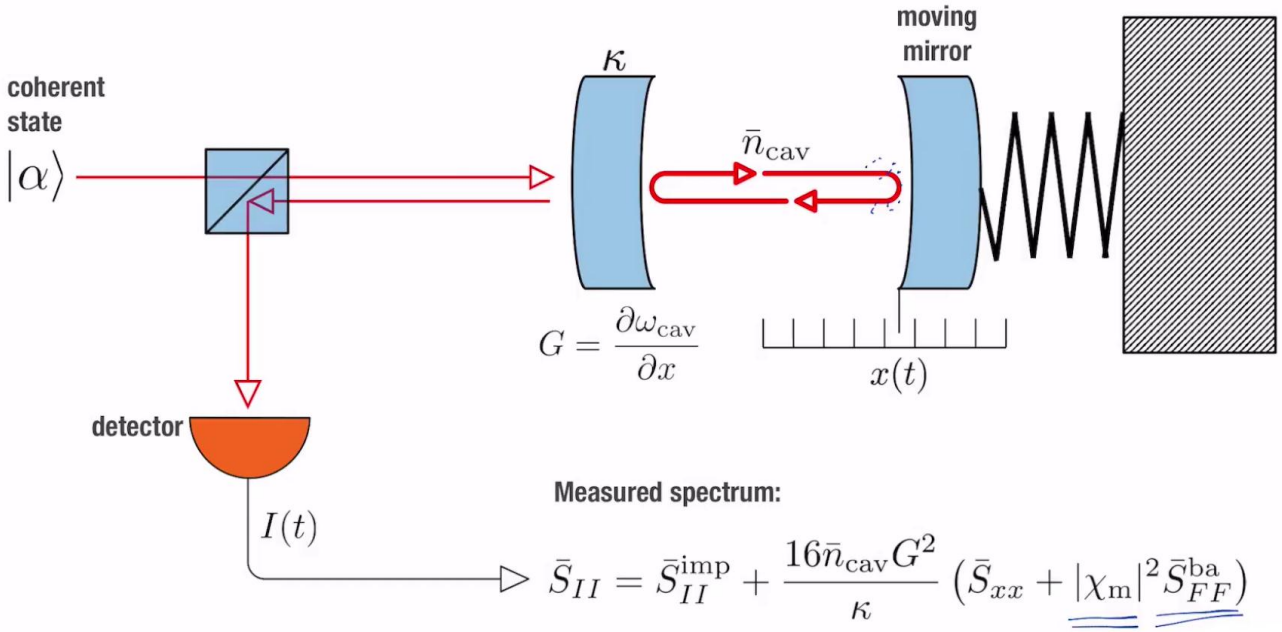
Now how does the measurement imprecision scale when I increase my probe strength, when, for example, I increase the number of cavity photons? To express that in a convenient way, what I can do is I can divide the photocurrent spectral density that absorbs this imprecision term. I can divide it by the transduction factor that's this first term here and thereby express the shot noise in my detector as a displacement spectral density, as an equivalent displacement spectral density. And this is obviously a noise that my measurement apparatus adds to the actual mechanical displacement so this will be an added noise expressed here in units of displacement. Now this added displacement the imprecision reduces when I'm using more light. This is in essence saying that my signal-to-noise ratio goes up when I use more light because the shot noise grows more slowly than my signal. So the added measurement noise indeed goes down with the probe strength.

Notes

Summary

3m 21s

MEASUREMENT BACKACTION



So as an experimentalist then I would be tempted to just use as much light as I can but that's not the full story because as we know in optomechanics, we are susceptible to the effects of radiation pressure. So the photons that are circulating here in the cavity exert a force on this moving mirror since they transferring a part of their momentum and now here we have to account for the fact that these photons really come in discrete packets so I really have a randomness in this process. I have random arrival times of the photons at the mirror and this gives a rise to a random force which I'm describing here with a spectral density as well. This is the backaction of my measurement and that, of course, translates also to mirror motion as prescribed by the mechanical susceptibility.

