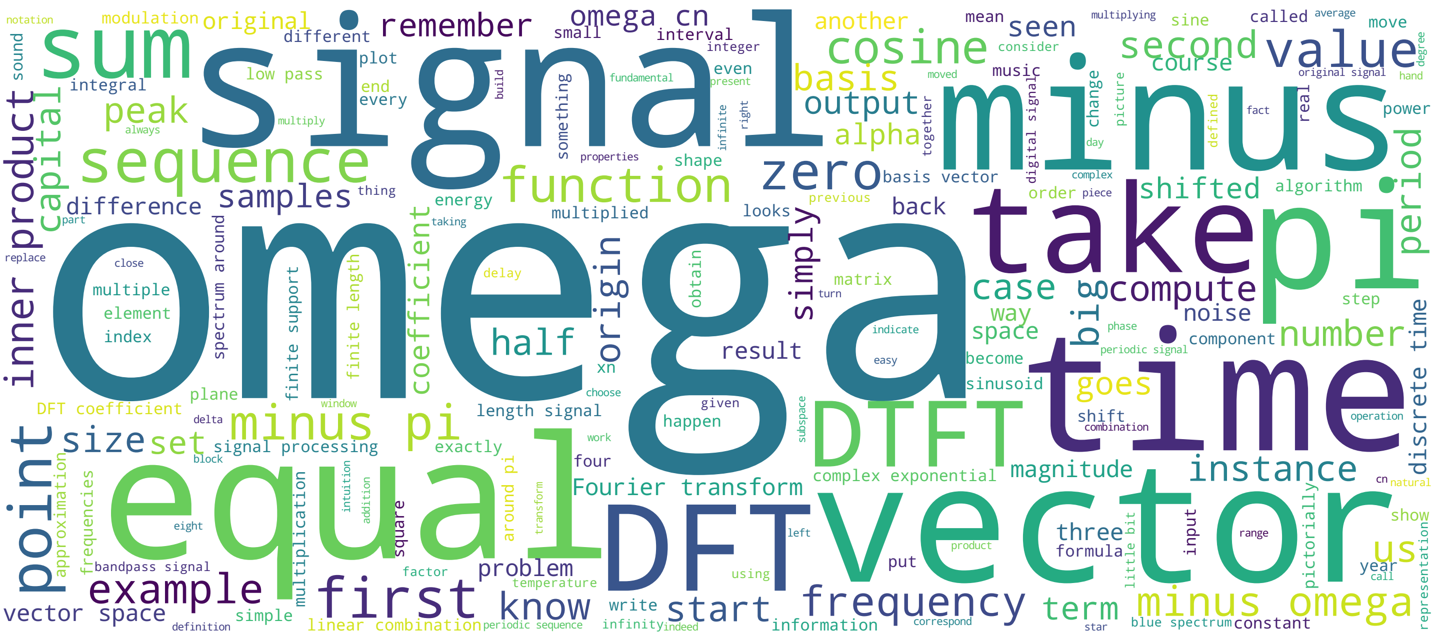
Classifying signals in frequency

Three broad categories according to where most of the spectral energy resides:

- lowpass signals (also known as "baseband" signals)
- highpass signals
- bandpass signals



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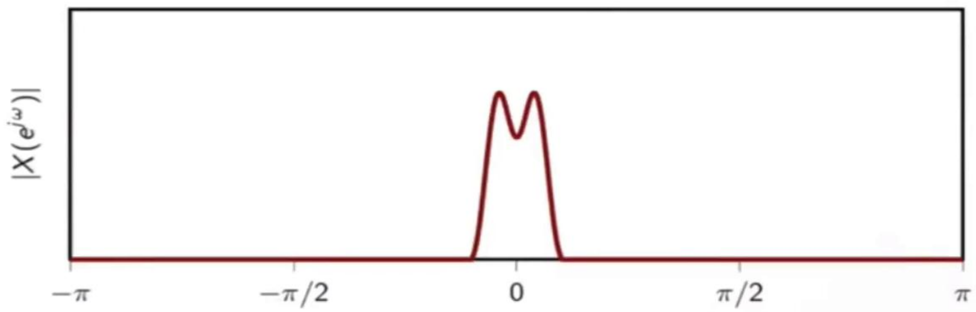
Video

Lowpass example



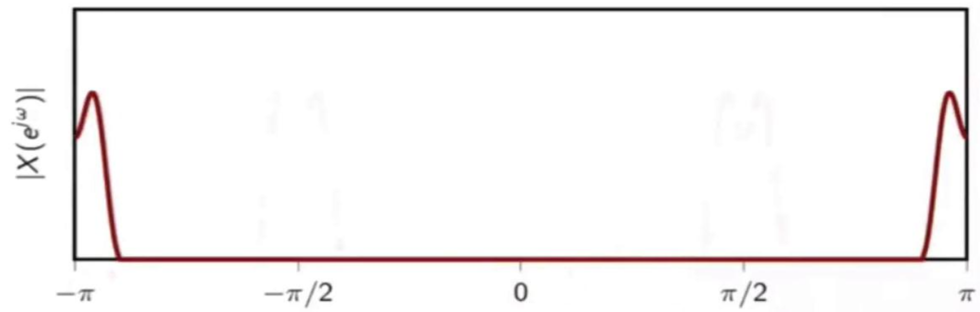
There are three broad categories of signals, depending on where the spectral energy actually is mostly concentrated. The easiest one and most natural one is low pass signals, sometimes called baseband signals. Then we have highpass signals, where the frequency content is mostly around high frequencies, and in between, you have bandpass signals. Now, there will be a difference between discrete time and continuous time signals, as we shall see. But these basic categories are present in both cases. Let's look at the low pass spectrum. As you can see, the energy is mostly concentrated around the origin, around zero, and there is no energy outside.

Notes

Summary

0m 00s

Highpass example



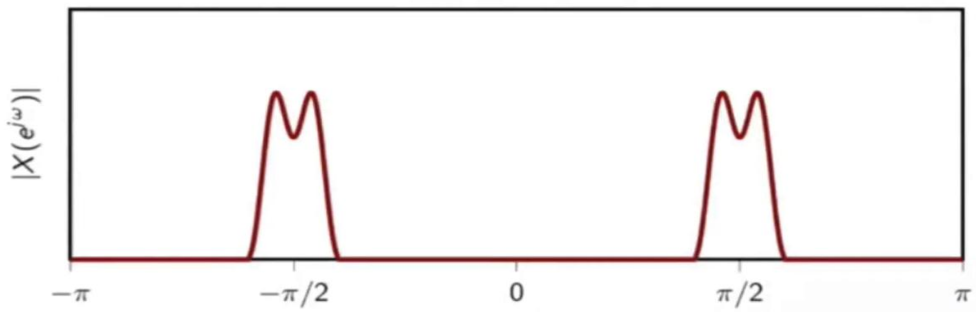
This is now a highpass signal. The energy is around pi or minus pi, and there is no energy around the origin.

