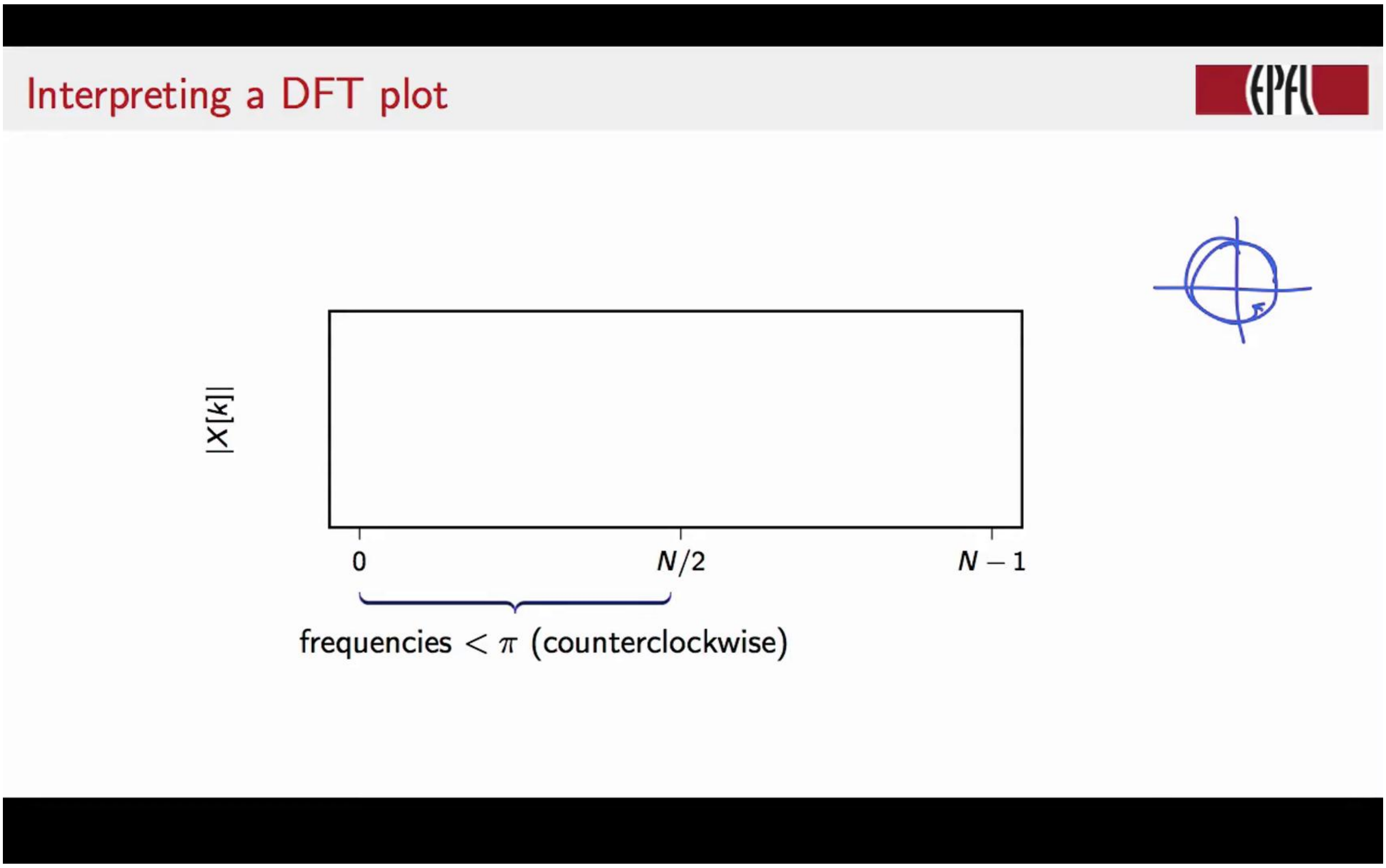
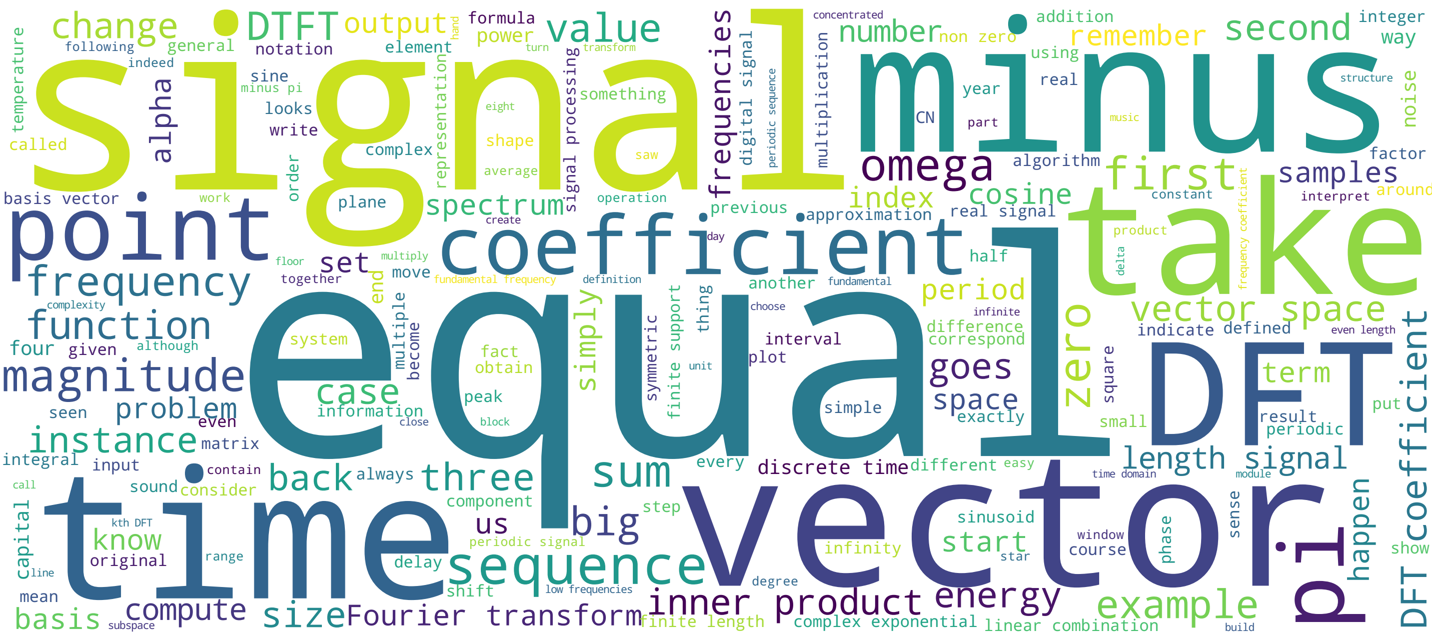
