ENTANGLEMENT II

Cavity Quantum Optomechanics

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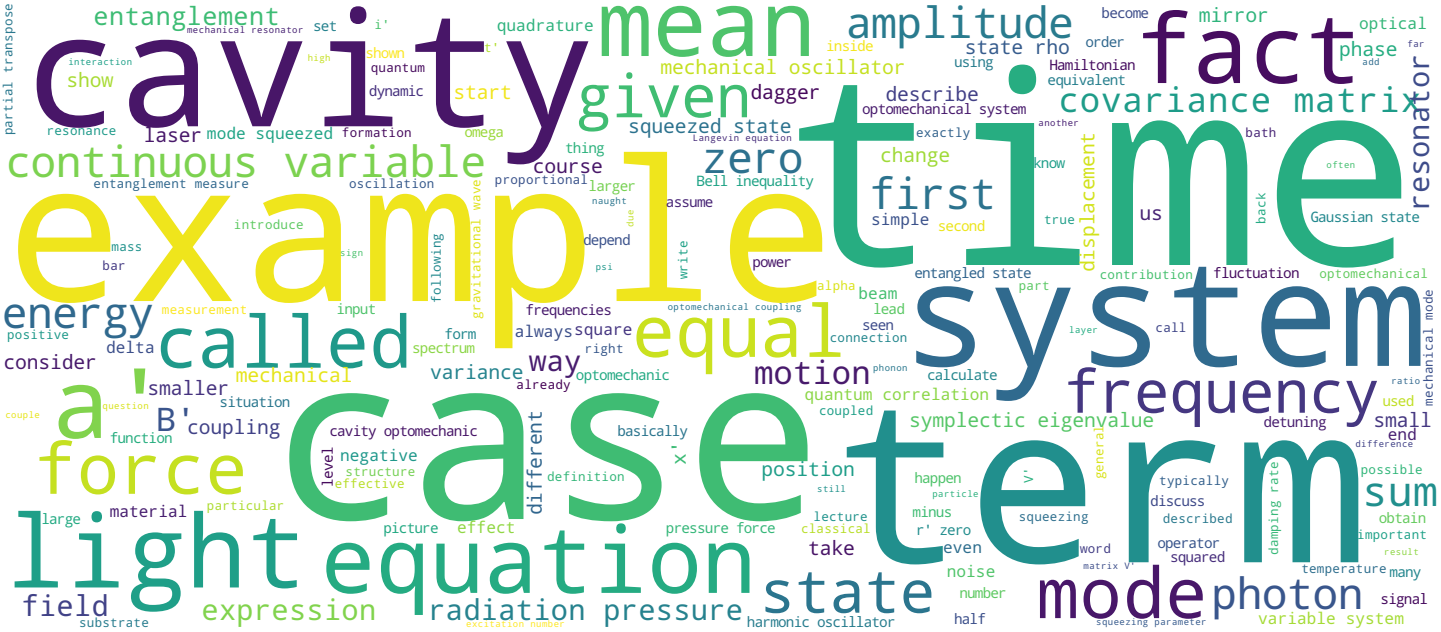

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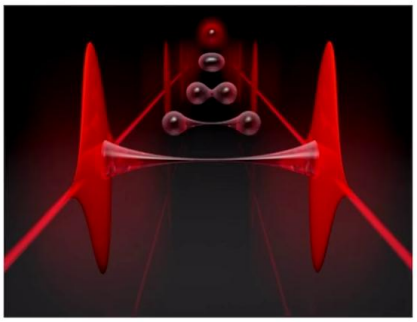
WELCOME & OUTLINE

- This is the second lecture on the concept of entanglement. In the first one we have provided the basic mathematical definitions and the main properties.

In this second part we shall apply them to cases of interest for cavity optomechanics:



- Entanglement in continuous variable (CV) systems
- Entanglement for CV Gaussian states
- Entanglement and quantum correlations

Welcome, I'm David Vitali and I will talk about entanglement in cavity optomechanical systems. This is the second lecture on the concept of entanglement. In the first one, we have provided the basic mathematical definitions and the main properties of the entanglement between two quantum system. In the second part, we shall apply the definitions and we have seen in the previous part two cases of interest for cavity optomechanics. In fact, we shall discuss about entanglement in continuous variable systems. We shall then specify to the Gaussian state case. And finally, we shall discuss a bit in general, the connection between the concept of entanglement and the wider concept of quantum correlations.

