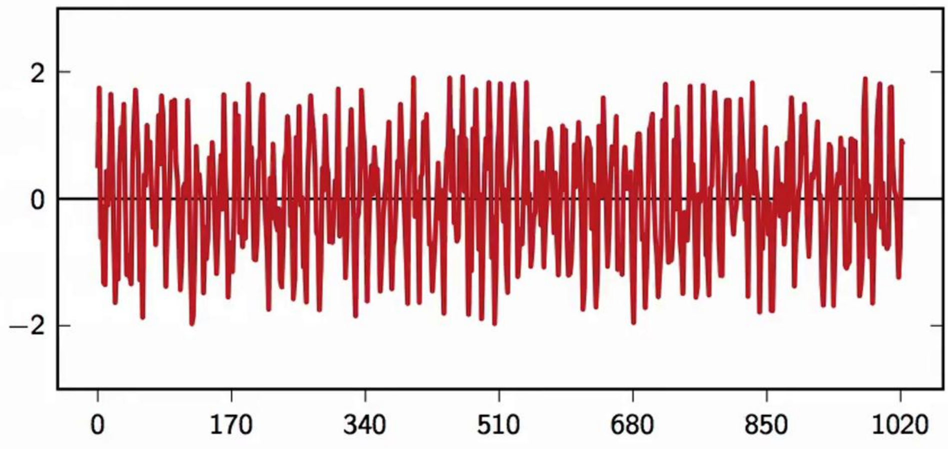
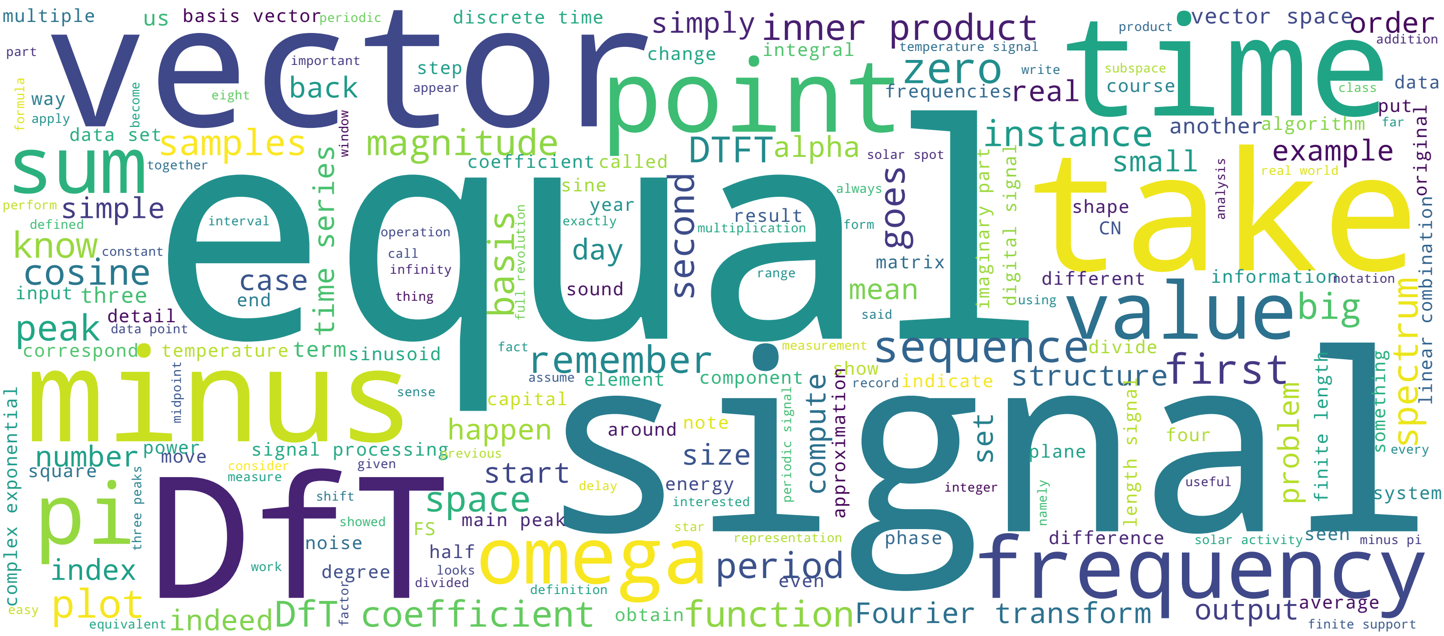
Mystery signal revisited





