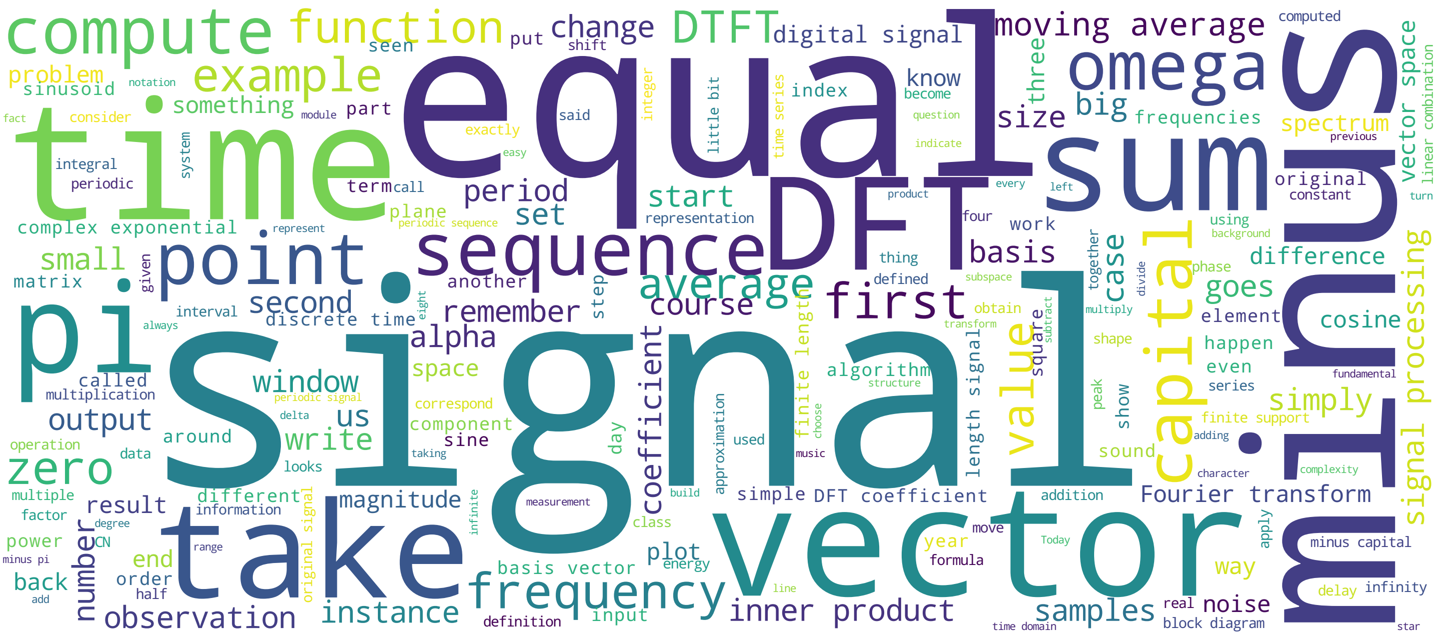
Digital Signal Processing

Signal of the Day: