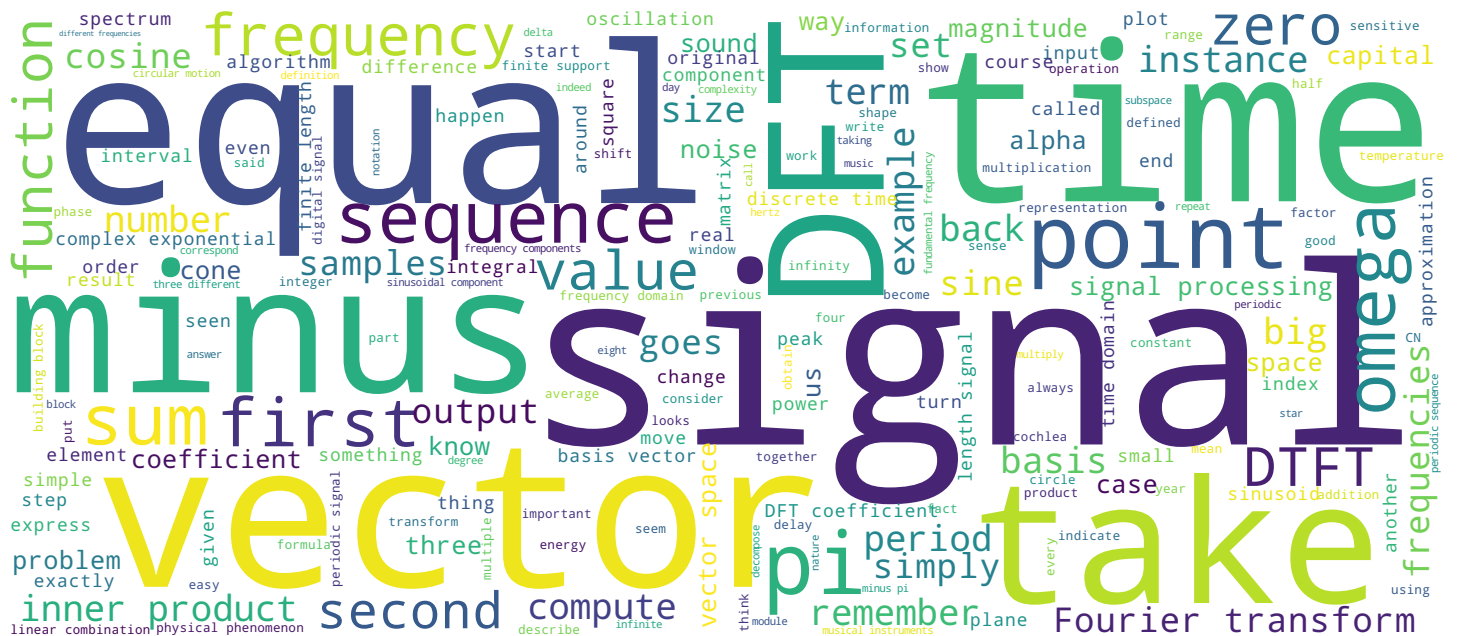




EPFL

- ▶ sustainable dynamic systems exhibit oscillatory behavior
- ▶ intuitively: things that don't move in circles can't last:
 - bombs
 - rockets
 - human beings...

In a nutshell, the goal of Fourier analysis and signal processing is to be able to express a signal in terms of basic sinusoidal components, in terms of sines and cosines. Now, before we tackle the mathematical details of such a transformation, it is important to ask ourselves why we choose sinusoidal components as the building blocks of our transform. Now, remember that the signal is, as we said, the description of a physical phenomenon evolving in time. It turns out that oscillations are really the fundamental pulse of nature, so to speak. These are some examples. If you think about your heart beating, you see that it has a periodic pattern that repeats. A train, for instance, would have an engine that makes wheels turn in a circular motion. Waves are another example of periodic ebb and flow that can be modelled in a sinusoidal fashion. Of course, musical instruments generate sound by vibrating at a certain fundamental frequency. It turns out that any sustainable dynamic system exhibits an oscillatory behaviour. Intuitively, it's rather easy to see that things that don't move in circles will sooner or later hit a dead end. Take bombs. An explosion is a clearly non-reversible phenomenon.