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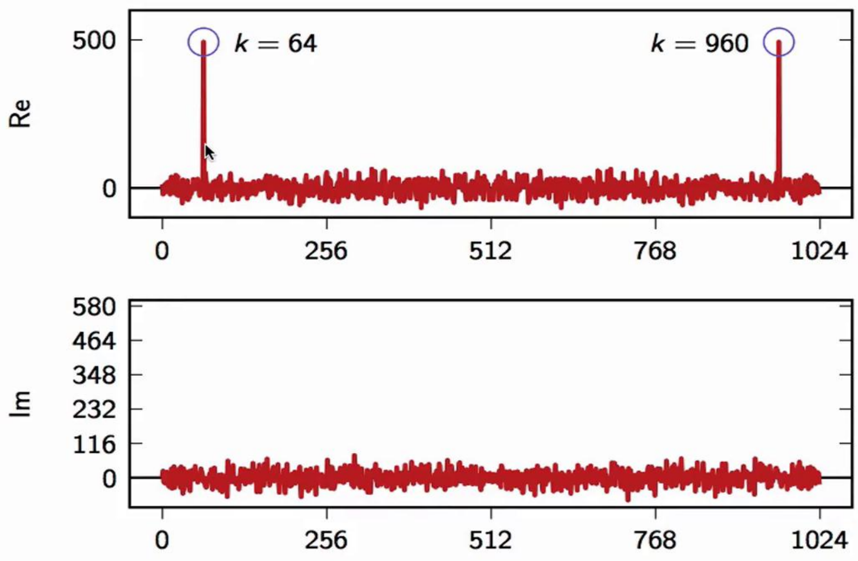
Mystery signal revisited



You remember that we tried to wet your appetite with respect to Fourier analysis by showing you a mystery signal, by showing you that by changing the basis in which the signal is represented, we all of a sudden could see some structure in the data. So now we can go back to that example and analyse it, applying what we know about DfT. So we take the DfT of the mystery signal, we plot the real and imaginary part of the DfT and we see that the structure that appears is a couple of spikes at symmetric positions in the spectrum. Now, we already know that this indicates the presence of a strong sinusoidal component and more importantly, a sinusoidal component that has a frequency, which is a multiple of the basic frequency for the space that the signal lives in. Now, if you look in detail at what happens in the spectrum, we see that the peaks are for K equal to 64 and for K equal to 960. We also see that the peaks appear only in the real part of the spectrum. And we remember that when this happens, the underlying sinusoid, the sinusoid represented by the speaks, is a cosine.

Notes

Summary

0m 00s

Mystery signal revisited

$$x[n] = \cos(\omega n + \phi) + \eta[n]$$

with

$$\begin{aligned} \phi &= 0 \\ \omega &= \frac{2\pi}{1024} 64 \end{aligned}$$

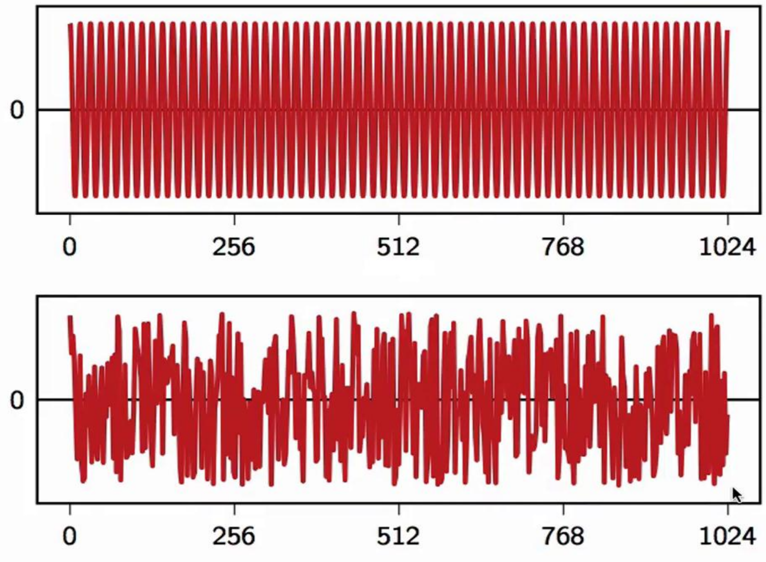
So from this simple visual inspection, we can already write our signal as such. There will be a cosine component and we will have to determine both the frequency and the initial phase of this cosine. And there will be another part that we can probably call a noise component, the sense that it doesn't have any structure. Since the imaginary part of the Fourier transform doesn't exhibit any particular peak, we can assume that the phase of the cosine is zero. And as far as the frequency is concerned, we know that the peak occurs at K equals 64. We are in a space of 1024 points and therefore Omega will be equal to 2 Pi over 1024. The basic frequency for the space times 64.