Notes

Summary

0m 46s

Bandpass example



Finally, the bandpass signal. In this case, it's concentrated around pi over two, at minus pi over two and pi over two. Since this is an example of a real spectrum, it has this symmetry that we have seen in the properties of the DTFT.

Notes

Summary

0m 56s

Sinusoidal modulation

$$\text{DTFT}\left\{x[n]\cos(\omega_c n)\right\} = \text{DTFT}\left\{\frac{1}{2}e^{j\omega_c n}x[n] + \frac{1}{2}e^{-j\omega_c n}x[n]\right\}$$
$$= \frac{1}{2}\left[X(e^{j(\omega-\omega_c)}) + X(e^{j(\omega+\omega_c)})\right]$$

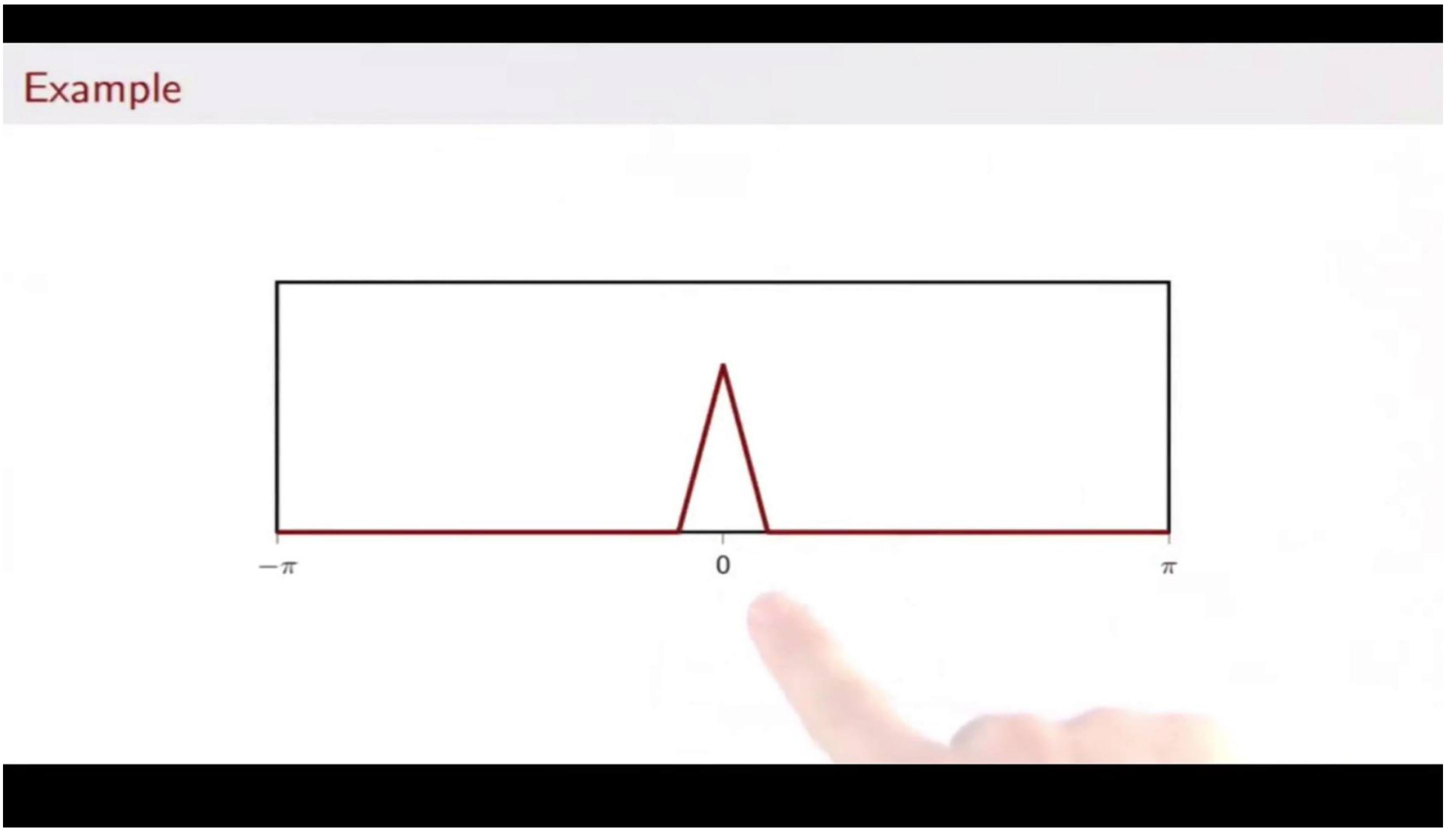
- usually $x[n]$ baseband
- ω_c is the *carrier* frequency

Consider now sinusoidal modulation. This is obtained by taking a signal xn and multiplying it by a cosine of omega c times n. What will this produce on the spectrum when we know x of n and its DTFT, capital X of e to the j omega? It is the DTFT of xn multiplied by a cosine of omega c times n, so that's the DTFT of using Euler's formula as usual of x of n multiplied by both e to the j omega cn and e to the minus j omega cn. This simply creates a double spectrum, namely, it's equal to one half capital X of e to the j omega shifted to omega c and shifted to minus omega c. Usually, we take xn as a baseband and omega c is called the carrier frequency. Now, to get an intuition for this formula, think of the following case. Think of x of n being the constant. We simply have the DTFT of cosine omega cn, which of course has these two peaks at omega c and minus omega c, as we know from the DTFT of a cosine function. This gives intuition, so if xn is a very narrow band, low pass signal, it looks a little bit like a constant. Then through modulation, it will be moved to these two peaks at omega c and minus omega c.

