

A black rotary telephone with a numeric keypad. The keypad has buttons for digits 1 through 9, 0, *, and #. Each digit button also has its corresponding letters (e.g., 2 has ABC, 3 has DEF). The button for 3 is highlighted with a blue border. Below the keypad is a label 'TEL.' followed by a blank space. The telephone has a coiled cord and a handset.



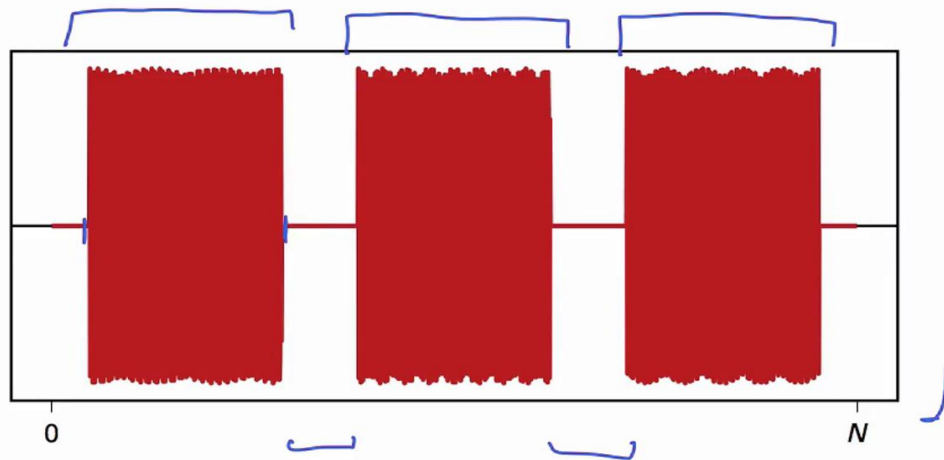
	1209Hz	1336Hz	1477Hz
697Hz	1	2	3
770Hz	4	5	6
852Hz	7	8	9
941Hz	*	0	#

Let's start with a very concrete example, and let's talk about dual-tone multi frequency dialing or DTMF for short. This is the way analog telephones work. Whenever you press a button here in the dial pad, you generate a sound which is composed of two sinusoids. The frequencies associated to the sinusoids are given by this matrix here. Here is the keypad of your telephone, the digits 0-9, plus the pound and the star sign, and when you press a button, say you press digit number four, you generate two sinusoids, one at 770 Hertz and another one at 1209 Hertz. These frequencies have been chosen, so there are co-prime, and also so that no sum or difference of two different frequencies correspond to one of the frequencies in the set. The idea behind this choice is to minimise the possibility of error when the telephone central office tries to decode the sequence of numbers that you have dialed on your phone.

Notes

Summary



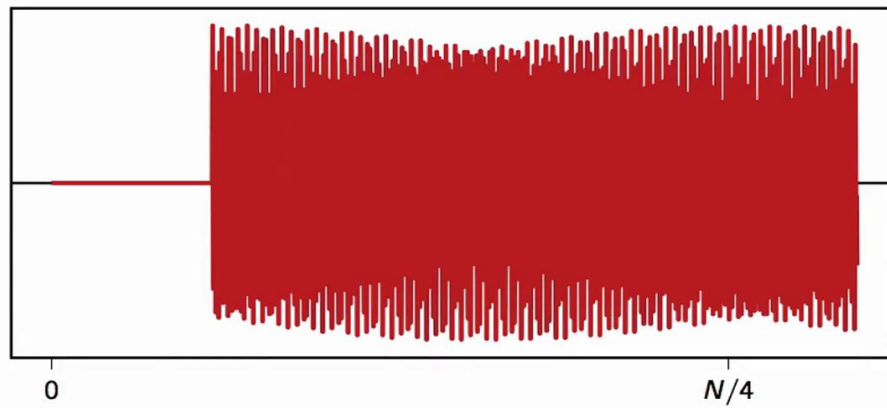


Here's, for instance, what happens if you dial 159 on your keypad? You generate a signal that sounds like this. Now, if we plot the signal in time, we get something that looks like this graph here. We see that there are indeed three bursts of sound with a pause in between, but it's absolutely impossible to understand which digit has been pressed just by looking at the time domain plot. We know when the digit has been started and when it ends, but we don't know its value.