Notes

Summary

1m 14s

Mystery signal unveiled



If we plot the two components separately at this point, we see that we have indeed a cosine that oscillates 64 times in 124 points. And we have an additional noise component that doesn't really seem to have any structure.

Notes

Summary

2m 05s

Solar spots

- sunspot number: $s = 10 \times \#$ of clusters $+ \#$ of spots
- data set from 1749 to 2003, 2904 months

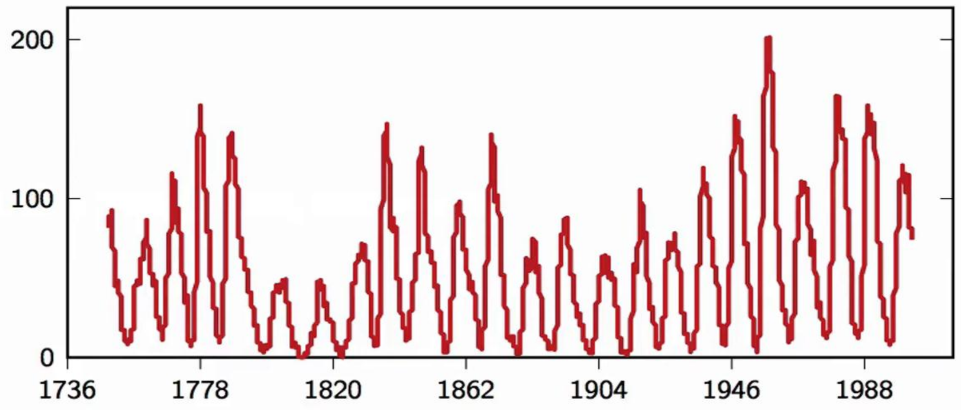
Let's now look at another signal, one that we showed in the introduction to this class, and that is a time series that maps the solar activity since the 1700s. Astronomers have noticed that solar activity could be related to what they call a sunspot number. The details are not really important, but fundamentally a measure of how many solar spots are at any given point in time on the face of the sun. We have data set that goes from 1749 to 2003, that is equivalent to 2900 months. Solar spots are computed every month.

Notes

Summary

2m 21s

Solar spots



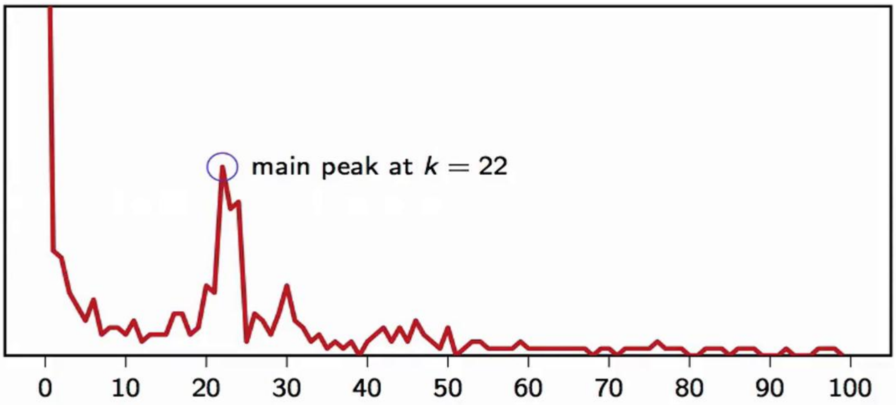
And as we plot this data, we see that there is an oscillatory behaviour. We are interested in finding out if there's any fundamental periodicity in solar activity. So we can take a four year transform.

Notes

Summary

2m 58s

Solar spots



The sunspot time series is a real signal. Now, if we're interested just in the magnitude of the DfT coefficients, you remember we need only show the first half of them. Even so, if you look at the first 1500 coefficients, you see that after the hundredth coefficient or so, their magnitude becomes too small to be relevant. So in this plot, for clarity, we just show the first hundred DfT coefficient magnitude. And indeed we can see that there is not just one peak, but a series of peaks. Now, what is the fundamental period? What is the most dominant mode of this time series? The main peak happens at K equal 22.