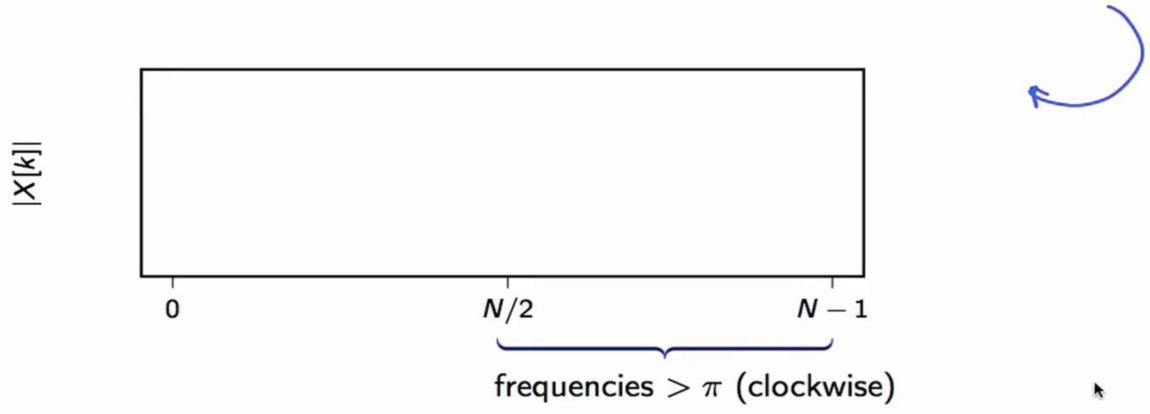




