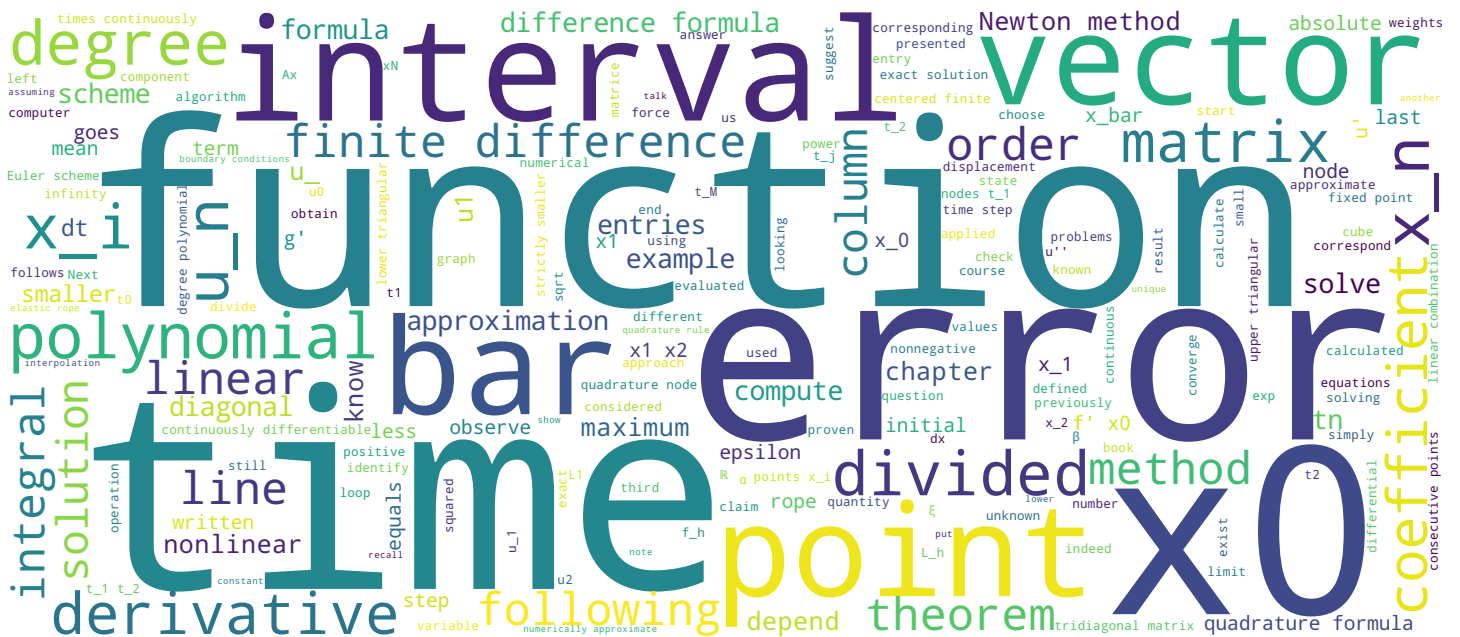


## Chapitre 10 : Résumé

# Introduction à l'analyse numérique

Prof. Marco Picasso

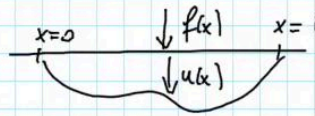


## Video



# Chap 10 : résumé

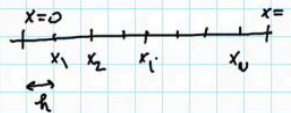
$$\begin{cases} -u''(x) = f(x) & 0 < x < 1 \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$



$$\frac{-u_{i-1} + 2u_i - u_{i+1}}{h^2} = f(x_i) \quad i = 1, \dots, N$$

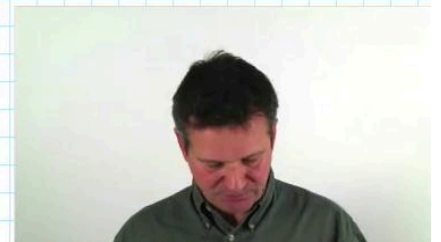
$$A\vec{u} = \vec{f}$$

$$\max_{1 \leq i \leq N} |u(x_i) - u_i| = O(h^2) \quad u \in C^4[0,1]$$



$$\begin{cases} -u''(x) + xu(x) = f(x) & 0 < x < 1 \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$

$$\vec{u} \text{ tq } \vec{F}(\vec{u}) = 0 \quad \vec{u}^n, \vec{u}^{n+1} \text{ tq } \mathcal{J}F(\vec{u}^n)(\vec{u}^n - \vec{u}^{n+1}) = f(\vec{u}^n)$$