Notes

Summary

3m 10s

Solar spots

- DFT main peak for $k = 22$
- 22 cycles over 2904 months
- period: $\dfrac{2904}{22} \approx 11$ years

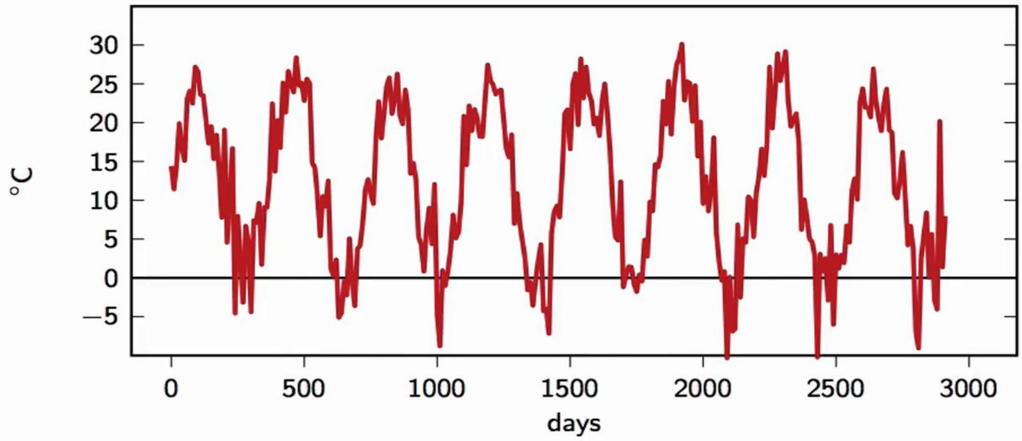
This tells us that there are 22 cycles over the 2904 data points that we showed in the previous picture. Therefore, the main period of the signal is 2904 over 22, which corresponds to approximately 11 years. Which means that solar sparked activity has an inherent periodicity of 11 years.

Notes

Summary

3m 50s

Daily temperature (2920 days)



We can perform the same analysis with a data set that records the daily temperature over a total of 2920 days.

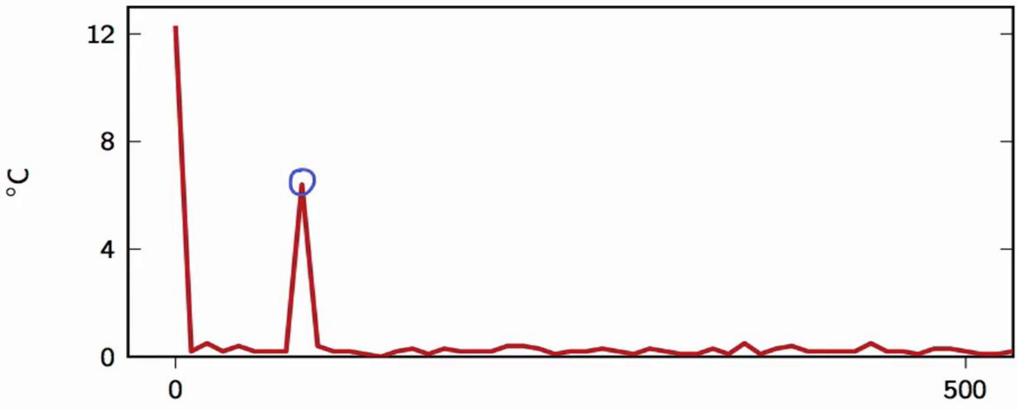
Notes

Summary

4m 12s

Daily temperature: DFT

$$X[0] = \sum_{n=0}^{N-1} x[n]$$



first few hundred DFT coefficients
(in magnitude and normalized by the length of the temperature vector)

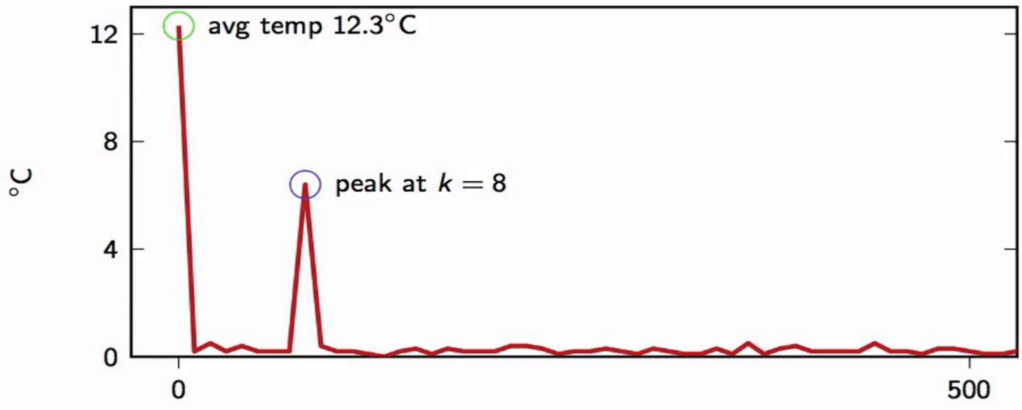
If we take the DfT of this time series, we see that there is actually a very pronounced periodicity in the spectrum. There is basically just one peak. And if we plot the normalised DfT coefficients, so if we divide the DfT coefficients by the length of the temperature vector, we can actually extract some information about the temperature values in this time series. Remember, for instance, the DfT coefficient for K equal to 0 is the nonnormalized average of all the data points, because x of 0 is simply the sum of all the points in the time series where N goes from 0 to N minus 1.