Notes

Summary





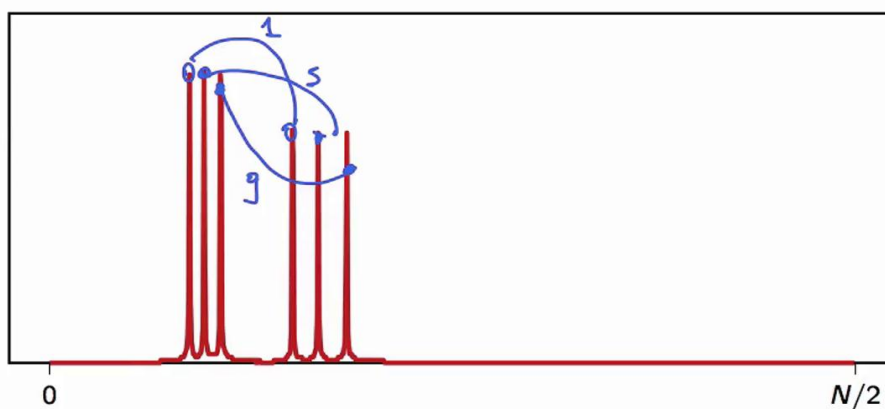
And even if we zoom in, it's very difficult to understand which frequencies make up this shape in time.

Notes

Summary



1-5-9 in frequency (magnitude)



On the other hand, if we take the DFT of the signal, we can see these frequencies very clearly. If you remember the DTMF matrix, each digit has a pair of frequencies associated to it, a low frequency and a high frequency. These will be the frequencies associated to the digit number one, these are the frequencies associated to number two, and these are the one associated to number three. But we cannot tell in which order these digits have been pressed.

Notes

Summary



1m 48s

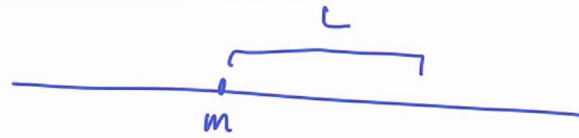
- ▶ time representation obfuscates frequency
- ▶ frequency representation obfuscates time

The time representation completely obfuscates the frequency content, so we know the timing, but we don't know the content. The frequency representation obfuscates the time information. We know the frequencies, but we don't know when they happen.

Notes

Summary





Idea:

- ▶ take small signal pieces of length L
- ▶ look at the DFT of each piece:

$$X[m; k] = \sum_{n=0}^{L-1} \underbrace{x[m+n]}_{\substack{\uparrow \\ \uparrow}} e^{-j \frac{2\pi}{L} nk}$$

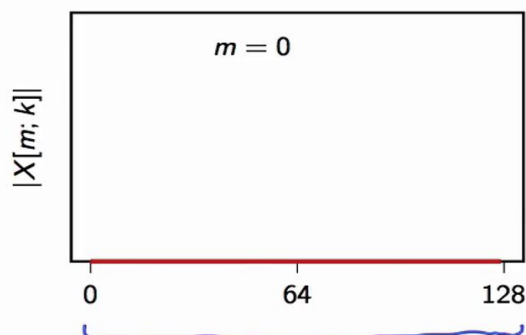
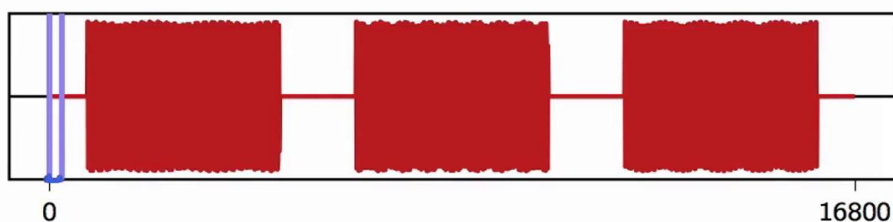
The idea behind the short time for you transform is the following. Instead of looking at the DFT of the whole signal in one go, we take small pieces of length L and we look at a DFT of each piece. The DFT coefficients now are indexed by two variables m is the starting point for the localized DFT and k is the DFT index for that chunk. This is your signal, you start here at m you take L points and you perform a DFT and then you move m to another point and compute another L point DFT and so on and so forth. The values of the STFT coefficients for M and K are given by the sum from zero to L minus one. A standard DFT over L points of the signal samples starting at m times e to the $-j \frac{2\pi}{L} nk$.

Notes

Summary



Short-Time Fourier Transform ($L = 256$)



Let's apply the strategy to the DTMF signal. We have 16,800 samples and we take window size of 256 points. Suppose we start at zero, we take therefore the DFT of this little chunk, the content of the chunk is pure silence. The DFT coefficients will be identical zero. Here in the lower plot, we show the first half of the DFT vector because the signal is real.

