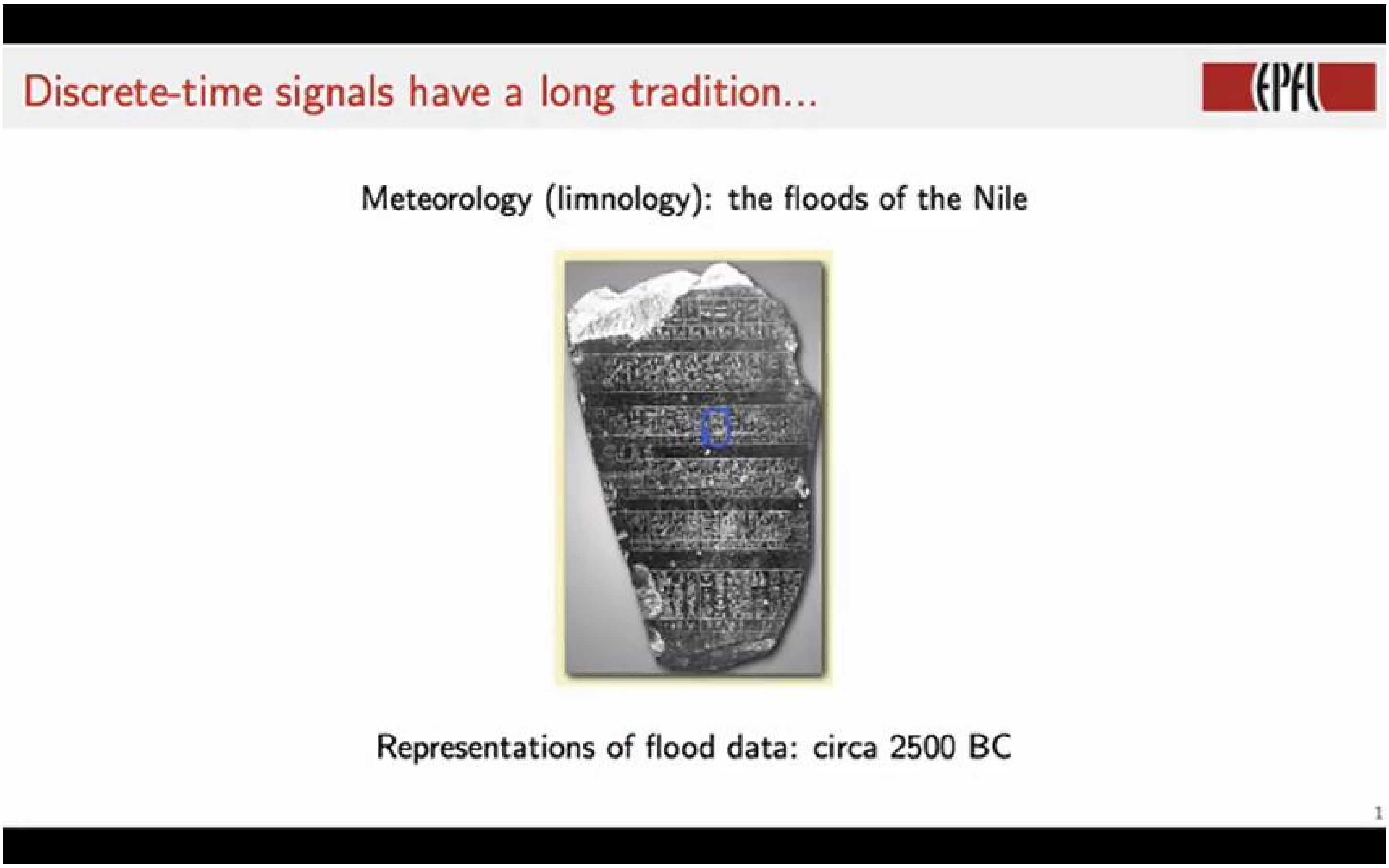
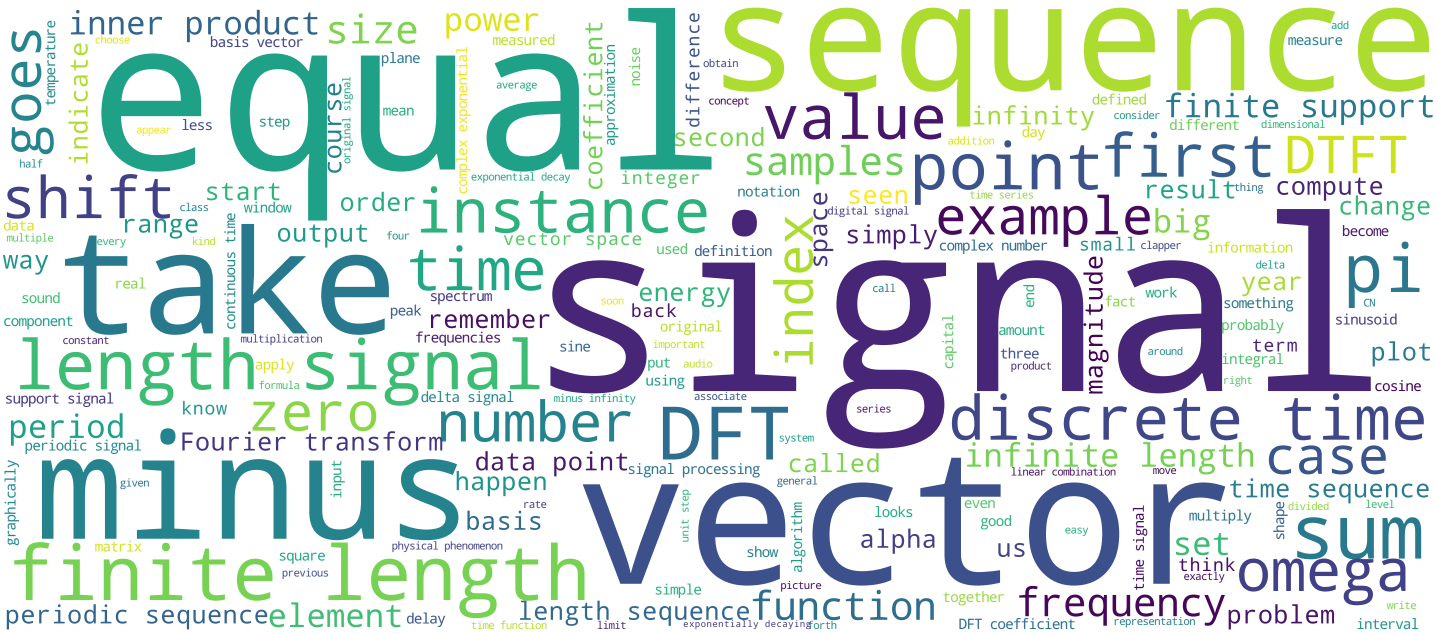
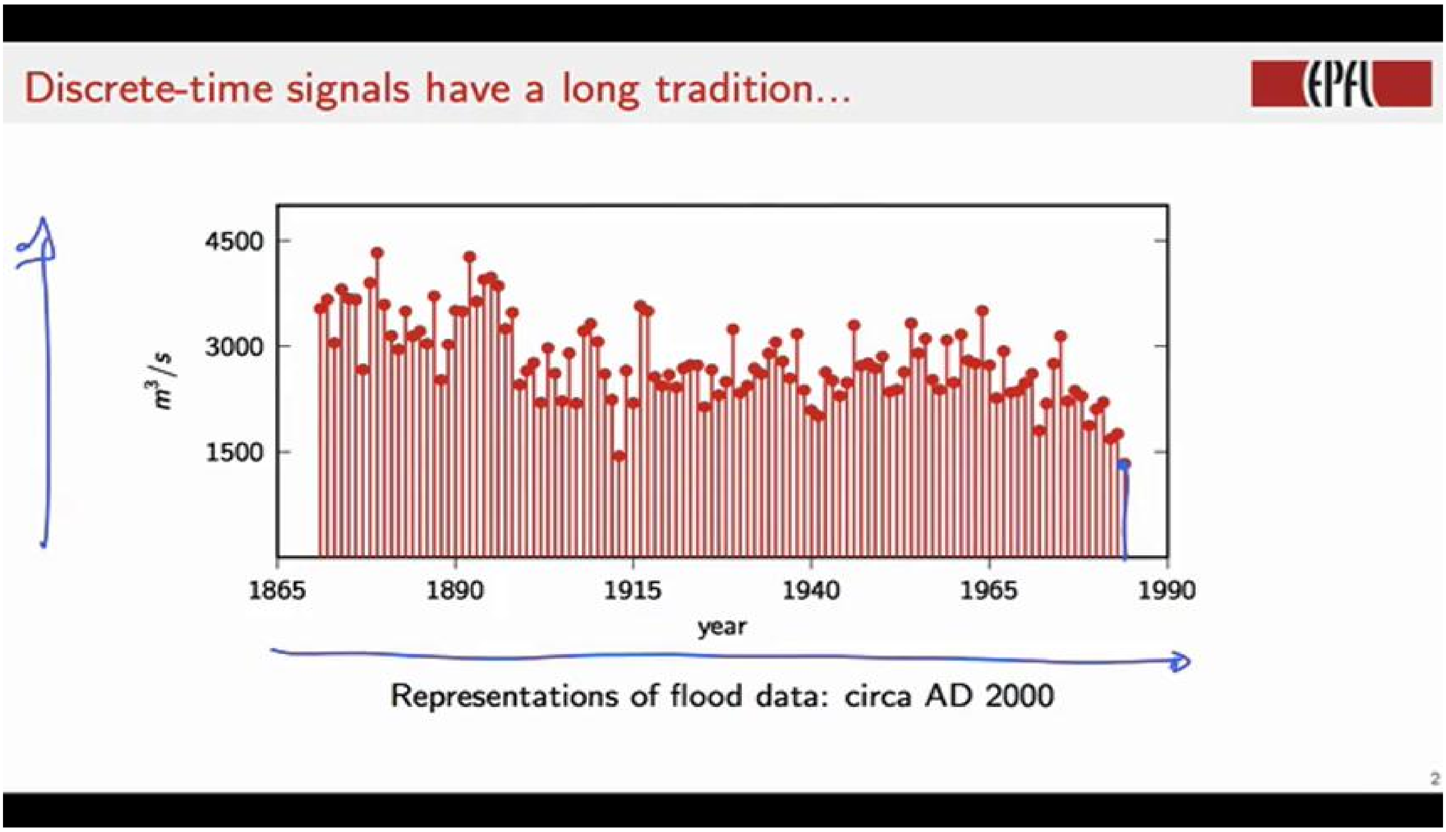