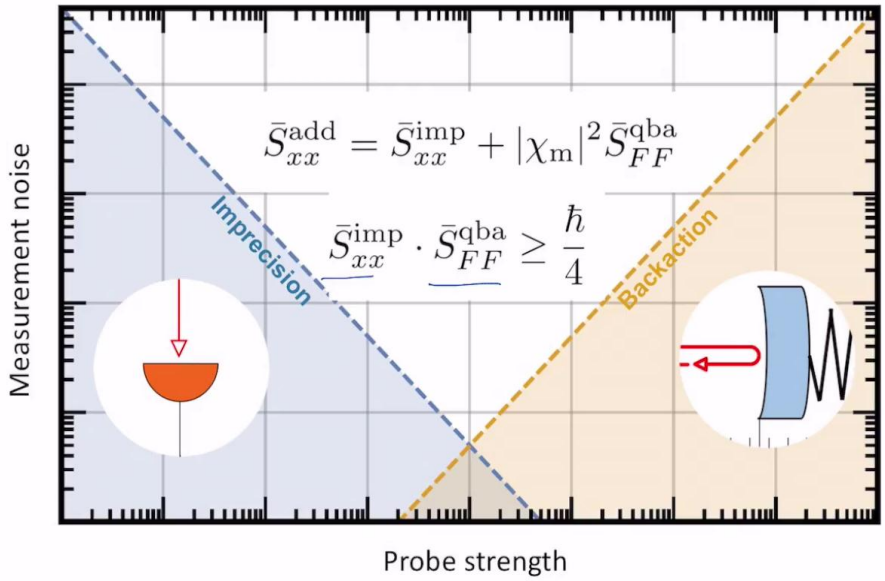
Notes

Summary

4m 45s

QUANTUM NOISE IN DISPLACEMENT MEASUREMENT

Heisenberg inequality bounds the product of imprecision and backaction



Braginsky, Vorontsov, Sov. Phys. Usp. **17**, 644 (1974)
Caves. PRL **45**. 75 (1980)

Now it will be hardly surprising that this force fluctuations here grow the more photons I have in the cavity and so if I plot the backaction here as a function of the probe strength, say, the photon number in the cavity then this grows linearly with the photon number and so if I look at my total added noise in the measurement which now has two contributions. The imprecision, that's the shot noise in the detection process and the quantum backaction which is a radiation pressure shot noise. I find that I have to balance these two processes and given that the imprecision term arises essentially from the quantum fluctuations of the phase quadrature of the light and the backaction term arises from the quantum fluctuations of the amplitude quadrature of the light. I actually have an additional constraint on these two terms. The imprecision noise and the backaction noise in the form of a product the imprecision backaction product, has to be larger than a certain bound in this case the reduced Planck constant over four. This is a variant of the Heisenberg inequality.

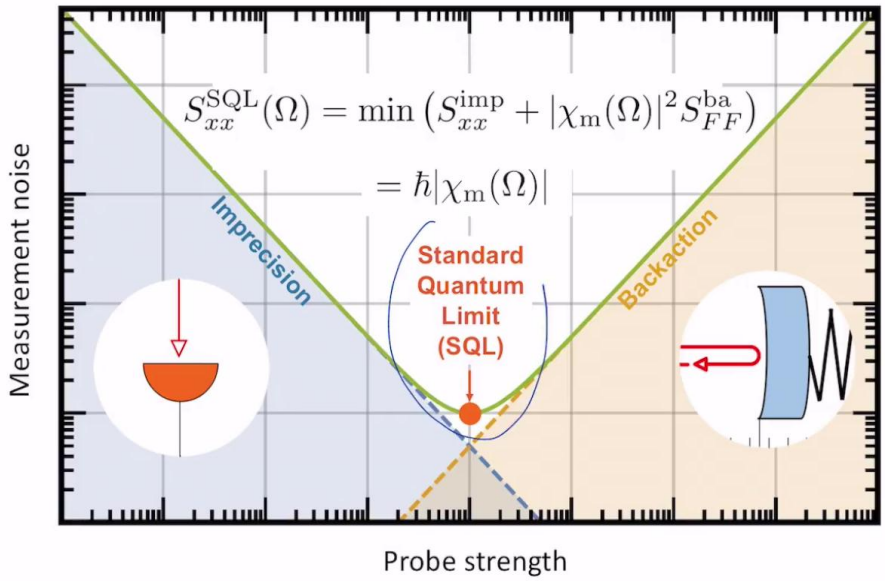
Notes

Summary

5m 46s

THE STANDARD QUANTUM LIMIT (SQL)

Optimum trade-off of imprecision and backaction yields the Standard Quantum Limit (SQL)



Braginsky, Vorontsov, Sov. Phys. Usp. **17**, 644 (1974)
Caves. PRL **45**. 75 (1980)

And we see here that the Standard Quantum Limit arises due to the quantum properties and basically commutation relations for the electromagnetic field that I'm using to probe the mechanical system so it's important to note that the Standard Quantum Limit has nothing to do with the quantization of say, the motion or the mass that we're interested in measuring. Now if I do this optimization and try to find the minimum of the sum of these two contributions under the assumption that these two contributions are uncorrelated, I find that this is simply the mechanical susceptibility magnitude times the Planck constant. So this is the Standard Quantum Limit.

