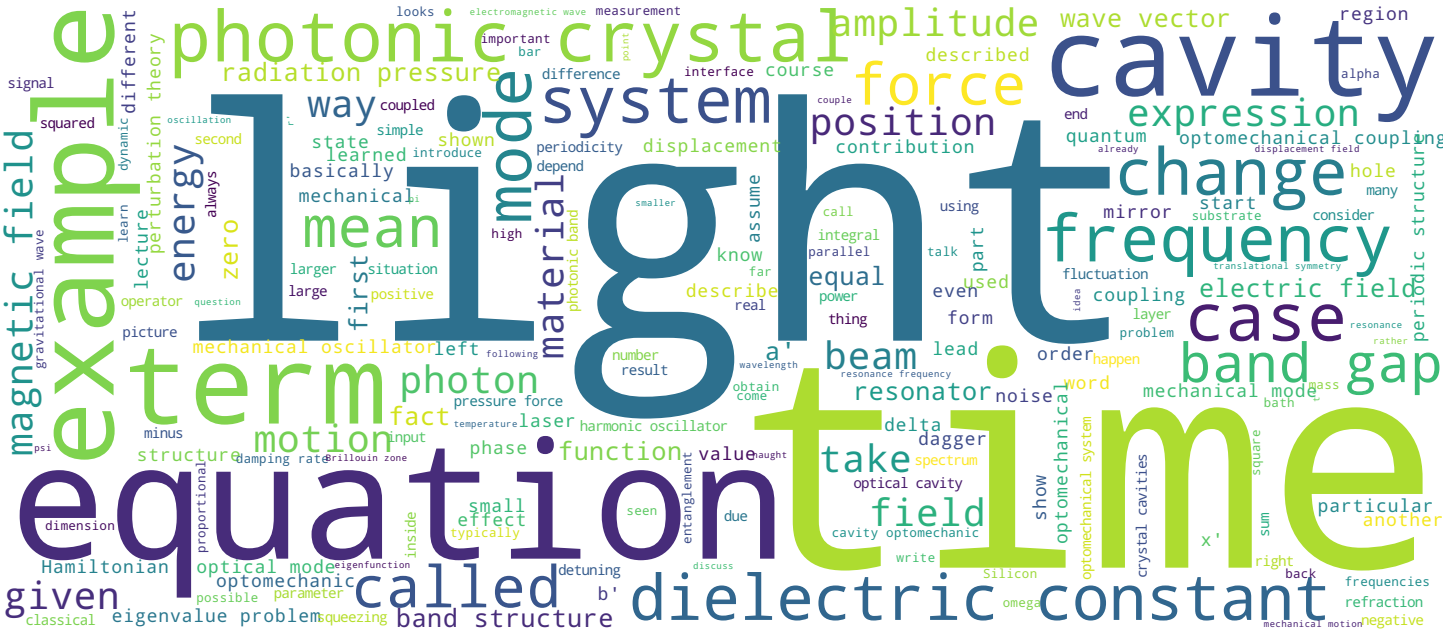
PHOTONIC CRYSTALS

Cavity Quantum Optomechanics

Paul Seidler
IBM Research Europe
Zurich, Switzerland

IBM **Research**

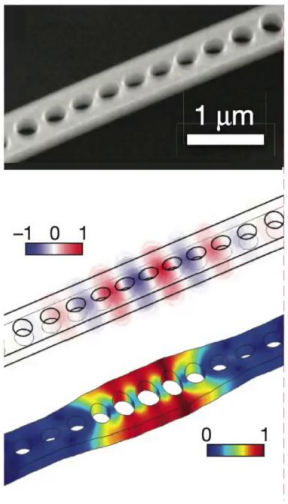


EPFL

OUTLINE

In this lesson, you will learn about photonic crystals, optical cavities based on photonics crystals, and their optomechanical behavior.

- **What are photonic crystals?**
- **Mathematical description of electromagnetic waves in periodic media**
- **Confining light: bandgaps and index guiding**
- **Mechanisms for optomechanical coupling**



J. Chan et al. *Nature* **478**, 90 (2011).

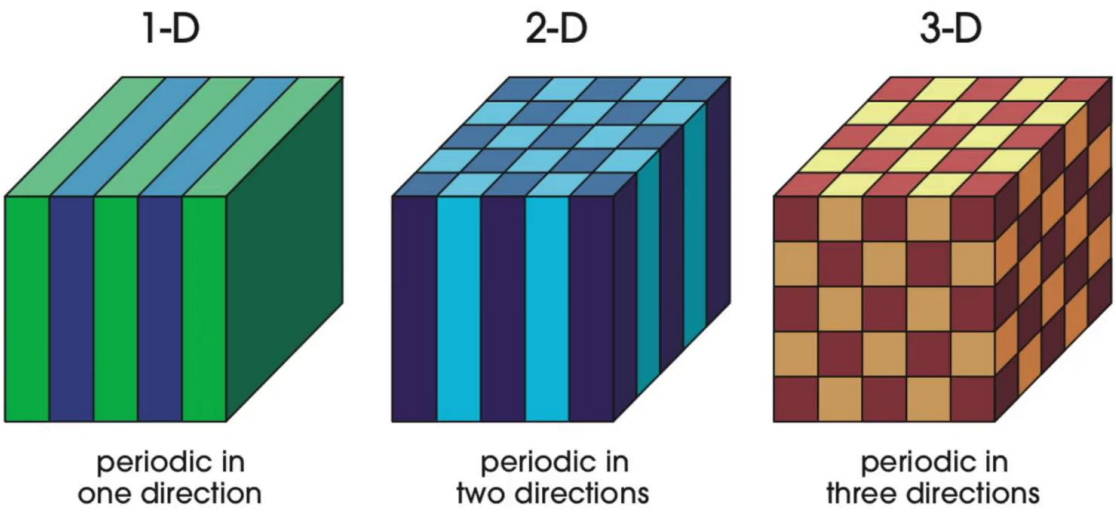
Welcome to this video on photonic crystal cavities. I'm Paul Seidler from the Zurich laboratory of IBM Research Europe. This video is part of the mooc, cavity optomechanics from the OMT network. Optical cavities based on photonic crystal structures have been employed extensively in optomechanics. They have been used to study a myriad of quantum phenomena with mechanical systems. In this lesson, you will learn about photonic crystals, optical cavities based on photonic crystals and their optomechanical behavior. The topics we will cover are: what are photonic crystals? The mathematical description of how electromagnetic waves behave in periodic media. We will talk about the confinement of light, in particular, how band gaps form and the role of index guiding. And finally, we will talk about mechanisms for optomechanical coupling in photonic crystal cavities.

Notes

Summary

0m 00s

WHAT ARE PHOTONIC CRYSTALS?



J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals, Molding the Flow of Light* (Princeton University Press, 2008).

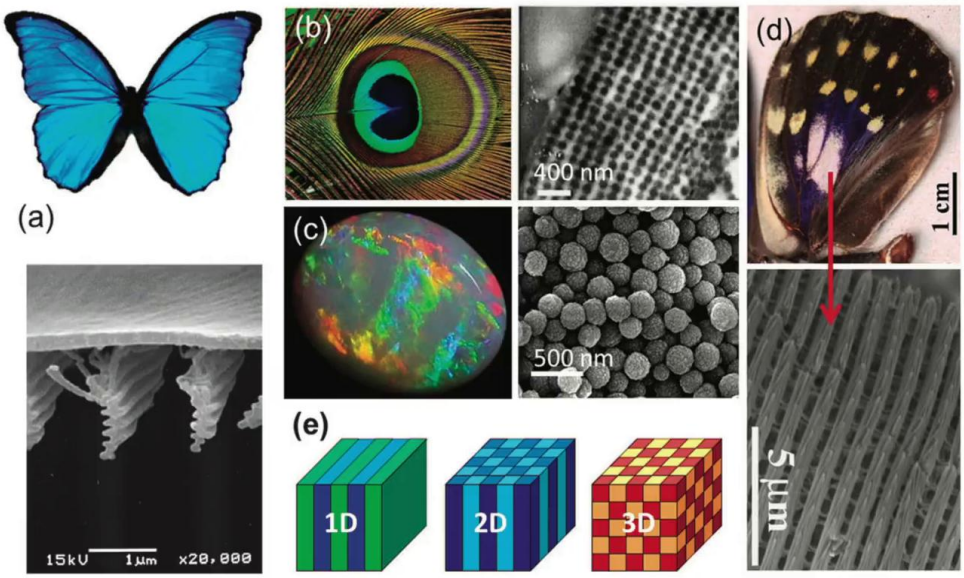
So what are photonic crystals? They're not crystals of light but rather material structures in which the permittivity or dielectric constant varies periodically. The periodicity can be in one dimension, two dimensions or all three dimensions. Scattering of light occurs at the interface between different dielectric media. This alters the propagation of electromagnetic waves and can lead, for example, to guiding of light or even complete reflection of light. This control of light is, in general, frequency-dependent. It does not, however, require perfect periodicity. The periodic structure must not extend to infinity and some roughness of the interfaces can be tolerated.