Notes

Summary

0m 00s

CONTINUOUS VARIABLE SYSTEMS

In cavity optomechanics, we deal with **mechanical resonators & cavity modes = continuous variable (CV) systems,** characterized by position/momentum or amplitude/phase quadratures

$$q_i = \frac{a_i + a_i^+}{\sqrt{2}} \quad p_i = \frac{a_i - a_i^+}{i\sqrt{2}} \quad \left[a_i, a_j^+\right] = \delta_{ij} \Rightarrow \left[q_i, p_j\right] = i\delta_{ij}$$

$$u^T = \left(q_1, p_1, q_2, p_2, ,..., q_N, p_N\right)$$

Fundamental object for characterizing the entanglement of a state ρ is the **covariance matrix (CM) *V***

$$V_{ij} = \frac{1}{2}\left\langle u_i u_j + u_j u_i \right\rangle_\rho - \left\langle u_i \right\rangle_\rho \left\langle u_j \right\rangle_\rho$$

Real, symmetric *2Nx2N* matrix

One can express the **Heisenberg uncertainty theorem** with it

$$V + \frac{i}{2}\Omega \geq 0$$

$$\left[u_i, u_j\right] = i\Omega_{ij}$$

Symplectic matrix

Linear canonical transformations *u' = Mu* preserve commutation rules $\Rightarrow M\Omega M^T = \Omega$, and V can be diagonalized by M $\Rightarrow MVM^T = D = \text{diag}(\nu_1, \nu_1, \nu_2, \nu_2, ..) \Rightarrow$ the Heisenberg uncertainty becomes

$$\nu_j \geq 1/2$$

The symplectic eigenvalues ν_j must be larger than ½ = vacuum (zero-point) quantum fluctuations

In cavity optomechanics, we deal with mechanical resonators and cavity modes of of electromagnetic field which are continuous variable systems which means that they are described by an infinite dimensional Hilbert space characterized, therefore, by position momentum conjugate pair of variables position momentum for mechanical oscillator or amplitude phase quadratures for electromagnetic modes. We shall describe this mode, in general, using notation similar to those of mechanical system. So the two quadratures ‘qi’ and ‘pi’ which are linear combination the annihilation and creation operators 'ai' of a given mode. And they are measured in this case so that the commutator is equal to 'i'. We also introduce a column vector 'u' of the all quadratures of the 'N' modes so a column vector of dimensions '2N'. A fundamental object for characterizing the entanglement of state rho is the covariance matrix in the continuous variable case. The matrix ‘V’ is defined as we show here and the averages are performed over the state rho, so just the covariance is as usually defined in statistical situation. 'V', the covariance matrix is a real symmetric matrix of dimension 'N'.

Notes

Summary

0m 55s

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The symplectic eigenvalues ν_j must be larger than ½ = vacuum (zero-point) quantum fluctuations

'V' is useful, for example, to express the Heisenberg uncertainty theorem in a very general way. As a positivity condition, for a matrix 'VN' by 'V' with the sum of the so-called symplectic matrix which is a compact way of expressing the commutation rules between the continuous variables. So 'V' plus 'i' over two omega has to be positive. This is a compact way of expressing the fact that variances of continuous variable should not to be too small. This can be also in equivalent way expressed in terms of the so-called symplectic eigenvalues of the covariance matrix 'V'. The symplectic eigenvalues are obtained by diagonalizing with a linear transformation 'M' which preserve the commutation rule so preserves the symplectic matrix omega. So by diagonalizing 'V' with 'M', one gets a diagonal matrix. Diagonal elements are coded with the symplectic eigenvalues and they must be larger than one-half equivalent to the positivity of the above matrix. Therefore, the Heisenberg uncertainty theorem is going to state that all the symplectic eigenvalues or their covariance matrix must be larger equal or larger than one-half which is the just the size of the zero-point vacuum quantum fluctuations.