Notes

Summary

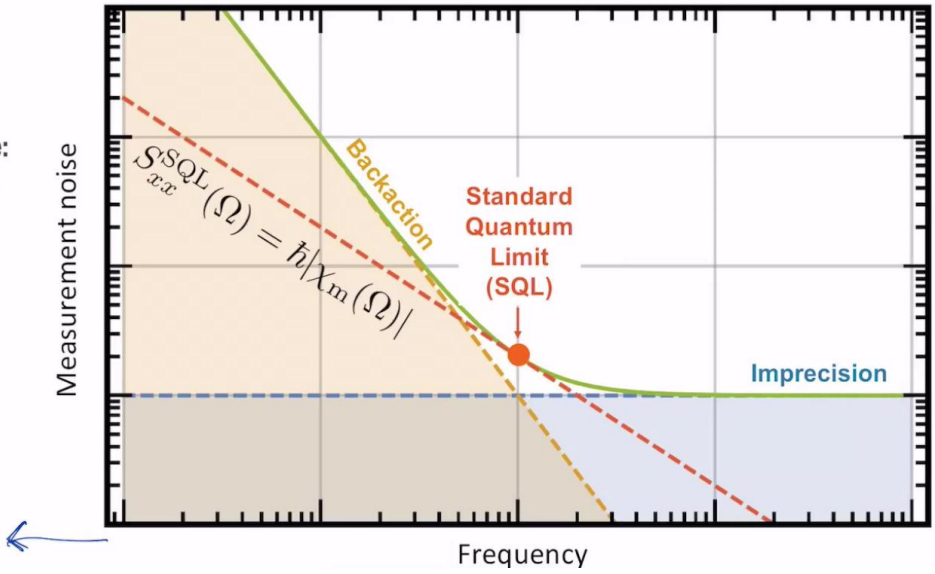
7m 21s

FREQUENCY DEPENDENCE OF THE SQL

At each Fourier frequency, we have

$$\bar{S}_{xx}^{\mathrm{SQL}}(\Omega) = \min \bar{S}_{xx}^{\mathrm{add}}(\Omega) = \min \left(\bar{S}_{xx}^{\mathrm{imp}} + |\chi_{\mathrm{m}}(\Omega)|^2 \bar{S}_{FF}^{\mathrm{ba}}\right) = \hbar|\chi_{\mathrm{m}}(\Omega)|$$

Close to a (here: low-frequency) mechanical resonance, backaction dominates



Far away from the mechanical resonance, imprecision dominates

Now I have written the Standard Quantum Limit here as in terms of the mechanical susceptibility. Now the mechanical susceptibility, of course, is a function of frequency and this explains another plot that you will very often see in the context of the Standard Quantum Limit where the measurement noise is shown as a function of Fourier frequency and more often and not in particular in the context of gravity wave detectors, it is assumed here that the mechanical resonances are at the low frequency end here so they're not really shown but you can basically assume that in the left part of this graph, we're closer to a mechanical resonance so we have a larger mechanical susceptibility, a larger response to forces and that means in the left part of this graph, we are actually dominated by backaction. First in the right part of the graph, we are far away from mechanical resonances so chi-m is a small is a small term and we are thereby dominated by the first part here, the imprecision noise. Now if the system operates perfectly at the quantum limit then again there will be a particular Fourier frequency where these two terms have an equal size and the sum of the imprecision and the backaction, the sum is shown here as the green curve, will exactly touch the Standard Quantum Limit at one particular Fourier frequency.

Notes

Summary

8m 22s

FREQUENCY DEPENDENCE OF THE SQL

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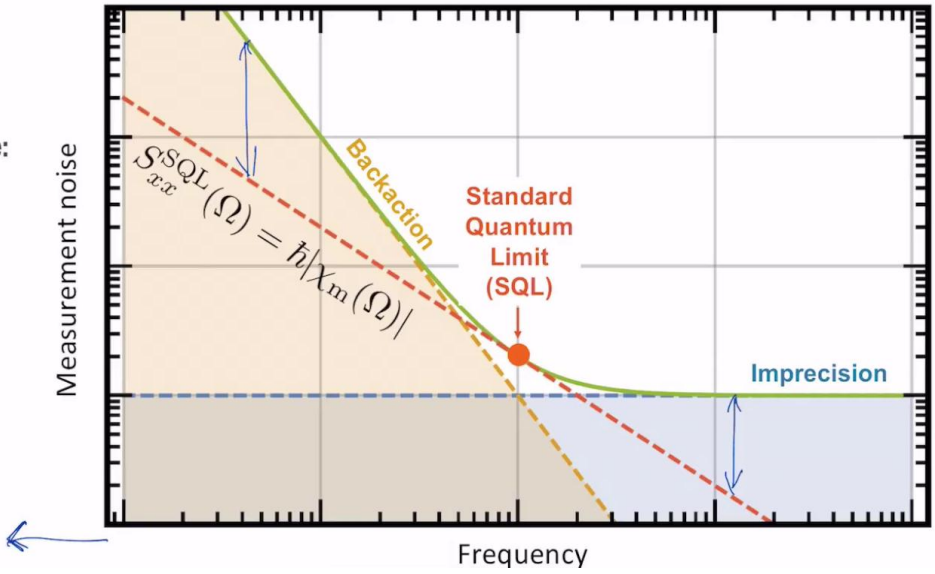


Far away from the mechanical resonance, imprecision dominates

At all other frequencies, we are above the Standard Quantum Limit here and here.