Search MOOC

Video

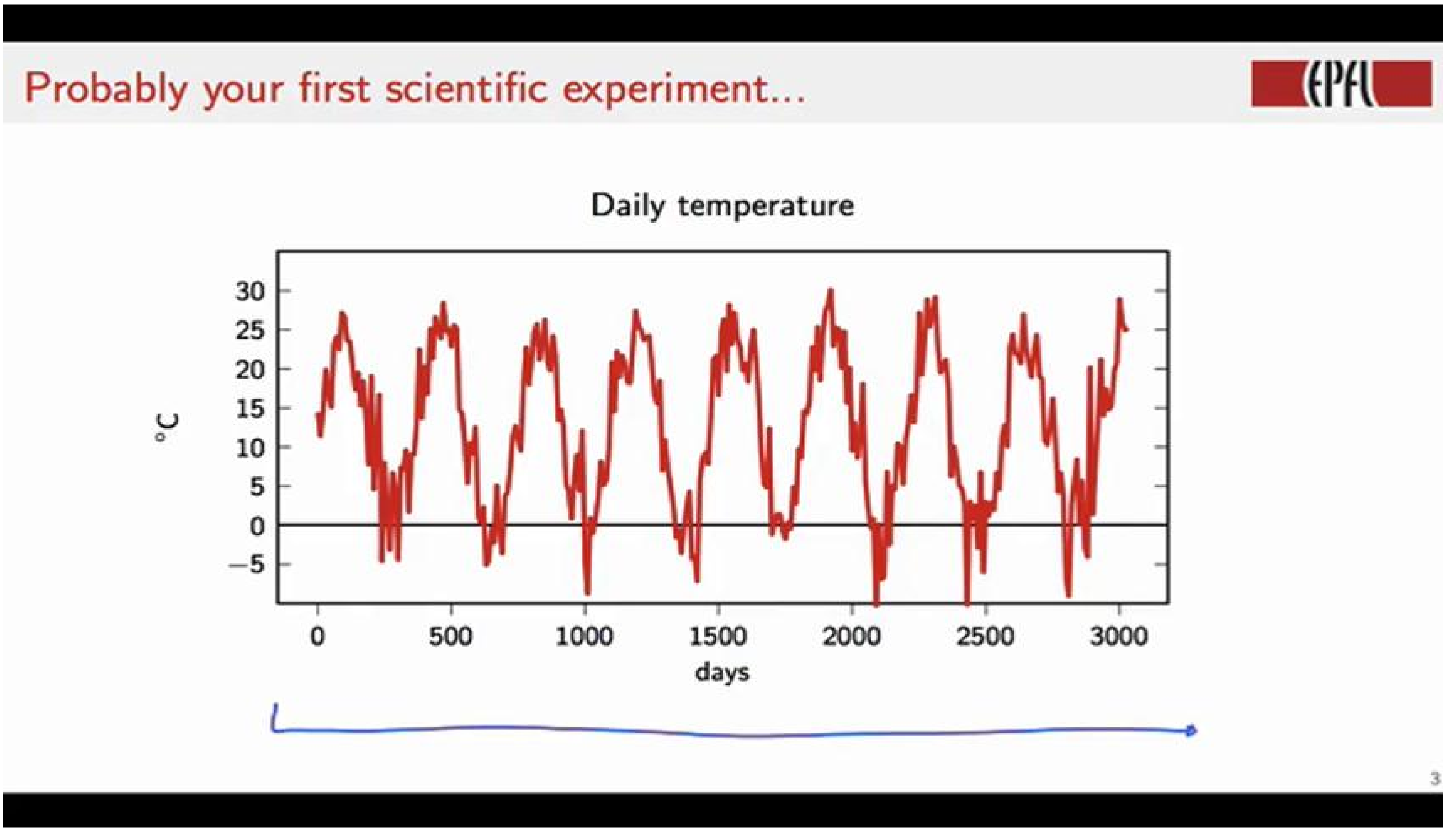


Perhaps the oldest known example of a discrete-time sequence is shown here in this photograph. This is called the Palermo Stone and it represents in hieroglyphs the level of the River Nile for a number of years. So each box here is a year with the level of the river for a number of years around 2,500 BC. In ancient Egypt, the fertility of the river banks really regulated the wealth of the state and so it was very important for the pharaohs to have a historical record of previous flood levels so that they could try and anticipate what the next year would bring. Here we have a discrete-time sequence that was used also for some kind of processing, namely a prediction on the fertility of the following agricultural season. Today, the same data would be represented like so. Here, we have a plot where on the horizontal axis, we have the years, and on the vertical axis, the flow of the river in cubic metres per second. Here, you see that each data point is represented using the so-called lollipop notation. So we have a stem that goes from the horizontal axis to the actual data value, and we use this notation where we want to make sure to indicate that the data is actually a discrete-time sequence and we have a countable set of values, not a continuous time function.

Notes

Summary

0m 00s

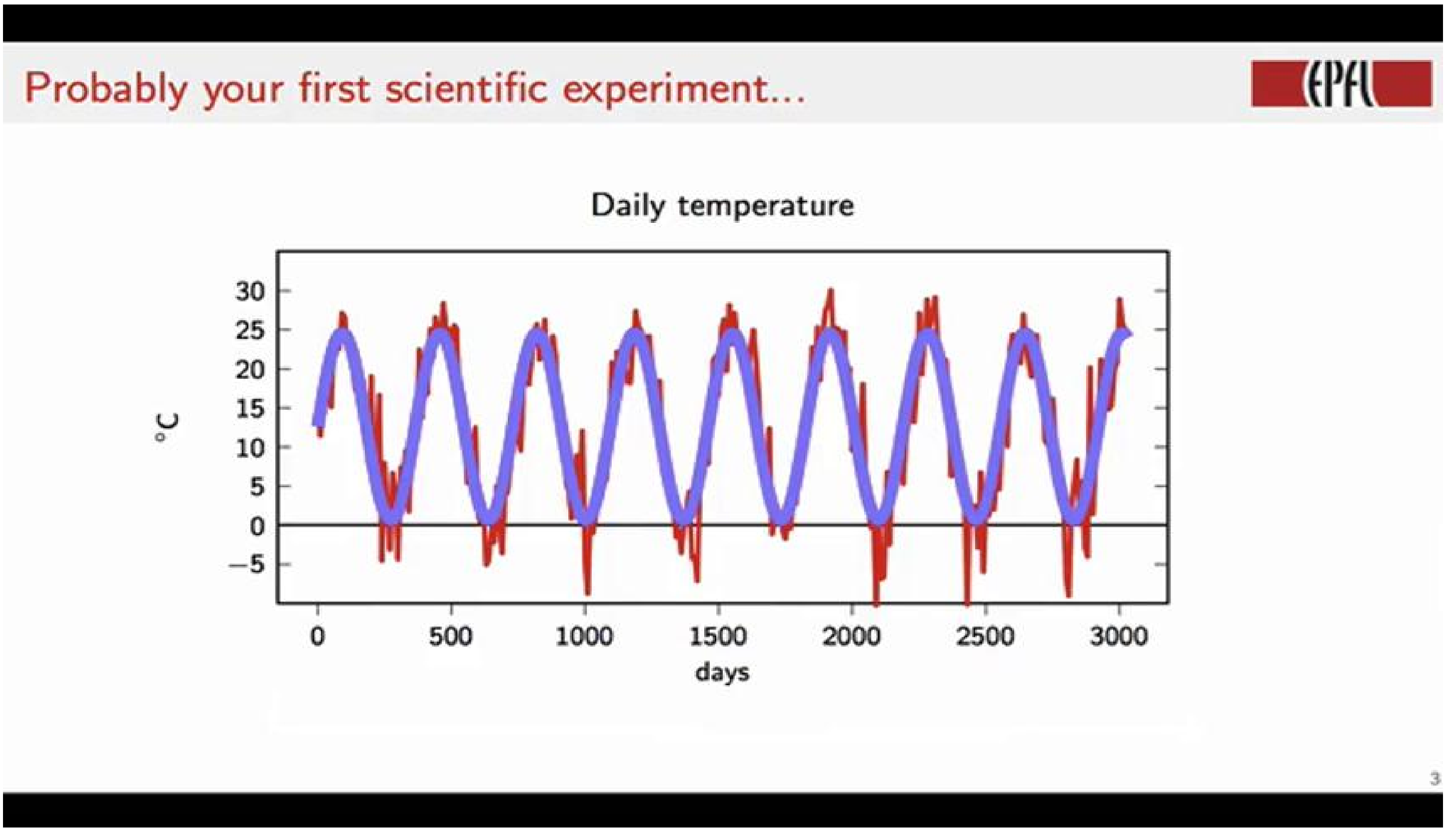


Sometimes, however, the number of data points is so high that the lollipop notation would be too cumbersome. So we forgo the stems. We just put the data points as dots, and they end up being so close that they give the impression of a continuous time function. Here, for instance, you have a plot of the daily temperature over 3,000 points, so 3,000 days, which is about 10 years. Here, you see that it looks like a continuous time function, but it's actually discrete measurements taken every day, and even visually, you can see that there is a periodic pattern.

Notes

Summary

1m 28s

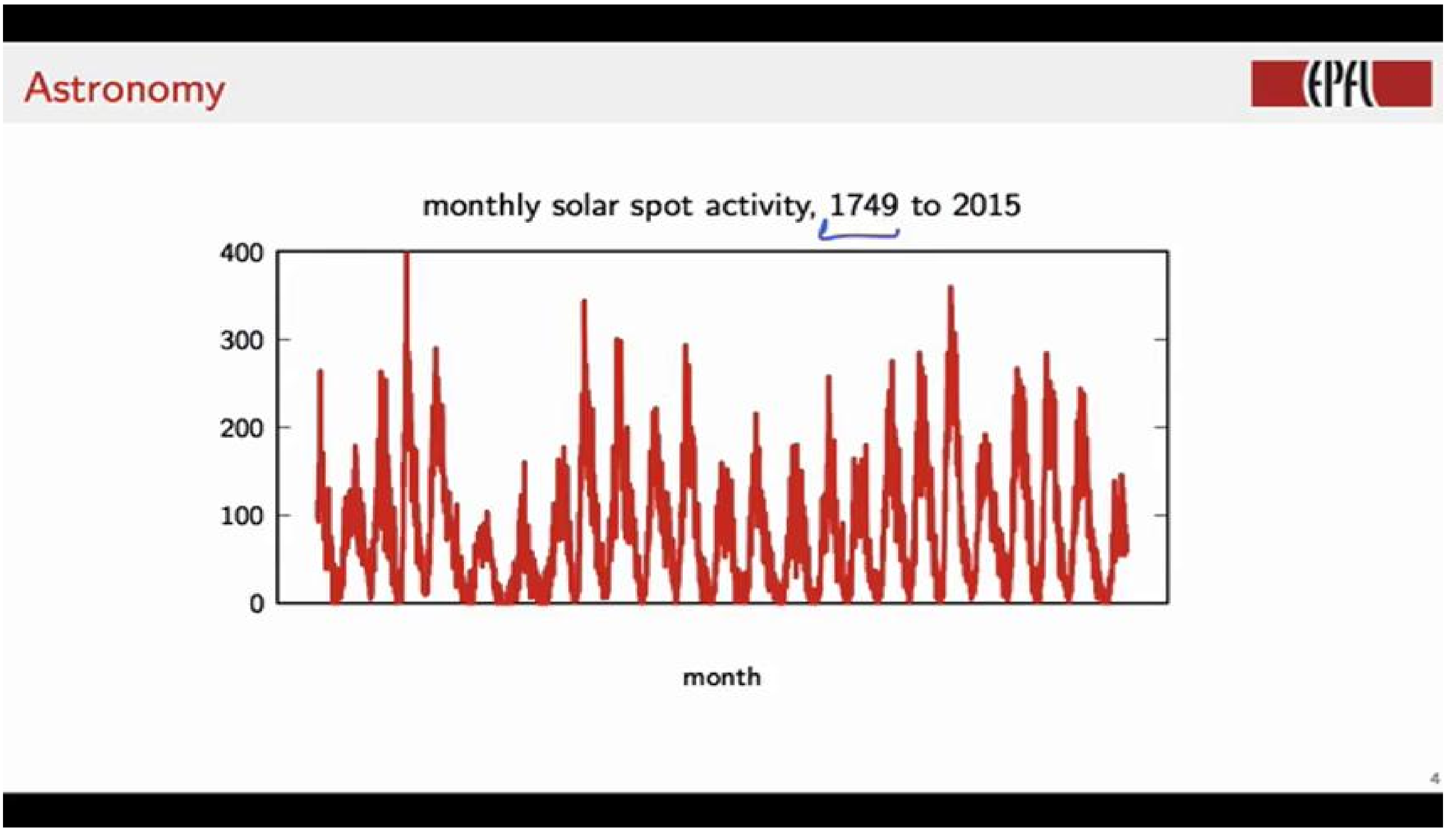


It's actually very easy to superimpose a sinusoid over this data set, and the sinusoid happens to have a period of 365 days which is of course consistent with the fact that the Earth repeats a seasonal pattern at each passage around the sun.