Now a quick summary of chapter 10. One dimension boundary value problems with the finite difference method. I considered the problem of the elastic rope -  $u''(x) = f(x)$ , where  $f$  is given force, and  $u$  is the displacement of the rope, this for  $x$  in the interval  $[0,1]$ . The boundary conditions are  $u(0) = 0$  and  $u(1) = 0$ . The rope is clamped at both ends. I presented a finite difference scheme, which was obtained using a centered finite difference formula of the second derivative. The scheme can be written:  $(-u_{i-1} + 2u_i - u_{i+1}) / h^2 = f(x_i)$ , for all points  $x_i$  uniformly distributed on the rope. Here  $h$  is the step size which separates two consecutive points. I write the scheme as a linear system,  $Au = f$ , with  $A$  the tridiagonal matrix with 2 on the diagonal and -1 above and below the diagonal, with a coefficient  $1/h^2$  in front of this tridiagonal matrix. We have proven that the error is  $O(h^2)$ , the maximum of the error between  $u(x_i)$ , which is the unknown exact solution, and its approximation  $u_i$  which can be calculated by computer, this error is in  $O(h^2)$ , this means that it is divided by 4 each time  $h$  is divided by 2, assuming that the solution  $u$  is four times continuously differentiable.

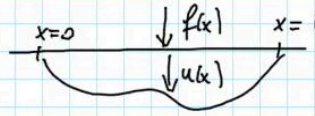
Notes

Summary



# Chap 10 : résumé

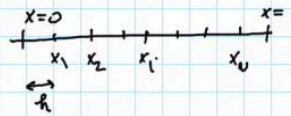
$$\begin{cases} -u''(x) = f(x) & 0 < x < 1 \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$



$$\frac{-u_{i-1} + 2u_i - u_{i+1}}{h^2} = f(x_i) \quad i = 1, \dots, N$$

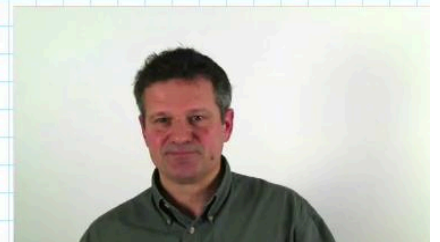
$$A\vec{u} = \vec{f}$$

$$\max_{1 \leq i \leq N} |u(x_i) - u_i| = O(h^2) \quad u \in C^4[0,1]$$



$$\begin{cases} -u''(x) + x(u(x))^3 = f(x) & 0 < x < 1 \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$

$$\vec{u} \text{ tq } \vec{F}(\vec{u}) = \vec{0} \quad \vec{u}^n, \vec{u}^{n+1} \text{ tq } D\vec{F}(\vec{u}^n)(\vec{u}^n - \vec{u}^{n+1}) = -\vec{F}(\vec{u}^n)$$



Notes

I then applied the same method to a nonlinear problem a nonlinear toy problem:  $-u''(x) + x u(x)^3 = f(x)$ .  $u(x)$  au cube ici, donc  $xu(x)$  au cube, et j'ai appliqué la même méthode. Instead of obtaining a linear system  $Au = f$ , I have a nonlinear system:  $F(u) = 0$  with  $F$  and  $u$  both vectors. We used Newton's method to approximate the solution to this problem, and this method was written as:  $u_n$  known, I will calculate  $u_{n+1}$ , which is such that  $Df(u_n)$  multiplied by  $u_n$  minus  $u_{n+1}$  equal to  $F(u_n)$ . I did not say it, but thus method is also  $O(h^2)$ . I suggest to perform numerical experiments during the exercise session.

Summary