Notes

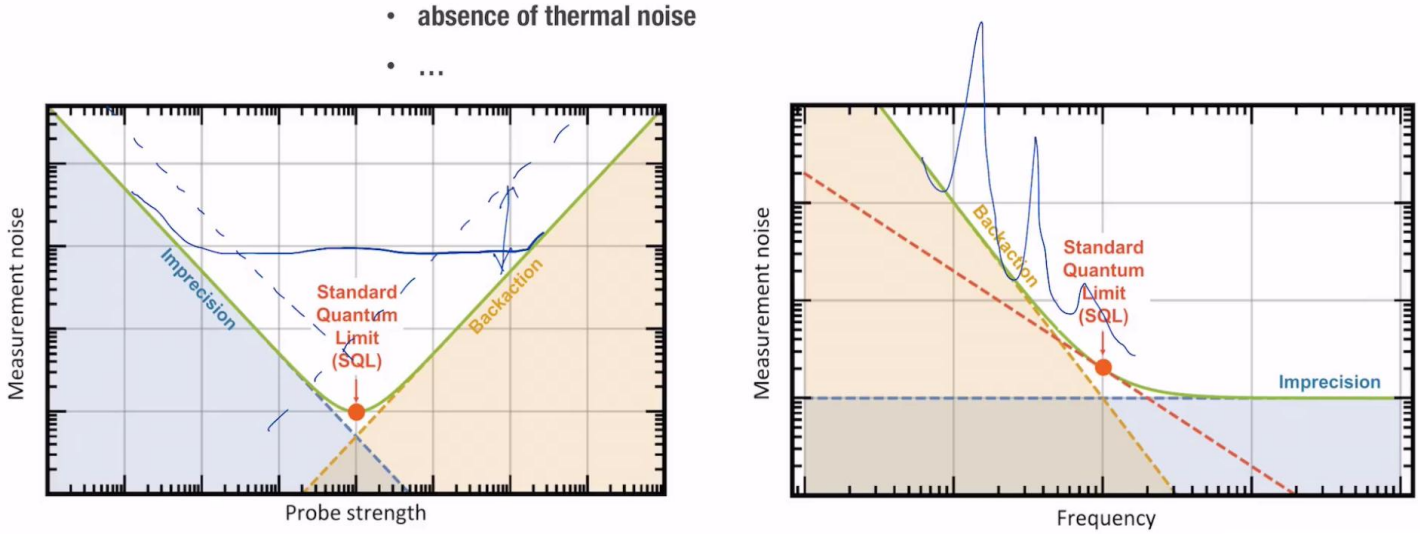
Summary

10m 13s

REACHING THE SQL IN PRACTICE

Access to the SQL requires an ideal measurement, including

- **unity detection efficiency**
- **pure coherent state of the probing light**
- **absence of thermal noise**
- **…**



Now what does it take to reach the Standard Quantum Limit in practice? We have made a number of idealizing assumptions. One is a unity detection efficiency. Typically my detector will have a finite detection efficiency then I will have more quantum noise in my detector because I will partly be measuring just a vacuum uncorrelated with mechanical motion, I will have more imprecision in my detector for a given signal so this would lead for to an offset like this, for example. Or I could have a laser which is not emitting a very pure coherent state but it has excess amplitude noise and that would lead to a increase in the backaction and so I would not be able to reach the Standard Quantum Limit. Another very common source of noise is a thermal noise in my mechanical system which would say, lead to a offset like this just an added noise that is completely independent of what I'm doing with the light but also this would prevent me from breaching the Standard Quantum Limit. Here on the right-hand side, if you plot it as a function of frequency I could have, for example, other mechanical resonances here that prevent me from breaching the Standard Quantum Limit.