Notes

Summary

2m 02s

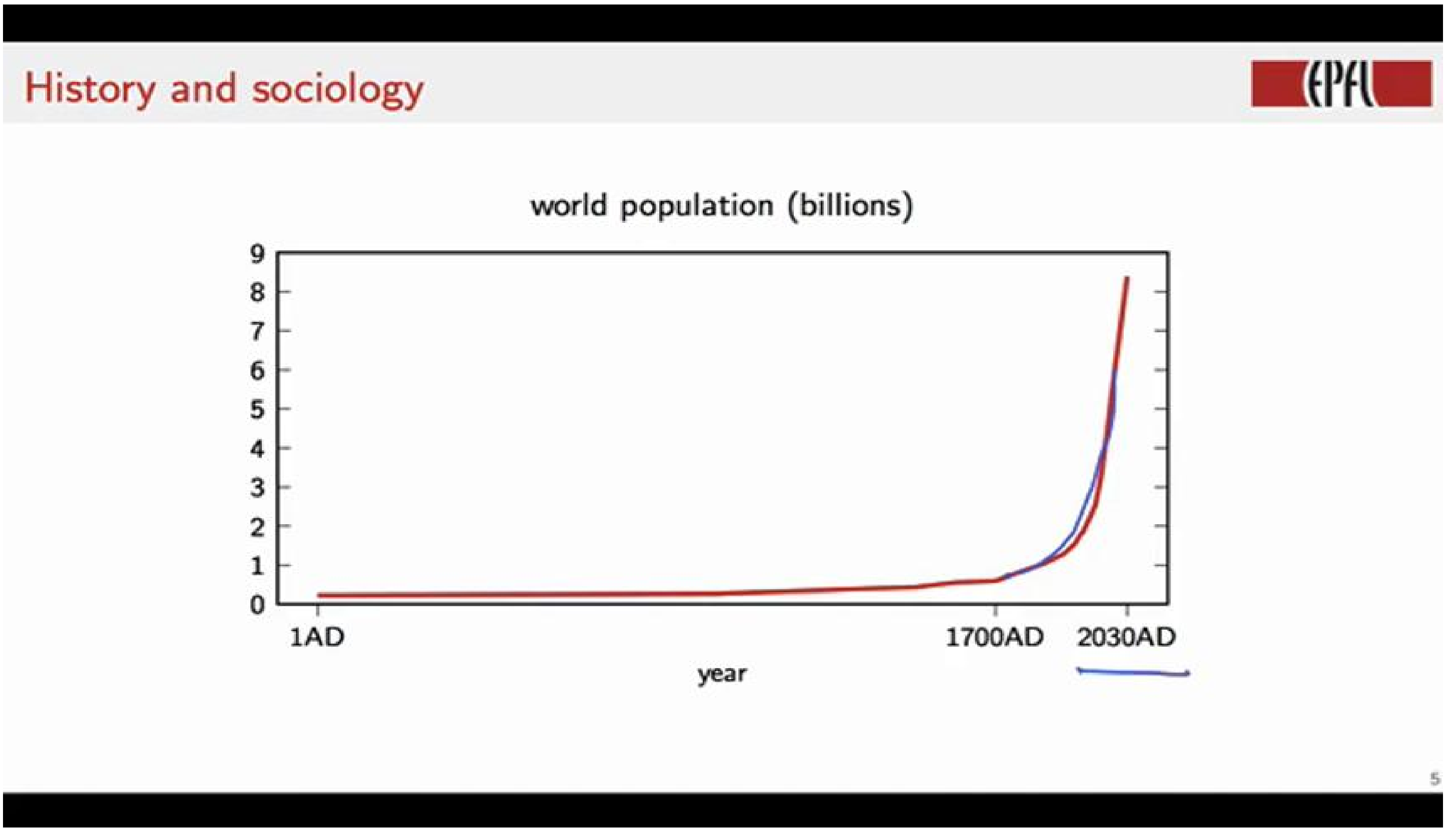


Astronomy uses a lot of discrete-time sequences. This is for instance a sequence of solar spots that measure the sun activity over time. The interesting thing about this series is that it has been kept for a very long time. This is a monthly time series, so there is a data point for each month, and it's been kept since 1749. So there's a lot of data points to work with.

Notes

Summary

2m 19s

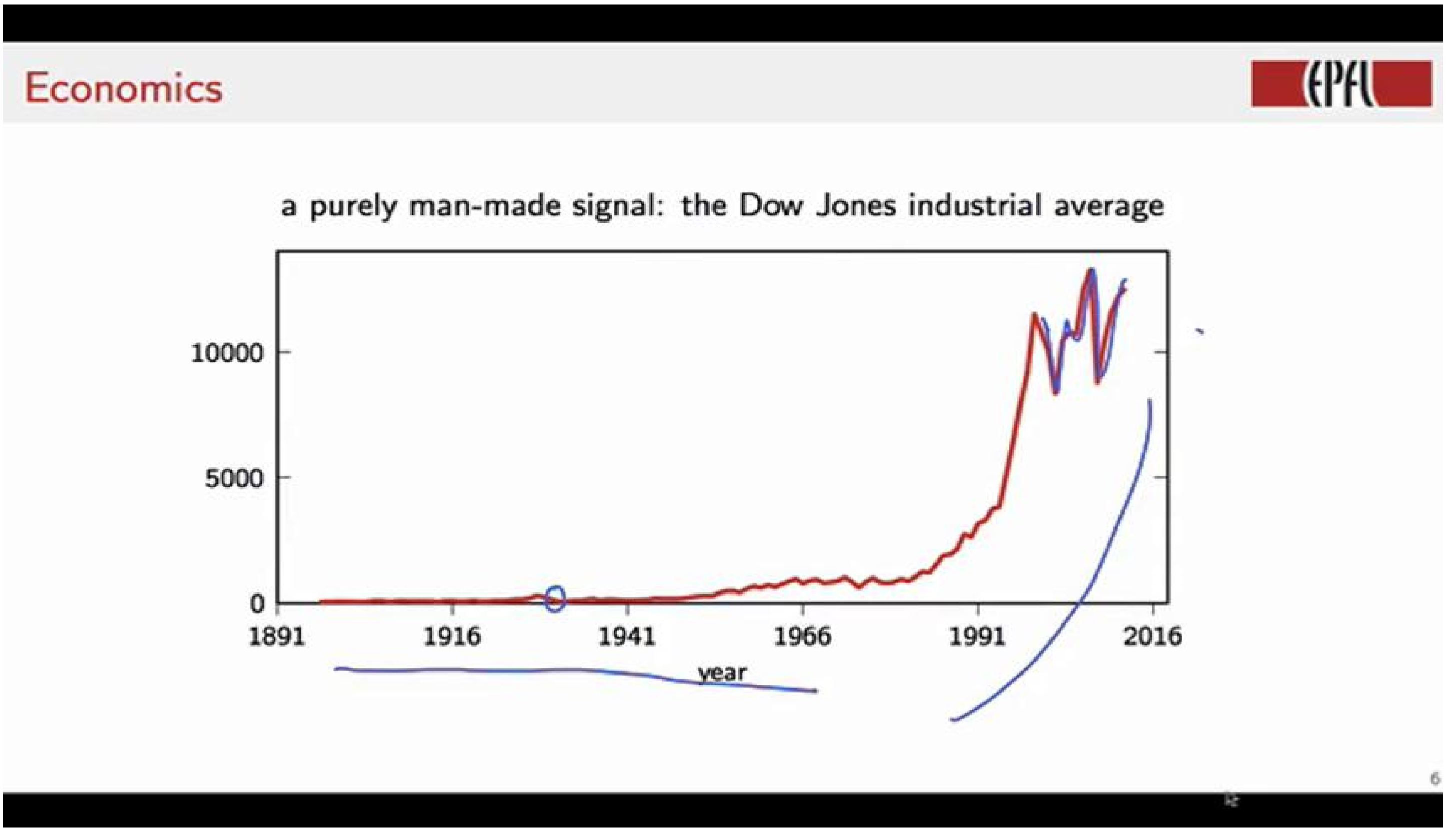


History and sociology are interested in discrete-time sequences as well. For instance, the world population can be measured, say, annually, Here, we have an estimate that starts from Year 1 AD up to a projection for year 2030. You can see that probably we're going to have a little problem here since the trend seems to be exponential.

Notes

Summary

2m 44s

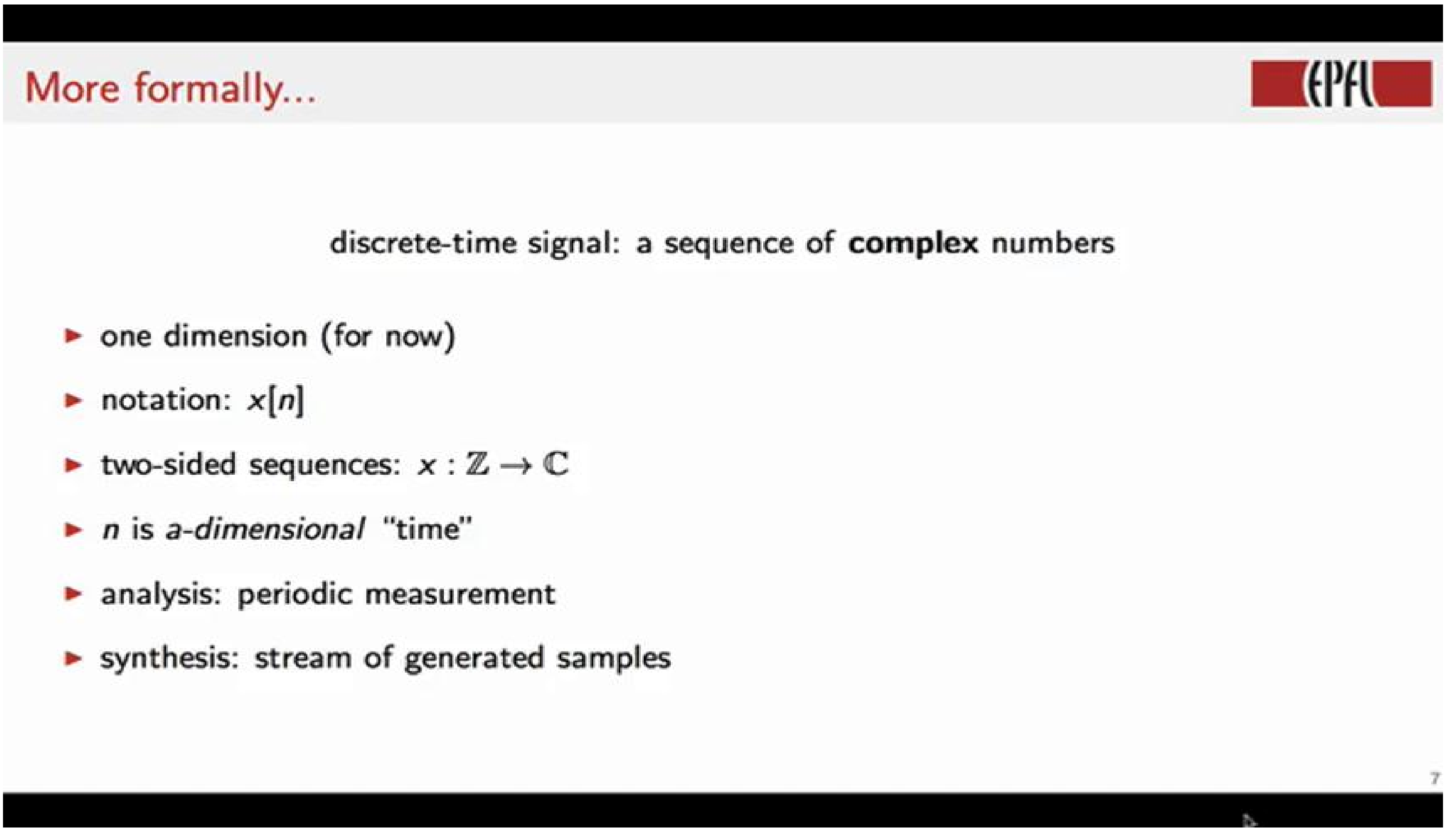


Other time series are completely manmade such as in economics. Here, you have the Dow Jones index which measures some sort of health state of the economy in certain circles. You can see once again that there are some worrisome trends. For instance, the Dow Jones has been pretty low for best part of last century and then it has grown again exponentially. But what is interesting is that probably you're familiar with the crash of 1929 which is this little dip here. Compare that which went down in history as some sort of major disaster with the kind of swings that we have today when the Dow is at such high levels. So we have seen some examples.

Notes

Summary

3m 06s



Now let's try to formalise the concept of discrete-time signal. For us, this is a sequence of complex numbers. It is a one-dimensional sequence, at least for now. The notation is x of n where n is in square brackets to indicate that n is an integer. It's a two-sided sequence so n goes from minus infinity to plus infinity, and it's a mapping, therefore, from Z, the set of integers, to C, the set of complex numbers. n is what we call an a-dimensional time. We can think of it as time if we want, but we have to make sure not to associate a physical unit to n. n is a-dimensional. It just sets an order on the sequence of samples. Discrete-time signals can be created by an analysis process where we take periodic measurements of a physical phenomenon— think of the floods of the Nile, if you want— or in a synthesis process where we use, say, a computer program to generate data point that simulates a physical phenomenon that we want to reproduce. We will see an example very soon.

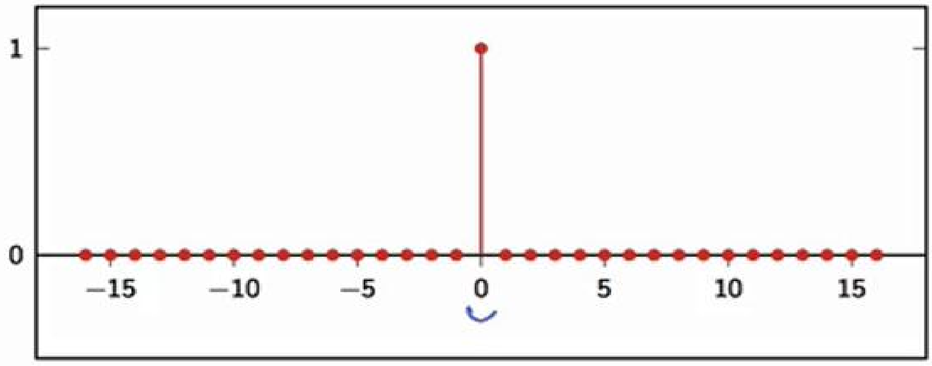
Notes

Summary

3m 51s

The delta signal

$$x[n] = \delta[n]$$



8

Let's now look at some prototypical signals that will appear again and again in this class. The simplest non-trivial signal that you can think of is a signal where every sample is equal to 0 except for n equal to 0 where the sample is equal to 1. This is called the delta signal, and it exemplifies a physical phenomenon that has a very short duration in time.