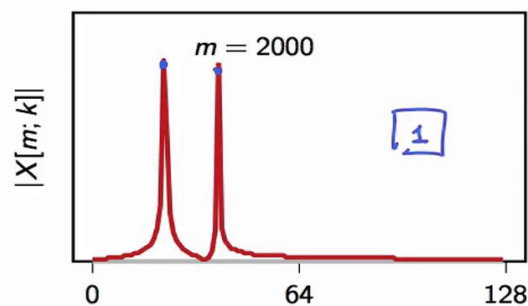
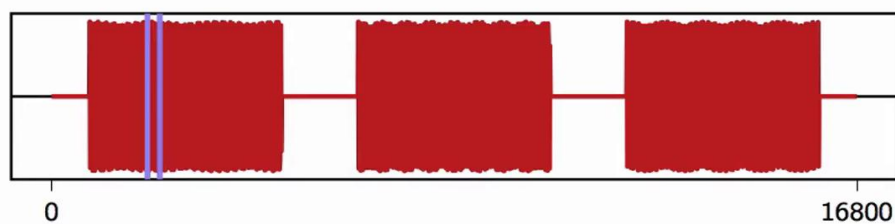
Notes

Summary



3m 34s

Short-Time Fourier Transform ($L = 256$)



Now we move the analysis window in the middle of the first sound burst and now the DFD coefficient, indeed show us the two frequencies associated to digit number one.

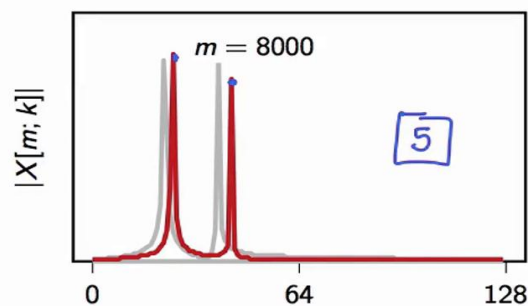
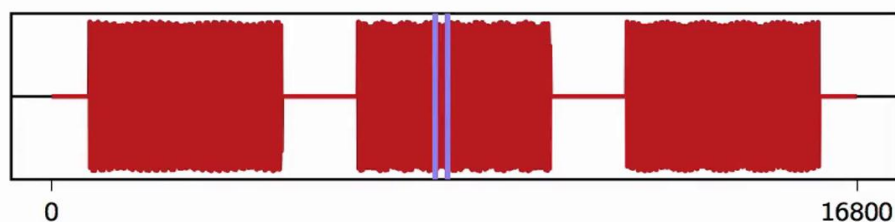
Notes

Summary



4m 04s

Short-Time Fourier Transform ($L = 256$)



We move the window to the second burst and we can see that now the DFT coefficient show the frequencies associated to number two, and you can compare this to the previous peaks, and you see that they have moved.

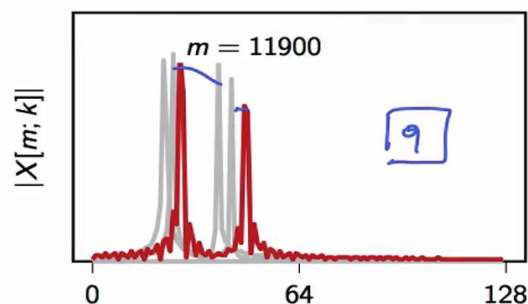
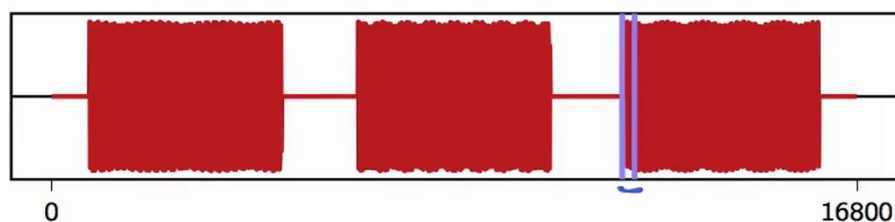
Notes

Summary



4m 16s

Short-Time Fourier Transform ($L = 256$)



Finally, when we move to the third burst of sound, we have the frequencies corresponding to digit number three. You can notice that the amplitudes of the peaks are not uniform in spite of the fact that the signal is of equal volume. The reason is because the position of the window influences the amount of energy that we capture for the DFT. Here, for instance, in this case, probably we're spending a little bit of silence before the onset of the waveform and therefore, the peaks are lower than in the former cases.

Notes

Summary



4m 30s