Notes

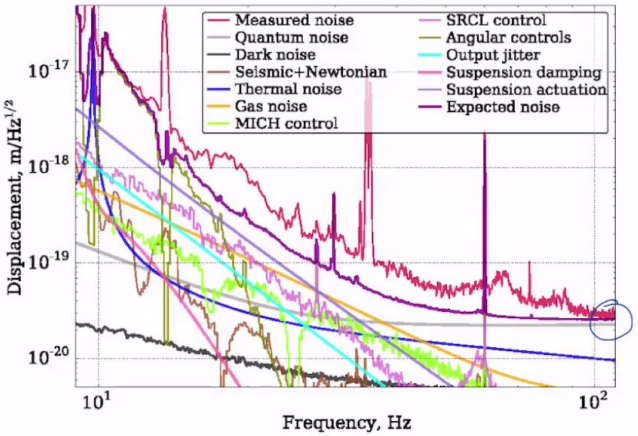
Summary

10m 21s

REACHING THE SQL IN PRACTICE

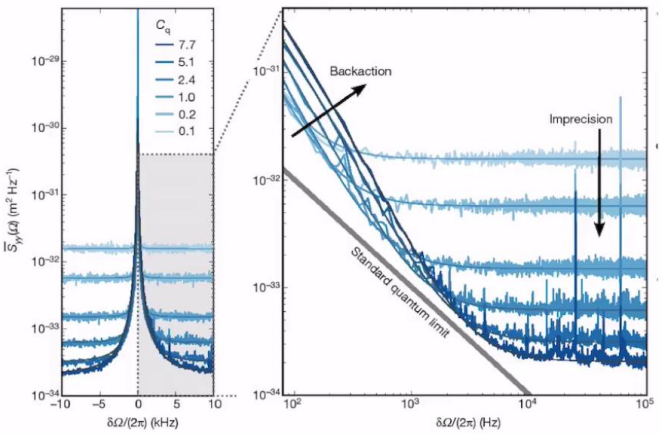
Advanced LIGO

Martynov et al., Phys. Rev. D **93**, 112004 (2016)



Cryogenic optomechanical system

Rossi et al., Nature **563**, 53 (2018)



So here are some examples how such spectra look like in the real world. On the left-hand side, you see a data set from advanced LIGO so the gravitational wave interferometer that made the first detection of a black hole merger and you see that there are indeed many many many contributions to the noise spectrum in the relevant band and while at high frequencies so this would be on that end quantum noise, this is the green line really dominates the measured noise at low frequencies where we would expect to see backaction. There are still a number of other contributions even though this instrument has one of the advanced most advanced measurement systems that we have in all of physics I may say. It's getting very close to being completely dominated by quantum noise and these data are already a little bit outdated so nowadays LIGO is even closer. On the right-hand side, you see data from a cryogenic optomechanical system where these concepts of quantum measurement are really explored in a smaller setting that also allows cryogenic cooling to reduce, for example, the thermal noise of the mechanical system and here you see that close to a mechanical resonance so the frequency zero here is exactly aligned with one mechanical resonance. Close to a mechanical resonance indeed one can see both the imprecision and the backaction and they're scaling as the laser power is changed.

Notes

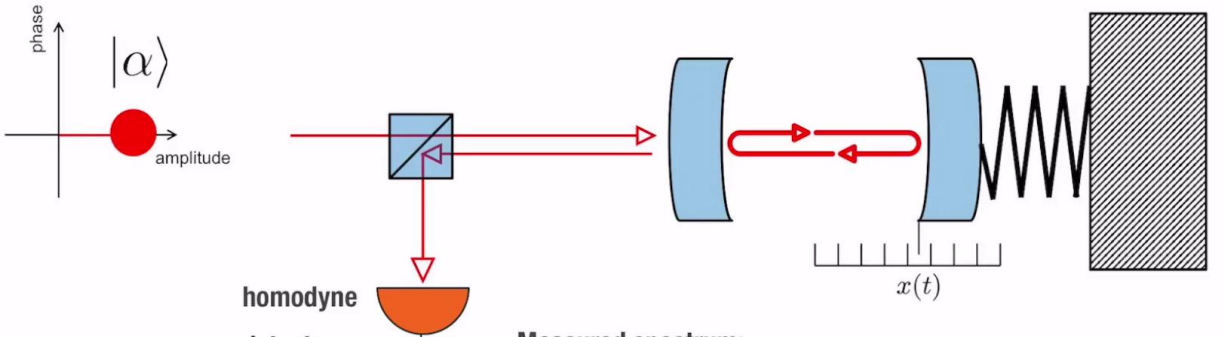
Summary

12m 01s

BEATING THE SQL

To overcome the SQL, make a quantum non-demolition (QND) measurement (V. B. Braginsky)

In the case of continuous position measurement: exploit correlations.



Measured spectrum:

$$\bar{S}_{II} = \bar{S}_{II}^{\mathrm{imp}} + \frac{16\bar{n}_{\mathrm{cav}}G^2}{\kappa}\left(\bar{S}_{xx} + |\chi_{\mathrm{m}}|^2 \bar{S}_{FF}^{\mathrm{ba}}\right)$$

phase ampl.

Braginsky, Unruh, Caves, Matsko, Vyatchanin, Kimble, Thorne, …

Now, is the Standard Quantum Limit a fundamental limit? Is it possible to overcome it at all or does quantum mechanics forbid this? Already in the 70s, Vladimir Braginsky of Moscow State University figured out that there are ways to actually overcome the Standard Quantum Limit. In the case of continuous position measurements, this can be done by exploiting quantum correlations. This will implement what Braginsky called a quantum non-demolition measurement that means a measurement that does not the, that does not destroy or demolish the information that we are interested in extracting from our quantum system. Now how does this work? The basic idea is that we have in our derivations assumed that the imprecision noise and the backaction noise are uncorrelated so, therefore, when we calculate the total added noise, we just add these two terms. But you have to keep in mind that these are fluctuations again essentially related to the phase and the amplitude quadrature of the light and they can be correlated. In particular, if we inject instead of a coherent state which is often represented in phase space by this uncertainty disk here with an equal amount of fluctuations in amplitude and phase.