Search MOOC

Video

Interpreting a DFT plot

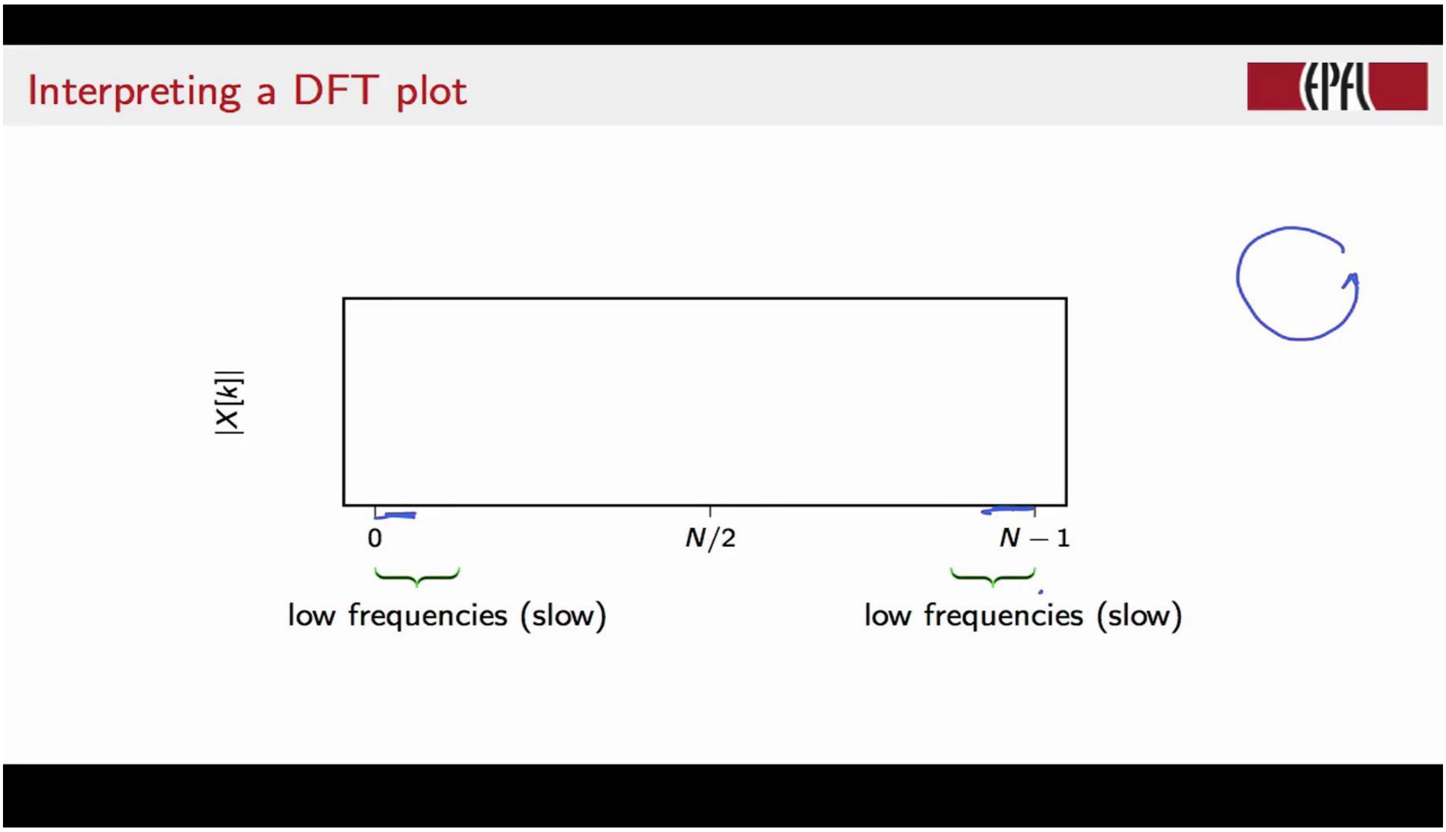


Now that we've seen some examples, we can give some general guidelines on how to interpret a DFT plot. First of all, you will have frequency coefficients from zero to begin minus 1, where N is the size of your vector space. The first N over 2 coefficient correspond to frequencies less than pi. We're talking about counterclockwise movement of the point on the complex plane. Frequencies from N over 2 to N minus 1 are frequencies that are larger than pi, and we interpret those as clockwise rotations in the plane.