Goethe's Temperature Measurement



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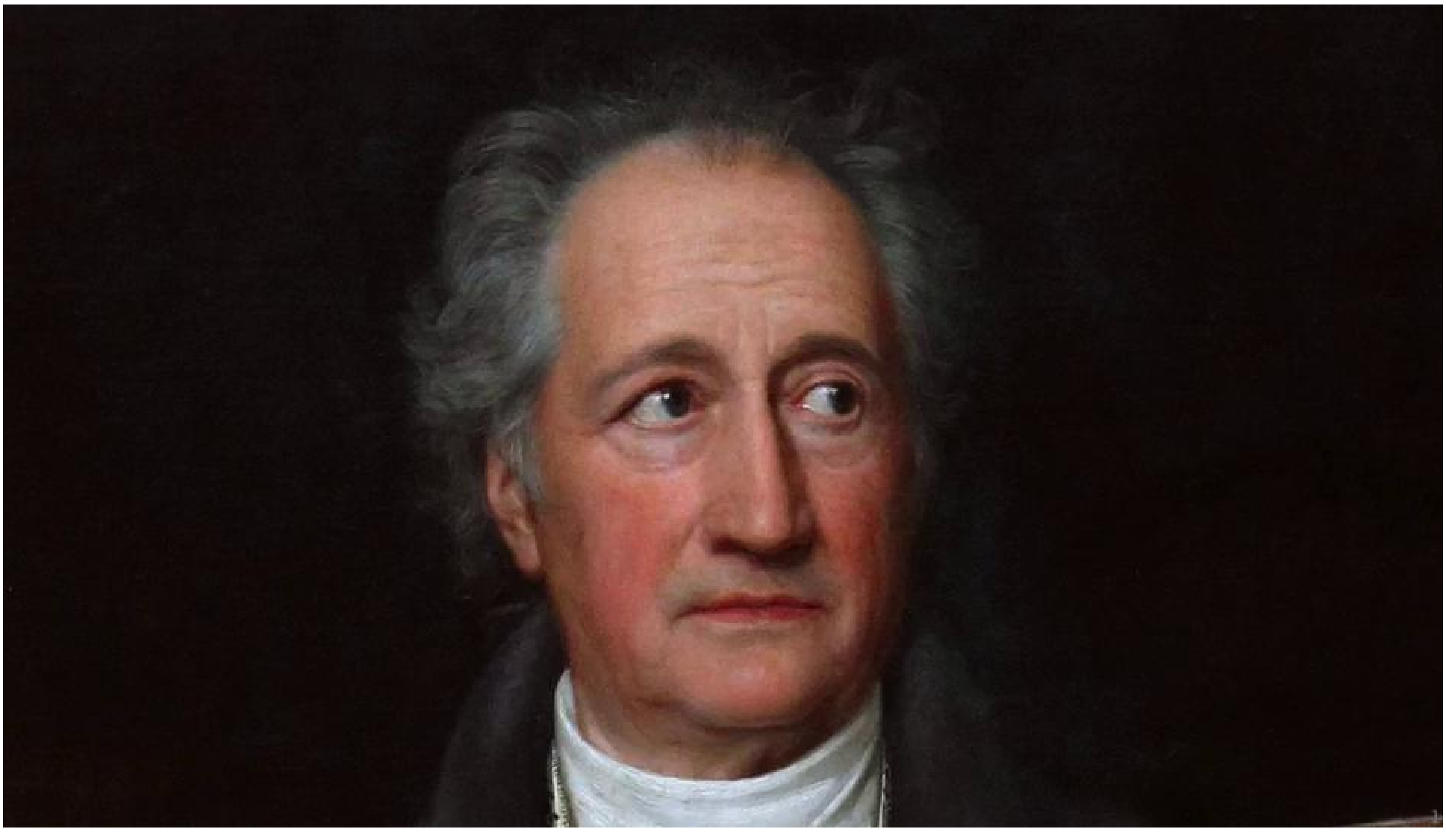
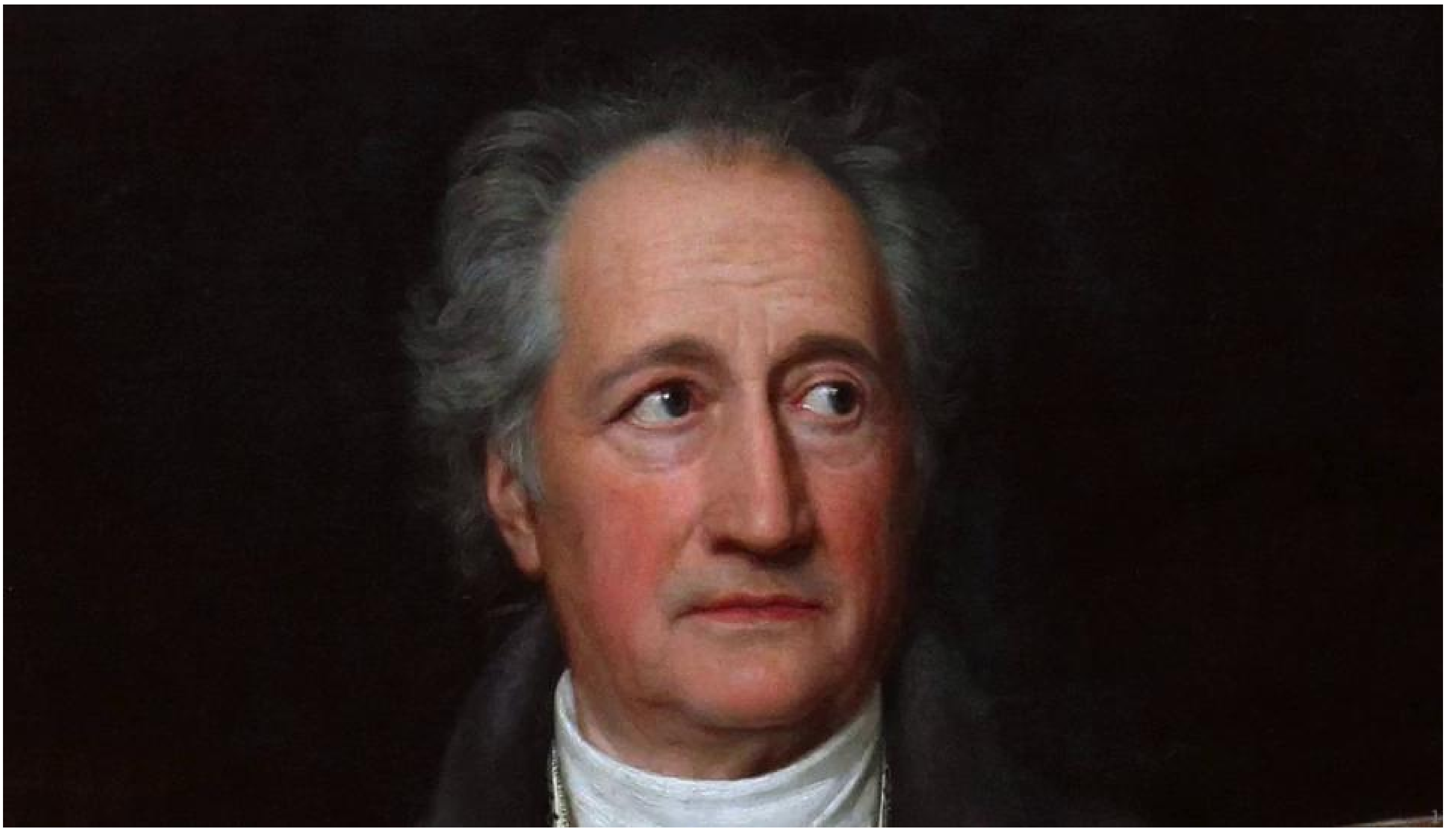