Notes

Summary

1m 04s

EXAMPLES FROM NATURE



E. Armstrong and C. O'Dwyer, *J. Mater. Chem. C*, **3**, 6109–6143 (2015).

This is evident from the examples you find in nature. The iridescent colors of some butterfly wings or peacock feathers or an opal are due to periodic structures in those materials. This is somewhat counter to the intuition that the color that we observe in nature is due to preferential absorption of certain wavelengths.

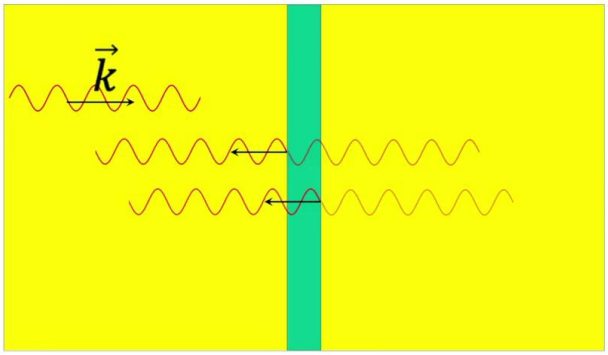
Notes

Summary

1m 53s

FRESNEL REFLECTION

REFLECTION IN A WINDOW



At normal incidence $R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2$

Impedance mismatch

We are all familiar with the experience of seeing a reflection in a window despite the fact that the glass is almost completely transparent by which I mean that it is only weakly absorbing. This is an example of scattering of light at the interface between different dielectric media. The phenomenon is known as Fresnel reflection. On the right hand side of this slide is a schematic of an electromagnetic wave propagating in a medium with one dielectric constant impinging on a layer with another dielectric constant like the situation with the window. Light is partially reflected and partially transmitted at both interfaces. For normal incidence the reflectivity is described by this equation where 'n1' and 'n2' are the indices of refraction of the two materials. It can be seen that the amount of reflection increases quadratically with the difference in the indices of refraction, in other words, with a difference in the speed of light in the two media. The reflection can this be thought of as resulting from an impedance mismatch between the two materials. We see in the photograph a second fainter image offset because we are viewing the reflection at an angle.

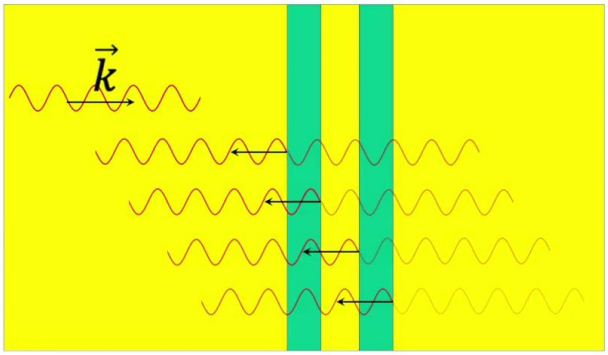
Notes

Summary

2m 25s

FRESNEL REFLECTION

REFLECTION IN A WINDOW



At normal incidence $R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2$

Impedance mismatch

The second image is due to this being a double-paned window. This is the beginning of a periodic structure. Depending on the index difference, the thickness of the layers and the frequency, the light reflected from the various interfaces will have different phase shifts and may interfere destructively or constructively. The same holds true for the transmitted light. The interference becomes most apparent when the layer thickness is on the order of the wavelength of the light in the material. This is the basis of controlling the propagation of light in a photonic crystal.

Notes

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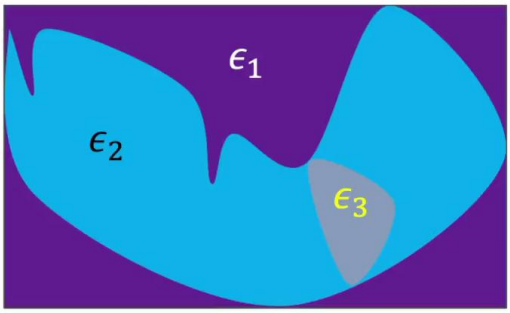
3m 39s

MATHEMATICAL DESCRIPTION

AN ARBITRARY DIELECTRIC STRUCTURE



ASSUMPTIONS:

No free charges or currents
⇒ light propagates but there are no sources of light.

Relative permitivity ϵ is:

- Linear (no non-linear optical effects)
- Macroscopic and isotropic
- Frequency-independent (no dispersion)
- Real and positive (no absorption)

For now, the structure does not vary with time.

MAXWELL'S EQUATIONS

$$\nabla\cdot(\epsilon(\vec{r})\vec{E}(\vec{r},t))=0 \qquad \nabla\times\vec{E}(\vec{r},t)+\mu_0\frac{\partial\vec{H}(\vec{r},t)}{\partial t}=0$$

$$\nabla\cdot\vec{H}(\vec{r},t)=0 \qquad \nabla\times\vec{H}(\vec{r},t)-\epsilon_0\epsilon(\vec{r})\frac{\partial\vec{E}(\vec{r},t)}{\partial t}=0$$

Let's look in more detail at the mathematical description of an electromagnetic field in a dielectric medium. We start with an arbitrary dielectric structure in Maxwell's equations. To make life simpler, we assume that there are no free charges and no free currents flowing. In other words, light can propagate but there are no sources of light in the material. We also assume that the permittivity or dielectric constant epsilon is linear, macroscopic, meaning we do not need to take into account the atomic structure and isotropic, that it has negligible dispersion over the frequency range in which we are interested and is real and positive. In other words, we have no absorption. These assumptions are not essential. For example, the formulas can be generalized to treat an isotropic media and perturbation theory can be used for optical nonlinearities. For the time being we also assume a static material system. For optomechanics though, we will have to look at changes in the structure. We now use the expression for the displacement field and magnetic flux density to write Maxwell's equations in terms of only the electric and magnetic fields. The result looks like this where I have now shown explicitly the dependence on position and time.

Notes

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4m 14s

A HERMITIAN EIGENVALUE PROBLEM

$$\nabla\cdot(\epsilon(\vec{r})\vec{E}(\vec{r},t))=0 \qquad \nabla\times\vec{E}(\vec{r},t)+\mu_0\frac{\partial\vec{H}(\vec{r},t)}{\partial t}=0$$

$$\nabla\cdot\vec{H}(\vec{r},t)=0 \qquad \nabla\times\vec{H}(\vec{r},t)-\epsilon_0\epsilon(\vec{r})\frac{\partial\vec{E}(\vec{r},t)}{\partial t}=0$$

Decompose fields into harmonic modes:

$$\vec{H}(\vec{r},t)=\vec{H}(\vec{r})e^{-i\omega t}$$
$$\vec{E}(\vec{r},t)=\vec{E}(\vec{r})e^{-i\omega t}$$

HERMITIAN EIGENVALUE PROBLEM

$$\underbrace{\nabla\times\left(\frac{1}{\epsilon(\vec{r})}\nabla\times\vec{H}(\vec{r})\right)}_{\text{differential operator}}=\underbrace{\left(\frac{\omega}{c}\right)^2}_{\text{eigenvalue}}\underbrace{\vec{H}(\vec{r})}_{\text{eigenfunction}} \qquad c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Constraint: $\nabla\cdot\vec{H}(\vec{r})=0$

Eigenvalues are real ⇒ ω is real

Eigenfunctions are orthogonal ⇒ optical modes are orthogonal

Now to use these equations, we decompose the electric and magnetic fields into harmonic modes. We know from Fourier analysis we can construct any time-dependence from harmonic modes. So this represents no limitation. Combining these equations and eliminating the electric field yields this equation where the speed of light 'C' has been introduced. What we have now is a Hermitian eigenvalue problem. The eigenfunction is the spatial dependence of the magnetic field with the constraint that its divergence is zero. The operator is this complicated differential operator consisting of first taking the curl of the magnetic field then dividing by the dielectric constant and then taking the curl again. As with all Hermitian eigenvalue problems, the eigenvalues are real consistent with the frequency being real. Also the eigenfunctions are orthogonal which means that the optical modes are spatially orthogonal. One could also have solved the equations for the electric field instead of the magnetic field but then one is left with a generalized eigenvalue problem and the expression is more complicated.