Notes

Summary

0m 01s



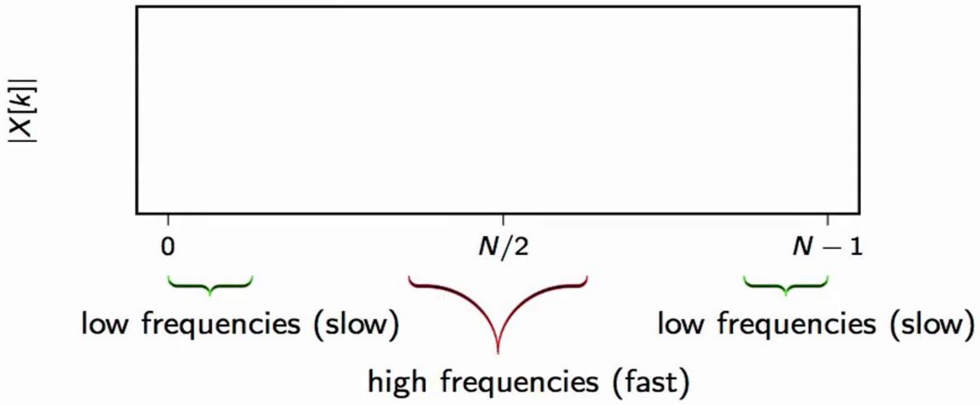
Frequencies in this band, close to 0 and close to n minus 1 are low frequencies in the sense that they indicate a slow rotation around the unit circle, either counterclockwise or clockwise here, whereas frequencies centred around N over 2 correspond to the fastest frequencies in the vector space, either clockwise or counter clockwise.

Notes

Summary

0m 39s

Interpreting a DFT plot



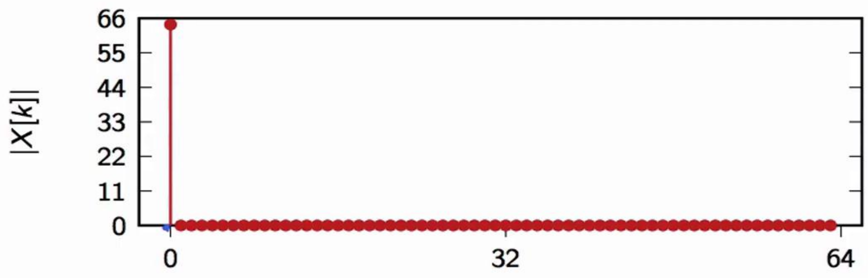
Notes

Summary

0m 54s

Interpreting a DFT plot

$x[n] = 1$ (slowest signal)



only lowest frequency

If we go back to the examples that we saw before, if we take x of n, the unit signal, so equal to 1 for all points, well, this is the slowest possible signal in the sense that it never really changes. It remains constant for the whole duration of its life. And correspondingly, its Fourier transform only contains the lowest frequency coefficient. It's not even a frequency in a sense, because k equal to 0 is absence of movement.

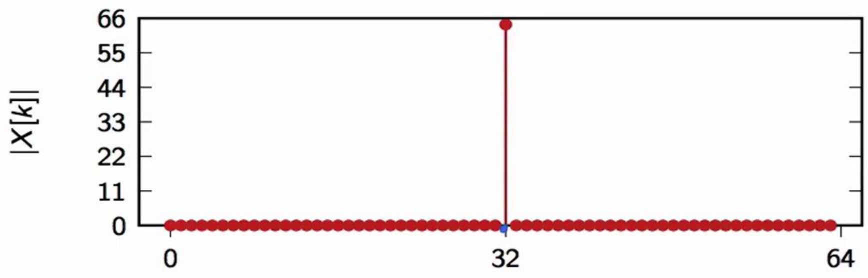
Notes

Summary

1m 05s

Interpreting a DFT plot

$x[n] = \cos \pi n = (-1)^n$ (fastest signal)



only highest frequency

Conversely, if we take X of n equal to cosine of pi n, which is equivalent to, say, minus one to the power of n, which is the fastest signal in discrete time, its DFT will only have one non-zero coefficient exactly at the highest frequency point.

Notes

Summary

1m 35s

Energy distribution

Recall Parseval's theorem : $\|\mathbf{x}\|^2 = \sum |\alpha_k|^2$

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

square magnitude of k-th DFT coefficient
proportional to signal's energy at frequency $\omega = (2\pi/N)k$

Now, if you recall Parseval's theorem, we know that the energy of a signal will not change if we change the underlying basis. Conservation of energy across domains. Now, the square magnitude of the kth DFT coefficient gives us an indication of the energy associated to the underlying oscillation that composes a signal. The signal's energy at a given frequency is proportional to the magnitude of the DFT coefficient at that point.

