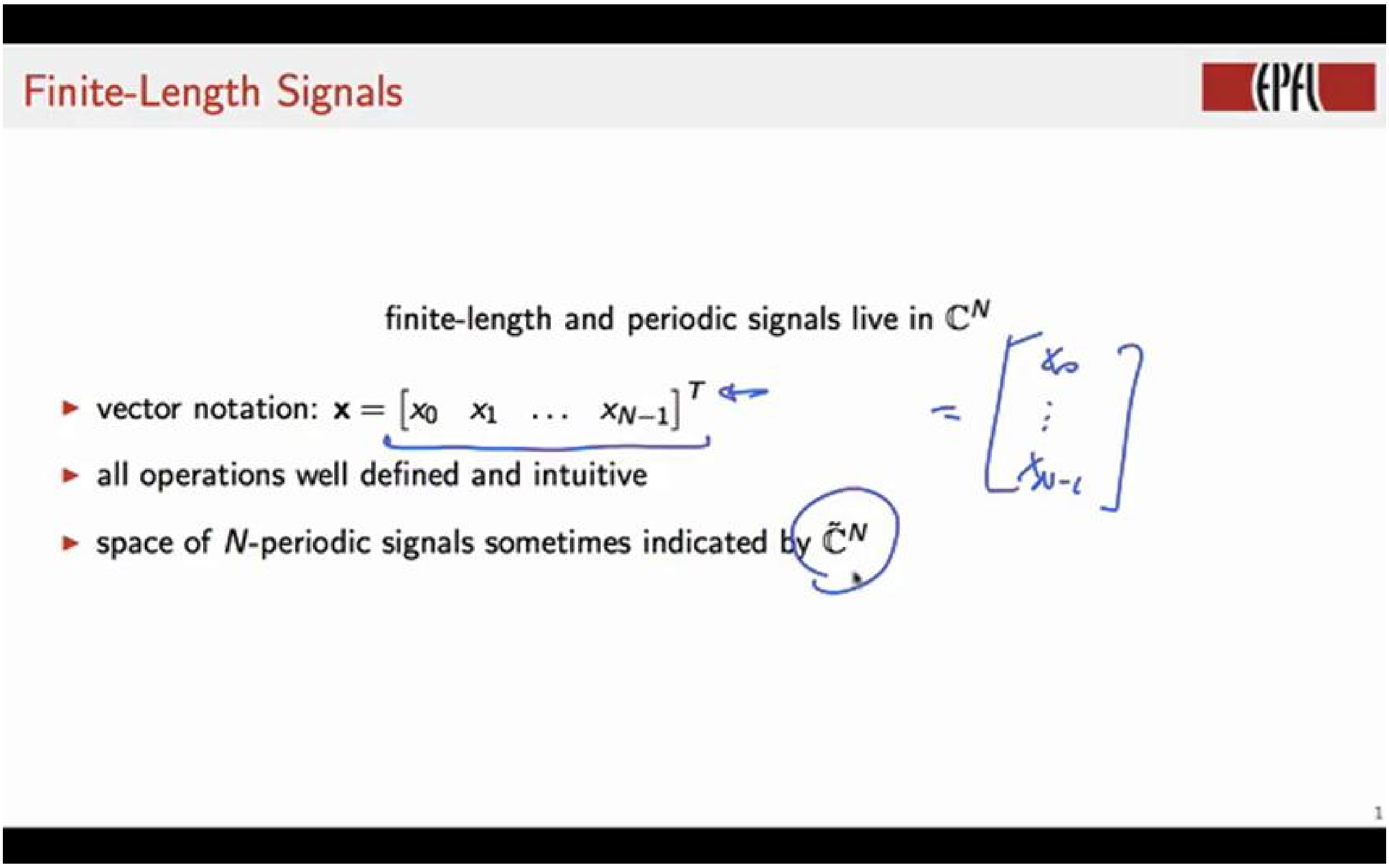

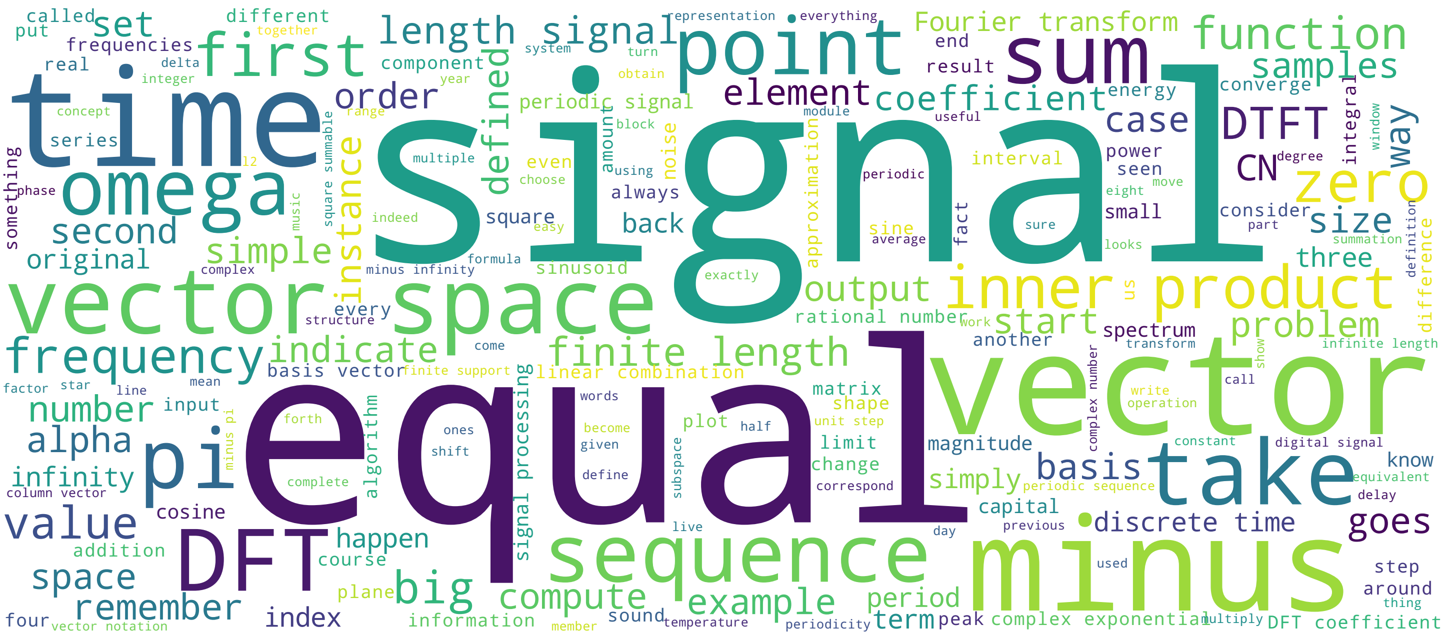

Search MOOC

Video

Inner product for signals

$$\underbrace{\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{n=0}^{N-1} x^*[n]y[n]}$$

well defined for all finite-length vectors (i.e. vectors in $\mathbb{C}^N$)