Every week, to complement the video lectures, we will have a look at an example of a particular signal that we call the signal of the day. Our aim is to show you cool, real life applications of signal processing. These signals come from a large variety of fields, environmental sciences, acoustics, image processing and more. Whatever your background is, there should be something for you. Moreover, the signal of the day is always related to the theory you're currently learning in the class, and thus illustrates how this sometimes complex theory could be put to work. We hope you enjoy these signals of the day as much as we had fun preparing them. Today, we are going to investigate the longest recorded signal of daily measurements. This signal consists of a series of daily mean temperature recorded in the city of Vienna in Germany. These measurements were originally initiated by one of the most famous German writers of all times, Johann Wolfgang Von Goethe. Although Goethe is nowadays better known for his writings, he had a keen interest in sciences, in particular meteorology. For example, he popularized the use of the so called Goethe's barometer, which measures atmospheric pressure.

Notes

Summary

0m 00s

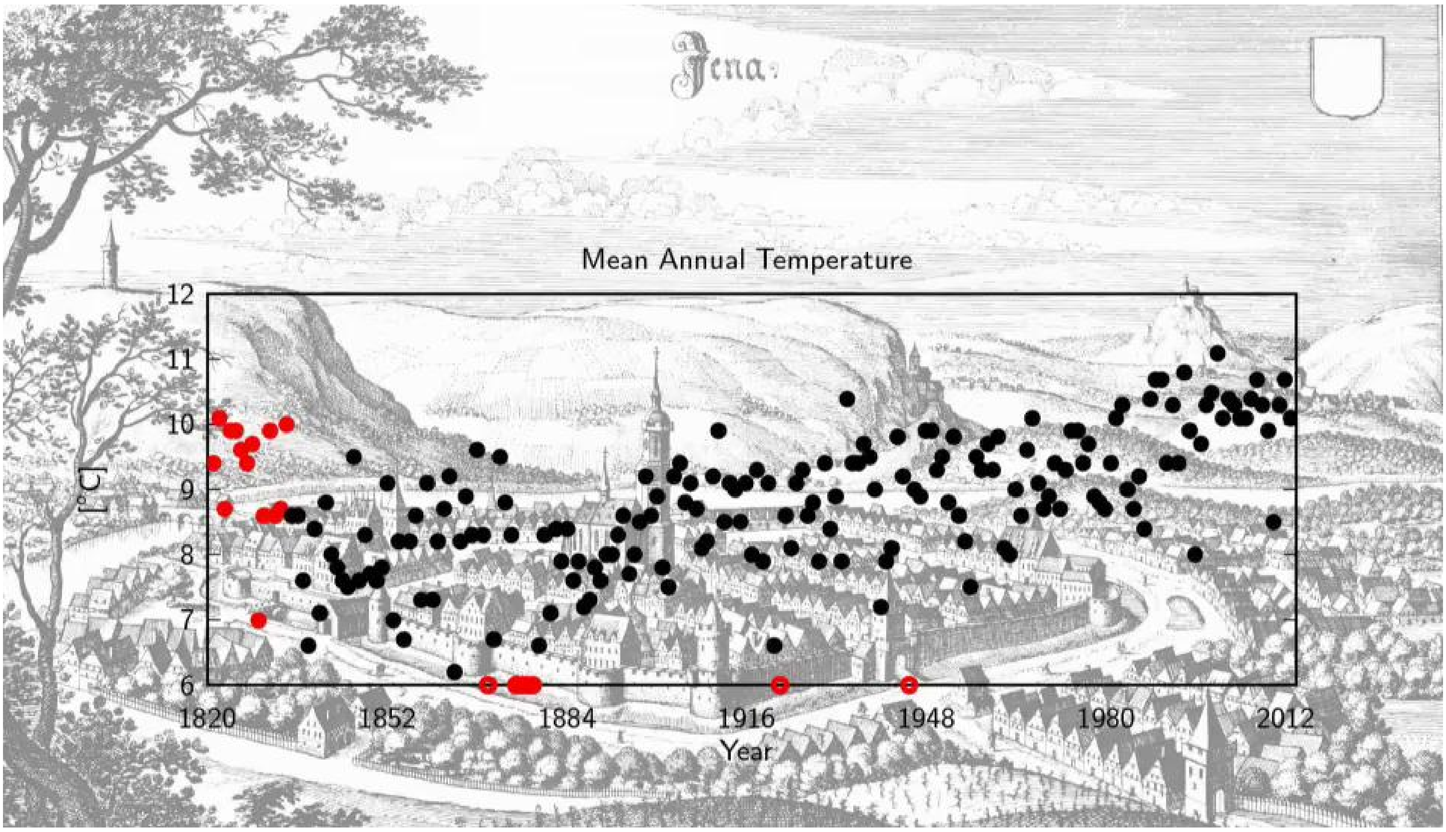


He also studied to consistently record the weather in his city Vienna, and thus can be seen as one of the first digital signal processing experts. Today, this heritage is preserved and extended by Professor Heimann at the Max Planck Institute, whom we would like to thank for providing us with the data.