Notes

Summary



0m 00s

- ▶ sustainable dynamic systems exhibit oscillatory behavior
- ▶ intuitively: things that don't move in circles can't last:
 - bombs
 - rockets
 - human beings...

It's impossible to bring the energy back, and so it's something that ends very quickly and very disruptively. Rockets, they burn fuel in a one-directional way and have to be jettisoned after they reach orbit. Human beings evolve, unfortunately, in just one direction towards old age. It would be nice if we could circle back to infancy, but that's a metaphysical digression that we will not take at this point.

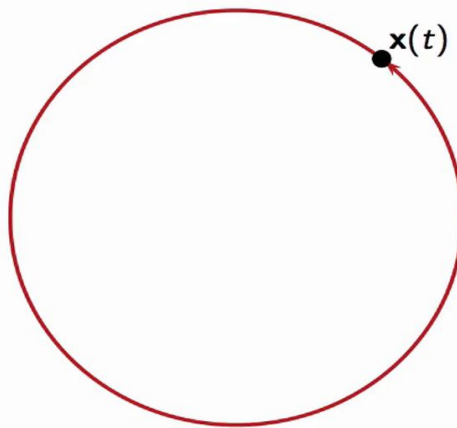
Notes

Summary



1m 19s

Period P
Frequency $f = 1/P$



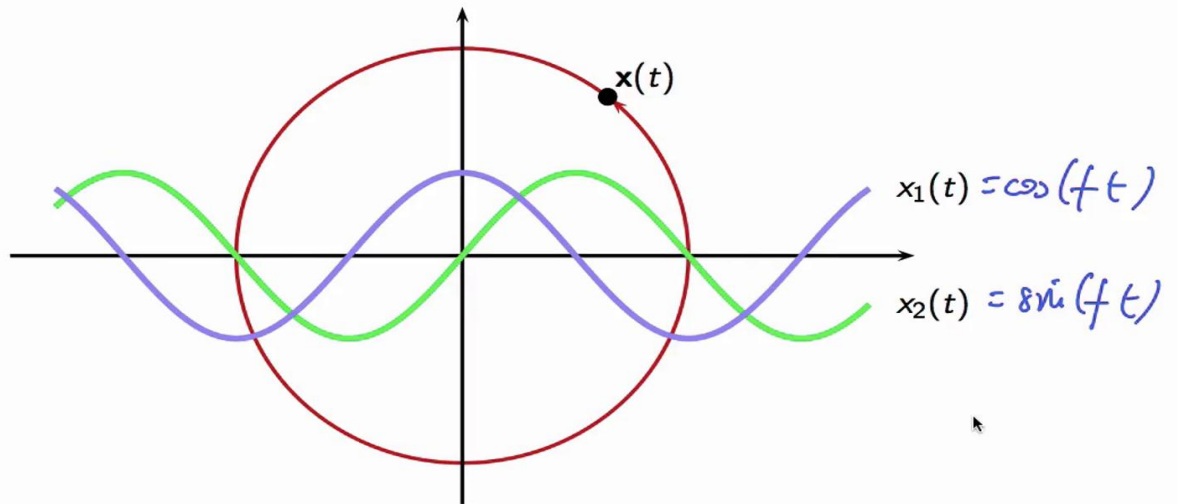
Oscillations are very easy to describe and parameterise. Let's take the very simple case of a two-dimensional circular motion. If you have a point on the plane and this point starts moving in circles, you can characterise this motion by taking its period P , which is the number of seconds it takes for this point to complete the full revolution, or equivalently, by its frequency f , which is just the reciprocal of the period and which indicates how many revolutions per second the point performs.

Notes

Summary



1m 45s



If you put coordinate systems around the centre of this oscillation, then you can express the instantaneous coordinates of this point on the plane via the very familiar sine and cosine functions. The DSP will be simply the cosine of ft and the ordinate will be the sine of ft , where f is the frequency of the oscillation.