Notes

Summary

1m 13s

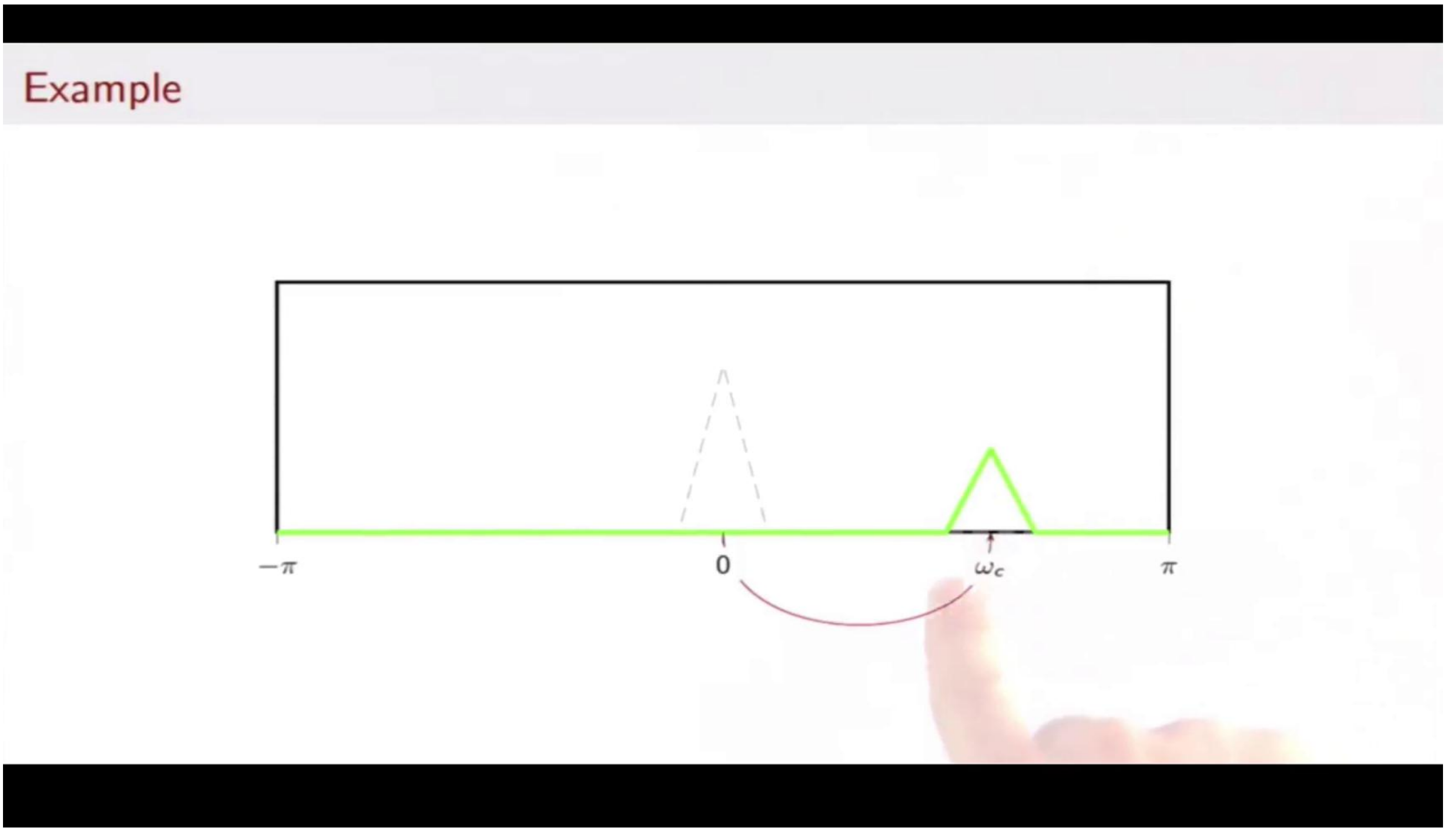


Let's do this pictorially. We start with a spectrum here. It's a triangular spectrum around the origin.

Notes

Summary

2m 44s

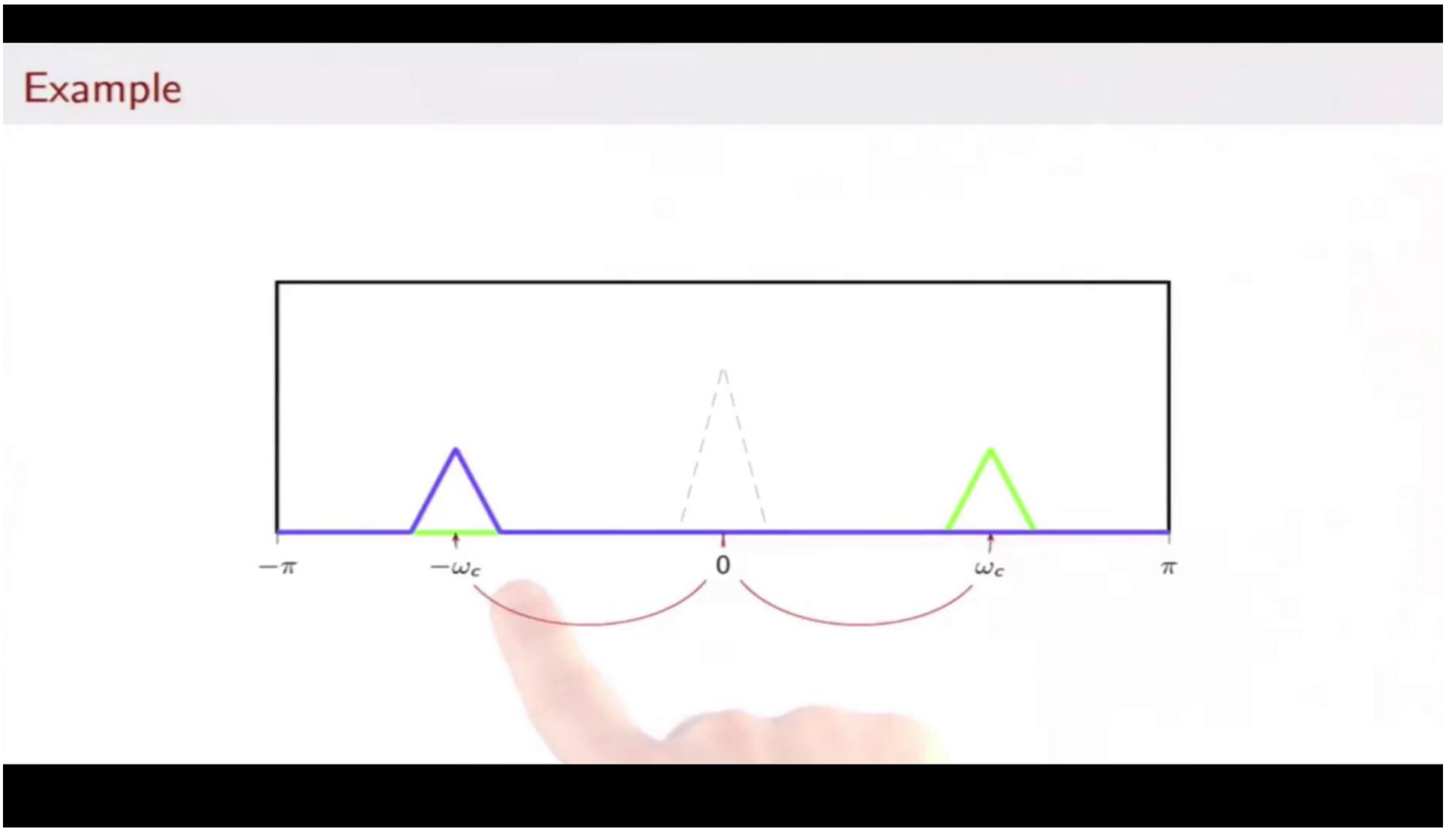


We move it to omega c multiplied by one half. That's the first green spectrum.