Notes

Summary

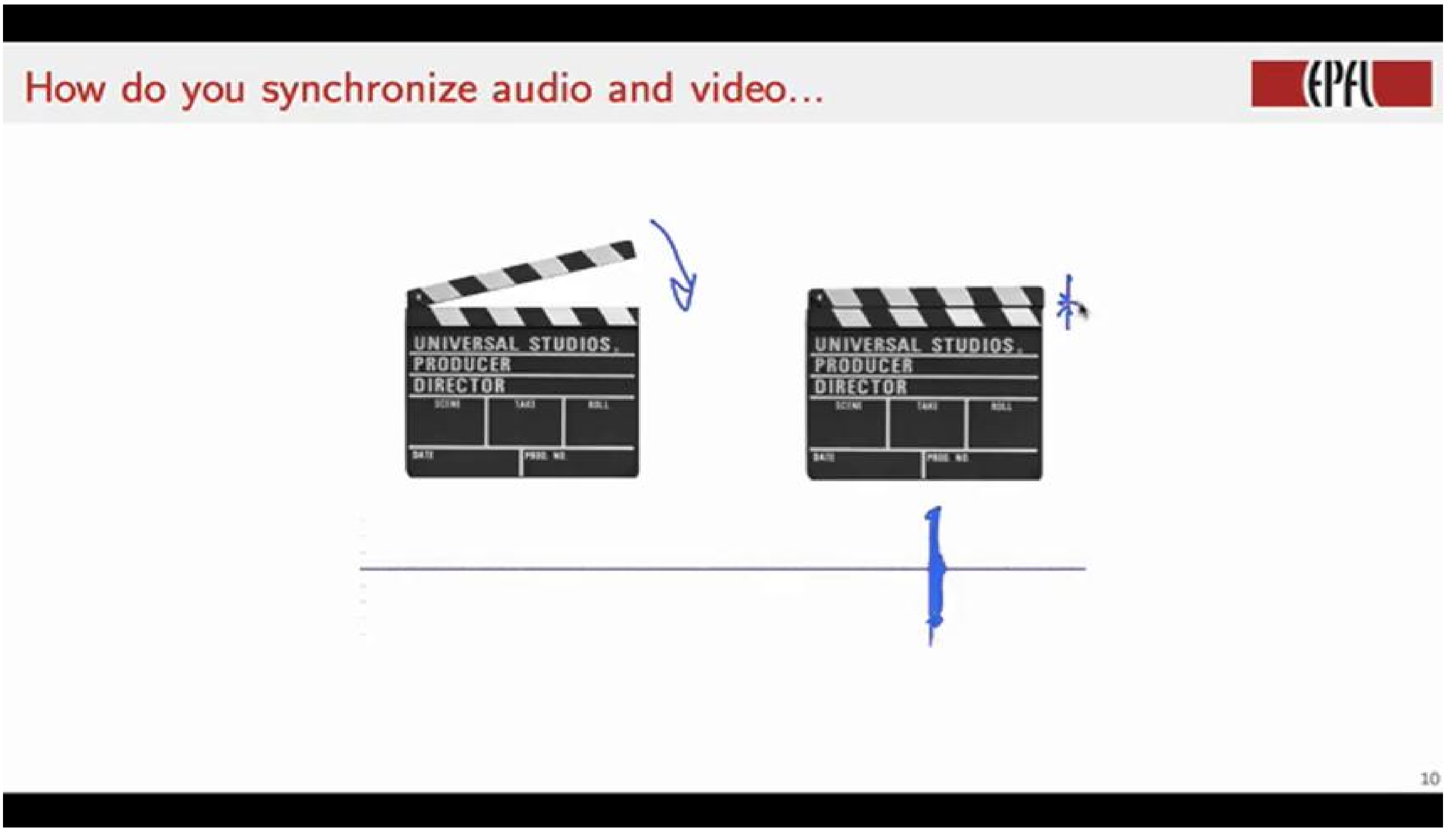
5m 01s

To help your memory, you can associate the delta signal to a clapper, the device that's used in the movie industry, although perhaps not in the mechanical form that you see here in this picture, to synchronise the audio and the video tracks. When you shoot a movie, the video and the audio are recorded on separate devices, and then you have to synchronise the two tracks together.

Notes

Summary

5m 25s

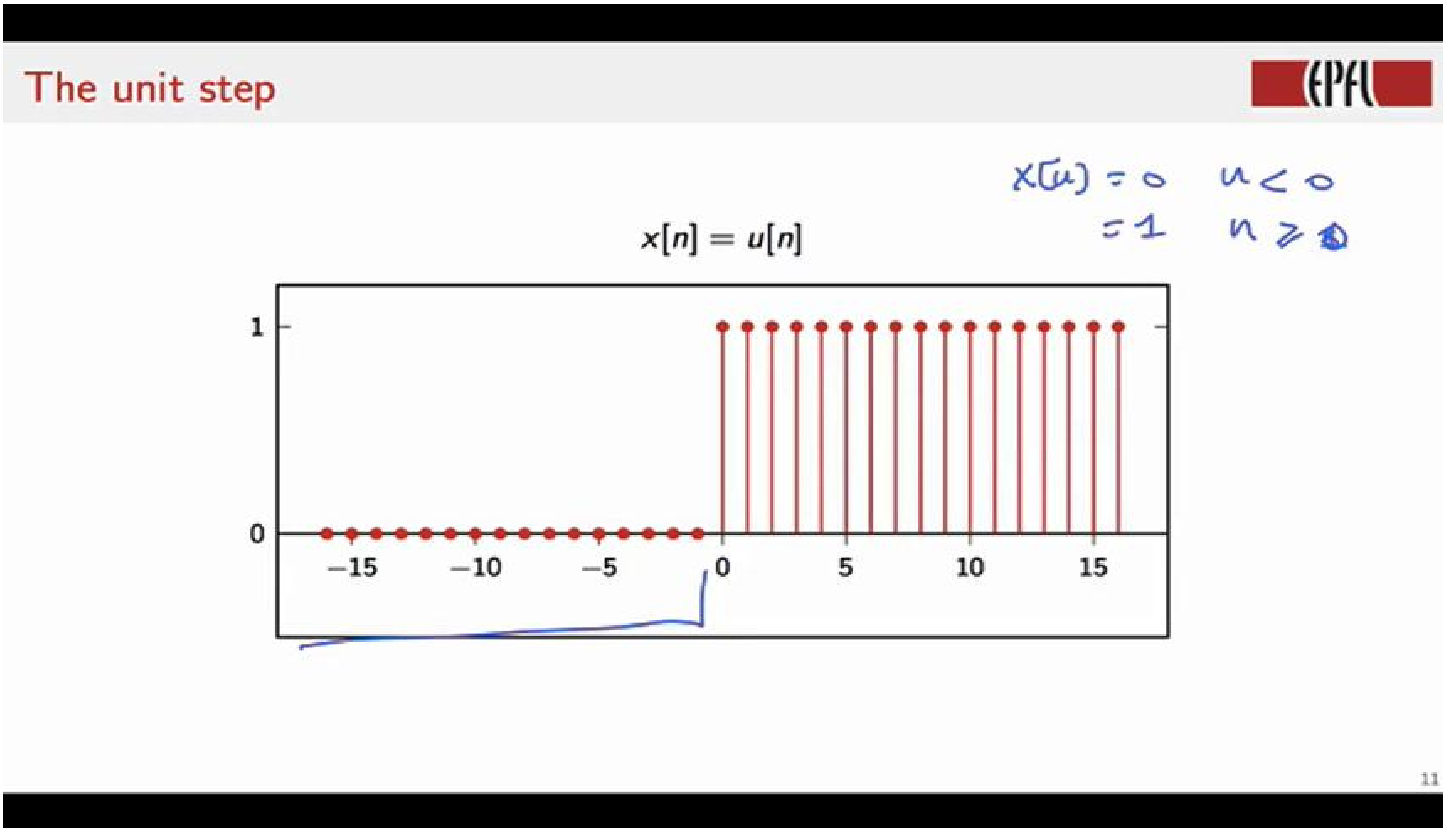


The way this is done is by filming the clapper and then having the top part of the clapper slam down on the bottom part. This will generate a very short instantaneous sound that on the audio track will look like a delta signal or a combination of positive and negative delta signals. When you need to synchronise audio and video, you will look for this pattern in the audio track. You will look for the delta and associate it to the frame where the top part of the clapper is hitting the bottom part.

Notes

Summary

5m 47s

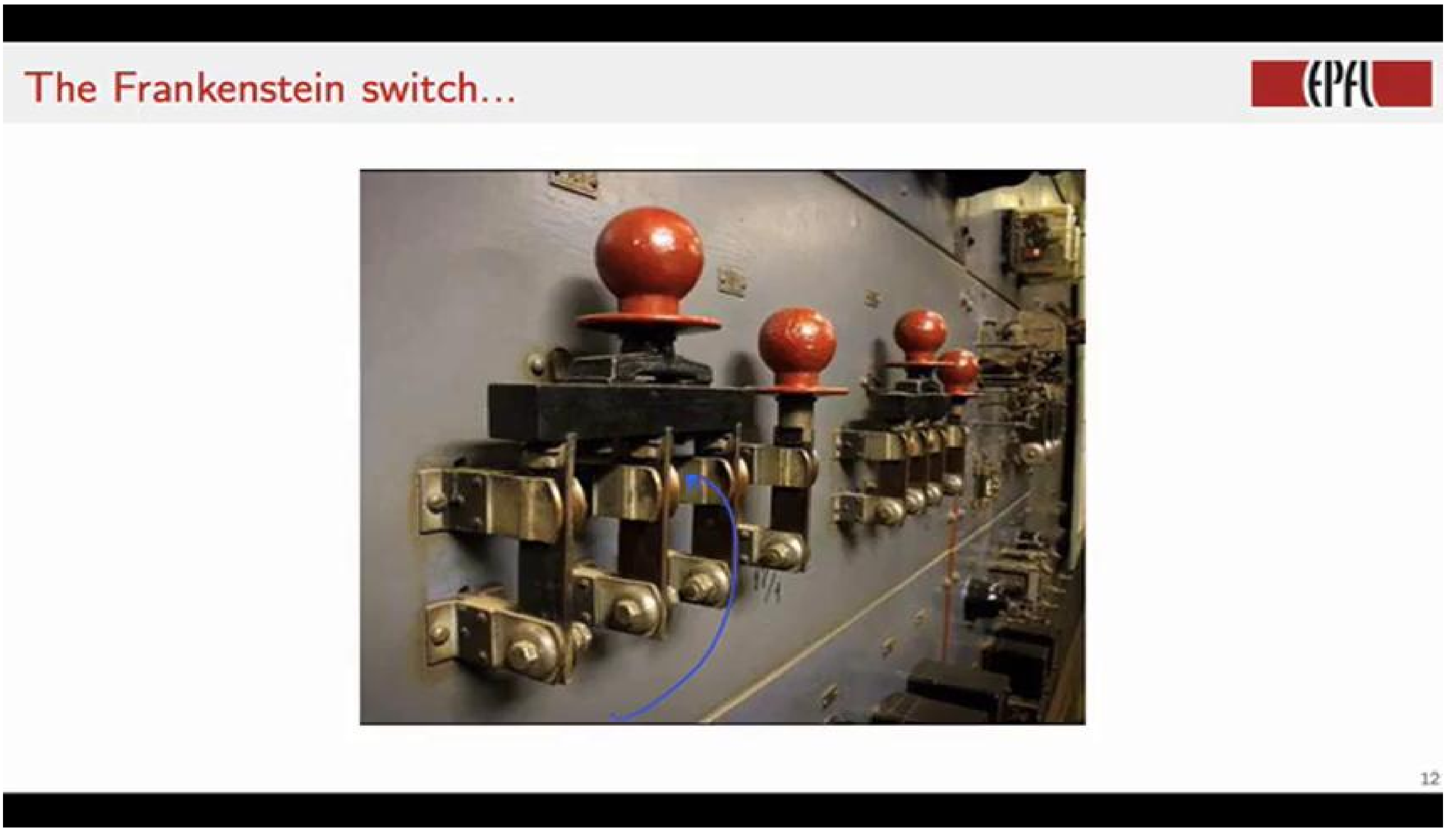


Another useful signal is the unit step. This is a signal that is zero for all negative values of the index. x of n was equal to 0 for n less than 0 and is equal to 1 for n greater than or equal to 0. This depicts a very simple phenomenon, the flipping of a switch.