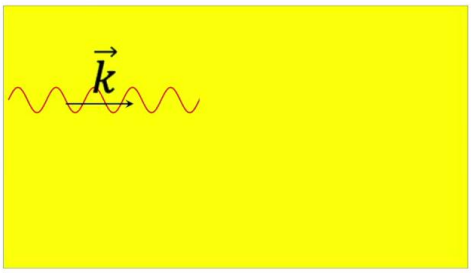
Notes

Summary

5m 38s

TRANSLATIONAL SYMMETRY

Homogeneous system:



Translational symmetry in all three directions means

$$\epsilon(\vec{r}) = \epsilon(\vec{r} + \vec{R}) \quad \text{for all } \vec{R}$$

which implies

$$\vec{H}(\vec{r}) = \vec{H}_0 e^{i\vec{k}\cdot\vec{r}}$$

Translation commutes with the differential operator

$\Rightarrow \vec{k}$ is conserved

We now want to figure out what the expression for the magnetic field ‘H’ is in a periodic structure. We start with a homogeneous system and consider translational symmetry. Translational symmetry in all three directions means that the dielectric constant has this form for all translation vectors capital ‘R’ which in turn implies that the magnetic field can be described as a constant vector times a plane wave. The translation operator commutes with our complicated differential operator so they have the same eigenfunctions, which means that the wave vector ‘k’ is conserved, that is, if a field pattern starts out with a particular wave vector ‘k’, it will maintain that value of ‘k’ at all future times.

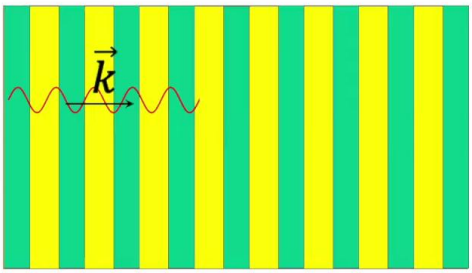
Notes

Summary

6m 50s

TRANSLATIONAL SYMMETRY IN PERIODIC MEDIA

Periodic system:



Real-space lattice: $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$

Reciprocal space lattice: $(\vec{b}_1, \vec{b}_2, \vec{b}_3)$

$$\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij}$$

$$\vec{k} = k_1\vec{b}_1 + k_2\vec{b}_2 + k_3\vec{b}_3$$

Translational symmetry in all three directions means

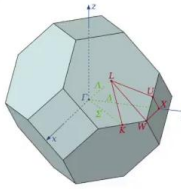
$$\epsilon(\vec{r}) = \epsilon(\vec{r} + \vec{R}) \qquad \text{for } \vec{R} = l\vec{a}_1 + m\vec{a}_2 + n\vec{a}_3$$

which implies

$$\vec{H}_{\vec{k}}(\vec{r}) = \vec{u}_{\vec{k}}(\vec{r})e^{i\vec{k}\cdot\vec{r}} \qquad \vec{u}_{\vec{k}}(\vec{r}) = \vec{u}_{\vec{k}}(\vec{r} + \vec{R})$$

Translation commutes with the differential operator

$\Rightarrow \vec{k}$ is conserved



Brillouin Zone

https://commons.wikimedia.org/wiki/File:Brillouin_Zone_(1st,_FCC).svg

Bloch's theorem

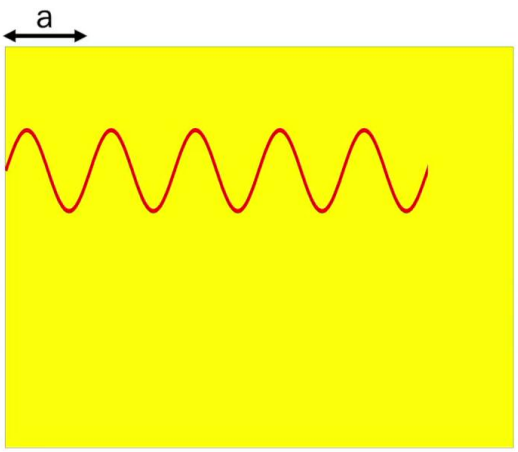
Now consider a periodic system. In three dimensions we can describe the periodicity with three vectors a1, a2 and a3 that are not necessarily at right angles to each other. For simplicity, the schematic here shows the case for periodicity in one dimension. Translational symmetry is now described by these equations where the vector capital 'R' is given by this expression for integer values of l, m and n. The expression for the spatial dependence of the magnetic field then becomes this equation where 'u' has a spatially periodic function for particular 'k' with the periodicity given by the capital 'R' vector and as before, the wave vector 'k' is conserved. In other words, a mode with wave vector 'k' defined by these equations propagates in a periodic medium indefinitely without scattering. This is Bloch's theorem, which may be familiar to you from electronic band structures in Solid State Physics and just like in electronic band structures, one can define a reciprocal lattice related in this way to the real lattice. The wave vector can be described by components in reciprocal space and the reciprocal space is represented by the so-called Brillouin Zone. The analogy to electronic band structures makes it clear where the terminology photonic crystal comes from.

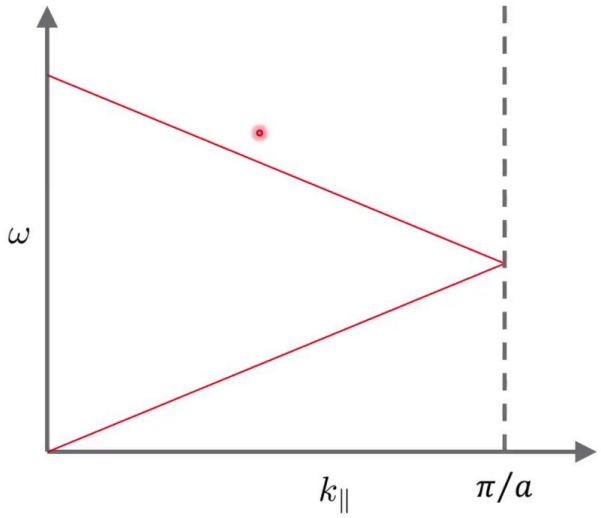
Notes

Summary

7m 38s

FORMATION OF A BANDGAP





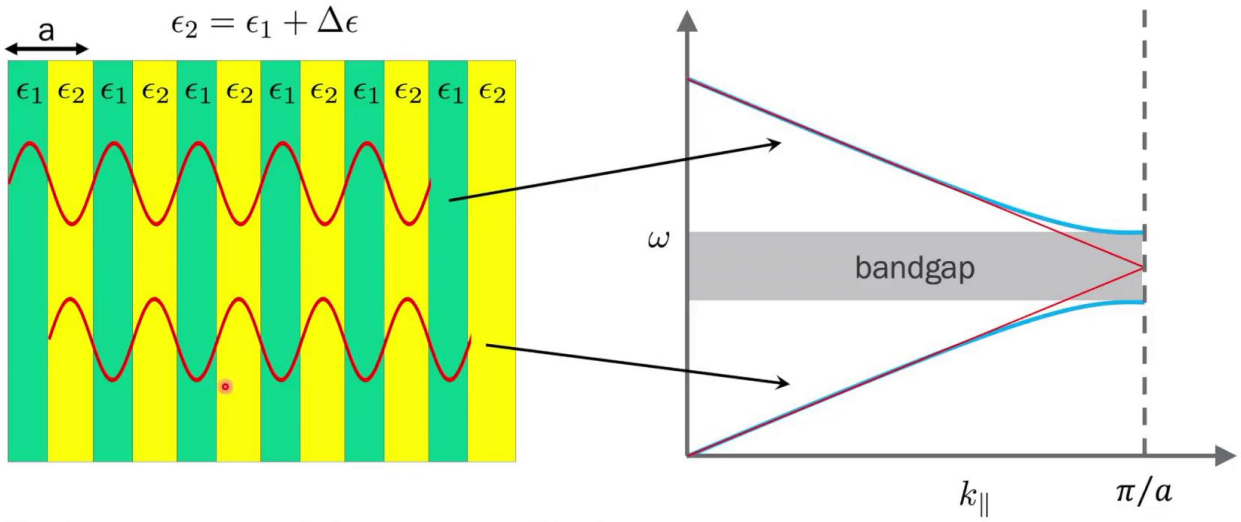
An important aspect of periodic systems is the possibility of forming a photonic band gap again similar to electronic systems. Consider first propagation of light in a homogeneous material. The band structure consist of a straight line, assuming no dispersion where the slope is determined by the index of refraction of the material. Here I'm using an artificial periodicity of length ‘a’ so that the band folds back on itself at ‘k’ equals pi over ‘a’ to create a higher band but this is just a mathematical construct.