Notes

Summary

2m 49s

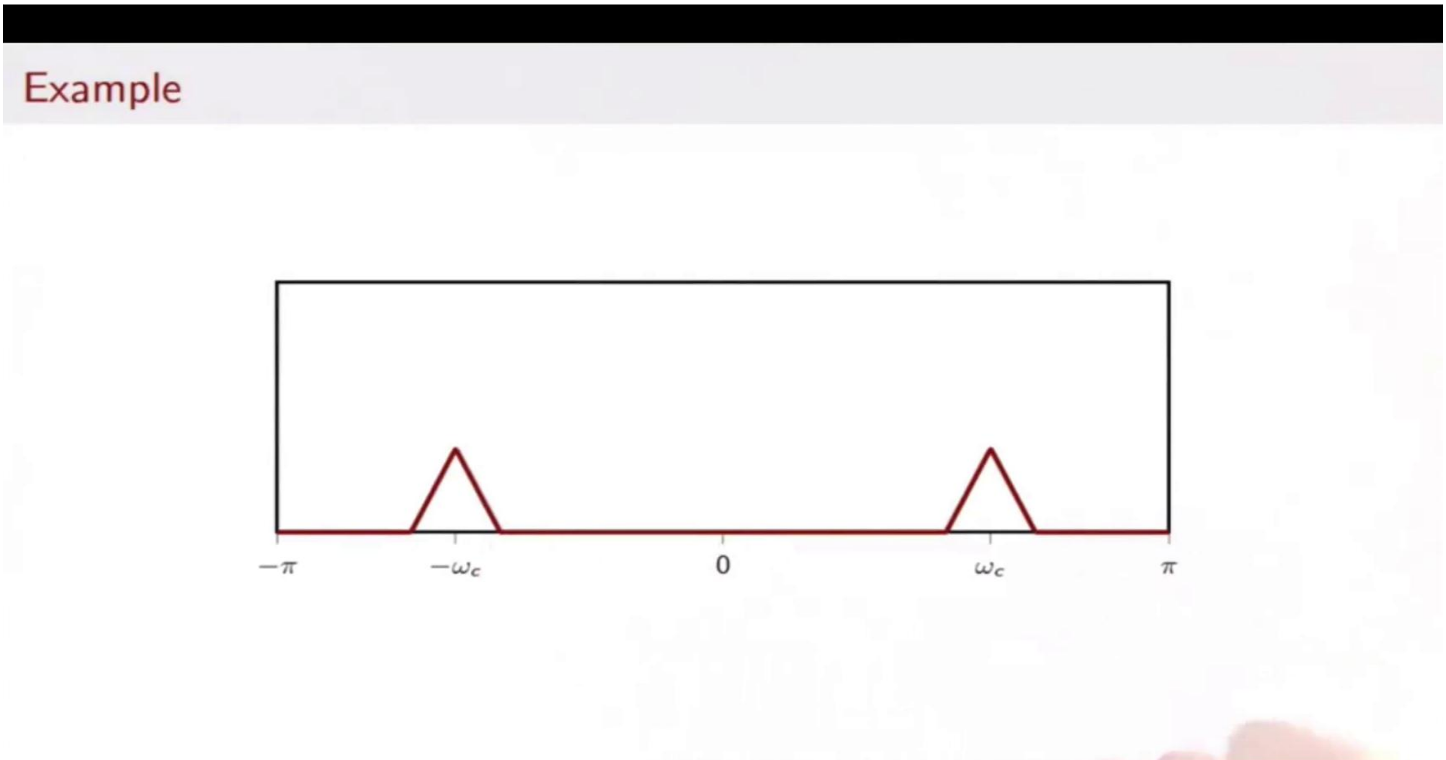


Then we move it to minus omega c. It's a blue spectrum also multiplied by one half.

Notes

Summary

2m 55s



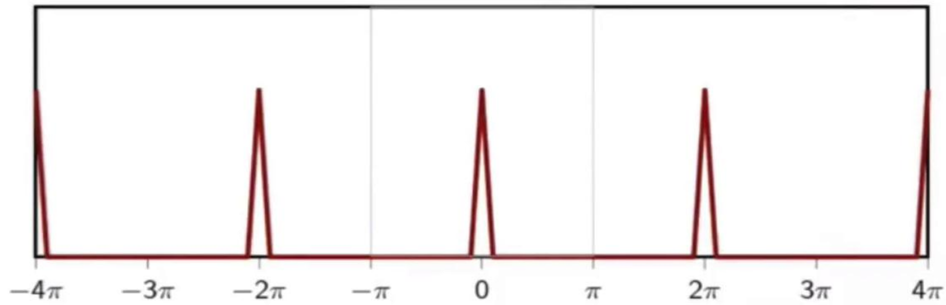
This is the result, the red spectrum now after modulation. The central peak has been moved into two half as big peaks at minus omega c and omega c.

Notes

Summary

3m 01s

Again, explicitly showing the periodicity of the spectrum



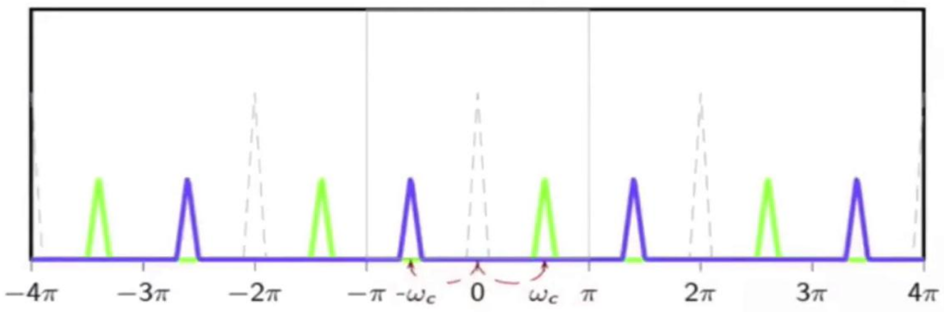
We know that the spectrum is 2 pi periodic.