Notes

Summary

6m 17s

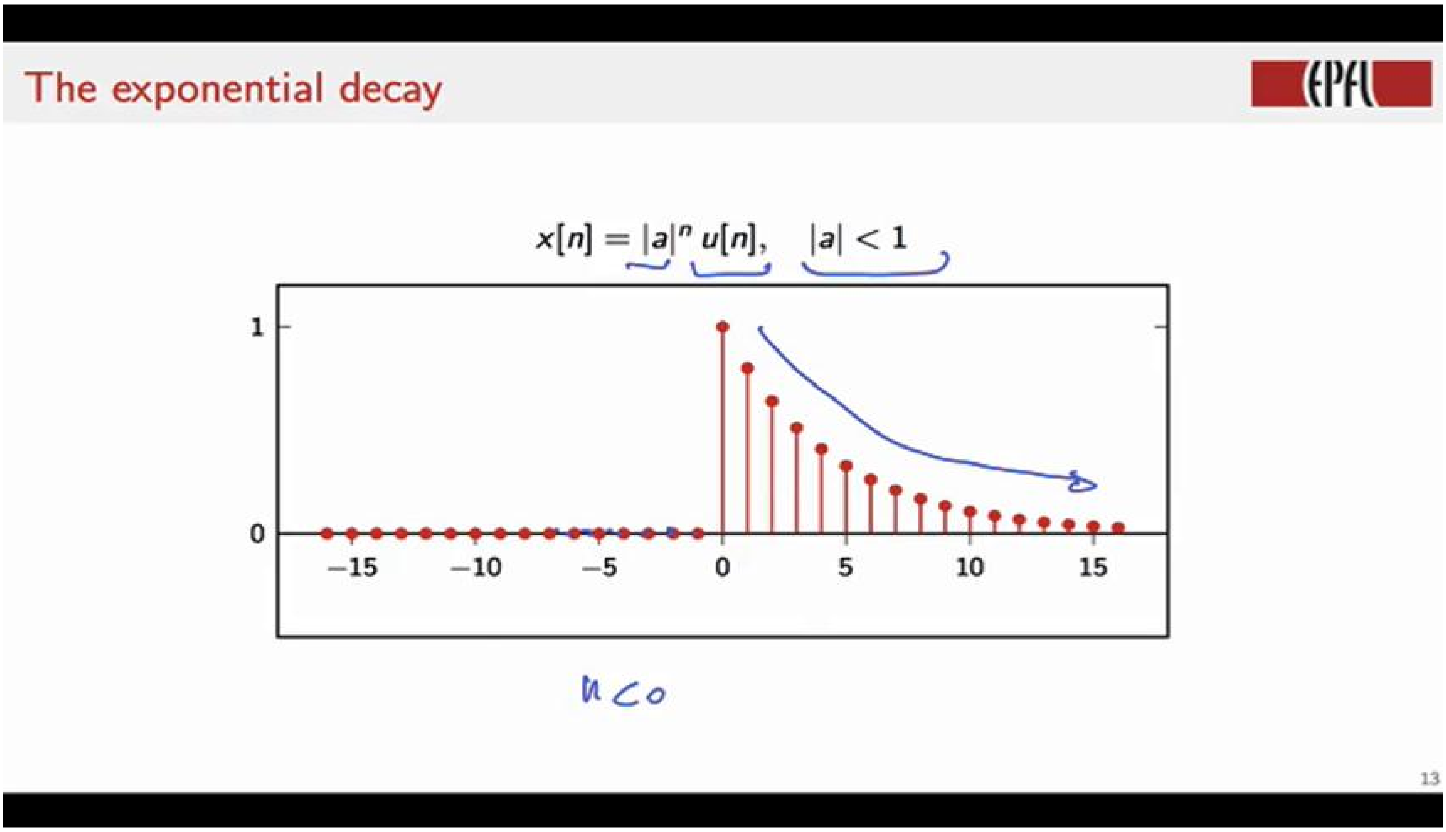


So think of a Frankenstein switch when this is pulled up, then the contact is made, and the signal will go from 0 to 1 and stay at 1 forever.

Notes

Summary

6m 40s



Another common signal is the exponential decay. We take a number a less than 1 in magnitude and we take successive powers of the absolute value of a. Because a is less than 1 in magnitude, successive powers will go down exponentially to 0, but of course will never reach 0 unless we go to infinity. In order to prevent the signal from exploding when n is negative, we multiply the signal by the unit step. So we basically force to 0 all values of the sequence for negative values of the index. The exponential decay captures the behaviour of a lot of physical systems.

Notes

Summary

6m 51s

For instance, it shows how your coffee cup gets cold.

Notes

Summary

7m 30s

How fast does your coffee get cold...

Newton's law of cooling:

$$\frac{dT}{dt} = -c(T - T_{\text{env}})$$

$$T(t) = T_{\text{env}} + (T_0 - T_{\text{env}})e^{-ct}$$

In practice:

- must have convection only
- must have large conductivity

15

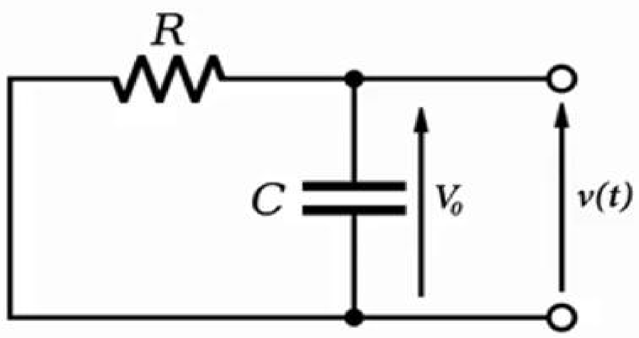
Newton's law of cooling says that the rate of change of the temperature of a body is proportional to the difference in temperature between the environment and the body itself. I,f you solve this differential equation you find out that the evolution of the temperature follows indeed an exponentially decaying trend. Of course, this is an idealised version of how a coffee gets cold because you should have only convection and large conductivity. But in general, this is a common behaviour for a lot of physical systems.

Notes

Summary

7m 34s

Also, how fast your capacitor discharges



$$v(t) = V_0 \, e^{-\frac{t}{RC}}$$

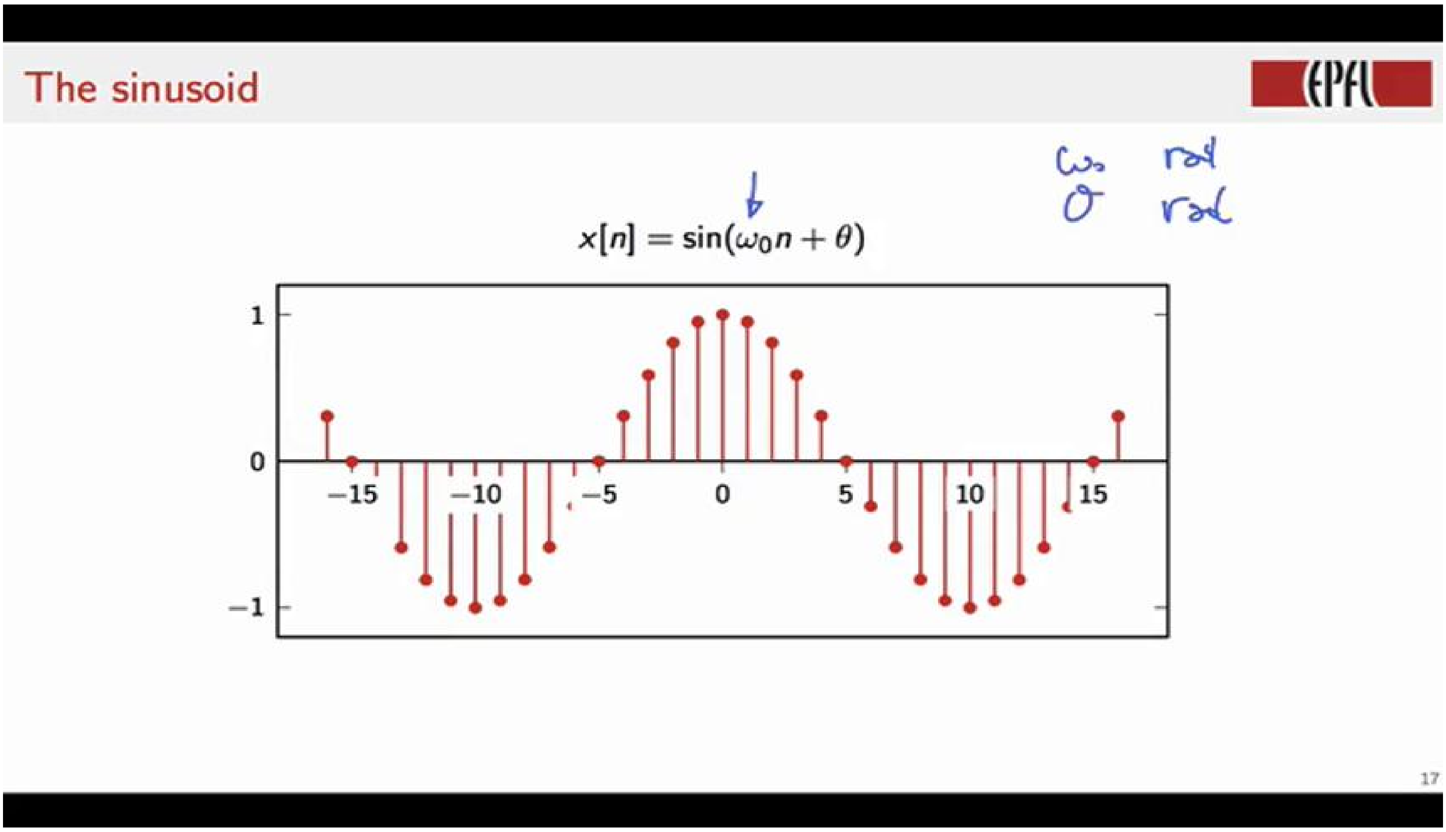
16

We have seen, for instance, that the rate of discharge of a capacitor in an RC circuit is also an exponentially decaying curve. In discrete-time, the exponential decay, a to the power of n models this kind of behaviour.

Notes

Summary

8m 06s



Finally, we have sinusoidal signals. Here we have, for instance, an example of using the sine function, the discrete-time sequence is simply the sine of an angular frequency Omega$_0$ times the index n plus an initial phase Theta. Omega$_0$ is measured in radians. Theta is measured in radians as well because n is a-dimensional. So the sum of Omega$_0$ n plus Theta is measured in radians.

Notes

Summary

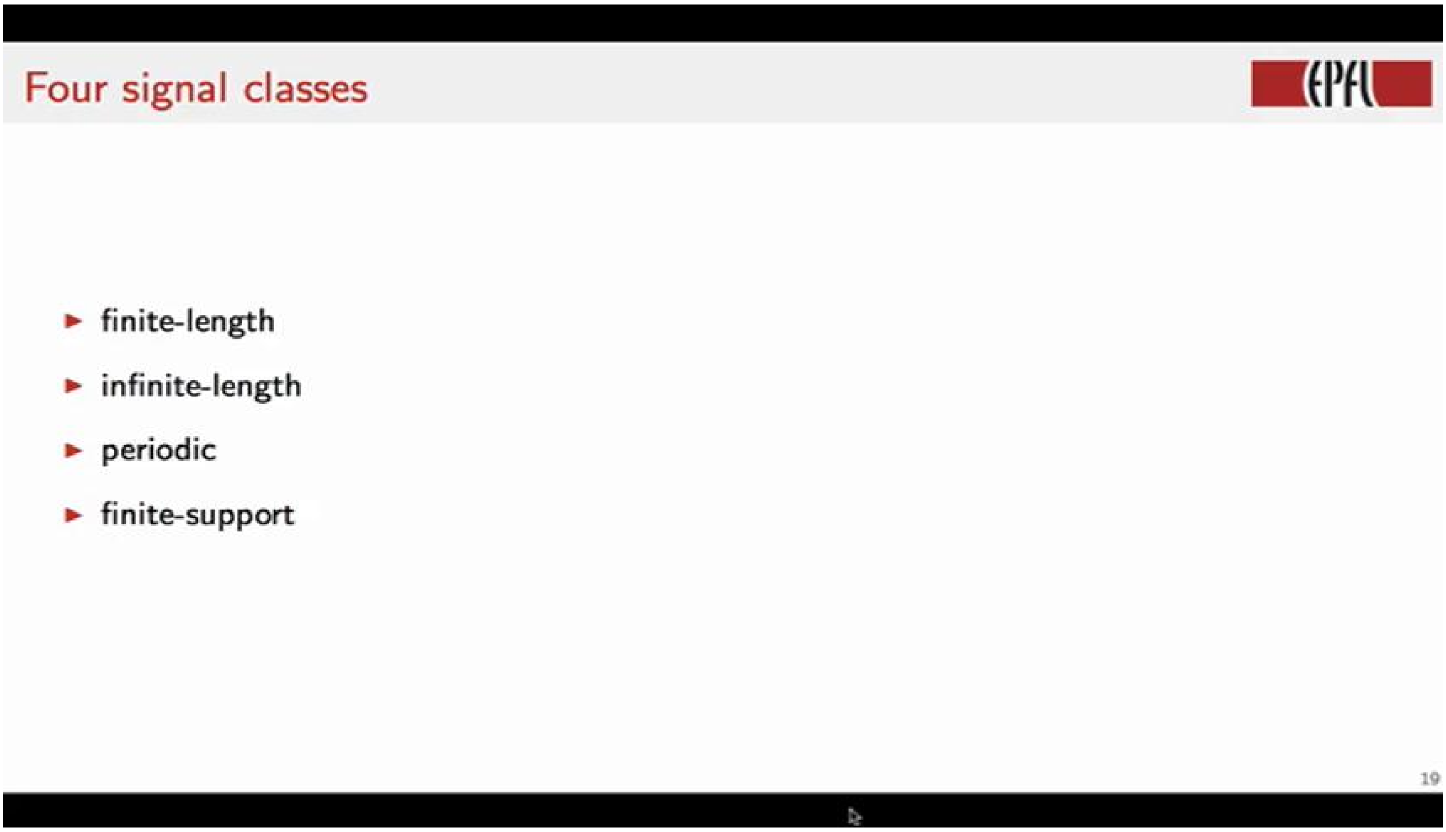
8m 22s

There is certainly no need to stress the importance of oscillatory behaviour in nature, your heartbeat, engines, the motion of the waves, the vibration of strings in musical instruments. But in signal processing, oscillations are particularly important because they are at the heart of Fourier analysis as we will see very soon.