Notes

Summary

9m 08s

FORMATION OF A BANDGAP



The lower energy mode is concentrated in the material with the higher dielectric constant.

Now assume we introduce a periodic variation in the dielectric medium with the same period ‘a’. For long wavelength that is small ‘k’ values, the light is relatively insensitive to the variation in dielectric constant and the band structure again looks more or less linear. This is also true for larger ‘k’ values as long as ‘k’ is far from pi over ‘a’ or an odd multiple of pi over ‘a’, say in this region. But when the wavelength is close to two times the period ‘a’, the phase of the field with respect to the periodic medium becomes important and the degeneracy at ‘k’ equals pi over ‘a’ is lifted and a band gap forms. Frequencies in the band gap do not propagate in this one-dimensional system and are completely reflected. Note that the oscillating function is drawn here are meant to represent the square of the field, the local energy density, not the electric or magnetic field itself. And the low frequency mode, that is, the lower energy mode is concentrated in the material with the higher dielectric constant.

Notes

Summary

9m 43s

RESTATING THE HERMITIAN EIGENVALUE PROBLEM

$$\nabla \times \left(\frac{1}{\epsilon(\vec{r})} \nabla \times \vec{H}(\vec{r}) \right) = \left(\frac{\omega}{c} \right)^2 \vec{H}(\vec{r}) \qquad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad \textbf{Constraint:} \quad \nabla \cdot \vec{H}(\vec{r}) = 0$$

$$\vec{H}_{\vec{k}}(\vec{r}) = \vec{u}_{\vec{k}}(\vec{r}) e^{i\vec{k}\cdot\vec{r}} \qquad \vec{u}_{\vec{k}}(\vec{r}) = \vec{u}_{\vec{k}}(\vec{r} + \vec{R})$$

$$\underbrace{(i\vec{k} + \nabla) \times \left(\frac{1}{\epsilon(\vec{r})} (i\vec{k} + \nabla) \times \vec{u}_k(\vec{r}) \right)}_{\text{differential operator}} = \underbrace{\left(\frac{\omega(\vec{k})}{c} \right)^2}_{\text{eigenvalue}} \underbrace{\vec{u}_k(\vec{r})}_{\text{eigenfunction}} \qquad \textbf{Constraint:} \quad (i\vec{k} + \nabla) \cdot \vec{u}_{\vec{k}}(\vec{r}) = 0$$

Returning to the Hermitian eigenvalue problem. We can use the expression for the spatial dependence of the magnetic field in terms of the spatially periodic function ‘u’ to restate the problem. The eigenvalue problem now takes this form with this particular constraint. The eigenfunction is now the function ‘u’ for each ‘k’.

Notes

Summary

10m 56s

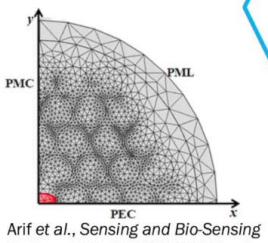
HOW IS THE EIGENVALUE PROBLEM SOLVED?

$$(i\vec{k}+\nabla)\times\left(\frac{1}{\epsilon(\vec{r})}(i\vec{k}+\nabla)\times\vec{u}_k(\vec{r})\right)=\left(\frac{\omega(\vec{k})}{c}\right)^2\vec{u}_k(\vec{r})$$

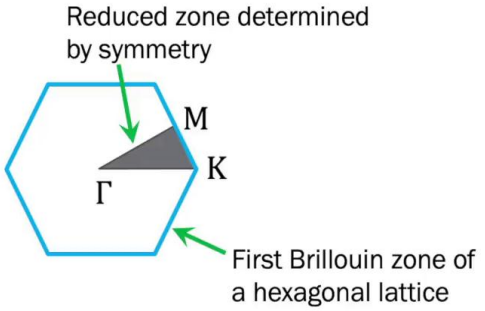
Constraint: $(i\vec{k}+\nabla)\cdot\vec{u}_{\vec{k}}(\vec{r})=0$

Goal: numerically solve for $\omega(\vec{k})$ for each mode

- **Limit range of $\vec{k}$ to the irreducible Brillouin zone**
- **Expand $\vec{u}_k(\vec{r})$ in a finite basis**
- **Employ an efficient iterative method**



Arif *et al.*, *Sensing and Bio-Sensing Research* **22**, 100252 (2019).



Example: finite-element method such as COMSOL Multiphysics®

The eigenvalue problem is generally not solvable analytically. So how is it solved? They solve numerically where the goal is to determine the frequency as a function of wave vector. In other words, we want to determine the band structure. The details of the numerical methods used are beyond the scope of this lecture. But to give you a feeling how it is done typically we limit the range of the calculation to the irreproducible Brillouin zone. That is the part that is unique and from which the rest of the Brillouin zone is determined by symmetry. The function 'u' is then expanded in a finite basis most commonly in terms of a finite element decomposition of the structure. An inefficient iterative method is used to determine the eigenvalues. This can be done with commercial finite element software such as Comsol Multiphysics.

Notes

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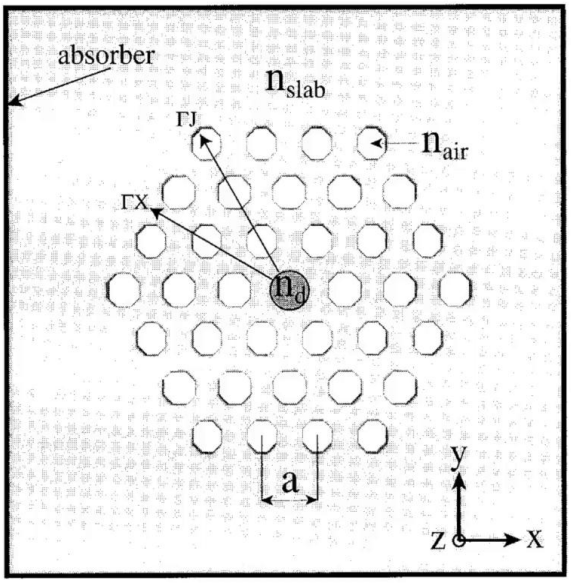
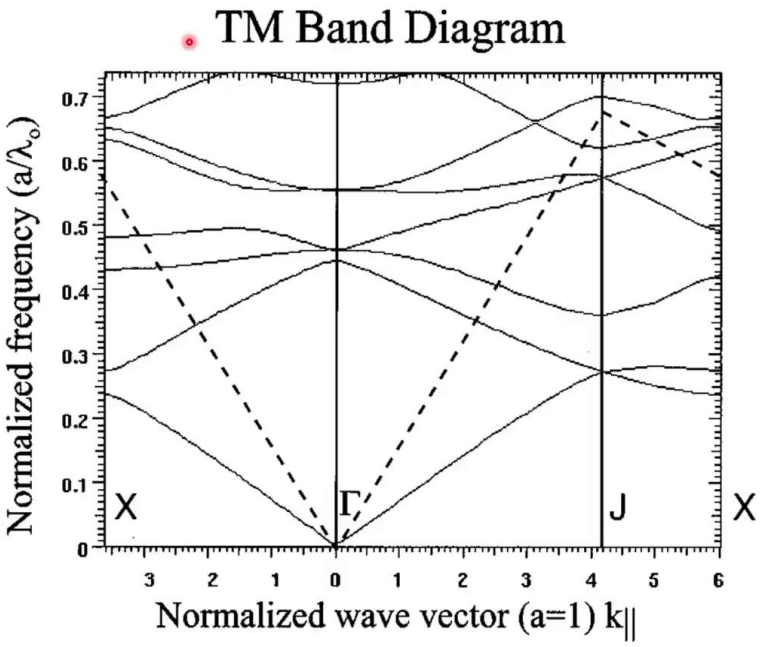
11m 19s

AN EXAMPLE: 2-D PHOTONIC CRYSTAL





Painter et al. "Defect modes of a two-dimensional photonic crystal in an optically thin dielectric slab," *J. Opt. Soc. Am. B* 16, 275 (1999).