Notes

Summary

4m 20s

Daily temperature: DFT



first few hundred DFT coefficients
(in magnitude and normalized by the length of the temperature vector)

So if you plot the normalised coefficients, the coefficient of zero would be the average temperature for the time series, which in this case happens to be 12.3 Celsius degrees. And then we remark that there is a peak in the DfT for K equal to 8.

Notes

Summary

5m 03s

Labeling the frequency axis

If we know the "clock" of the system T_s (see Module 2.2)

- fastest (positive) frequency is $\omega = \pi$
- sinusoid at $\omega = \pi$ needs two samples to do a full revolution
- time between samples: $T_s = 1/F_s$ seconds
- real-world period for fastest sinusoid: $2T_s$ seconds
- real-world frequency for fastest sinusoid: $F_s/2$ Hz

So if we sum up what we learned from the DfT about the temperature signal, we know that the average value of the temperature is the 0th DfT coefficient normalised, so 12.3 degrees Celsius. The main peak is at K equal to 8, for a value of 6.4 degrees. What that means is there are eight cycles of the temperature signal over the entire duration of our data set, which is 2920 days. And so the period is 2900 divided by 8, which happens to be 365 days. So indeed, the temperature has a yearly periodicity, as we all know very well. Now, the value of the DfT main peak is 6.4 degrees Celsius. Now, if we have a sinozoid of the form, A cosine of Omega N, and we take the DfT of the signal, we will have a peak in magnitude for some index K that has a value of A over 2. Remember the definition of the DfT of a cosine function. So from this we can say that the temperature excursion around the average is twice the value of the DfT main peak. And so we can say that the yearly temperature at the point of measurement for the year was an average of 12.3 degrees, plus or minus 12.8. So this is the second example in which in order to find the real world frequency of a certain DfT component, we take the total length of the signal and we divide by the location of the peak.

Notes

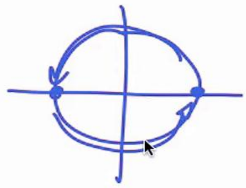
Summary

5m 19s

Labeling the frequency axis

If we know the "clock" of the system T_s

- fastest (positive) frequency is $\omega = \pi$
- sinusoid at $\omega = \pi$ needs two samples to do a full revolution
- time between samples: $T_s = 1/F_s$ seconds
- real-world period for fastest sinusoid: $2T_s$ seconds
- real-world frequency for fastest sinusoid: $F_s/2$ Hz