Notes

Summary

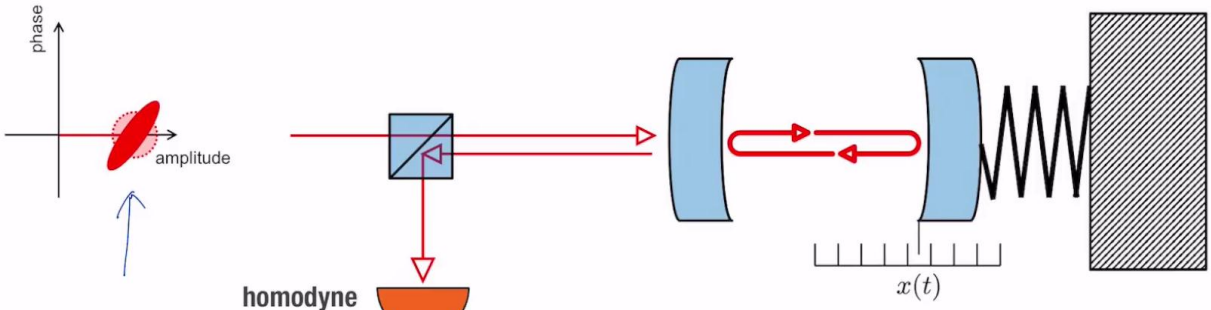
13m 54s

BEATING THE SQL

To overcome the SQL, make a quantum non-demolition (QND) measurement (V. B. Braginsky)

In the case of continuous position measurement: exploit correlations.

For example, by injecting squeezed light, or by detecting the correlations generated by optomechanical interaction.



Measured spectrum:

$$\bar{S}_{II} = \bar{S}_{II}^{\mathrm{imp}} + \frac{16\bar{n}_{\mathrm{cav}}G^2}{\kappa}\left(\bar{S}_{xx} + |\chi_{\mathrm{m}}|^2\bar{S}_{FF}^{\mathrm{ba}}\right) + \cdots \bar{S}_{IF}$$

Braginsky, Unruh, Caves, Matsko, Vyatchanin, Kimble, Thorne, …

If instead of such a coherent state we inject a squeezed state, a state that has correlations between amplitude and phase and thereby the uncertainty disk is not a disk but it's actually an ellipse, which is only possible if I have correlations between the amplitude and phase, then I get another term out here with a prefactor which I will not derive that reflects the correlations between the imprecision and the backaction. And this term can be both positive and negative and the idea here is to design the experiment in such a way that this term becomes negative so that the added noise is lower than the added noise at the SQL.

Notes

Summary

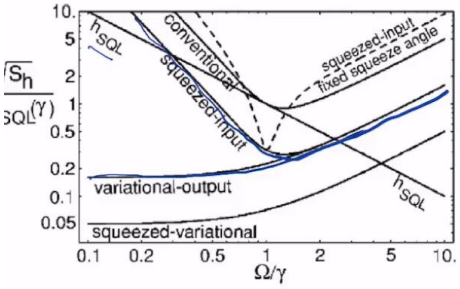
15m 37s

BEATING THE SQL OF CONTINUOUS POSITION MEASUREMENT

Theory:

For broadband enhancement, correlations must have tailored frequency dependence

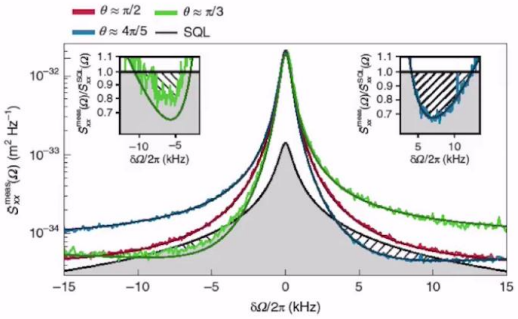
Kimble et al., Phys. Rev. D **65**, 022002 (2001)



Experiment:

Sub-SQL position measurement in a cryogenic optomechanical system by homodyne detection of cavity output

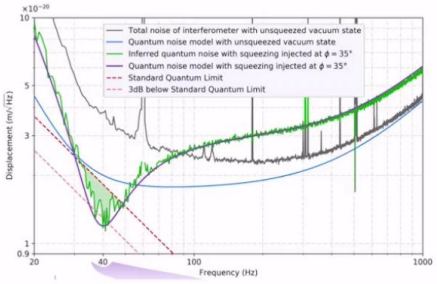
Mason et al., Nature Physics **15**, 745 (2019)