Notes

Summary

1m 22s



Let us first have a look at the signal. In the background, you see the city of Vienna. In the foreground, you see a sequence of daily measurements. For convenience of representation, we have computed from the daily time series of observations the annual mean temperature. First, see how this signal really corresponds to our idea of a digital signal. It consists of a series of measurements taken at regular intervals that we call samples. Moreover, each sample has a discrete amplitude, which is related to the precision of the thermometer that was used. These are the two characteristics of digital signals. Furthermore, there are periods in the course of history where the signal was not regularly recorded, for example, in 1870 or 1945. We will simply ignore these missing points in our computation. Moreover, in the period from 1821 to 1834, the overall amplitude of the signal is somehow higher, which was corrected by a change of equipment in 1835. Therefore, in all our further processing, we will simply ignore that early period.

Notes

Summary

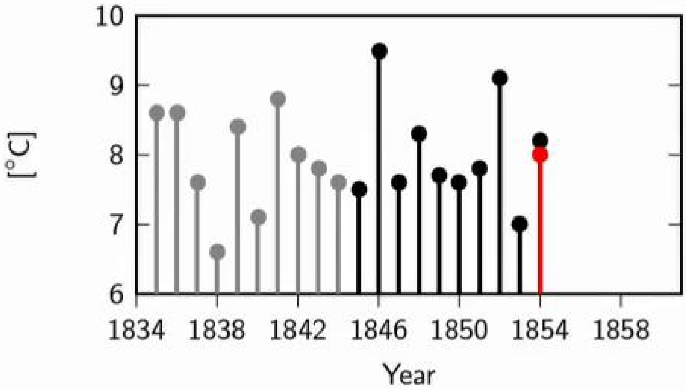
1m 45s

Smoothing the Time-Series

- Moving average :

$$y[n] = \frac{1}{N} \sum_{m=0}^{N-1} x(n-m)$$

N: window of last observations over which average is computed



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We are going to apply a very simple processing method to this signal. Compute a moving average. At any time, small n, it is simply the average of the last capital N observations, where capital N is the window of observation over which the average is computed. The result is a smoothed version of the original signal as differences are averaged out by this operation. Let us have a look at how this works in practice. Consider a smaller portion of the signal depicted here. We would like to compute the moving average in 1854. Let us take arbitrarily a window of capital N is equal to 10 observations which are highlighted in black. To compute the moving average in 1854, we simply take the average of these 10 samples. You can see the average is slightly smaller than the actual value in 1854.

Notes

Summary

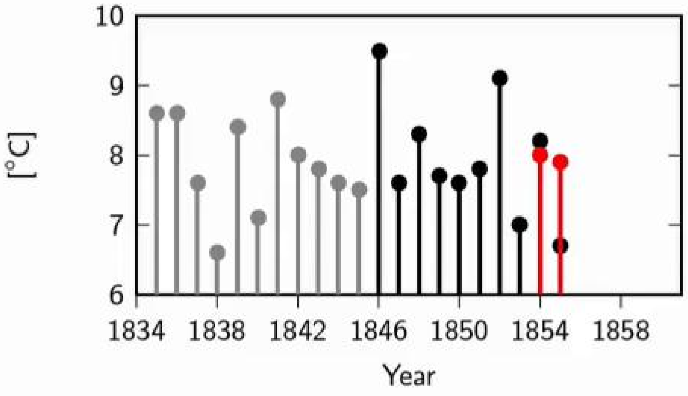
3m 03s

Smoothing the Time-Series

- Moving average :

$$y[n] = \frac{1}{N}\sum_{m=0}^{N-1} x(n-m)$$

N: window of last observations over which average is computed



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In 1855, we make a new observation. We slide the window of observation by removing the oldest observation in 1845 and adding the most recent observation in 1855. Then we compute the moving average in 1855 by taking the average of samples over these observations. Again, we plot the resulting value in red. This process carries on until we have reached the end of the signal.