Notes

Summary

3m 13s

Again, explicitly showing the periodicity of the spectrum

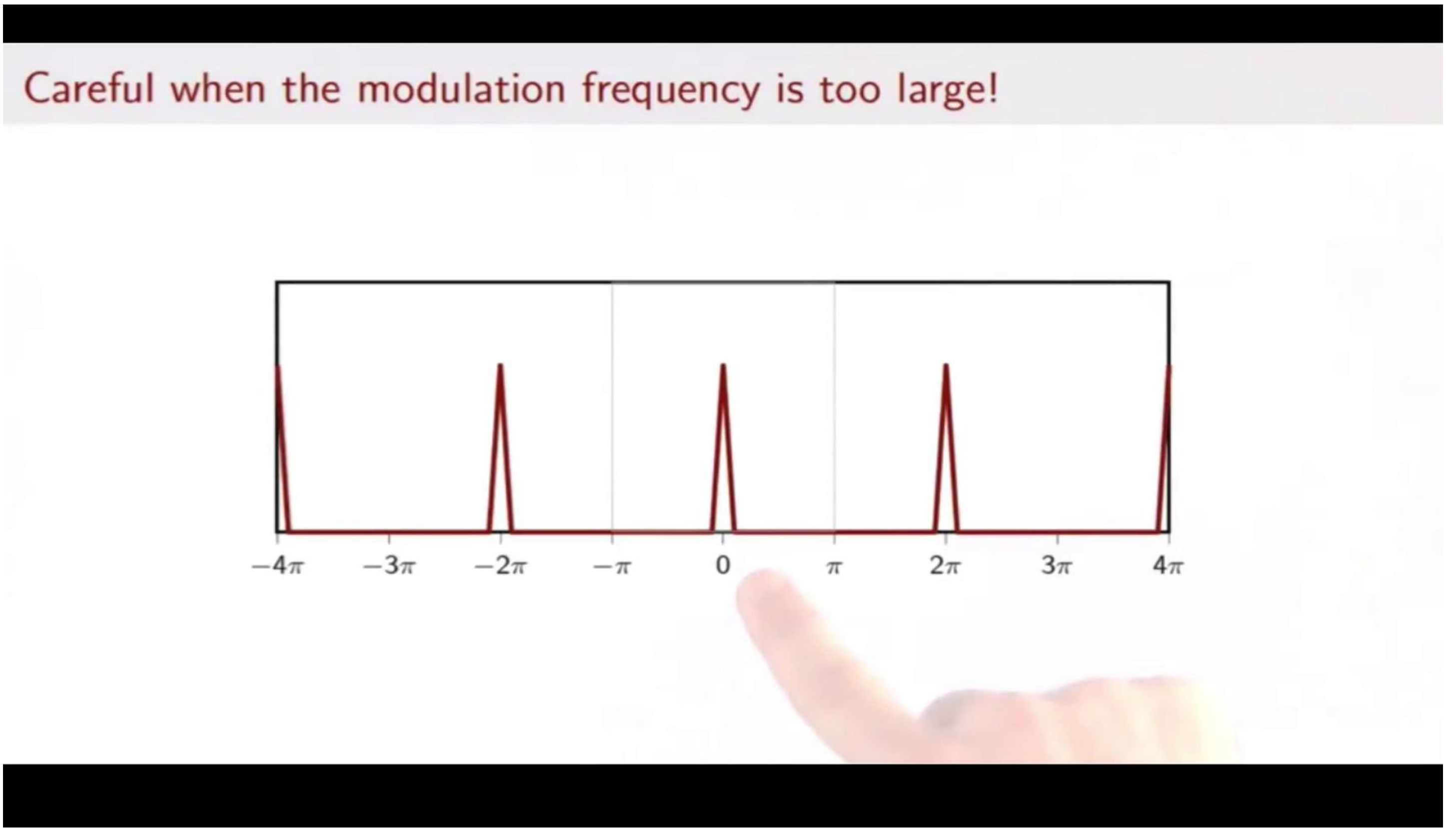


Let's show a few periods here for minus 4 pi to plus 4 pi, shifted to omega c, green spectrum, shifted to minus omega c, blue spectrum and the resulting red spectrum.

Notes

Summary

3m 15s

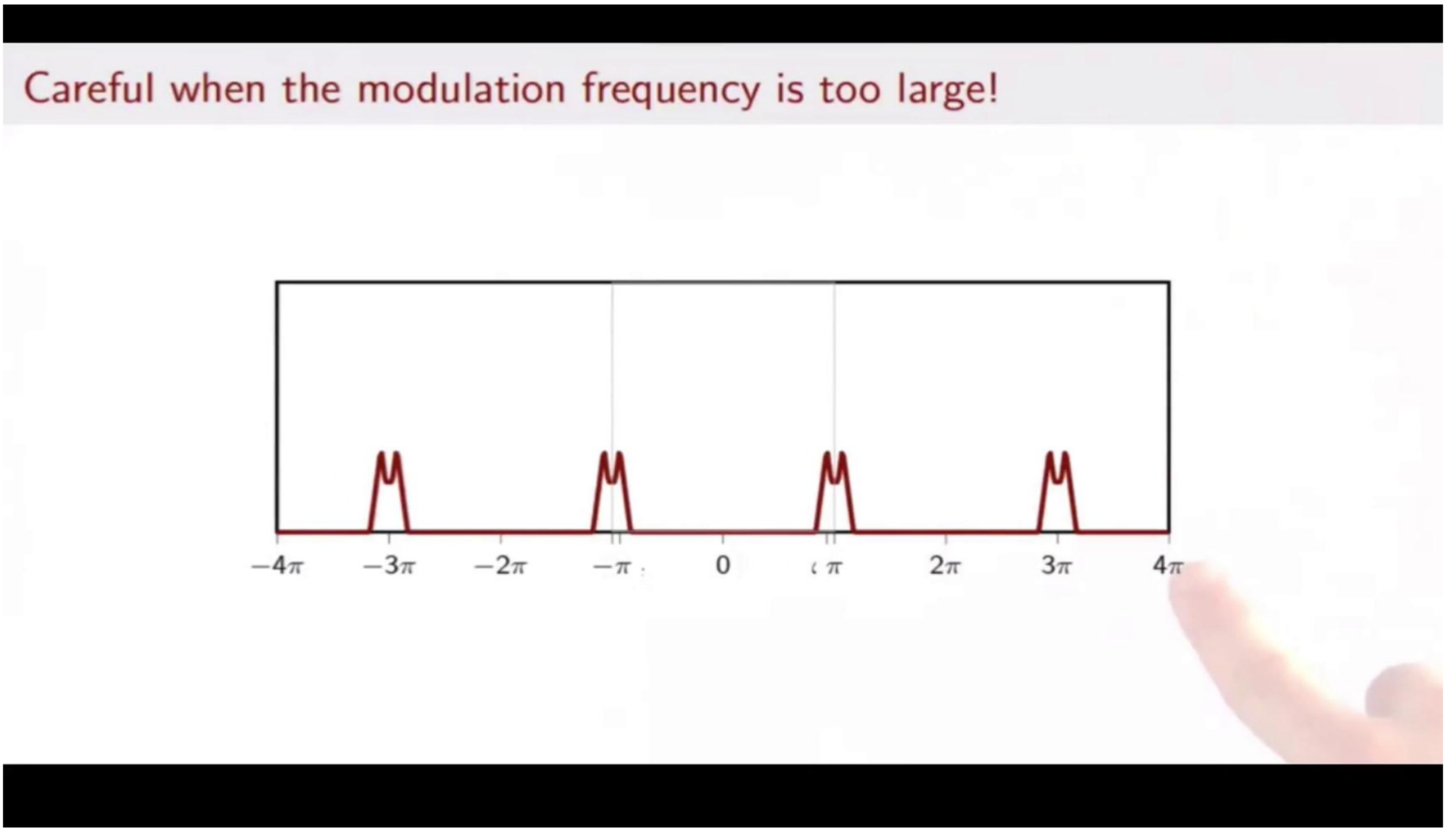


Now one has to be careful if the modulation frequency grows beyond a certain point and demonstrates this again pictorially. Here omega c is very close to pi, the maximum frequency close to minus pi, the blue spectrum.