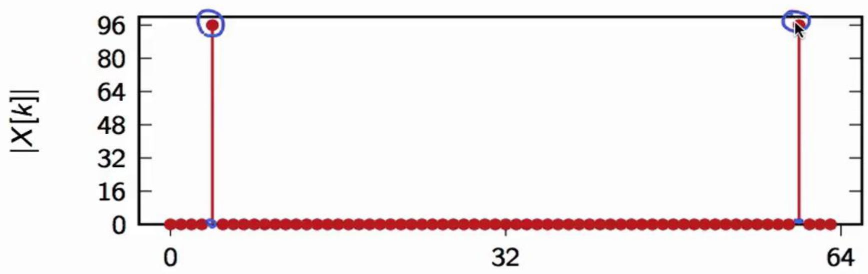
Notes

Summary

1m 52s

Interpreting a DFT plot

$x[n] = \underbrace{3\cos(2\pi/16\, n)}$ (sinusoid)



energy concentrated on single frequency
(counterclockwise and clockwise combine to give real signal)

If we go back to one of the examples that we worked out before, and we take a sinusoid with a frequency that is a multiple of the fundamental frequency for the space, we see that the magnitude is non-zero only in two points, and that indicates that the energy of the signal is concentrated at these two frequencies and nowhere else.

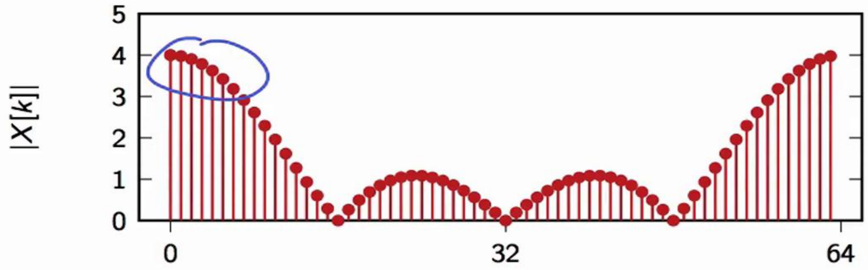
Notes

Summary

2m 24s

Interpreting a DFT plot

$$x[n] = u[n] - u[n-4] \text{ (step)}$$



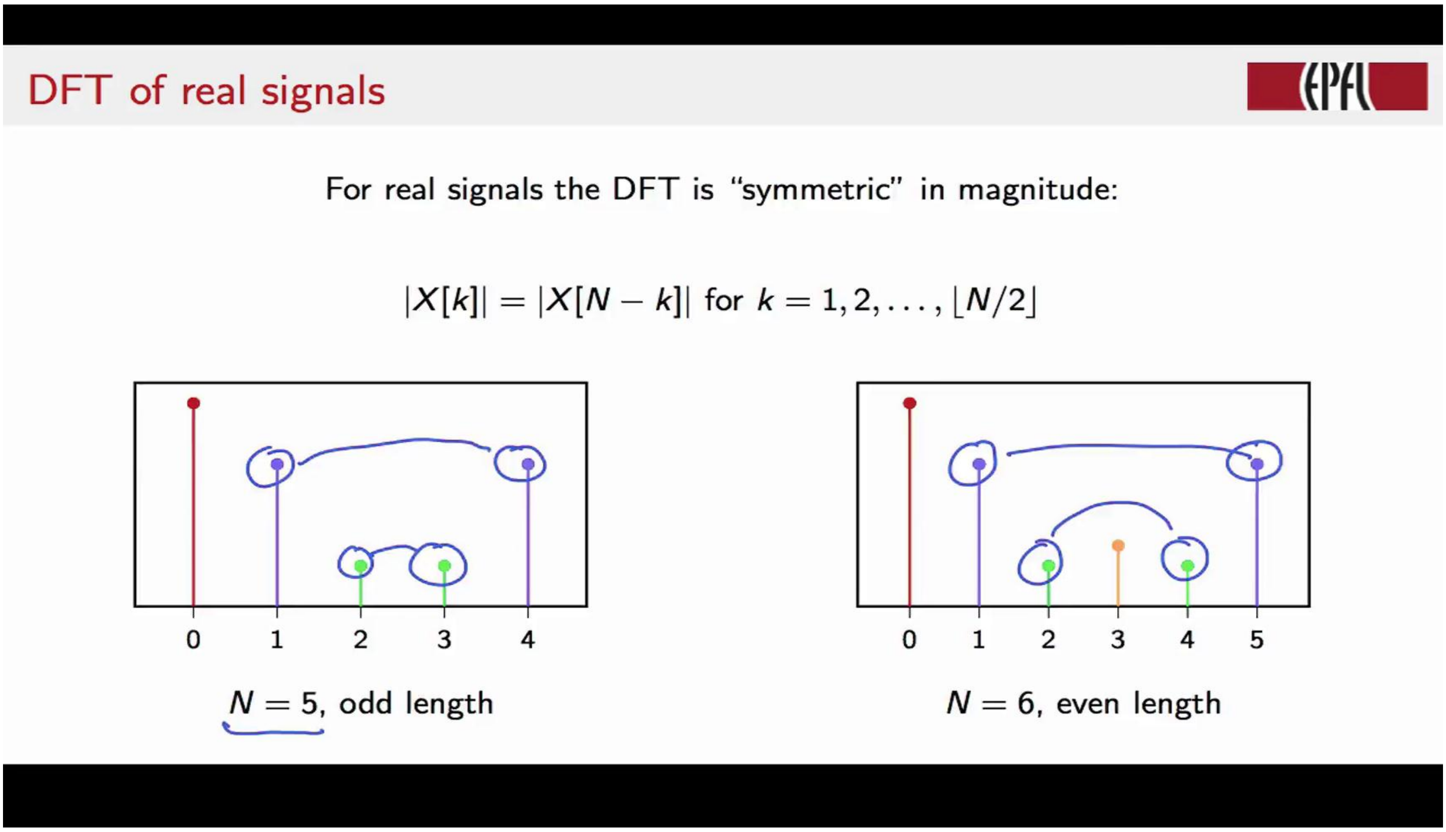
energy mostly in low frequencies

If we take the DFT of the unit step, on the other hand, we see that although most of the energy is concentrated on the low frequencies, there will be energy all across the spectrum. We'll need energy at all frequencies to create a step.

Notes

Summary

2m 44s



Finally, let's try and formalise something that you might have noticed already, and that is the fact that the DFT of real signals is symmetric in magnitude. Now symmetric here is in quotes, because when we're dealing with finite length vectors, symmetry depends on the parity of the vector. For odd length signals, we have a situation; for even-length signal, we have another situation. In general, what we say is that the magnitude of the kth DFT coefficient