Notes

Summary

8m 51s

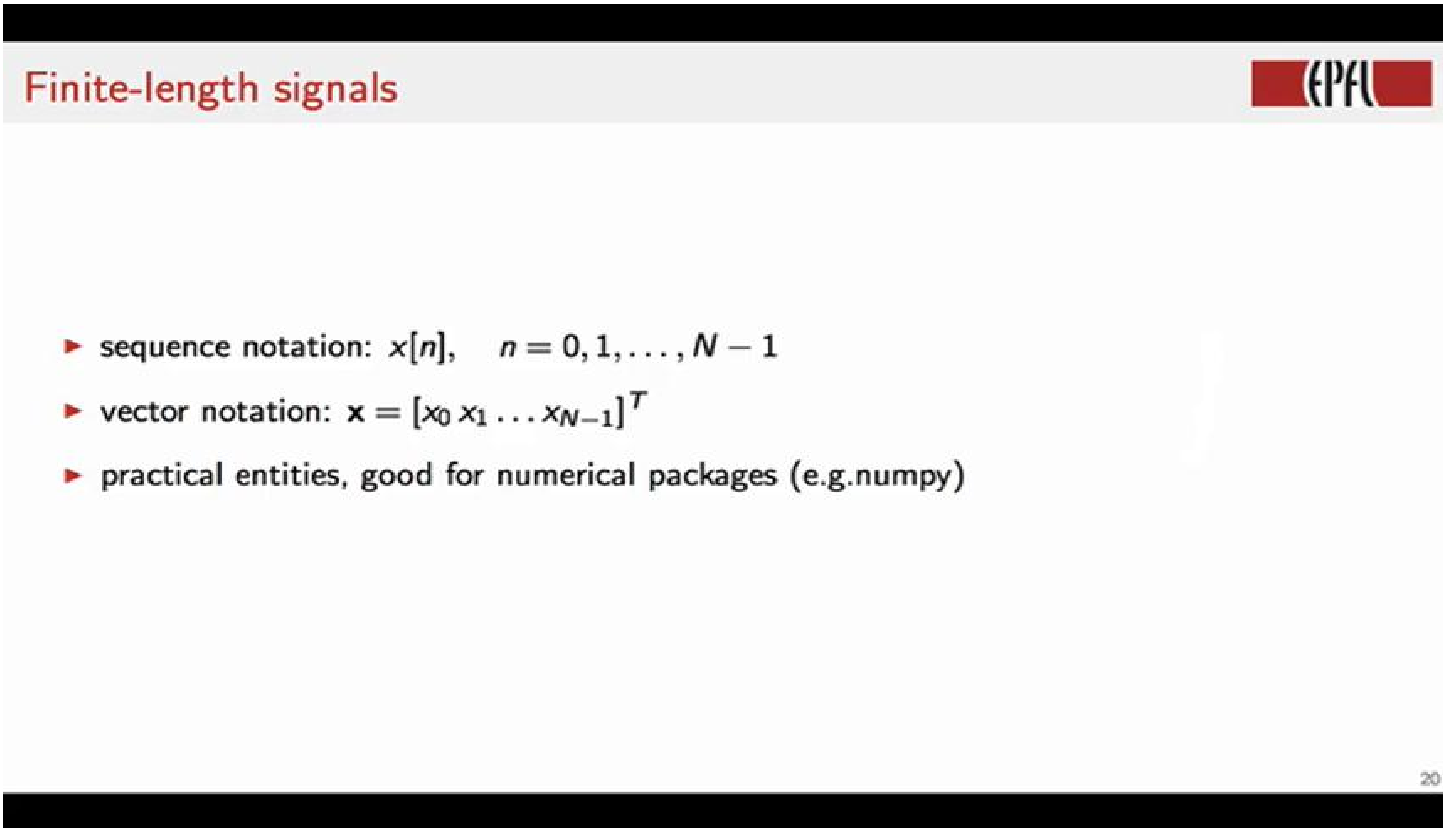


It is useful to divide discrete-time signals into four classes: finite-length signals, infinite-length signals, periodic signals, and finite support signals. We will now look at them in turn.

Notes

Summary

9m 12s



Finite-length signals are signals that contain only N samples. We indicate them with the notation x of n as for standard sequences, but we always specify the range of the index n that goes from 0 to N minus 1. Sometimes we will also use vector notation. In this case, the signal is a column vector like so. The connection between finite-length signals and vectors will be clear very soon in one of the future lectures. Finite-length signals are very practical entities and they're good for numerical packages. You will always deal with array of data where the size of the array is finite. However, it's not practical to develop the entire signal process in theory concentrating only on finite-length signals because the length gets in the way.

Notes

Summary

9m 25s

Infinite-length signals

- sequence notation: $x[n], \quad n \in \mathbb{Z}$
- abstraction, good for theorems

21

Infinite-length signals are standard sequences where the index n ranges over the entire set of integers from minus infinity to plus infinity. These are of course abstract entities because they contain potentially an infinite amount of data, but they're very good for theorems and results that do not depend on the length of the data.

Notes

Summary

10m 22s

Periodic signals

- N-periodic sequence: $\tilde{x}[n] = \tilde{x}[n + kN], \quad n, k, N \in \mathbb{Z}$
- same information as finite-length of length N
- "natural" bridge between finite and infinite lengths

Periodic sequences are sequences where the data repeats every N samples. We indicate that with the notation tilde of x. The symbol here indicates periodicity explicitly. The relationship for a periodic sequence of period N is that x of n is equal to x of n plus k times N for all integer values of n and k. The amount of information contained by a periodic sequence is exactly equivalent to the amount of information contained by a finite-length signals of length N. So somehow periodic sequences are a natural bridge between finite and infinite-lengths. They have a finite amount of information, but they have an infinite-length.

Notes

Summary

10m 46s

Finite-support signals

- Finite-support sequence:

$$\bar{x}[n] = \begin{cases} x[n] & \text{if } 0 \leq n < N \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbb{Z}$$

- same information as finite-length of length N
- another bridge between finite and infinite lengths

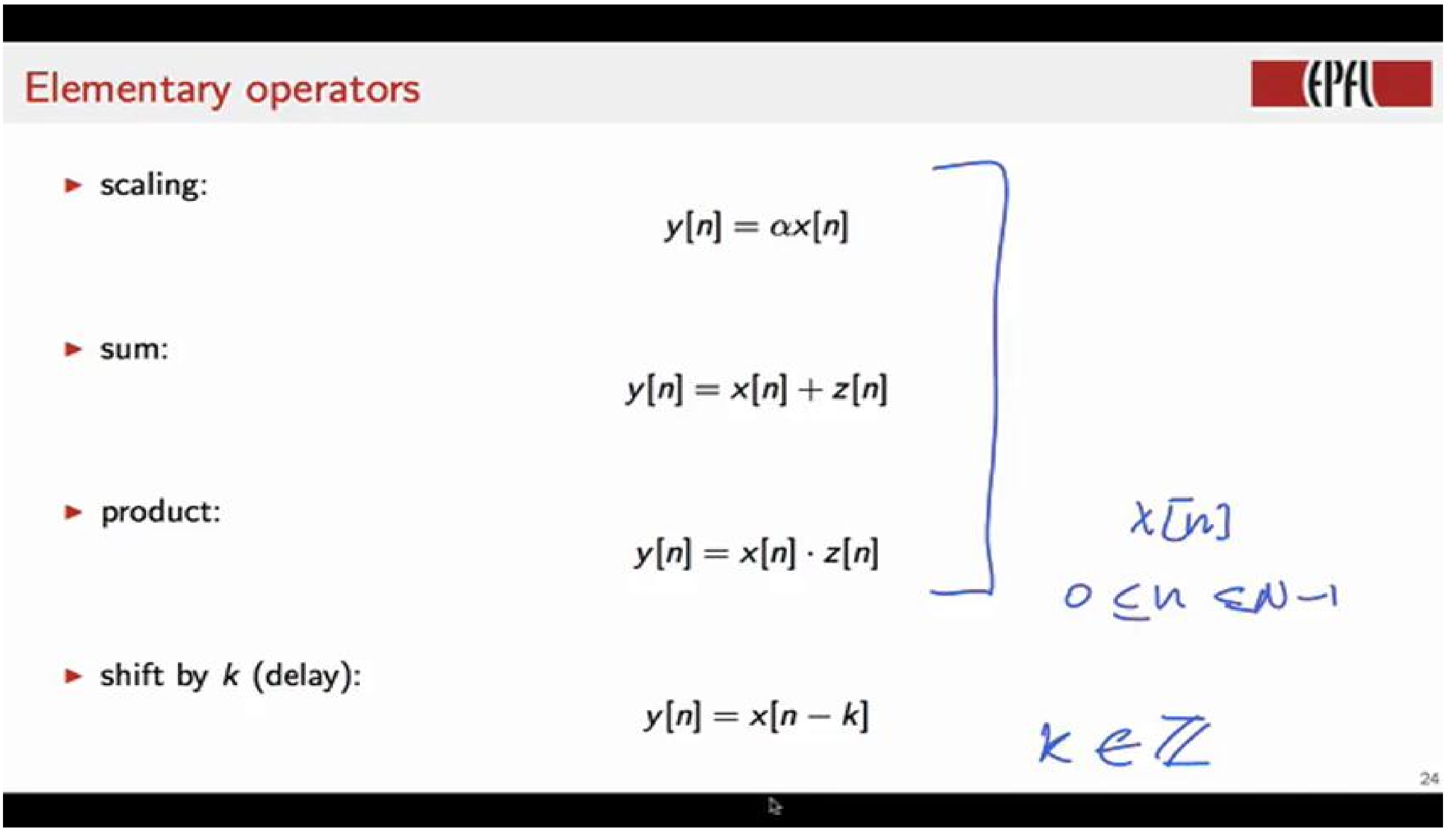
23

Finally, finite support signals are infinite-length sequences with only a finite number of non-zero samples. We will indicate this with the notation bar x, and that means that the support of the signal is compact. The amount of information of a finite support sequence is the same as a finite-length sequence of length N, and they constitute another bridge between finite and infinite-length sequences. In a way, we can always embed a finite-length sequence into an infinite-length sequence either by periodising the finite-length sequence, so turning that into a periodic signal, or by turning into a finite support signal by appending zeros before and after the interval 0, N minus 1.