Notes

Summary

2m 30s

NPT ENTANGLEMENT CRITERION FOR CV SYSTEMS

Since $\rho = \rho^{+}$, $\rho^{T} = \rho^{*}$, and therefore **for CV systems transposition = time inversion**

$$\rho^{T_B} \leftrightarrow (q_A, p_A, q_B, p_B)^T \rightarrow \Lambda_B (q_A, p_A, q_B, p_B)^T = (q_A, p_A, q_B, -p_B)^T$$

NPT state $\rho^{T}_{B} < 0 \Rightarrow$ **it violates Heisenberg theorem**

$$\widetilde{V} = \Lambda_B V \Lambda_B$$

$$\Lambda_B = Diag(1,1,1,-1)$$

$$\widetilde{V} + \frac{i}{2}\Omega < 0 \Leftrightarrow \exists j \,|\, 0 < \widetilde{\nu}_j < 1/2$$

NPT sufficient criterion for entanglement for CV systems in terms of the **PT symplectic eigenvalues** of $\widetilde{V}, \widetilde{\nu}_j$

Special case of 1x1 systems (A and B with one mode only)

$$V = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

$$\widetilde{\nu}_{-} = \sqrt{\frac{\Sigma(V)}{2} - \sqrt{\left(\frac{\Sigma(V)}{2}\right)^2 - Det(V)}}$$

$$\Sigma(V) = Det(A) + Det(B) - 2Det(C)$$

NPT sufficient entanglement criterion (Simon)

$$0 < \widetilde{\nu}_{-} < 1/2 \Leftrightarrow 4Det(V) + 1/4 < \Sigma(V)$$

Let us now apply the entanglement criterion entanglement measure to the case of continuous variable system. Let's start with entanglement criteria and let's start with the NPT, the Non-positive Partial Transpose criteria. For continuous variable system, transposition is equal to time inversion because transposition is equivalent to take the complex conjugate and therefore, at the level of variables partial transposition of the system 'B' is equivalent to change the sign of the momentum 'pB', as we can see here. Therefore, an NPT state with an opposite partial transpose is certainly non-feasible so here 'B' is negative and therefore, it must violate Heisenberg theorem and therefore, if one considers the covariance matrix of the partial transpose state will be equal V-tilde which is transformed by the operation lambda 'B' which describes now the partial transpose so the change the sign of the momentum. V-tilde must violate Heisenberg theorem as we can see here. So it must be plus 'i' over two omega must be negative which is equivalent to say, that at least one PT symplectic eigenvalue exists which is smaller than one-half. So this is just the NPT sufficient criterion in terms of the PT symplectic eigenvalues nu-tilde.

Notes

Summary

4m 05s

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NPT sufficient entanglement criterion (Simon)

$$0 < \widetilde{\nu}_{-} < 1/2 \Leftrightarrow 4Det(V) + 1/4 < \Sigma(V)$$

In the special case of one times one subsystem which 'A' and 'B' have both only one mode. 'V' is a four by four matrix and one has only one negative symplectic relevant symplectic eigenvalue which can be less than one-half and we expressed this here in terms of a quantity related to the sub-matrices A, B, C, sigma and determinant. And so everything is quite easily computable.

Notes

Summary

5m 30s

NONLINEAR WITNESSES FOR CV SYSTEM

Again for the special case of 1x1 systems (A and B with one mode only)

Duan et al. sufficient criterion for entanglement in CV systems (sum criterion):
the state ρ is entangled if $\exists$ a≠0 such that

$$Var\left(|a|q_A+\frac{q_B}{a}\right)+Var\left(|a|p_A-\frac{p_B}{a}\right)<a^2+\frac{1}{a^2}$$

$$Var(q)=Tr\left[q^2\rho\right]-\left\{Tr\left[q\rho\right]\right\}^2$$

Mancini et al. sufficient criterion for entanglement in CV systems (product criterion):
the state ρ is entangled if $\exists$ a≠0 such that

$$Var\left(|a|q_A+\frac{q_B}{a}\right)\times Var\left(|a|p_A-\frac{p_B}{a}\right)<\left(a^2+\frac{1}{a^2}\right)^2$$