will be equal to the magnitude of the coefficient computed in big N minus k, and this is valid for k that goes from 1 to n over 2 in floor. So in the case of odd-length signal, what we have is the following. The coefficient in zero is free, the coefficient in one will be equal to the coefficient in big N minus 1. Here we have an N equal to 5, so this will be equal in magnitude to the coefficient in four, and the coefficient in 2 will be equal in magnitude the coefficient in three. Now, if you take n equal to 6, so an even length space, the coefficient in zero will be 3, the one in one will be equal to the one in five in magnitude, the one in two will be equal to the one in four in magnitude, and the one in k equal to three will be three.

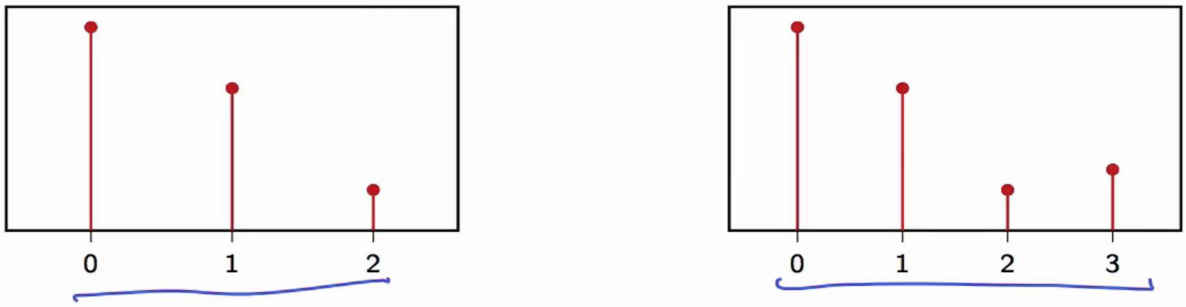
Notes

Summary

3m 00s

DFT of real signals

For real signals, magnitude plots need only $\lfloor N/2 \rfloor + 1$ points



$N = 5$, odd length

$N = 6$, even length

What that means is that for real valued signal, the magnitude of the DFT is completely specified by only the floor of n over 2 plus 1 coefficient. So for n equal to 5, for instance, we only need three coefficients, and for n equal to 6, we only need four coefficients. The reason why this happens is because when we take the DFT of a real signal, although we end up with complex coefficients, the coefficients will have to combine together to give back a real signal. Coefficient will be complex conjugate across the n over 2 index.

Notes

Summary

4m 20s