Notes

Summary

3m 30s

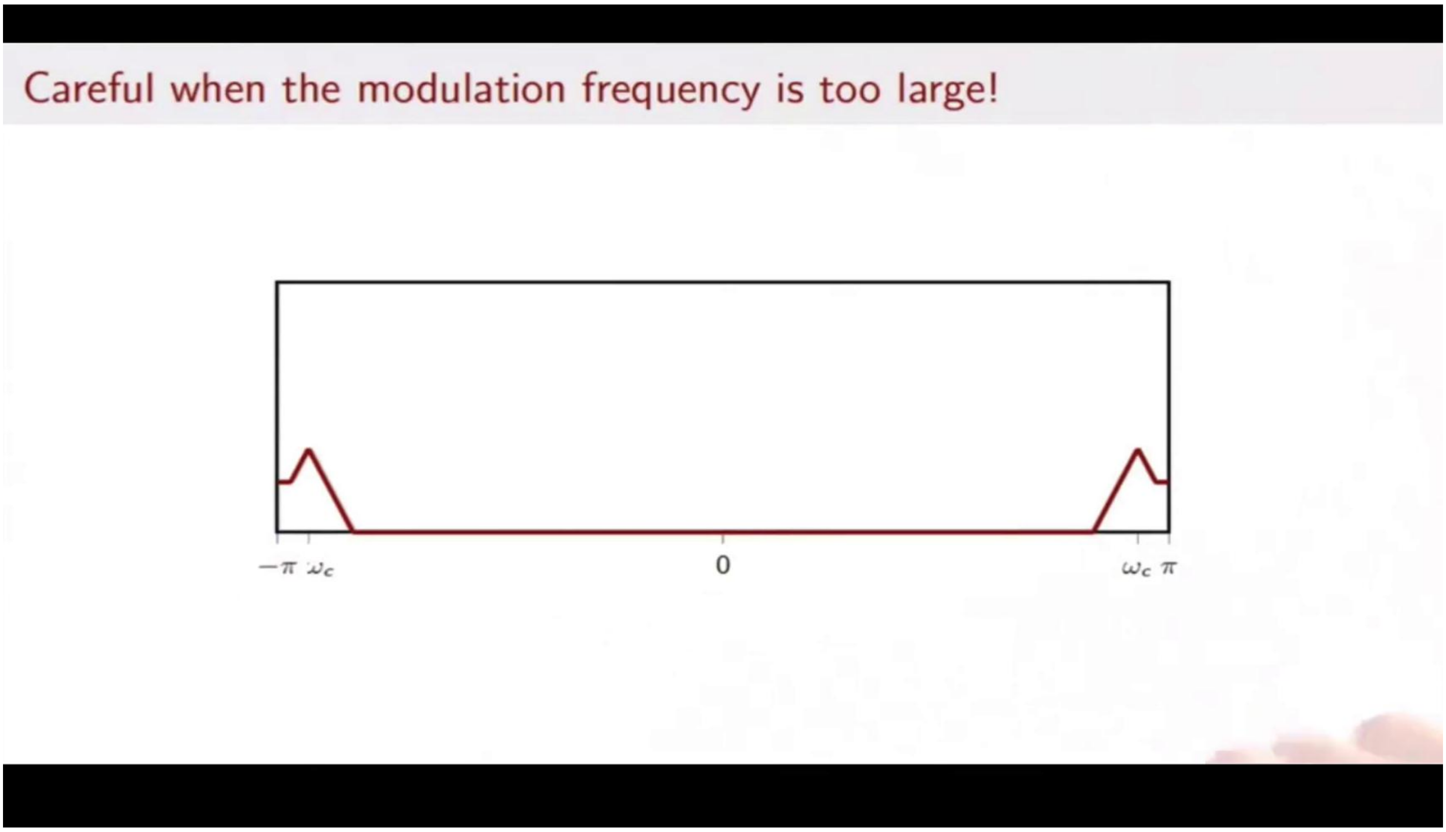


We see now that we have a funny-looking spectrum around plus or minus pi and plus or minus 3 pi. This is not exactly what we had expected.

Notes

Summary

3m 47s



If we blow it up, we can see that we don't have the triangular spectrum anymore. We have a piece of the triangles and something funny around minus pi and pi.

Notes

Summary

3m 58s

Sinusoidal modulation: applications

- voice and music are lowpass signals
- radio channels are bandpass, in much higher frequencies
- modulation brings the baseband signal in the transmission band
- demodulation at the receiver brings it back

Let us look at some applications of what we have just learned about signal modulation. For example, voice and music are typically lowpass signals. They don't have infinitely high frequencies because anyways, they wouldn't be heard by the human hearing system. Radio channels on the other hand, are bandpass signals because we need to modulate them high up. Otherwise, there is too much interference or too much loss in transmission. Modulation is a process of bringing a baseband signal, for example, a voice signal into the transmission band for radio transmission. Demodulation is the inverse or the dual of modulation and it will bring back the signal from a bandpass down to the baseband.

Notes

Summary

4m 09s

Sinusoidal demodulation

just multiply the received signal by the carrier again

$$y[n] = x[n]\cos(\omega_c n) \qquad Y(e^{j\omega}) = \frac{1}{2}\left[X(e^{j(\omega-\omega_c)}) + X(e^{j(\omega+\omega_c)})\right]$$