It detects more entangled states than the sum criterion

NPT criterion and nonlinear witnesses provide **general, operational entanglement criteria for CV systems**

Another sufficient criterion is obtained using the witnesses in this and often in continuous variables, one make use of non-linear witnesses so non-linear functions of the state rho. Again, we focus only to the case of one times one systems again 'A' and 'B' with only one mode. There are two quite useful and used sufficient criteria. One is the Duan, Giedke, Cirac and Zoller criteria also called the sum criterion. According to each state rho is entangled if there is an nonzero real value 'a' exists such that the sum of the variances shown here which has linear combination of qA qB pA pB is less is smaller than a-square plus one over a-square so this is just enough so if we find in this appropriate combination of sum of variances, we are sure that the state is entangled. Another sufficient criterion is the so-called product criterion by Mancini and others which consider the same combination of quadratures but consider the sum of the two variances which has to be smaller than the square of the other quantity so a-square plus one over a-square squared. We provide also the second criterion because usually this product criterion detects more entangled state because given two positive numbers, it's easier to get the product of these two numbers smaller than a given number other than the sum.

Notes

Summary

6m 03s

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NPT criterion and nonlinear witnesses provide **general, operational entanglement criteria for CV systems**

However, in general the NPT criterion and nonlinear witnesses provide general operational entanglement criteria for continuous variable systems.

Notes

Summary

7m 41s

THE CASE OF GAUSSIAN CV STATES

The state of a CV system is Gaussian when **its Wigner function *W(u)***, or equivalently its Fourier transform, the (symmetrically ordered) characteristic function ***χ(ξ)***, **is Gaussian. Often obtained in optomechanics when linearized Langevin eqs. apply**

$$\chi(\xi) = \exp\left[-\frac{1}{2}\xi^T \cdot V \cdot \xi + id \cdot \xi\right]$$

V = covariance matrix, and *d* =<*u*>, fully characterize Gaussian states. **Entanglement properties depend upon *V* only**

$$W(u) = \frac{1}{(2\pi)^N \sqrt{Det(V)}} \exp\left[-\frac{1}{2}u^T \cdot V^{-1} \cdot u\right]$$

For *N* modes and *d = 0*

Important theorem in the Gaussian case

$$\tilde{V} + \frac{i}{2}\Omega < 0 \Leftrightarrow \exists j \,|\, 0 < \tilde{\nu}_j < 1/2$$

NPT entanglement criterion **is necessary and sufficient** for entanglement for **Gaussian bipartite *1xN* CV states**. It detects **ALL** Gaussian entangled states

2x2 Gaussian bound entangled states exist (also experimentally found)

Let us now specialize what we've said up to now to the case of Gaussian continuous variable states which are relevant in optomechanics because they are often obtained whenever we, a treatment based on linearized Langevin equations apply. State of continuous variable system is Gaussian when its Wigner function 'W' or equivalently Fourier transform, which is the so-called symmetrically ordered characteristic function, is Gaussian so for example, for the characteristic function chi, this is the general form of a Gaussian state expressed in terms of the covariance matrix 'V' which appears in the exponential. And also in terms of a vector 'd' which is just the vector of mean values. These two quantities so the first order and second order moments fully characterize Gaussian states. However, entanglement properties depend only upon 'V' the covariance matrix. So usually one make a displacement and which is equivalent to assume that all the mean values are zero and also one ends up with the Wigner function. I'll show you here. We are showing here in terms of the inverse of the covariance matrix.

Notes

Summary

7m 50s

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There is an important theorem for the Gaussian case which is the fact that the NPT entanglement criterion which we've seen before, not only is sufficient but also necessary in the case of Gaussian bipartite case of the form one times 'N' so for Gaussian bipartite systems in which one of the two parties has only one mode. So it means that in this special case of Gaussian one times 'N' CV states, we are able to detect all Gaussian entangled states with the NPT criterion so by finding a symplectic eigenvalue which is smaller than one-half. However, so this means also that two times two for example, Gaussian bound entangled state exist and actually they have been already also experimentally demonstrated in continuous variable case.