Notes

Summary

4m 07s

$$\boxed{y[n] = \frac{1}{N}\sum_{m=0}^{N-1} x(n-m)}$$

$$\frac{1}{N}\sum_{m=1}^{N} x(n-m) = y(n-1)$$

$$= \frac{1}{N}x[n] + \underbrace{\frac{1}{N}\sum_{m=1}^{N-1} x(n-m) + \frac{1}{N}x[n-N]}_{y[n-1]} - \frac{1}{N}x[n-N]$$

$$= y[n-1] + \frac{1}{N}\left(x[n] - x[n-N]\right)$$

We can devise a recursive procedure to compute this moving average. Let us start with the definition of the moving average here in the first equation. This is a sum from N is equal to 0 to capital N minus 1 of x N minus M. The entire sum divided by capital N to compute the average. Now we take out the first term of this sum here. That's the term for M is equal to 0. What is left is the sum from M is equal to 1 to capital N minus 1. Now we do something that looks silly. We add x N minus capital N and we subtract it right away. So the sum of these two is 0, so nothing has changed. But then we recognize that adding this changes this sum into the sum M is equal to 1 to capital N of x N minus M of course, divided by capital N. This entire sum here is simply y of small n minus 1. This is what we write here. We have three terms. We have y N minus 1. We have this character here, this character, and we have a new way to write this recursive averaging.

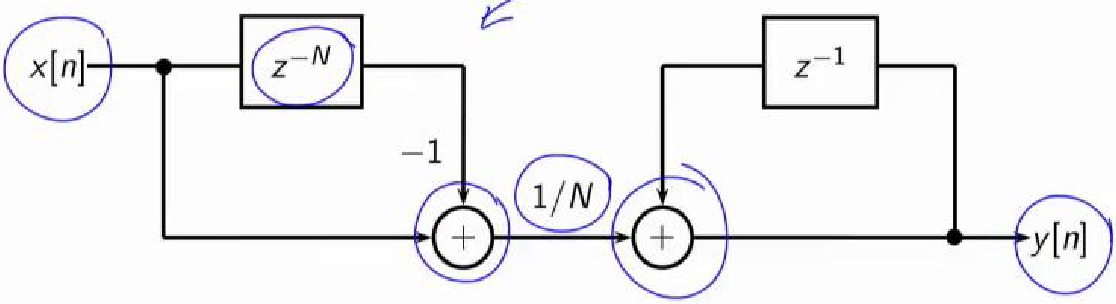
Notes

Summary

4m 44s

A Recursive Method (Ctd.)