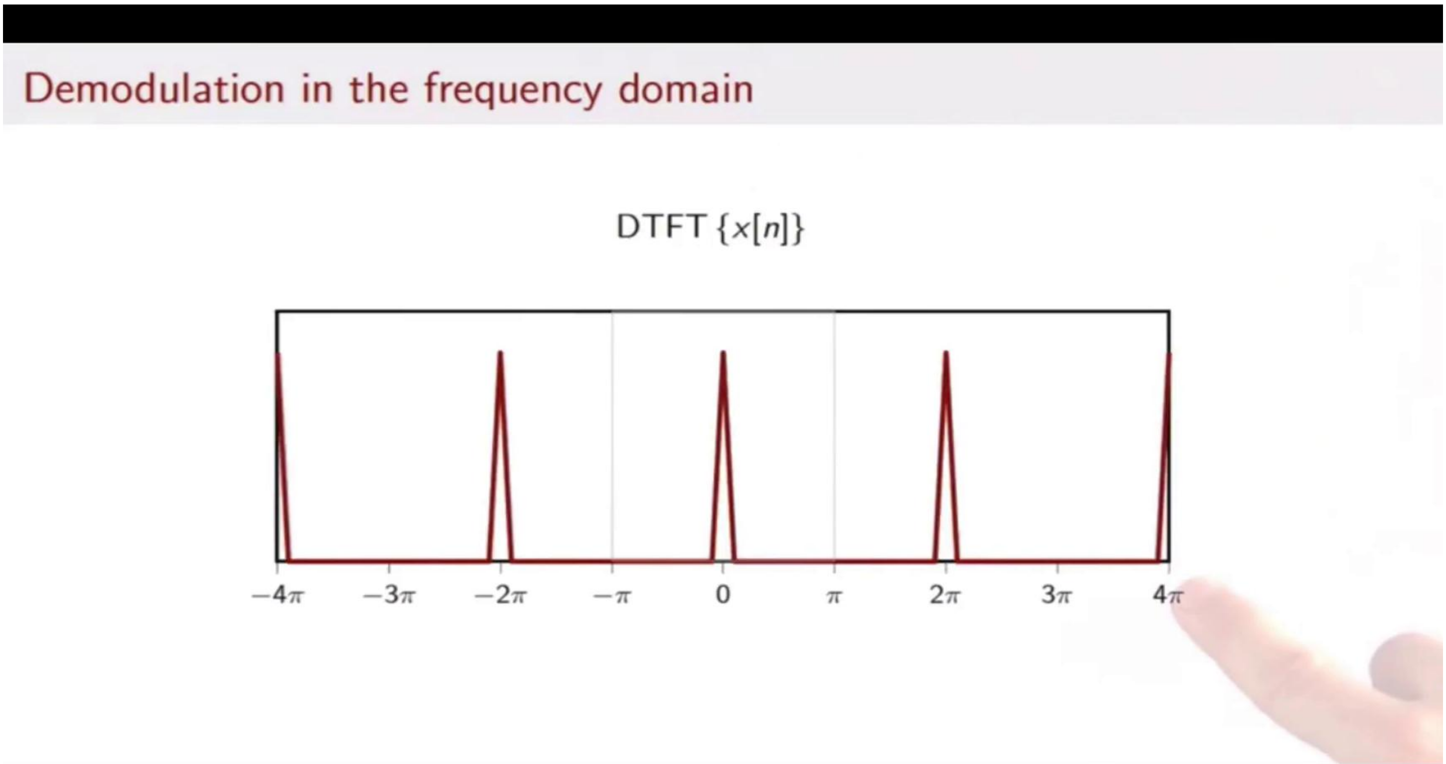
$$\begin{aligned}\text{DTFT}\{y[n]\cdot 2\cos(\omega_c n)\} &= Y(e^{j(\omega-\omega_c)}) + Y(e^{j(\omega+\omega_c)}) \\ &= \frac{1}{2}\left[X(e^{j(\omega-2\omega_c)}) + X(e^{j(\omega)}) + X(e^{j(\omega)}) + X(e^{j(\omega+2\omega_c)})\right] \\ &= X(e^{j(\omega)}) + \frac{1}{2}\left[X(e^{j(\omega-2\omega_c)}) + X(e^{j(\omega+2\omega_c)})\right]\end{aligned}$$

Let us look at this demodulation process. It is simply done by multiplying the received signal by the same carrier again. We have yn is xn times cosine of omega cn. Its spectrum, we have seen before, capital Y e to the j omega is a combination of the two spectras shifted to omega c and minus omega c. The DTFT of yn multiplied by two cosine of omega cn, while it's going to be the combination of capital Y shifted to omega c and to minus omega c. Then we replace the formula we just had before. We have four terms, one shifted by two omega c, another one by minus two omega c and two terms that are actually at the origin. We have indeed capital X e to the j omega plus one half and two modulated versions at two omega c and minus two omega c.

Notes

Summary

4m 55s



Let's do this pictorially. The DTFT of xn is shown here. It's a triangle spectrum around the origin.

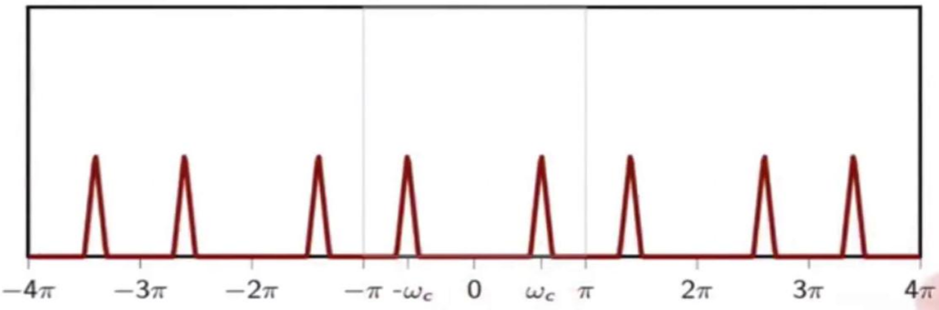
Notes

Summary

5m 55s

Demodulation in the frequency domain

$$\text{DTFT}\{y[n]\} = \text{DTFT}\{x[n]\cos\omega_c n\}$$



Then its modulated version has two peaks at minus omega c and plus omega c.