Let's look at an example taken from an early paper in the field. The structure examined was a two-dimensional photonic crystal consisting of a hexagonal array of holes in a dielectric layer as shown here on the left. The materials indium, gallium, arsenide, phosphide have interest for its potential use in making low-power integrated semiconductor lasers. The photonic band structure is on the right where gamma 'X' and 'J' are points in the Brillouin zone. 'TE' stands for Transverse Electric which means that the light is polarized with the electric field in the plane of the structure. One can see that there are several bands with various dispersion relations for the different crystal directions. For this polarization, there is a band gap for normalize frequencies between 0.28 and 0.35. In this frequency range, no light propagates. In contrast, for 'TM', Transverse Magnetic polarized light, the band structure shows no band gap. All frequencies are possible. Note that this is in contrast to the one-dimensional system where there is always a band gap.

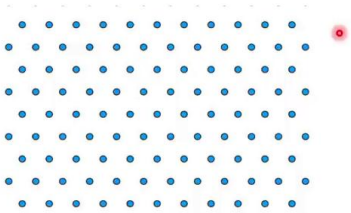
Notes

Summary

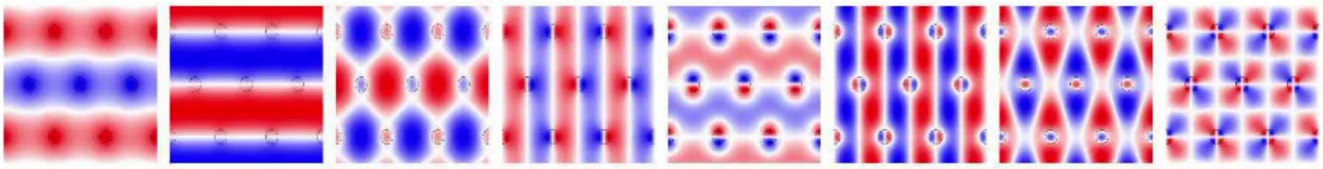
12m 15s

WHAT DO THE MODES LOOK LIKE?

Hexagonal lattice



E_z field component for the first eight modes at the M point



Andonegui et al. "The finite element method applied to the study of two-dimensional photonic crystals and resonant cavities," *Optics Express* **21**, 4072 (2013).

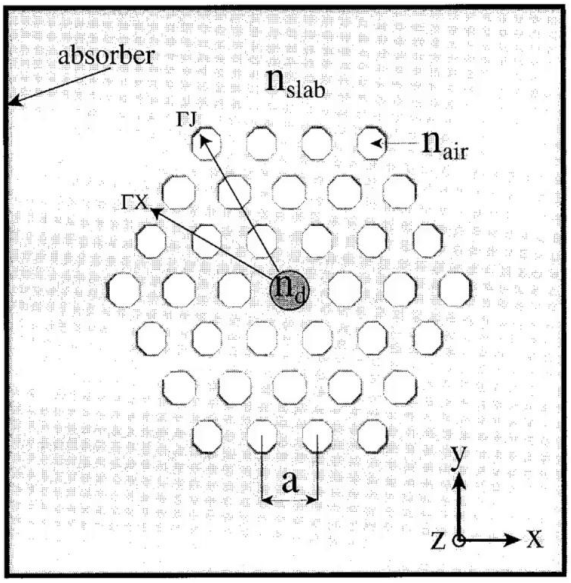
And to give you a feeling for what optical modes look like in a photonic crystal, here's an example from a later publication showing the first eight modes at the end point in a hexagonal matrix made of cylinders rather than holes. The color scale is for the component of the electric field in the z-direction, that is, in and out of the plane of the photonic crystal. Blue and red corresponds to the sign of the electric field. As mentioned, the lattice is made of cylinders rather than holes. As you can see, guiding of light in a photonic crystal does not require a continuous structure.

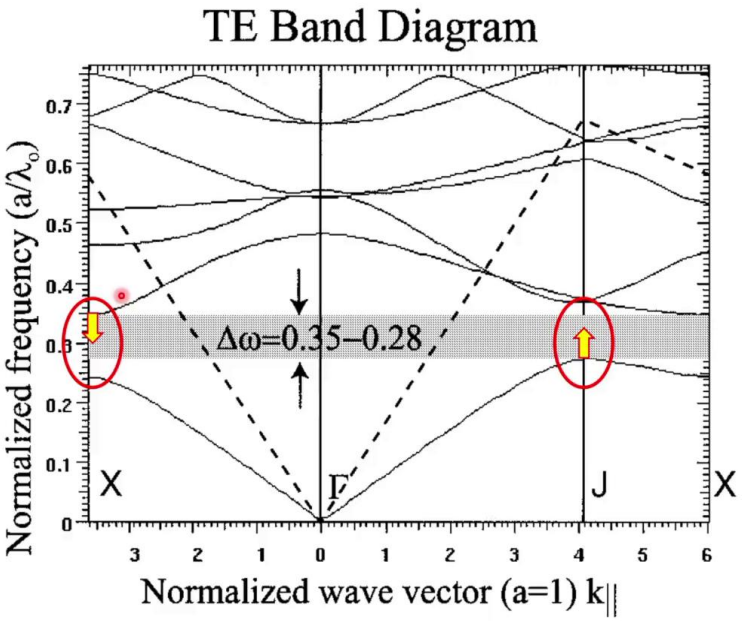
Notes

Summary

13m 28s

“DEFECT” MODES





Painter et al. “Defect modes of a two-dimensional photonic crystal in an optically thin dielectric slab,” *J. Opt. Soc. Am. B* 16, 275 (1999).

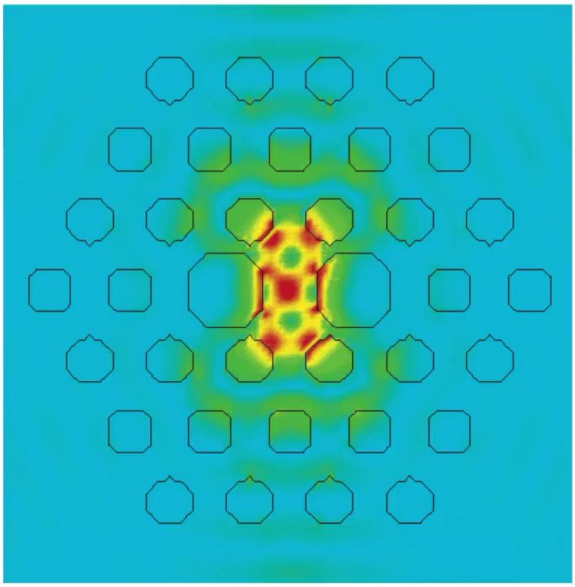
So far, I've only talked about periodic structures and how light propagates or is blocked by a band gap in such structures. What if we want to build an optical cavity? The cavity can be formed by deviating from periodicity. In other words, by intentionally creating a defect in the periodic structure. Coming back to this earlier paper. We see in the drawing on the left that the central hole is labeled differently. The idea was to create an intentional defect by changing the index of refraction or dielectric constant in this hole to disrupt the periodicity. In this way, a localized mode is created that either is pushed up from the bottom of the band gap or pull down from the top of the band gap depending on the sign of the change in refractive index. Deviations from the periodic structure can also be made by changing the size of the hole or changing multiple holes to confine optical modes in a more complicated fashion.

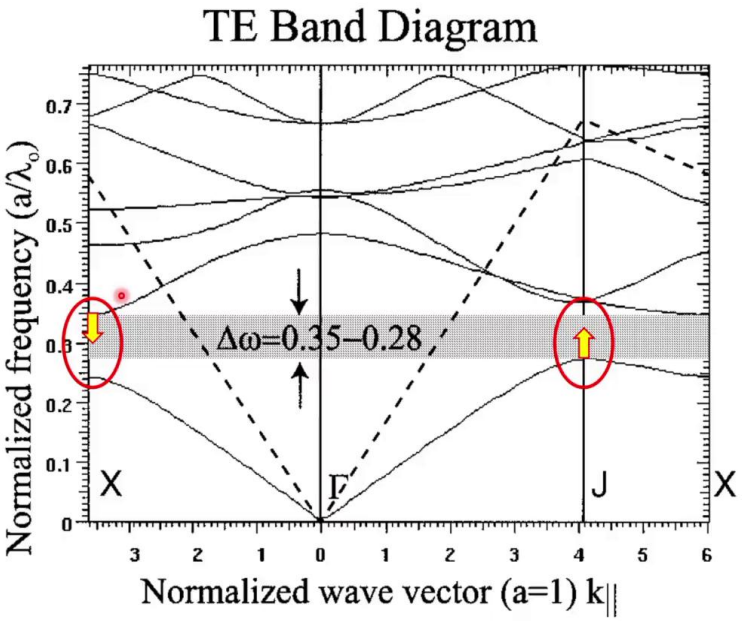
Notes

Summary

14m 06s

HOW TO MAKE A CAVITY: "DEFECT" MODES





Painter et al. "Defect modes of a two-dimensional photonic crystal in an optically thin dielectric slab," *J. Opt. Soc. Am. B* **16**, 275 (1999).
Painter et al. "Two-dimensional photonic band-gap defect mode laser," *Science* **284**, 1819 (1999).