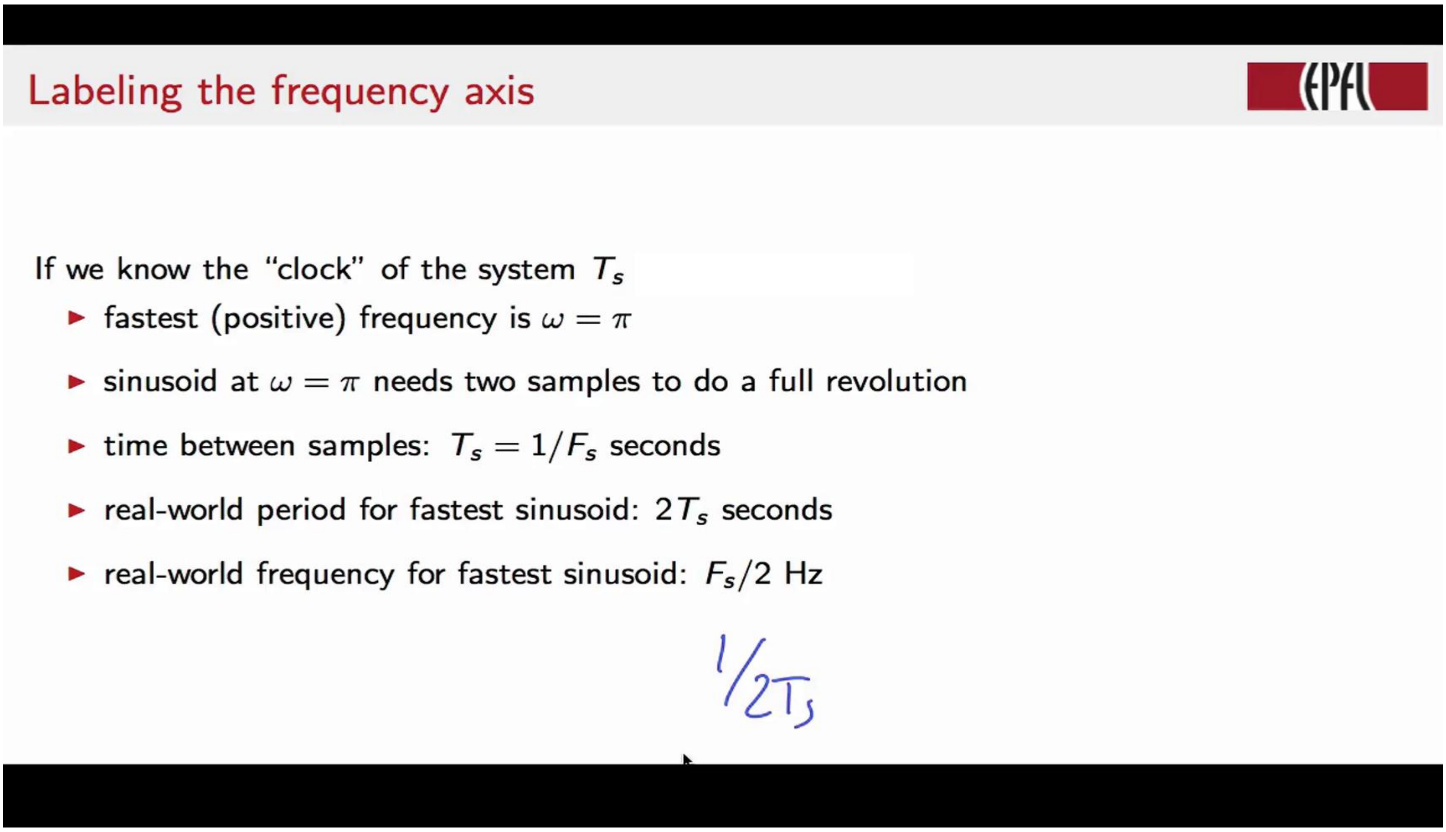


This is actually a general method to label the frequency axis of a DfT plot and it will be very useful in the future when we start analysing generic signals that have been sampled in a variety of contexts. So if you remember a few lessons ago, we informally introduced the notion of a clock for a digital signal processing system. And this is really equivalent to associating a certain time interval TS, measured in actual physical seconds between successive samples in the signal. So for instance, for the solar spot signal we had a TS of one month, whereas in the temperature signal the TS was equal to one day. We will see that for audio signal TS becomes very small because we will need to take at least 8,000 samples per second. Now, if we have a value for TS which is determined by the experimental set up, we can reason like so; the fastest positive frequency in a digital signal is Omega equal to Pi. So a sinusoid at that frequency needs two samples to do a full revolution, remember the unit circle. Something that moves at a speed of Pi will be here at instant n, here at n plus 1, here at n plus 2. Two samples to complete the full revolution.

Notes

Summary

6m 56s



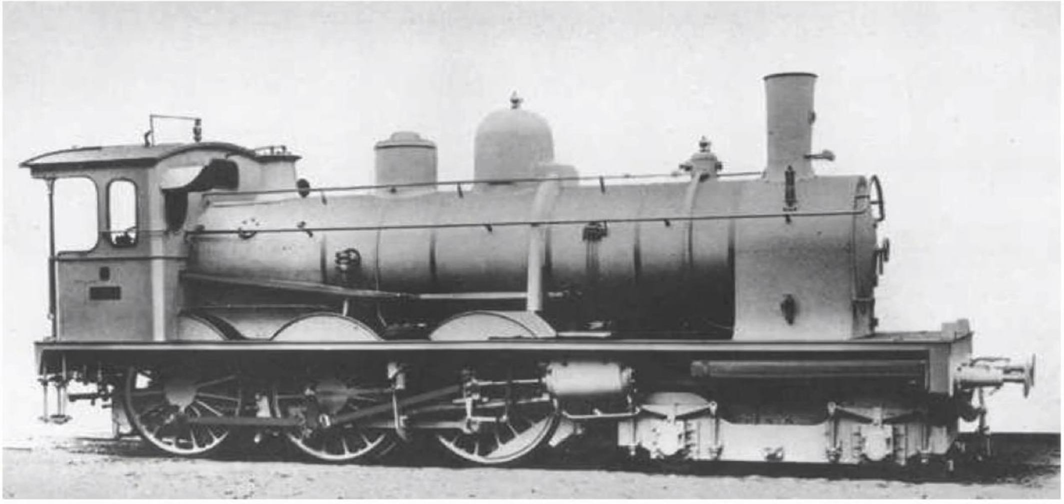
Now, the clock TS, can also be expressed as one over FS, where FS is the frequency of the system. This is a standard relationship between period and frequency for any system. If the real world period for the fastest sinozoid in a digital system is two times TS measured in seconds, the real world frequency for the fastest sinusoid is FS over 2. So the maximum frequency once we fixed the period between samples is FS over 2 or equivalently 1 over 2TS.