Notes

Summary

9m 12s

TWO-MODE SQUEEZED STATE

Paradigm CV 1x1 entangled state: two-mode squeezed (TMS) state (with squeezing *r*), e.g. generated by parametric amplifiers

Gaussian with CM

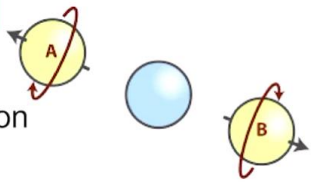
$$V_{TMS} = \frac{1}{2}\begin{pmatrix} I_2 \cosh r & \sigma_z \sinh r \\ \sigma_z \sinh r & I_2 \cosh r \end{pmatrix}$$

Schmidt decomposition in the Fock basis

$$|\psi\rangle_{TMS} = \sum_{n=0}^{\infty} \frac{(\tanh r)^n}{(\cosh r)^2} |n\rangle_A |n\rangle_B$$

Any 1x1 pure Gaussian state can be transformed with local Gaussian operations into a TMS. Moreover

$$Var(q_A - q_B) = Var(p_A + p_B) = e^{-2r} \qquad Var(q_A + q_B) = Var(p_A - p_B) = e^{2r}$$



When $r \to \infty$ a TMS tends to the Einstein, Podolski, Rosen **(EPR) state**, simultaneous eigenstate of q_A-q_B and p_A+p_B. The quadratures (and the photon number) of system A are **strongly correlated** with those of system B

A TMS provides also a clear example of the **physical meaning of entanglement as a nonlocal quantum correlation, stronger than any classical one**

Let us now consider a very special case of Gaussian state which is the two-mode squeezed state. We spend few words on it because this is just the paradigm continuous variable entangled state. It is characterized by only a single parameter 'r', the squeezing parameter 'r'. Its Gaussian continuous covariance matrix is simple. It just is shown here so in terms of the blocks, two by two blocks. I-two is just the identity matrix in dimension to a sigma sine 'hr'. It's just the Pauli matrix. In this case since the state is pure, there is also a very simple Schmidt decomposition which is just in the Fock basis of the excitation number. As we can see they have the two modes 'A' and 'B' have the same excitation number so a very simple Schmidt decomposition. Already from here we can see that there is a strong correlation between the the excitation number because they are always the same so a strong connection in the Fock basis. But however, this strong correlation between the two systems is also present at the level of variables because if one calculates from using the above shown covariance matrix 'V' for a two-mode squeezed state, the variance of the relative distance 'qA' minus 'QB' is equal to the variance of the total moment is equal to the exponential minus two 'r'.

Notes

Summary

10m 06s

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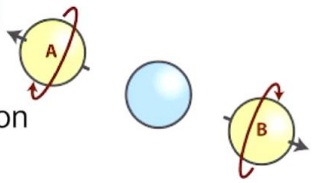

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So it can become, it tends to zero for 'r' squeezing. At the same time, the other non-commutative quantities is that the I have variance which diverges, this means that for 'r' going to infinity tend to have an energy with infinite system with state with infinite energy, sorry. Which tends the way over to the well-known Einstein–Podolsky–Rosen state, which is the simultaneous eigenstate of relative distance and total momentum. In this state, therefore, not only photons but also excitation number but also the quadratures are strongly correlated. So knowing the quadrature bond system, I can have especially the knowledge of the quadrature of another system just because the variance tends to zero. So the two-mode squeezed state just for this strong evident quantum correlation properties is just the clear example of the physical meaning of entanglement as a non-local quantum correlation. Just because non-local because the two parties can be separated as much as we want so this connection remain even if two parties are well, very distant.