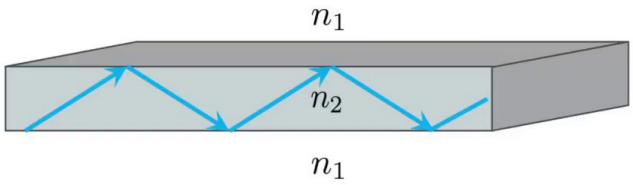
One example is shown here where the central hole is missing and the diameter of two adjacent holes has been increased. Because the frequency of the defect mode is in the band gap, it is strongly confined in the photonic crystal. If the index contrast is large and the periodic portion of the structure is sufficiently extensive, the confined optical mode will decay slowly resulting in a narrow optical resonance.

Notes

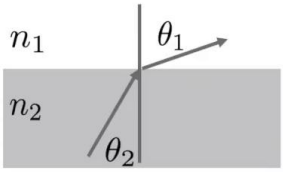
Summary

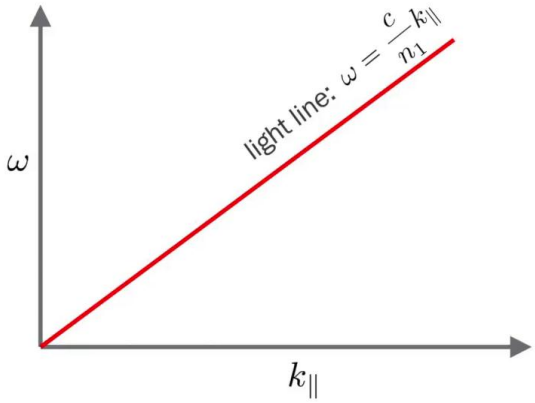
15m 06s

INDEX GUIDING



Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$





$$k_\parallel = |\vec{k}| \sin\theta \qquad |\vec{k}| = \frac{n\omega}{c}$$

$$\omega = \frac{c}{n_1}|\vec{k}| = \frac{c}{n_1}\sqrt{k_\parallel^2 + k_\perp^2}$$

$k_\parallel$

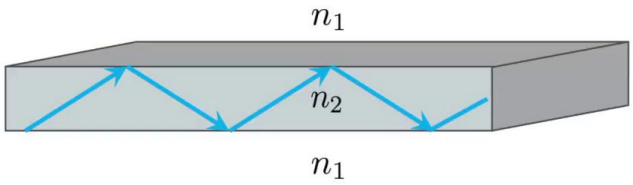
Now this is not the only way light can be confined. From ray optics, we are familiar with the phenomenon of total internal reflection. Consider a slab of dielectric material surrounded by a material with a lower index of refraction. We know from Snell's law that for large enough angles of incidence, light passing from a high index material to a low index material will be totally internally reflected. But this ray optics picture doesn't hold when the dimensions are on the scale of the wavelength of the light. What do we do then? We return to our symmetry arguments. Because of translational symmetry, the parallel component of the wave vector 'k' is maintained at the interface. Indeed Snell's law is just a result of the conservation of the parallel component of the wave vector and the frequency. Now rearranging these equations we can express the frequency omega in terms of the parallel and perpendicular portion of the wave vector. Once again, we plot frequency versus k-parallel and consider two regimes. The region above and the region below the so-called light line defined by the speed of light in the lower index material times k-parallel.

Notes

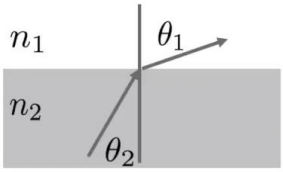
Summary

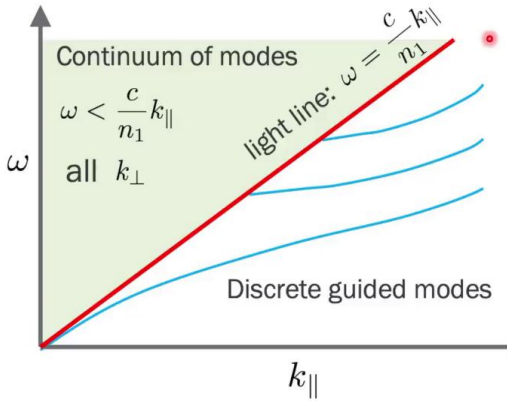
15m 33s

INDEX GUIDING



Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$





$$k_\parallel = |\vec{k}| \sin\theta \qquad |\vec{k}| = \frac{n\omega}{c}$$

$$\omega = \frac{c}{n_1}|\vec{k}| = \frac{c}{n_1}\sqrt{k_\parallel^2 + k_\perp^2}$$

$$k_\perp = \pm i\sqrt{k_\parallel^2 - \frac{n_1^2\omega^2}{c^2}}$$

- Evanescent decay outside waveguide
- Approaches ray optics picture of total internal reflection at large $k_\parallel$

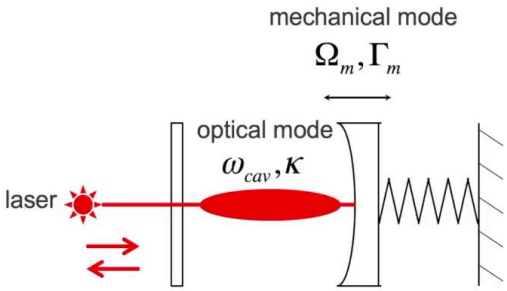
As you can see from this equation on the right, above the light line a real value of omega exists for all values of k-perpendicular and we have a continuum of modes. The light is not confined by the slab and can be radiated into the low dielectric medium. In contrast, below this line k-perpendicular is imaginary where the light decays evanescently outside the waveguide. It is guided. Because of the finite thickness of this slab, the mode structure is discrete. But for large values of k-parallel, there are more and more modes and eventually one approaches the ray optics picture of total internal reflection for a continuum of angles.

Notes

Summary

16m 52s

OPTOMECHANICS



$$G = \frac{\partial \omega}{\partial x} = \frac{\omega}{L}$$

So now that we know how to confine light, let's finally talk about optomechanics. As you have learned in other lectures, the light in an optical cavity can be coupled to its mechanical motion. Here I show a typical schematic of a Fabry-Perot cavity with one movable mirror. The optomechanical coupling is described by the change in resonance frequency with the displacement of the mirror given here by capital 'G'.

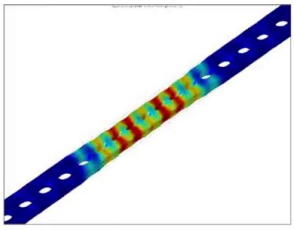
Notes

Summary

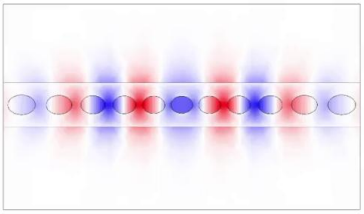
17m 39s

THE EFFECT OF SMALL PERTURBATIONS

Mechanical displacement



Optical mode



$$\nabla \times \left(\frac{1}{\epsilon(\vec{r})} \nabla \times \vec{H}(\vec{r}) \right) = \left(\frac{\omega}{c} \right)^2 \vec{H}(\vec{r}) \quad \Leftrightarrow \quad \nabla \times \nabla \times \vec{E}(\vec{r}) = \left(\frac{\omega}{c} \right)^2 \epsilon(\vec{r}) \vec{E}(\vec{r})$$

Hermitian eigenvalue problem — Generalized eigenvalue problem

Perturbation theory:

$$\Delta\omega \approx -\frac{\omega}{2} \frac{\langle E|\Delta\epsilon|E\rangle}{\langle E|\epsilon|E\rangle} = -\frac{\omega}{2} \frac{\int_V \Delta\epsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3\vec{r}}{\int_V \epsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3\vec{r}}$$

But how does this relate to what happened to the photonic crystal cavity where we might have a beam with a series of holes and the mechanical mode involves some sort of breathing motion of the structure. The way to get at this problem is through perturbation theory. If we consider our original eigenvalue problem in terms of magnetic field, the application of perturbation theory is complicated by the combination of the dielectric constant with the curl operations. It is more convenient to use the equivalent equation in terms of electric field, which, as I mentioned before, is now a generalized eigenvalue problem because of the appearance of operators on both sides of the equation. We can, nevertheless, apply standard perturbation theory resulting in the following expression for the change in frequency associated with a small change in dielectric constant. If the change in dielectric constant is isotropic, this reduces to a straightforward ratio of integrals. While a full derivation is beyond the scope of this lecture, this equation can be interpreted intuitively.