Notes

Summary

11m 36s

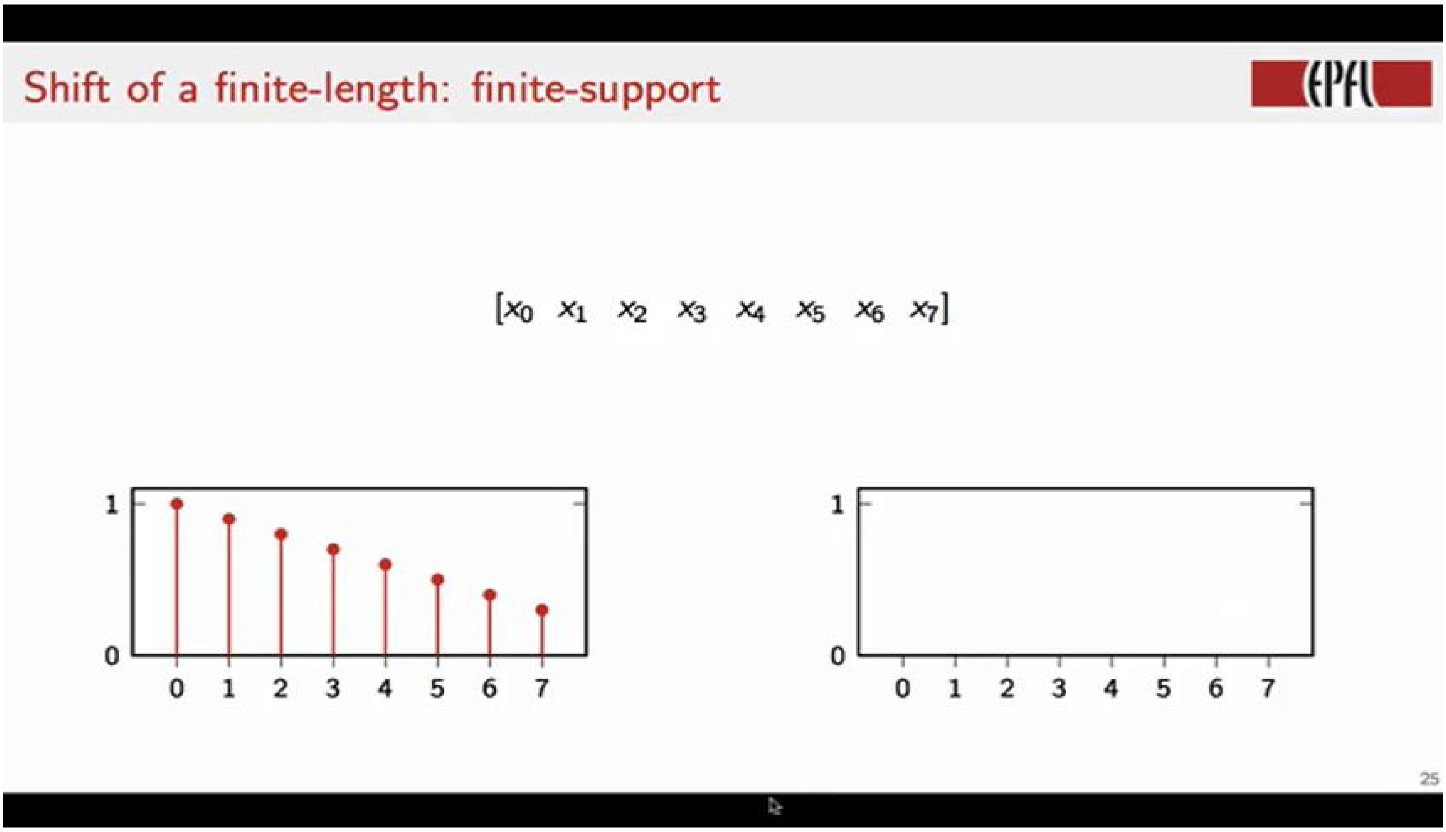


Elementary operators for signals include scaling where we take a sequence and we multiply each element in the sequence by a factor Alpha that belongs to the field of complex numbers. We can sum two signals together where we take a sequence and we add to each element of the sequence the corresponding element of the second sequence. The product is like the sum except that we multiply each element in the first sequence by the corresponding element in the second sequence. Finally, the shift by k, where we anticipate or delay a signal by shifting the sequence by an integer number of samples k, k belongs to Z. The definitions of the first three operators is valid for all classes of signals. In the case of the shift, however, we have to be careful when we apply a shift to a finite-length signal. Remember that for a finite-length signal, the index in x of n can only range between 0 and N minus 1. Now if we choose k too large or too small, we can easily send the argument here outside of the prescribed bounds. In order to apply a shift to a finite-length signal, we have to decide how to embed that signal into an infinite-length sequence.

Notes

Summary

12m 32s

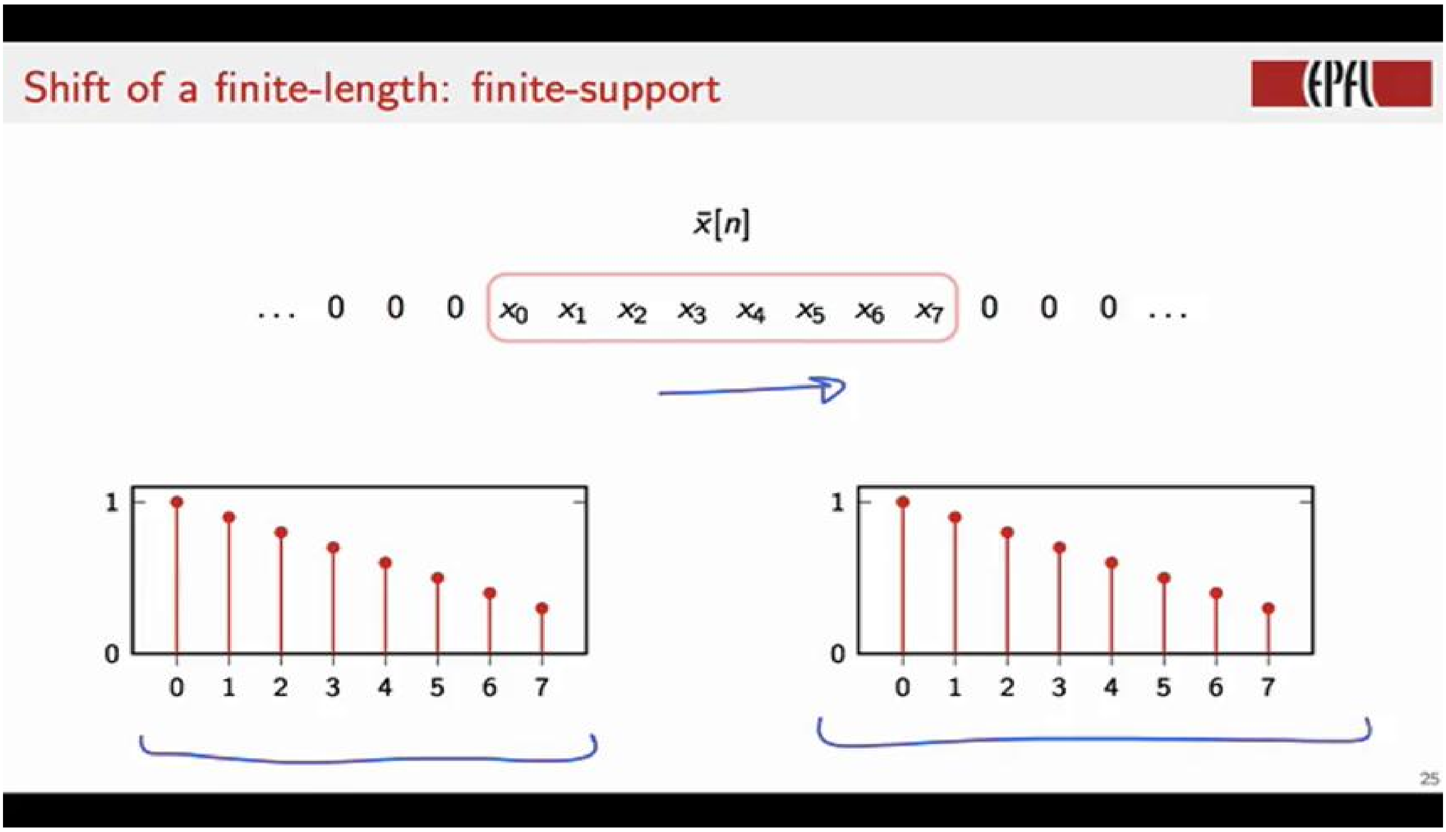


We have two types of shifts according to the embedding that we choose. Imagine we embed the finite-length signal into a finite support sequence.

Notes

Summary

13m 52s



In that case, it's as if we were appending and prepending zeros outside of the range of the signal. So in this case, when we shift the signal, say, suppose we're shifting towards the right, zeros will be pulled in into the range that is valid for the finite-length signal and we will lose the last points in the signal. Here graphically, we see what happens. Here's the original signal imagined embedded into a finite support signal and here is the result of the shift by 1, 2, 3, and so on and so forth.

Notes

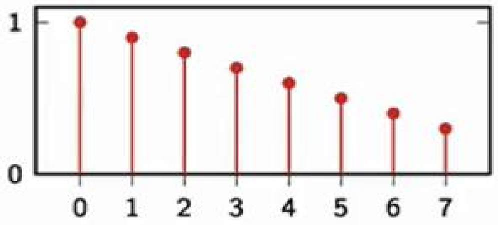
Summary

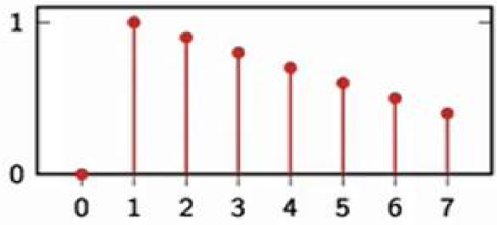
14m 00s

Shift of a finite-length: finite-support

$$\bar{x}[n-1]$$

$$\ldots \quad 0 \quad 0 \quad 0 \quad \boxed{0 \quad x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6} \quad x_7 \quad 0 \quad 0 \quad \ldots$$