Experiment:

Sub-SQL quantum noise in advanced LIGO by injecting squeezed light (thermal noise subtracted)

Yu et al., arXiv:2002.01519 (2020)



This idea has been around for quite some time. This paper here from 2001 gives a nice comparison of several approaches to implement this. You see as a dashed line here what I have just explained this is what you would get for sensitivity enhancement as compared to the SQL if you have a fixed squeeze angle meaning that the angle of rotation of this uncertainty ellipse is the same at any Fourier frequency. It was then soon understood that in order to be really useful what you would like to have is you would like to have a squeezed state whose rotation angle varies with Fourier frequency then you can get this curve which I have just redrawn here. It was also shown that it's not important whether you inject a squeezed state or whether you use the optomechanical phase amplitude correlations that arise in the optomechanical system itself by just arranging the homodyne detector in such a way to be sensitive to these correlations. Then you can actually get the same enhancement. This is known as the variational output method. Again here in order to get such a broadband enhancement, what you would have to do is you would have to have different homodyne detection angles for different Fourier frequencies which can also be done.

Notes

Summary

16m 37s

BEATING THE SQL OF CONTINUOUS POSITION MEASUREMENT

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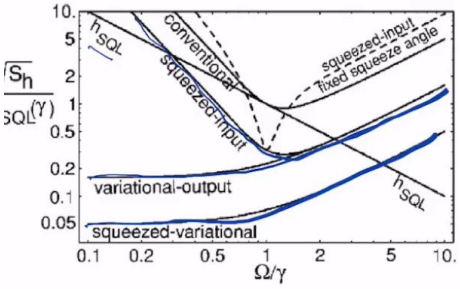
For broadband enhancement, correlations must have tailored frequency dependence

Kimble et al., Phys. Rev. D **65**, 022002 (2001)



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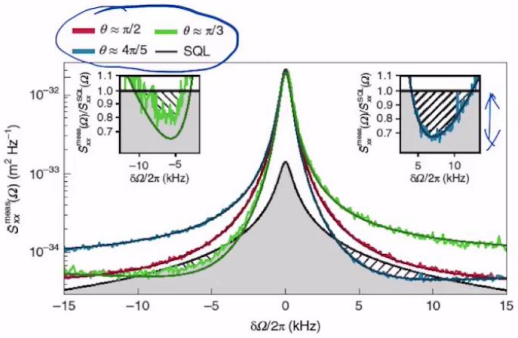

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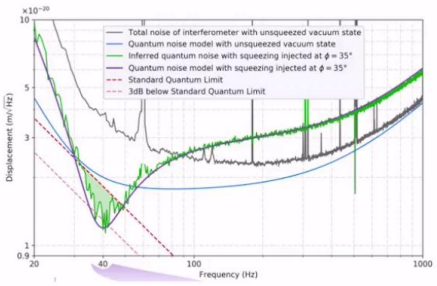


And finally, these two approaches can also be combined to give an even better enhancement. Now these experiments have been done. You see here in the middle a recent experiment where the variational method was implemented meaning that the phase of the homodyne detector was arranged in such a way to exploit optomechanical correlations between phase and amplitude arising directly in the optomechanical system and you see indeed that it was possible here to achieve a sensitivity below the SQL. This is again a cryogenic optomechanical system so no thermal noise had to be subtracted. This system really achieves the sub SQL performance and very recently there's also an experiment here from again the LIGO detectors where they show that if you inject squeezed light into advanced LIGO again at this stage only with a fixed squeezing angle and if you then subtract thermal noise because these just these detectors are not cryogenic so for now thermal noise is still too high but if you subtract the thermal noise then you can also see in this curve here that it's possible to actually reach below the SQL which is the red dashed line.

Notes

Summary

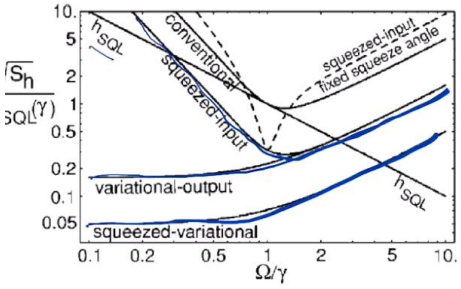
18m 18s

BEATING THE SQL OF CONTINUOUS POSITION MEASUREMENT

Theory:

For broadband enhancement, correlations must have tailored frequency dependence

Kimble et al., Phys. Rev. D **65**, 022002 (2001)