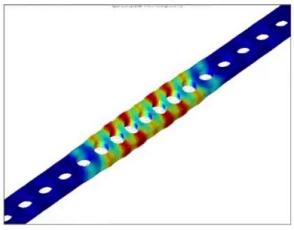
Notes

Summary

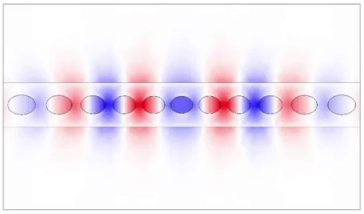
18m 05s

THE EFFECT OF SMALL PERTURBATIONS

Mechanical displacement



Optical mode



$$\nabla \times \left(\frac{1}{\epsilon(\vec{r})} \nabla \times \vec{H}(\vec{r}) \right) = \left(\frac{\omega}{c} \right)^2 \vec{H}(\vec{r}) \quad \Longleftrightarrow \quad \nabla \times \nabla \times \vec{E}(\vec{r}) = \left(\frac{\omega}{c} \right)^2 \epsilon(\vec{r}) \vec{E}(\vec{r})$$

Hermitian eigenvalue problem — Generalized eigenvalue problem

Perturbation theory:

$$\Delta\omega \approx -\frac{\omega}{2} \frac{\langle E | \Delta\epsilon | E \rangle}{\langle E | \epsilon | E \rangle} = -\frac{\omega}{2} \frac{\int_V \Delta\epsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3\vec{r}}{\int_V \epsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3\vec{r}}$$

$$\Delta\epsilon \approx \epsilon \frac{2\Delta n}{n} \qquad \frac{\Delta\omega}{\omega} \approx -\frac{\Delta n}{n} \cdot (\text{fraction of } \epsilon|\vec{E}|^2 \text{ in perturbed region})$$

Considering that the index of refraction is the square root of the dielectric constant for non-magnetic materials, we can write the change in dielectric constant as the integrand in the numerator here is non-zero only in those regions where the index of refraction changes and if we assume for the sake of argument that this change is uniform, we can take it outside the integral. In this case, the fractional change in frequency is equal to the fractional change in index of refraction multiplied by the fraction of the electric field energy in the perturbed region. The minus sign is a result of a higher refractive index leading to a lowering in the energy of the mode.

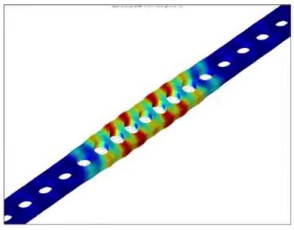
Notes

Summary

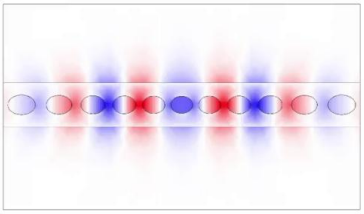
19m 11s

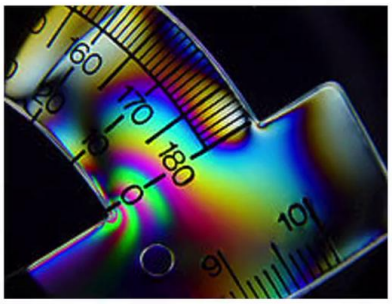
MECHANISMS FOR OPTOMECHANICAL COUPLING: PHOTOELASTIC EFFECT

Mechanical displacement



Optical mode





$$\frac{\partial \omega}{\partial x} \approx -\frac{\omega}{2} \frac{\langle E | \frac{\partial \epsilon}{\partial x} | E \rangle}{\int_V \epsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3\vec{r}}$$

$$\frac{\partial \epsilon}{\partial x} = -\epsilon_0 n^4 p_{ijkl} S_{kl}$$

photoelastic tensor — strain tensor

$$\approx \frac{\omega}{2} \frac{\int_V Re\left[\sum_{i,j} E_i^*(\vec{r}) \epsilon_0 n^4(\vec{r}) p_{ijkl} S_{kl} \vec{E}_j(\vec{r})\right] d^3\vec{r}}{\int_V \epsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3\vec{r}}$$

In practice what leads to a change in the dielectric constant? One important mechanism is the photoelastic effect, that is the change in dielectric constant with strain. The perturbation theory expression for the photoelastic effect looks like this. Note that ‘x’ here is not a displacement in a particular direction but rather a generalized coordinate parameterizing the displacement field of the mechanical mode. The expression for the change in dielectric constant with displacement involves the product of two tensors which means the full expression is somewhat complicated. Fortunately in practice, the dielectric material usually has certain crystal symmetry resulting in a small number of terms to be calculated.

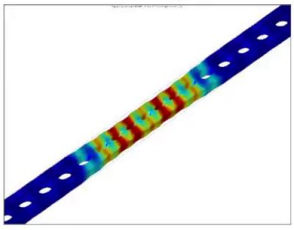
Notes

Summary

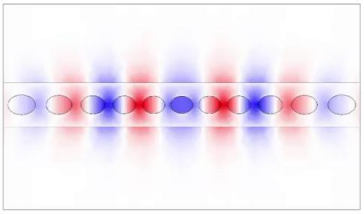
19m 57s

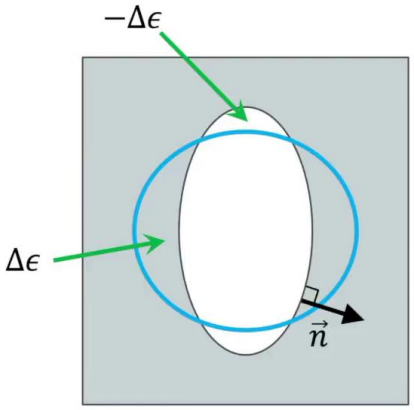
MECHANISMS FOR OPTOMECHANICAL COUPLING: MOVING BOUNDARY EFFECT

Mechanical displacement



Optical mode





Field discontinuity at dielectric boundary
⇒ perturbation theory more complicated

continuous at interface

$$\frac{\partial\omega}{\partial x} \approx -\frac{\omega}{2}\frac{\int_S(\vec{q}(\vec{r})\cdot\vec{n}(\vec{r}))[(\epsilon_1-\epsilon_2)|\vec{E}_{\parallel}(\vec{r})|^2-(\frac{1}{\epsilon_1}-\frac{1}{\epsilon_2})|\epsilon\vec{E}_{\perp}(\vec{r})|^2]dS}{\int_V \epsilon(\vec{r})|\vec{E}(\vec{r})|^2 d^3\vec{r}}$$

Johnson et al. *Phys. Rev. E* 65, 066611 (2002).