Notes

Summary

8m 14s

Example: train whistle



Let's take a concrete example. Suppose I give you an audio file that records a train whistle and I'm asking you to find out which notes make up the whistle. Of course your idea is to take the DfT of this file and find out the peaks in the Fourier transform which will probably correspond to the fundamental frequencies of the notes that are being played.

Notes

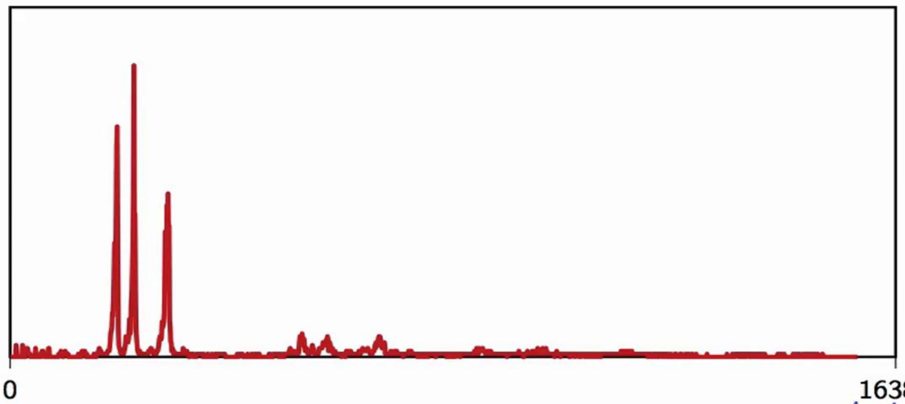
Summary

8m 49s

Example: train whistle

32768 samples (the "clock" of the system $F_s = 8000$Hz)

$T_s = \frac{1}{8000}$ s



So in order to do so, you need some extra information and namely you need to know the sampling frequency of this file, namely the time between successive samples. So the sampling frequency is FS equal to 8,000Hz, which means that TS is 1 over 8,000 seconds. This is the time between samples. And also the file contains 32,768 samples. We will see later why we often choose signal lengths that are a power of two. So if you take the DfT of this file and plot its magnitude, you see that there are three peaks that probably corresponds to three notes being played. Now, in order to label the axis, remember what we said. The highest frequency for the digital system corresponds to half the inherent sampling rate. And on the DfT vector, this point will correspond to the midpoint in the vector, namely the point where K is equal to N over 2. So here, indeed, we have the midpoint of the DfT vector, and therefore this will correspond to a frequency of 4 khz.

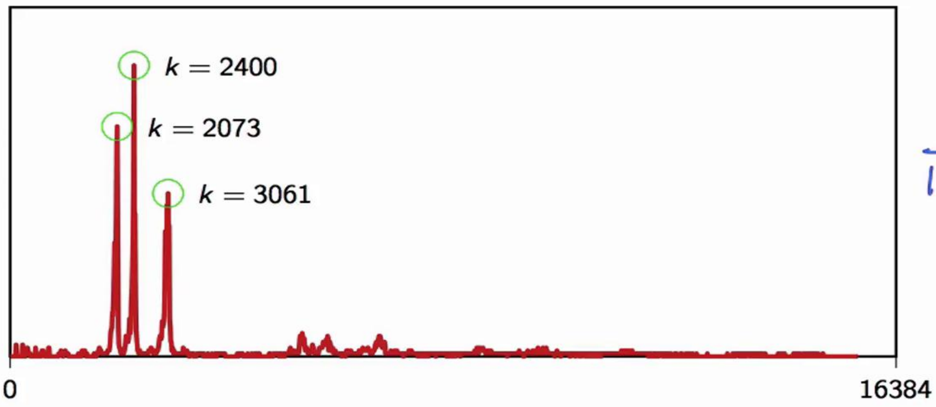
Notes

Summary

9m 14s

Example: train whistle

32768 samples (the "clock" of the system $F_s = 8000\text{Hz}$)