Notes

Summary

11m 45s

ENTANGLEMENT AND QUANTUM CORRELATIONS

Entanglement manifests the nonlocal nature of quantum mechanics (QM).
Bell theorem first pointed out the **incompatibility** between QM and **"local realism"** (as defined by Bell). Quantum systems have properties which may be influenced by other space-like separated systems. This is quantified by violation of a class of "**Bell inequalities**" involving set of observables

However, the connection between nonlocality, quantum correlations and entanglement is far from being fully clarified

Again, for **pure states** the situation is easy: **entanglement ⇔ violation of a Bell inequality**

For **mixed states**: **Bell inequality violation ⇒ entanglement** (it is an "entanglement witness"), but **the reverse is not true.** Entanglement is a weaker correlation

Recent definition of intermediate levels of quantum correlations:, e.g., **quantum steering**, the possibility of "steering" the quantum state of a distant system A from local measurements at B: for mixed states: **Bell inequality violation ⇒ steering ⇒ entanglement** (reverse not true)

So in this way, we can also add also few more words on the connection between entanglement and quantum correlations and nonlocality. In fact, the Bell's theorem was the first to point out the incompatibility between the quantum mechanics, the so-called local realism as just defined by Bell. Because quantum system of properties which maybe influenced by other space like separated system. And is what usually quantified by Bell in terms of violation of a class of Bell inequalities, which always involves pairs of set of observables. This connection is still a matter of, still studied in the literature because the connection between nonlocality, quantum correlations and entanglement is far from being clarified. Because, in fact again, for pure states the situation is easy. Entanglement is full equivalent to the violation of the Bell inequality so we can always find a Bell inequality which is violated by an entangled state. Again for mixed state, the situation is much more involved because Bell inequality violation, it's only implies entanglement but the reverse is not true so essentially it acts as a sort of entanglement weakness. Therefore, it means that entanglement is a weaker correlation than violation with an inequality.

Notes

Summary

12m 59s

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This vision is not even simple because recently people has also found intermediate levels of quantum correlation between Bell inequality, Bell nonlocality and entanglement. For example, one has quantum steering. Steering has been first introduced by Schrödinger. It means the possibility of steering, the quantum state of a distance system 'A' by performing local measurement on 'B'. Steering is just halfway between Bell inequality, Bell nonlocality and entanglement because Bell inequality violation implies steering which in turn implies entanglement and the reverse are not true.

Notes

Summary

14m 19s

1X1 CV GAUSSIAN STATES AND ENTANGLEMENT MEASURES

1. Logarithmic negativity: $E_N(\rho_{AB}) = \max\{0, -\log 2\tilde{\nu}_-\}$

Sum of EPR variances

Physical meaning of $E_N(\rho)$ $2\tilde{\nu}_- = \min_{LOCC} \frac{Var(q_A - q_B) + Var(p_A + p_B)}{2}$ ⇒ $E_N(\rho_{TMS}) = 2r$

2. Entanglement of formation (eof):

$$E_F(\rho_{AB}) = \cosh^2 r_0 \log(\cosh^2 r_0) - \sinh^2 r_0 \log(\sinh^2 r_0) = S(\rho_{TMS}(r_0))$$

r_0 the solution of a set of V-dependent nonlinear equations, and has the following physical meaning: Any CM V can be decomposed as $\boldsymbol{V = V_{pure}(r) + V_{noise}}$ and r_0 is the mimimum possible squeezing in such a decomposition = **minimum amount of squeezing needed to prepare** ρ_{AB}

The above E_F is useful also for **generic non-Gaussian 1x1 CV states** ρ: E_F of the Gaussian state with **the same V**, **$E_F(\rho_G)$, provides a lower bound of the exact eof, $E_F(\rho)$.**

$$E_F(\rho_G) \leq E_F(\rho) \quad \forall \rho$$

This is not true if we use E_N.

Let us finally conclude with some information about entanglement measure for the case of Gaussian states and we shall specialize to the case of one times one system. The logarithmic negativity we already defined in the first lecture is simply given by this expression, the maximum between zero and minus the log of two times the symplectic eigenvalue and from here one can also obtain a very easy clear meaning of the logarithmic negativity. It's just the minimum of the sum of EPR variances so if one minimizes the sum of the standard two variances appear in the two-mode squeezed state, it minimizes locally by making all the possible rotations on 'A' and 'B' sides. One get exactly the two times the symplectic eigenvalue. As a consequence of that so the log negative is just coincide that with essentially the squeezing parameter, in fact, for a two-mode squeezed state, exactly two times the squeezing parameter. For the one times one CV Gaussian state, it's also possible to calculate the entanglement of formation. This expression is given here which coincide exactly with the entanglement entropy of the of an effective two-mode squeezed state with an effective squeezing parameter 'r' zero.