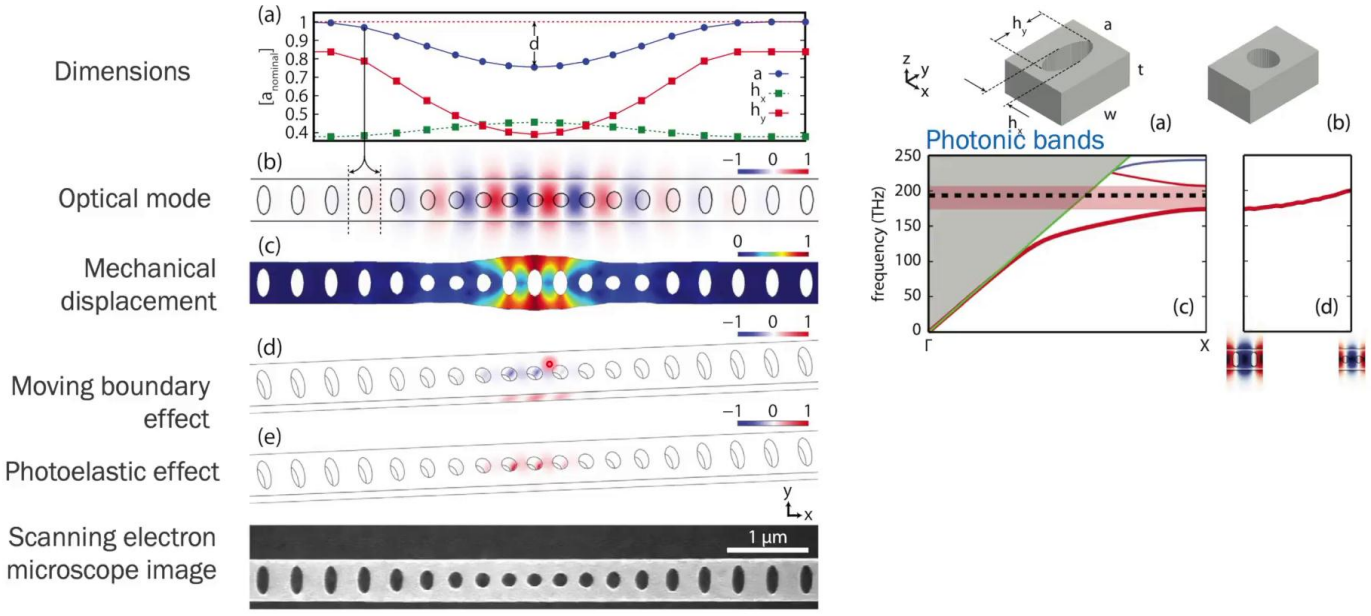
A second important mechanism for optomechanical coupling and photonic crystal cavities is the so-called moving boundary effect. This contribution more closely resembles the cartoon of a Fabry-Perot cavity with a moving mirror. It describes in a general sense the distortion of the structure, for example, the holes in a photonic crystal. There is one problem though. The field discontinuity at the dielectric boundary invalidates the standard approach. There is, however, a correction which looks like this. It makes use of the field components that are continuous at the interface; namely, the parallel component of the electric field and the perpendicular component of the displacement field. The integral in the numerator is a surface integral over the boundary between the two materials where ‘q’ is the displacement field and 'n' is the surface normal. There are other effects that can be treated by perturbation theory such as heating due to thermal absorption or the optical curl nonlinearity. Their achievement is straightforward using the standard perturbation theory and is left as an exercise.

Notes

Summary

20m 45s

BRINGING IT ALL TOGETHER: AN EXAMPLE



Chan et al. "Optimized optomechanical crystal cavity with acoustic radiation shield," *Appl. Phys. Lett.* **101**, 081115 (2012).

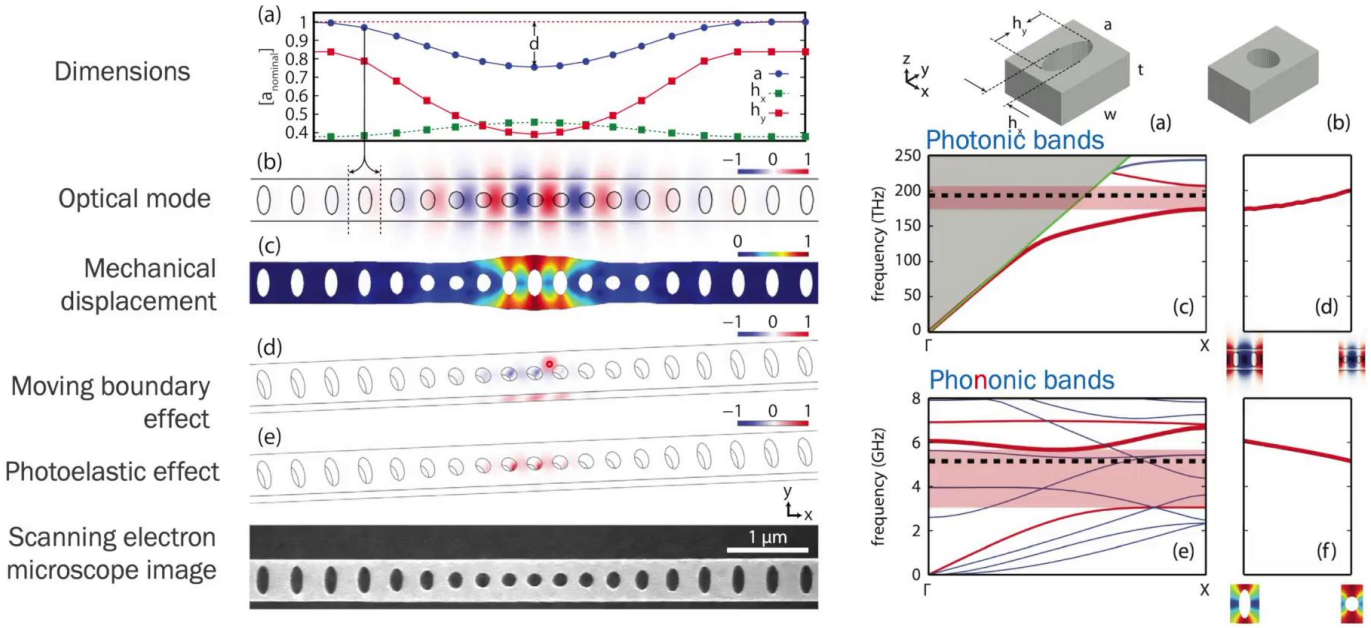
We can now bring together everything we have learned in a single example of a quasi-one-dimensional photonic crystal cavity. The system consists of a beam of silicon with a series of periodic holes that form a band gap for TE polarized light propagating along the axis of the beam. A confined optical mode is produced by the intentional defect formed by a change in the dimensions of the holes as well as the period. Confinement perpendicular to the axis of the beam is provided by index guiding. The mechanical mode of interest is a breathing mode in which the center portion of the structure expands and contracts distorting and straining the silicon. There is thus a contribution to the optomechanical coupling from both the moving boundary effect and the photoelastic effect. The color codes here show the contribution to the integrals for the respective effects. The photonic band structure of the periodic hole portion of the system looks like this. You can see that there are very few modes below the light line. The dashed line corresponds to the position of the confined mode at the center of the cavity.

Notes

Summary

21m 56s

BRINGING IT ALL TOGETHER: AN EXAMPLE



Chan et al. "Optimized optomechanical crystal cavity with acoustic radiation shield," *Appl. Phys. Lett.* **101**, 081115 (2012).

The right-hand panel shows how the frequency of the confined mode is tuned as the holes are smoothly transformed from the periodic portion of the structure to the central defect. Something I haven't talked about yet is the fact that the variation in the periodic hole structure also changes the mechanical modes and forms a partial phononic band gap that simultaneously confines the mechanical mode in the region of the optical mode, an important feature for enhancing the optomechanical coupling.

Notes

Summary

23m 09s

CONCLUSION / SUMMARY

SUMMARY OF KEY POINTS:

- Periodic media have a photonic band structure: $\omega(\vec{k})$
- Electromagnetic waves can propagate in a periodic medium without scattering (wavevector $\vec{k}$ conserved)
- With an appropriate dielectric structure $\epsilon(\vec{r})$, a photonic bandgap can form, i.e. certain frequencies ω do not propagate ⇒ can build optical cavities
- Several mechanism for optomechanical coupling: $G = \frac{\partial \omega}{\partial x}$

CONCLUSION:

- Photonic crystal cavities offer a versatile playground for cavity optomechanics.

FURTHER READING:

- J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, Photonic Crystals, Molding the Flow of Light (Princeton University Press, 2008).

With that we come to the summary. In this lesson, you learned that periodic media have a photonic band structure. In particular, you learned that electromagnetic waves can propagate in a periodic medium without scattering. In other words, the wave vector is conserved. By designing the system properly, a photonic band gap can be created that confines the light at certain frequencies allowing one to build an optical cavity and finally, you learned about several mechanisms for optomechanical coupling in photonic crystal cavities. I think you can see that photonic crystal cavities offer a versatile playground for cavity optomechanics. To learn more, I encourage you to look at this book from Professor Joannopoulos and co-authors where the concepts I have presented are discussed in more detail.

Notes

Summary

23m 44s