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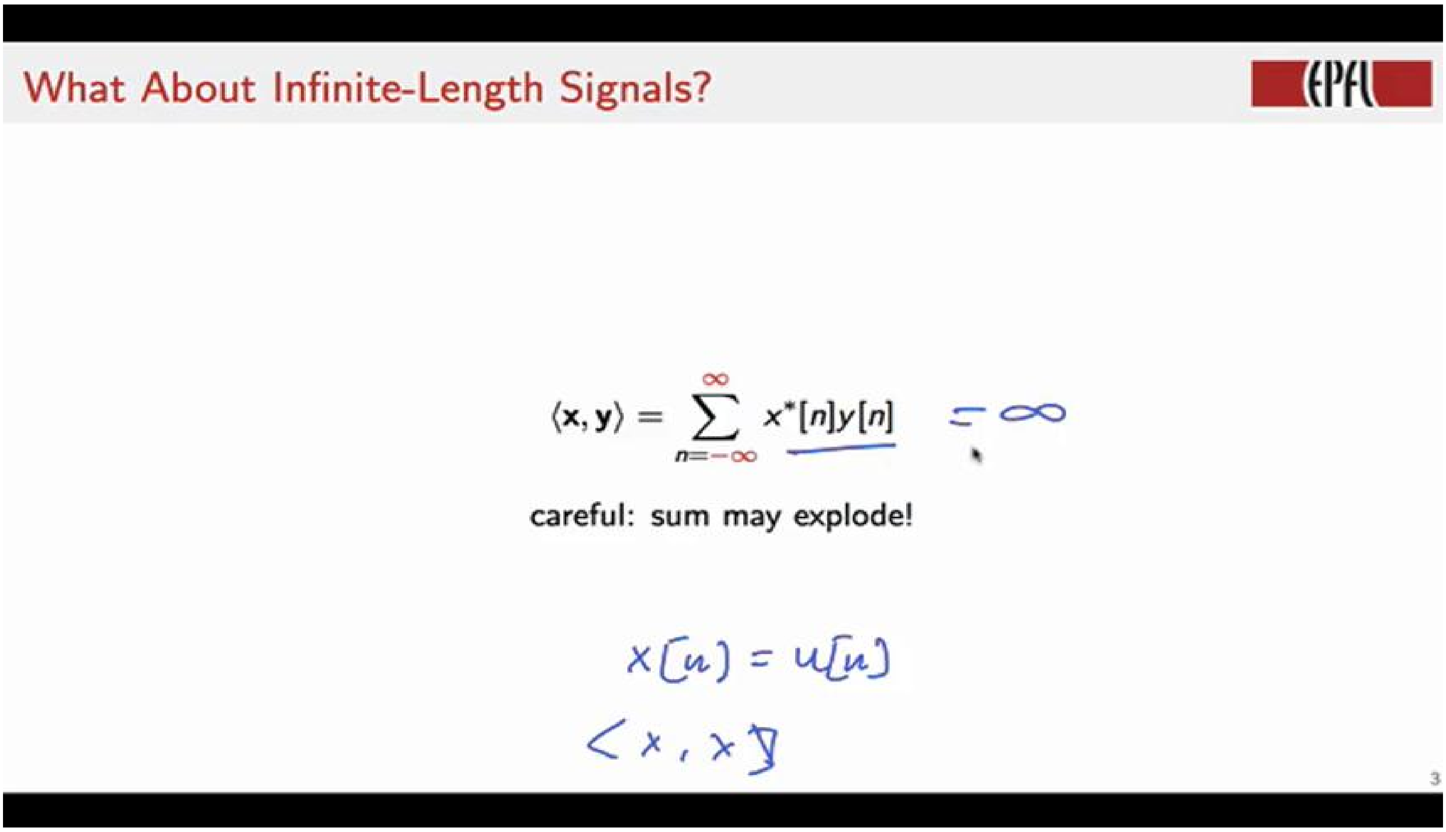
So how are vector spaces going to help us in dealing with discrete time signals? Well, by now it should be self-evident that finite-length signals live in CN. CN is a space of length and tuples of complex numbers in vector notation. Therefore, we will indicate a finite-length signal by the column vector x0, x1 up to x big N minus 1. The transpose operator here indicates that this is a column vector, so it's actually like so. All operations are well-defined and intuitive. We have addition and scholar multiplications defined for CN. The space of n periodic signals is also equal to CN because the amount of information of a periodic signal is equivalent to the amount of information of a finite-length signal of length N. Sometimes, to stress explicitly the periodicity of the signal, we will indicate the vector space of periodic sequence with the notation tilde C of N like so. In CN, the inner product is defined like this. It's the sum for small n that goes from zero to big N, minus one of the conjugate of the elements of the first tuple times the corresponding element of the second tuple. This is well defined for all finite-length vectors. So all vectors that live in CN and therefore all finite-length and periodic signals.

Notes

Summary

0m 00s



What about infinite length signals? We could extend the concept of CN to c infinity, but we would run into a problem. The inner products defined as now an infinite summation for an index that goes from minus infinity to plus infinity may not explode even for simple signals as x of n equal to the unit step. If you take the self inner product of this, you would get an infinite number of ones here in the summation and this would be equal to infinity.

Notes

Summary

1m 35s

What About Infinite-Length Signals?

$$\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{n=-\infty}^{\infty} x^*[n]y[n]$$

We require sequences to be *square-summable*: $\sum |x[n]|^2 < \infty$

Space of square-summable sequences: $\ell_2(\mathbb{Z})$

3

In order to make sure that this doesn't happen, in order to make sure that the inner product is always well-defined, we will define a vector space for infinite-length sequences by requiring that all sequences in this vector space are square-summable. Namely, the sum of the squares of the elements in the sequence is less than infinity. If you remember the definition of energy of a signal, this is equivalent to requiring that all sequences that live in this vector space have finite energy. This is the space of square summable sequences and we indicate this with the notation l2 of z.