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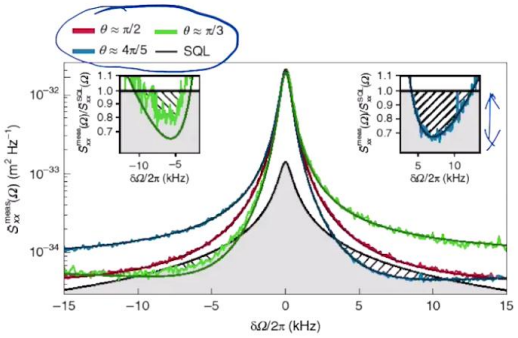

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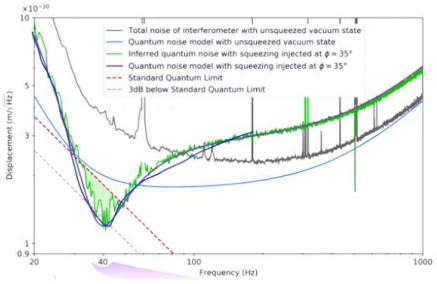

So these methods are very promising also for real world detectors such as advanced LIGO and, therefore, methods to realize this broadband squeezing input are actually developed and a big challenge here is really to get this frequency dependent rotation but this is just an engineering challenge which is taken on and in a good way to actually further improve the event rates in gravity wave detectors so in order not to be hindered by the SQL. There are many more ways in which the SQL can be beaten also in the context of displacement measurements but we won't have time to discuss them here.

Notes

Summary

19m 43s

LITERATURE

V. B. Braginsky, F. Ya. Khalili:
"Quantum Measurement"
Cambridge University Press (1992)

A. A. Clerk,M. H. Devoret, S. M. Girvin, F. Marquardt, R. J. Schoelkopf:
"Introduction to quantum noise, measurement, and amplification"
Review of Modern Physics 82, 1155 (2010)

A. Schliesser, T. J. Kippenberg:
"Cavity Optomechanics with Whispering Gallery Mode Microresonators"
Advances in Atomic, Molecular and Optical Physics 58, 207 (2010)

M. Aspelmeyer, T. J. Kippenberg, F. Marquardt:
"Cavity Optomechanics"
Review of Modern Physics 86, 1391 (2014)

Some of that you may be able to find in the literature which I'm listing here and in these papers you will, in particular, also find more derivations and more background and context on the SQL.

Notes

Summary

20m 36s

LITERATURE

V. B. Braginsky, F. Ya. Khalili:
"Quantum Measurement"
Cambridge University Press (1992)

A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, R. J. Schoelkopf:
"Introduction to quantum noise, measurement, and amplification"
Reviews of Modern Physics 82, 1155 (2010)

D. G. Matsko, A. A. Savchenkov, N. Yu, L. Maleki:
"Whispering-gallery-mode resonators for precision oscillators"
Proceedings of the 2006 IEEE International Frequency Control Symposium and Exposition, 2006., 2006, 1-6

I. S. Grudinin, A. B. Matsko, L. Maleki:
"Fluctuations in whispering gallery mode resonators"
Proceedings of the 2006 IEEE International Frequency Control Symposium and Exposition, 2006., 2006, 7-11

I. S. Grudinin, A. B. Matsko, L. Maleki:
"Fluctuations in whispering gallery mode resonators"
Proceedings of the 2006 IEEE International Frequency Control Symposium and Exposition, 2006., 2006, 7-11

A. B. Matsko, A. A. Savchenkov, D. Strekalov, V. S. Ilchenko, L. Maleki:
"Optical Hydrogen Maser"
Physical Review Letters 95, 143904 (2005)

T. J. Kippenberg, K. J. Vahala:
"Cavity Optomechanics: Back-Action at the Mesoscale"
Science 321, 1172 (2008)

F. Marquardt, S. M. Girvin:
"Optomechanics"
Physics 2, 40 (2009)

I. Wilson-Rae, N. Nooshi, W. Zwerger, T. J. Kippenberg:
"Theory of Ground State Cooling of a Mechanical Oscillator Using Dynamical Backaction"
Physical Review Letters 99, 093901 (2007)

F. Marquardt, J. P. Chen, A. A. Clerk, S. M. Girvin:
"Quantum Theory of Cavity-Assisted Sideband Cooling of Mechanical Motion"
Physical Review Letters 99, 093902 (2007)

C. Genes, D. Vitali, P. Tombesi, H. Gigan, M. Aspelmeyer:
"Ground-state cooling of a micromechanical oscillator: Comparing cold damping and cavity-assisted cooling schemes"
Physical Review A 77, 033804 (2008)

M. Aspelmeyer, S. Gröblacher, K. Hammerer, N. Kiesel:
"Quantum optics and quantum information experiments with nanomechanical oscillators"
Journal of the Optical Society of America B 27, A189 (2010)

M. Metcalfe:
"Applications of cavity optomechanics"
Applied Physics Reviews 1, 031105 (2014)

M. Aspelmeyer, T. J. Kippenberg, F. Marquardt:
"Cavity Optomechanics"
Review of Modern Physics 86, 1391 (2014)

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Summary

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