$$y(n) = \boxed{y(n-1)} + \boxed{1/N\,(x(n) - x(n-N))}$$

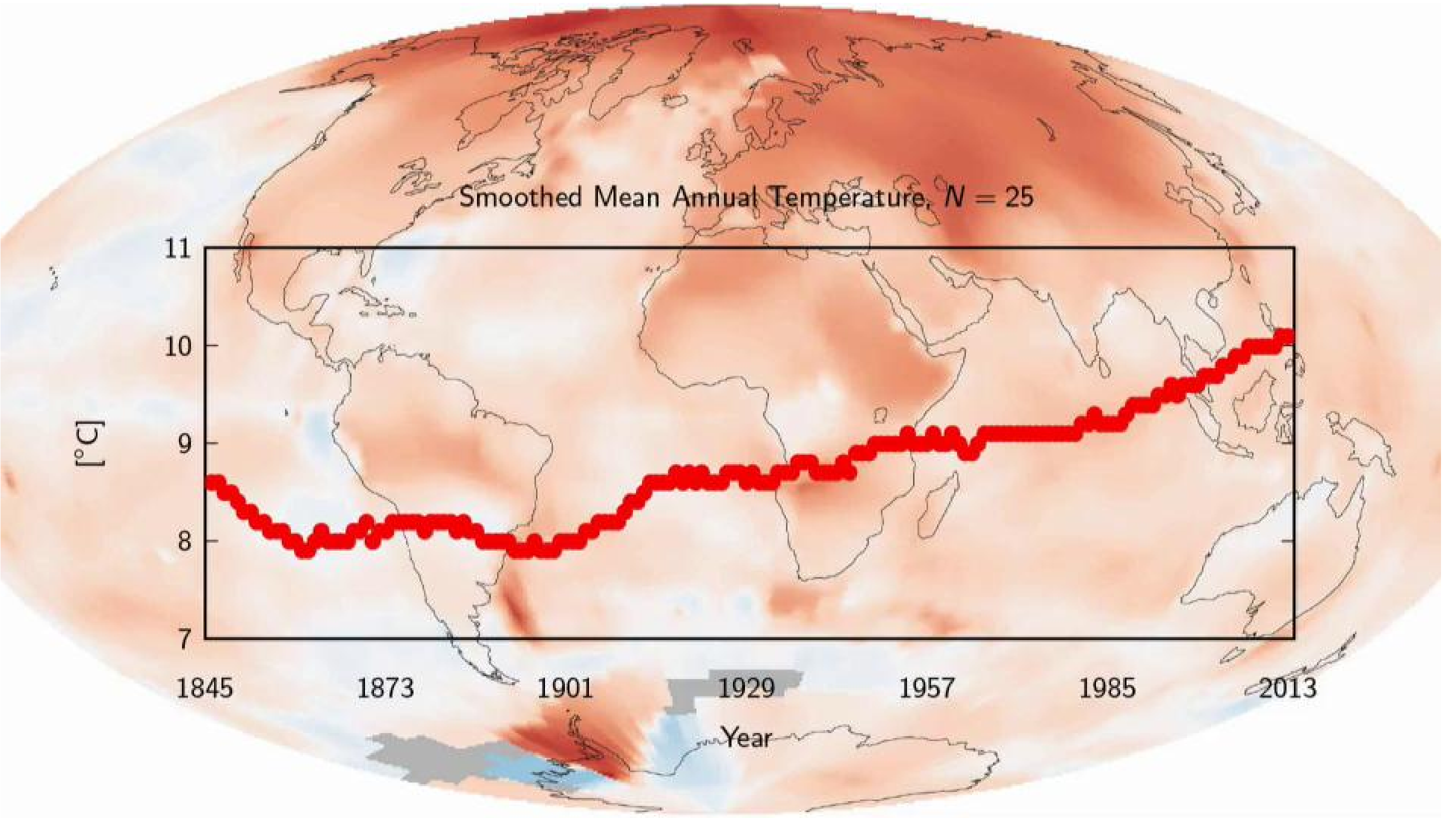


We can represent this procedure using a block diagram. Let's write the equation again. We said y N is equal to, y N minus 1 plus 1 over N x N minus x small n minus capital N. This part here we can implement like this. We take x N, we delay it by z N minus 1. That's a delay term by capital N samples. We subtract it from x N. This is this part. Then we scale it by y O N. Finally, y N, the output is the previous output, which is y of N minus 1 added to what just came out from the x N minus capital N, which is done here. We had seen block diagrams like this in the Karplers strong example where we had actually similar operations going on. At the same time, we're going to spend more time later in the class to study such block diagrams, so don't get too worried if they look a little bit too complex at this point in time.

Notes

Summary

6m 07s



Let us now apply this moving average to our signal with a window of 25 years to reveal long term trends. We observe a clear increase in temperature over the course of time, especially in the second part of the 20th century. This is a perfect illustration of the effect of global warming. The only question is where does global warming come from? This is a subject of hot, indeed, hot political debate. What I want to say here is that the moving average is a very simple tool, but it's used all over. For example, in finance to average out small variations in financial time series, but also in environmental sensing like this temperature example, or in other application s of signal processing to various areas of science and engineering.

Notes

Summary

7m 38s