Notes

Summary

2m 07s

Infinite-Length Signals

"well-behaved" infinite-length signals live in $\ell_2(\mathbb{Z})$

- vector notation: $\mathbf{x} = \left[\ldots \quad x_{-2} \quad x_{-1} \quad x_0 \quad x_1 \quad x_2 \ldots\right]^T$
- many interesting signals not in $\ell_2(\mathbb{Z})$ unfortunately ($x[n] = 1, x[n] = \cos(\omega n)$, etc)

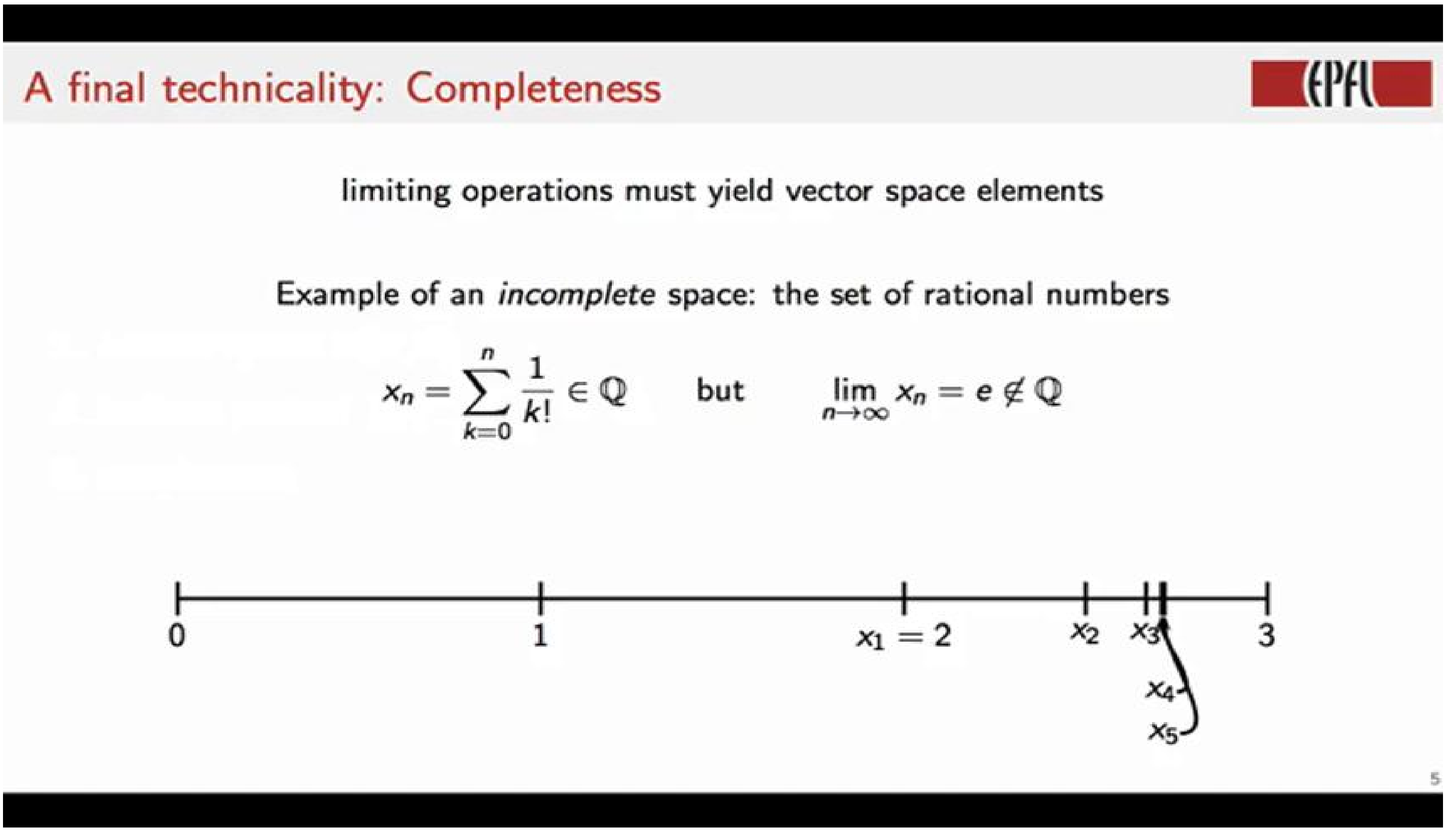
4

In l2 of z, everything is well. We can still use the same vector notation that we used for finite-length signals, so we now have an infinite column vector. However, many interesting signals are not in l2 of z, the unit step, the constant, sinusoidal functions, and so on and so forth. We will learn how to deal with the [inaudible 00:03:07] sequences without losing the vector formalism in due course.