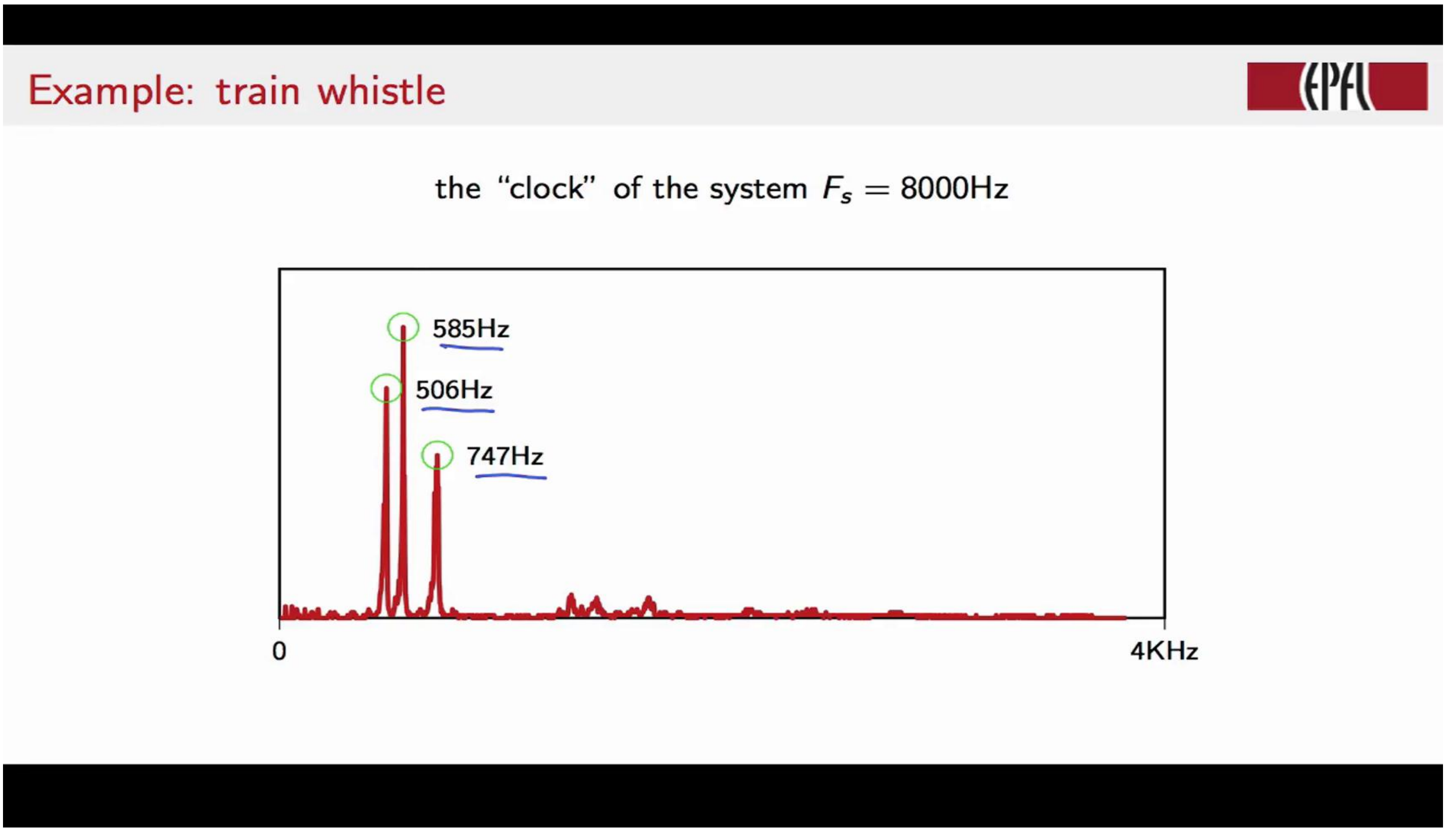
$$\frac{4\text{kHz}}{16384} \cdot K$$

We also have three peaks that take place for different values of K. To find the real frequency there, we just apply a linear mapping from zero to 4 khz. And so the frequency of any intermediate point will be 4 khz divided by the total number of points multiplied by the index K.

Notes

Summary

10m 21s



With this rule, we can find the value in hertz corresponding to the three peaks in the DfT. And the values are shown here in the figure. And if you look up the closest note in the western tuning that these frequencies correspond to, you obtain that the train whistle is nothing but a simple B minor chord.

Notes

Summary

10m 40s

Example: train whistle

if we look up the frequencies:

B minor chord

Notes

Summary

10m 55s