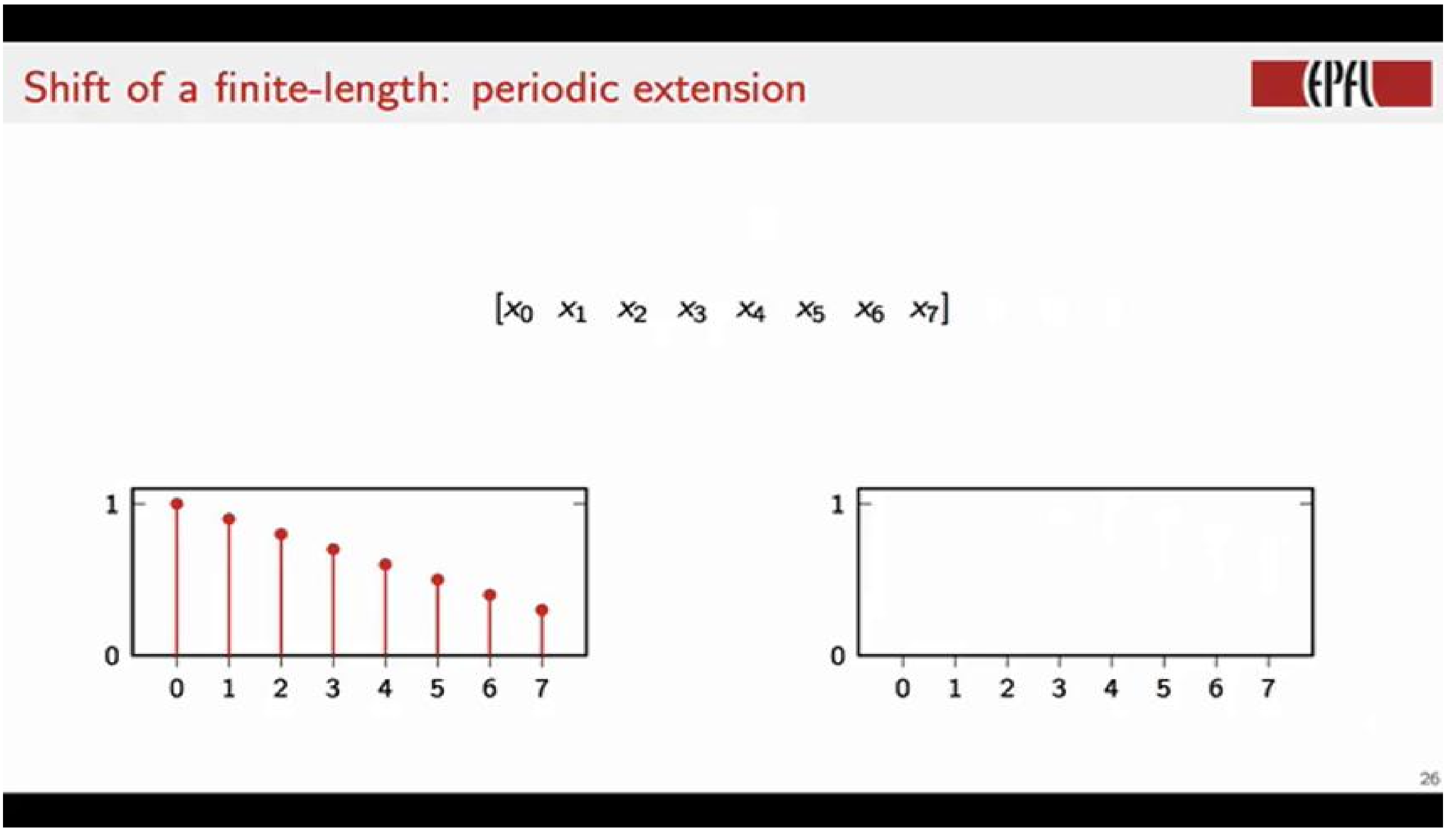


25

Notes

Summary

14m 33s

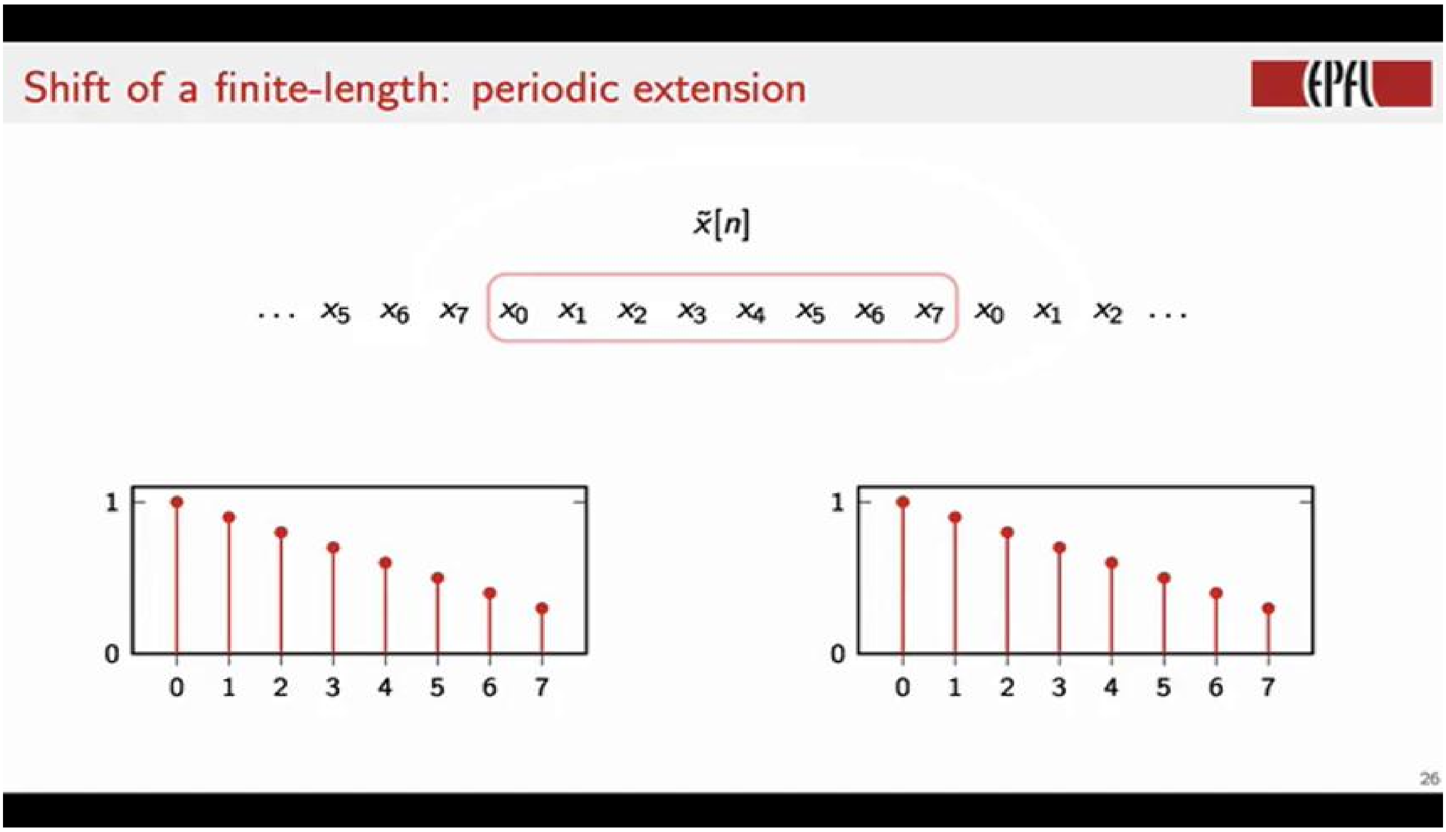


As we shift, we pull in zeros and we lose data. Conversely, if we imagine a periodic extension, a periodisation of the original sequence, the shift will become a circular shift.

Notes

Summary

14m 33s

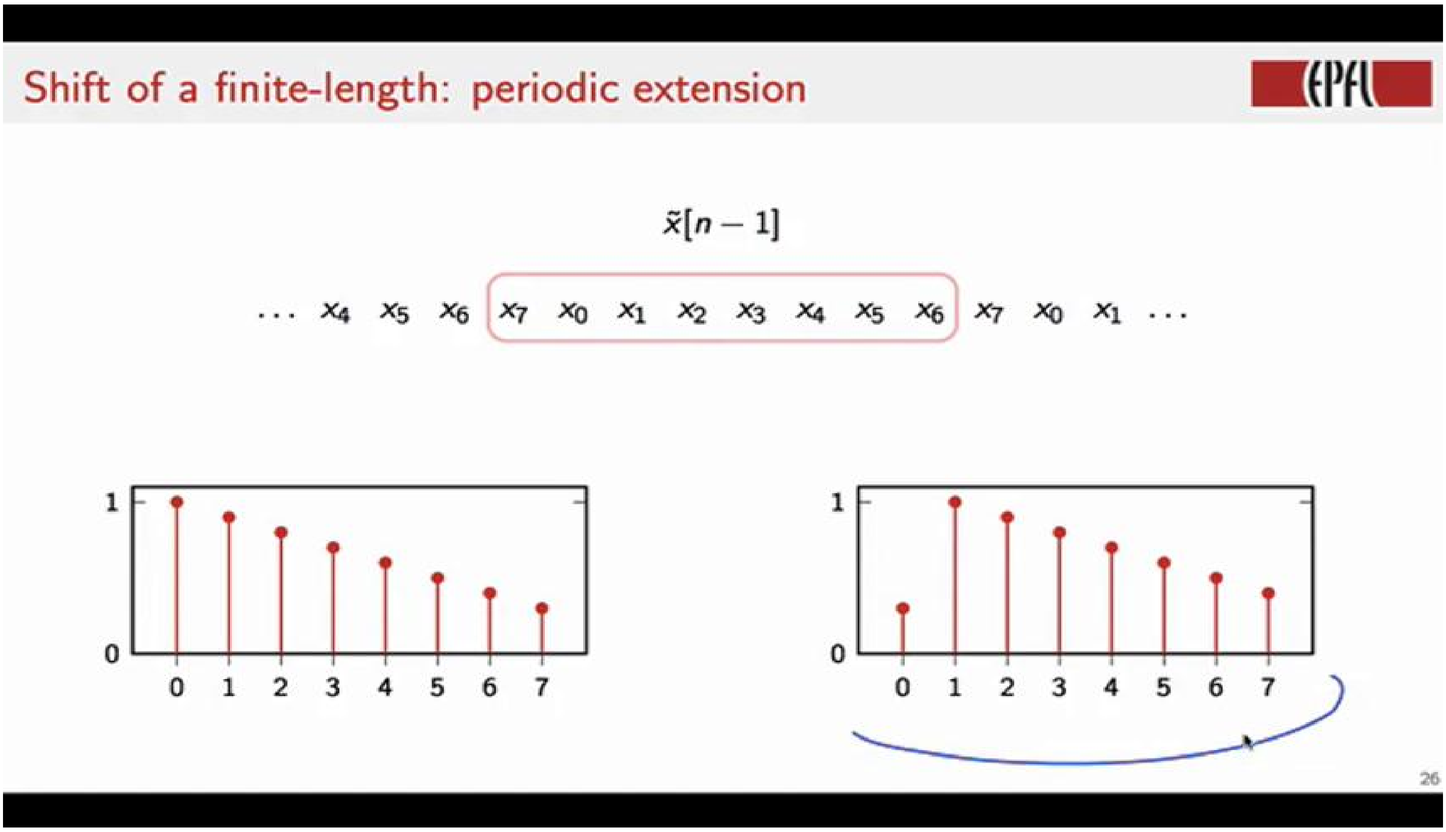


If we shift, say, towards the right, what goes out here will come back on the other side.

Notes

Summary

14m 45s

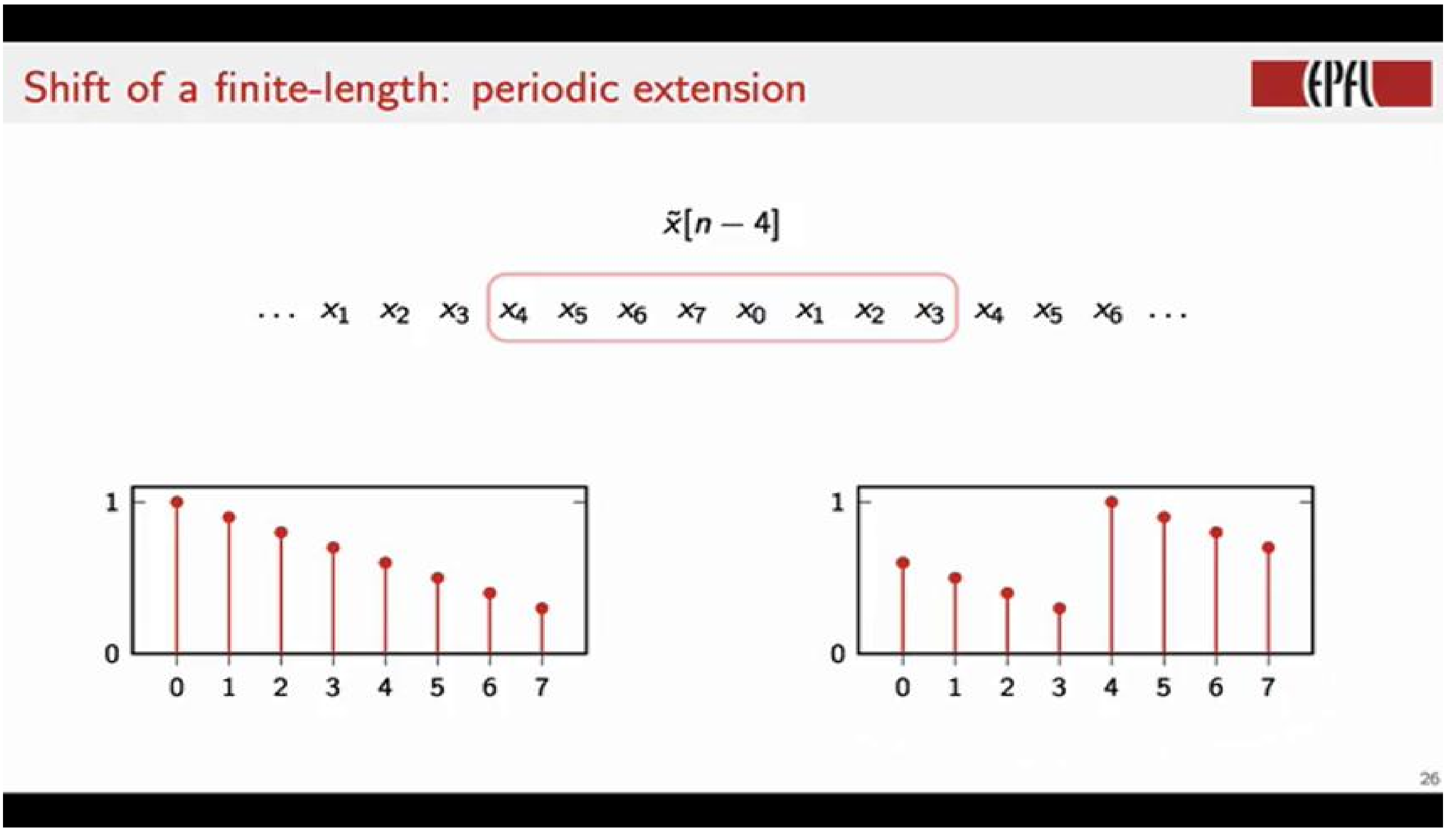


The result, as you can see here graphically, is that we're circulating the data around the support of the signal.

Notes

Summary

14m 51s



We will see later that the periodic extension, and therefore the circular shift, is actually the natural way to interpret the shift for a finite-length signal.

Notes

Summary

15m 00s

Energy and power

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$1\,\Omega$

$u[n]$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

27

We also have a definition of energy for a discrete-time signal. This is a sum for all elements in the sequence of the square magnitude of the elements. If you think of the signal's values as voltages across a 1-Ohm resistor then you can see that this definition of energy is consistent with the physical interpretation of energy. Many sequences have an infinite amount of energy, the unit step, for instance. If you do the sum, you will see that E_x goes to infinity. To describe the energetic properties of the sequences, we use the concept of power.

Notes

Summary

15m 09s

Energy and power

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P_x = \lim_{N\to\infty} \underline{\frac{1}{2N+1}} \underbrace{\sum_{n=-N}^{N} |x[n]|^2}$$

27

The power is the rate of production of energy for a sequence and it is defined as the limit for N that goes to infinity of the local energy computed over a window of size 2N plus 1 divided by the size of the window.

Notes

Summary

15m 49s

Energy and power: periodic signals

$$E_{\tilde{x}} = \infty$$

$$P_{\tilde{x}} \equiv \frac{1}{N} \sum_{n=0}^{N-1} |\tilde{x}[n]|^2$$

28

Take for instance periodic sequences. Periodic sequences have infinite energy because we're summing the values in one period an infinite number of times but their power, if you work it out, is equal to the energy over one period divided by the length of the period.

Notes

Summary

16m 07s