Notes

Summary

2m 47s

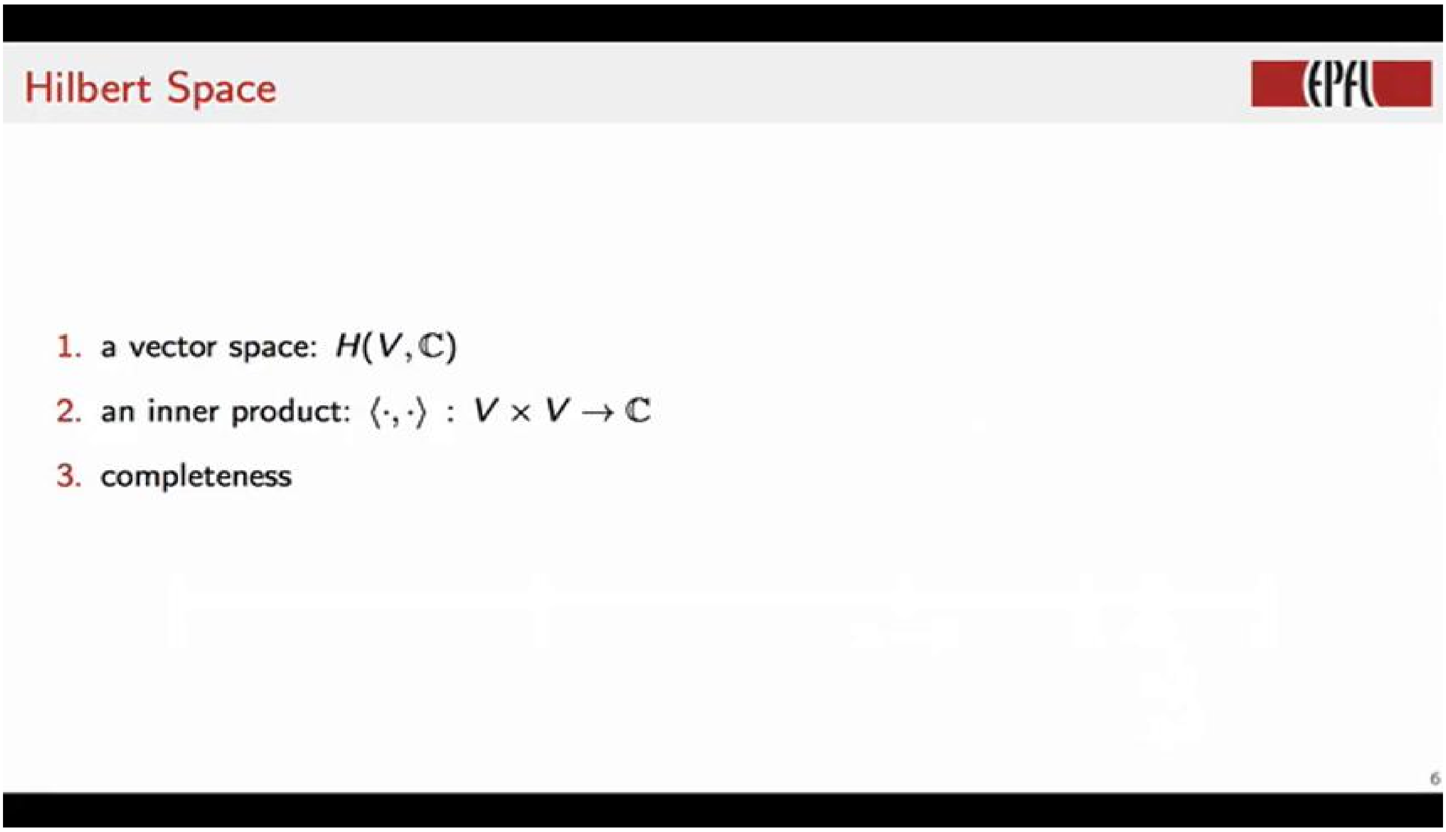


Finally, at last technicality. We have defined vector spaces in such a way that the standard operations, scalar, multiplication and addition do not take us outside of the vector space. In other words, any linear combination of vectors is a member of the vector space. Now, if we have an infinite sequence of vectors in the vector space that converges to a limit, we want this limit to be in the vector space as well. If a vector space is closed under the limiting operation, we say that the vector space is complete. Now, this is a technical requirement that is never really used explicitly in practical signal processing, but will be useful to prove some fundamental results, such as the sampling theorem. It's hard to come up with a simple example of a noncomplete vector space, but to give you an idea of what completeness entail, you can consider, for instance, the set of all rational numbers. Q, we can build converging sequences of rational numbers that do not converge to a rational number themselves. Famously, the series for the exponential function which is written here converges to the number e, in spite of fact that each member of the series is a rational number.

Notes

Summary

3m 12s



When a vector space equipped with an inner product is also complete, we call the vector space a Hilbert space.

Notes

Summary

4m 26s