Notes

Summary

15m 03s

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Sum of EPR variances

Physical meaning of $E_N(\rho)$ $2\tilde{\nu}_- = \min_{LOCC} \frac{Var(q_A - q_B) + Var(p_A + p_B)}{2}$ ➡ $E_N(\rho_{TMS}) = 2r$

2. Entanglement of formation (eof):

$$E_F(\rho_{AB}) = \cosh^2 r_0 \log(\cosh^2 r_0) - \sinh^2 r_0 \log(\sinh^2 r_0) = S(\rho_{TMS}(r_0))$$

r_0 the solution of a set of V-dependent nonlinear equations, and has the following physical meaning: Any CM V can be decomposed as $\boldsymbol{V = V_{pure}(r) + V_{noise}}$ and r_0 is the mimimum possible squeezing in such a decomposition = **minimum amount of squeezing needed to prepare ρ_{AB}**

The above E_F is useful also for **generic non-Gaussian 1x1 CV states** ρ: E_F of the Gaussian state with **the same V**, **$E_F(\rho_G)$, provides a lower bound of the exact eof, $E_F(\rho)$.**

$$E_F(\rho_G) \leq E_F(\rho) \quad \forall \rho$$

This is not true if we use E_N.

'r' zero is hard to find so the expression is simple, what is difficult is to find 'r' zero. Because there is a solution of a set of nonlinear equation which depends on the explicit elements of 'V'. However, the physical meaning is clear because for a given state, 'V' can always be decomposed as a sum of two terms, the covariance matrix 'V-pure r' associated to an effective two-mode squeezed state plus a noise term which is positive. 'r' zero is the minimum possible squeezing in such a decomposition so 'r' zero is just the minimum amount of squeezing which is needed to prepare the state rho-AB. Moreover, this is another important property of the so-called Gaussian entanglement of formation that we have written here. Because it is useful also for non-Gaussian one times one CV states. In fact, for a given generic state rho, if you know Gaussian, one can always calculate this this Gaussian entanglement of formations which means associated to the Gaussian approximation on this state so using the same covariance matrix 'V'. Now it has been proved that this Gaussian entanglement of formation provides a lower bound to the exact entanglement formation of the full exact state rho so we have this inequality which is true for any state rho if you use the entanglement of formation as an entanglement measure.

Notes

Summary

16m 24s

1X1 CV GAUSSIAN STATES AND ENTANGLEMENT MEASURES

1. Logarithmic negativity: $E_N(\rho_{AB}) = \max\{0, -\log 2\tilde{\nu}_-\}$

Sum of EPR variances

Physical meaning of $E_N(\rho)$ $2\tilde{\nu}_- = \min_{LOCC} \frac{Var(q_A - q_B) + Var(p_A + p_B)}{2}$ ➡ $E_N(\rho_{TMS}) = 2r$

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$$E_F(\rho_G) \leq E_F(\rho) \quad \forall \rho$$

This is not true if we use E_N.

For example, this is not true, if you instead would have used the log negativity. This just use the fact that as we've seen, entanglement formation is a real entanglement measure and log negativity is not. It's only an entanglement monotone.

Notes

Summary

17m 50s

SUMMARY

- We have seen how one can establish if a state is entangled or not (entanglement criteria) and how to quantify it.
- We have focused onto the case of continuous variable systems, e.g. mechanical or electromagnetic modes, which occurs in cavity optomechanics and provided specific formulas and example for these systems.
- We have also briefly discussed the (nontrivial and still investigated) connection between entanglement and the wider concept of quantum correlations

So in conclusion, we have seen how one can establish if a state is entangled or not and how to quantify it. And then here in the second part, we have focused onto the case of continuous variable systems, mechanical or electromagnetic modes which occurs in cavity optomechanics and we also provided specific formulas and examples for these systems. We also briefly discussed the non-trivial and still investigated connection between entanglement and the wider concept of quantum correlation.

Notes

Summary

18m 04s