Notes

Summary



2m 16s

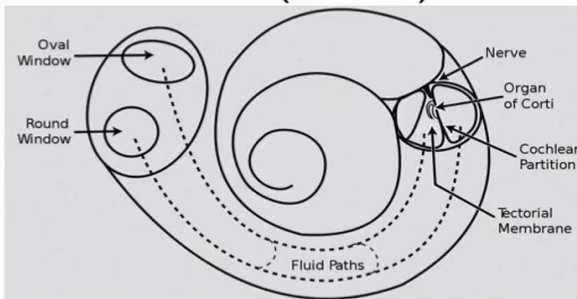
You too can detect sinusoids!



U. Kishin

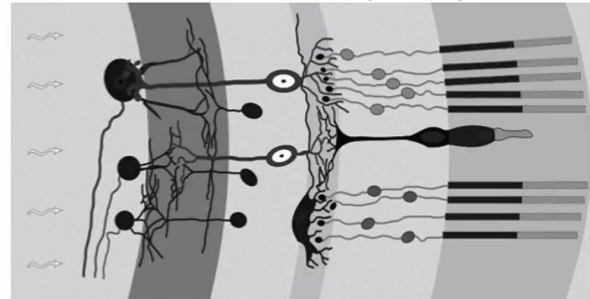
the human body has two receptors for sinusoidal signals:

cochlea (inner ear)



- ▶ air pressure sinusoids
- ▶ frequencies from 20Hz to 20KHz

rods and cones (retina)



- ▶ electromagnetic sinusoids
- ▶ frequencies from 430THz to 790THz

We claim that sinusoids are the building blocks of nature. Indeed, to drive the point home even more effectively, I will remind you that you yourself and your body have two specialised sinusoidal detectors: the cochlea in your inner ear that detects air pressure sinusoids at frequencies from 20 hertz to 20 kilohertz, and you have rods and cones in your retina in your eye that detect electromagnetic sinusoids with frequency from 430 terahertz to 790 terahertz. This is the range of frequencies for light in the visible spectrum. Very briefly, the cochlea is a snail-like structure inside your ear. But if you unroll it, it's like a cone, like this. Along this cone, you have hundreds of hair cells, and each hair cell will react to a specific frequency in the sounds that you hear. In a sense, your cochlea is performing an instantaneous Fourier transform—as we will see shortly—of the sound waves that are hitting your eardrum. In the eye, you have three types of cones. One is sensitive to red and one is sensitive to green. Another is sensitive to blue. So three different frequencies in the light spectrum and different colour sensations are given by different amounts of energy captured by the three different types of cones as processed by the brain.

Notes

Summary



2m 40s

- ▶ humans analyze complex signals (audio, images) in terms of their sinusoidal components
- ▶ we can build instruments that “resonate” at one or multiple frequencies (tuning fork vs piano)
- ▶ the “frequency domain” seems to be as important as the time domain

To go back to our original intuition, it seems that, for instance, humans analyse complex signals in terms of the frequency components. We can build instruments that resonate at given frequencies: musical instruments, tuning forks. Although, as we say, signals seem to be the evolution of a physical phenomenon in time, and very often we describe them as such, there seems to be a different domain, the domain of frequencies, that is just as good for describing physical phenomena as the time domain.

Notes

Summary



4m 00s

can we decompose any signal into sinusoidal elements?

yes, and Fourier showed us how to do it *exactly*!

analysis

- ▶ from time domain to frequency domain
- ▶ find the contribution of different frequencies
- ▶ discover “hidden” signal properties

synthesis

- ▶ from frequency domain to time domain
- ▶ create signals with known frequency content
- ▶ fit signals to specific frequency regions

The fundamental question is, can we decompose any signal into sinusoidal elements? The answer is yes. Fourier showed us how to do it exactly. From the analysis point of view, what we will do is move from the time domain to the frequency domain. We will decompose the signal into different frequencies that constitute the signal. By doing so, we will be able to discover hidden signal properties. The converse of this process is going back from the frequency domain to the time domain to synthesise a signal from its frequency components. We can use that to generate signals that have a known frequency content, and we can use that in signal processing to fit signals to specific frequency regions.

Notes

Summary