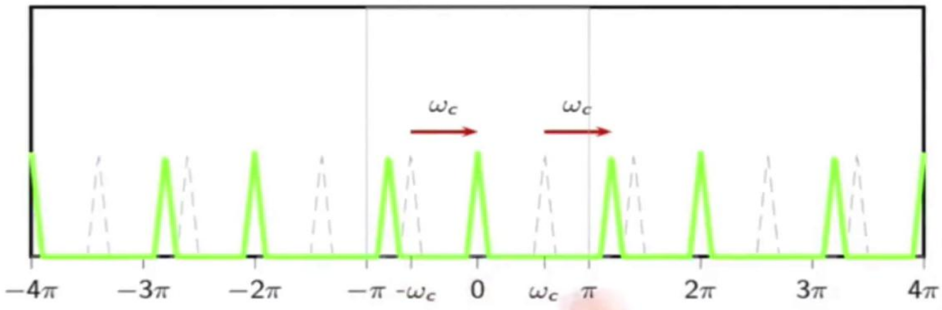
Notes

Summary

6m 04s

Demodulation in the frequency domain

DTFT $\{y[n] \cos \omega_c n\}$

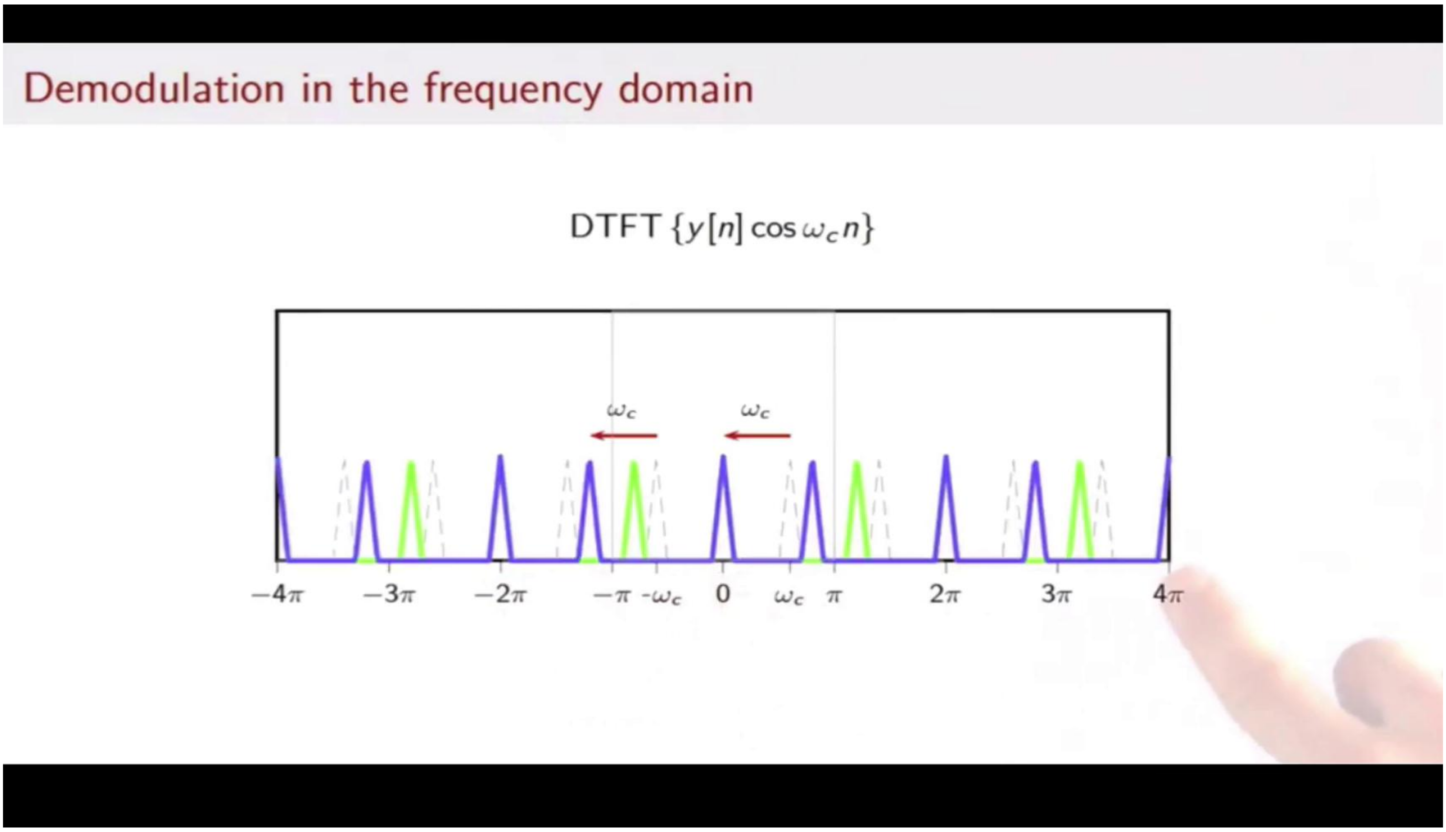


Then y n multiplied by cos omega cn has two shifted version, one to the right by omega c.

Notes

Summary

6m 11s

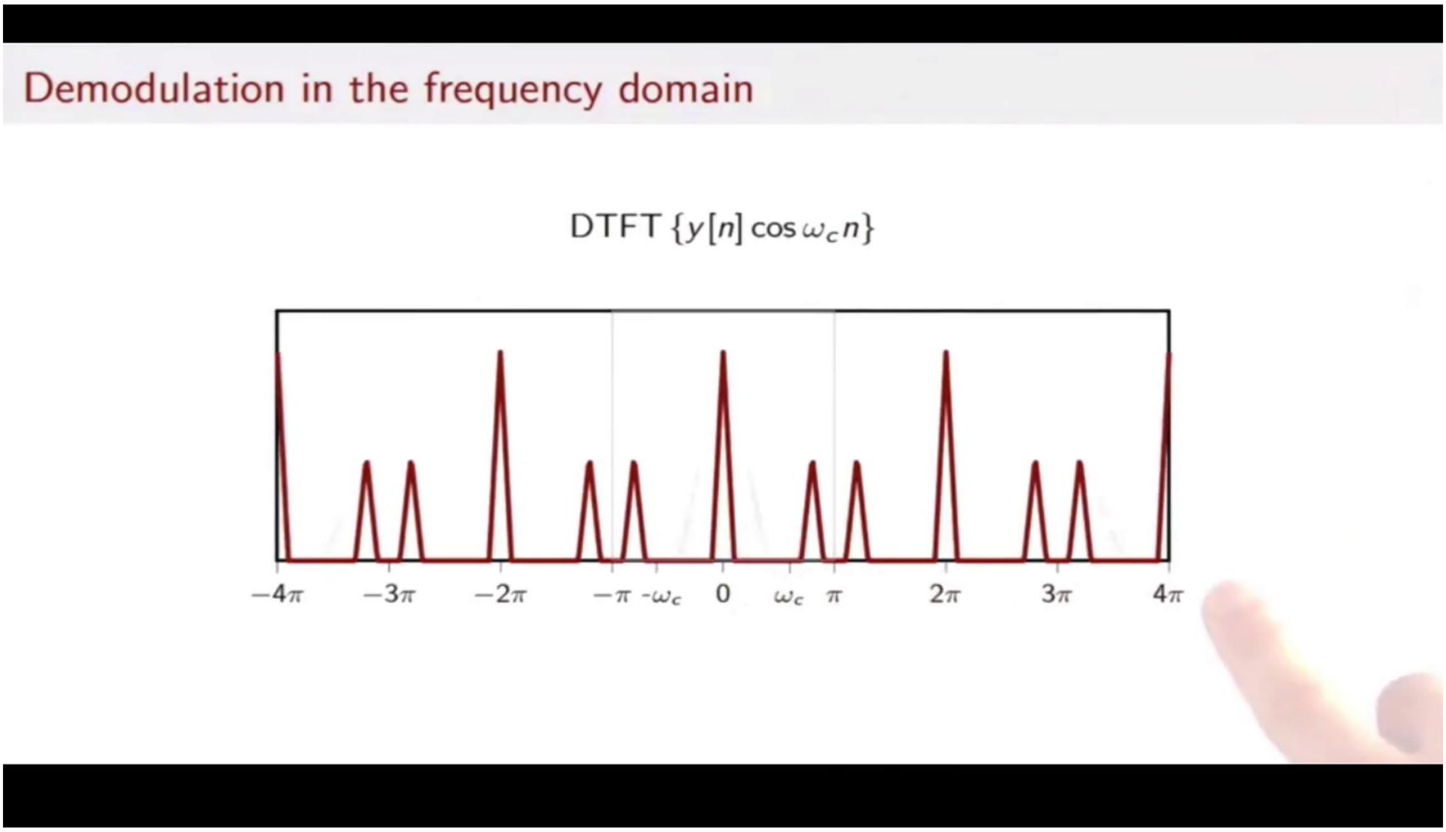


It's a green one, one to the left by omega c. It's a blue one.

Notes

Summary

6m 19s

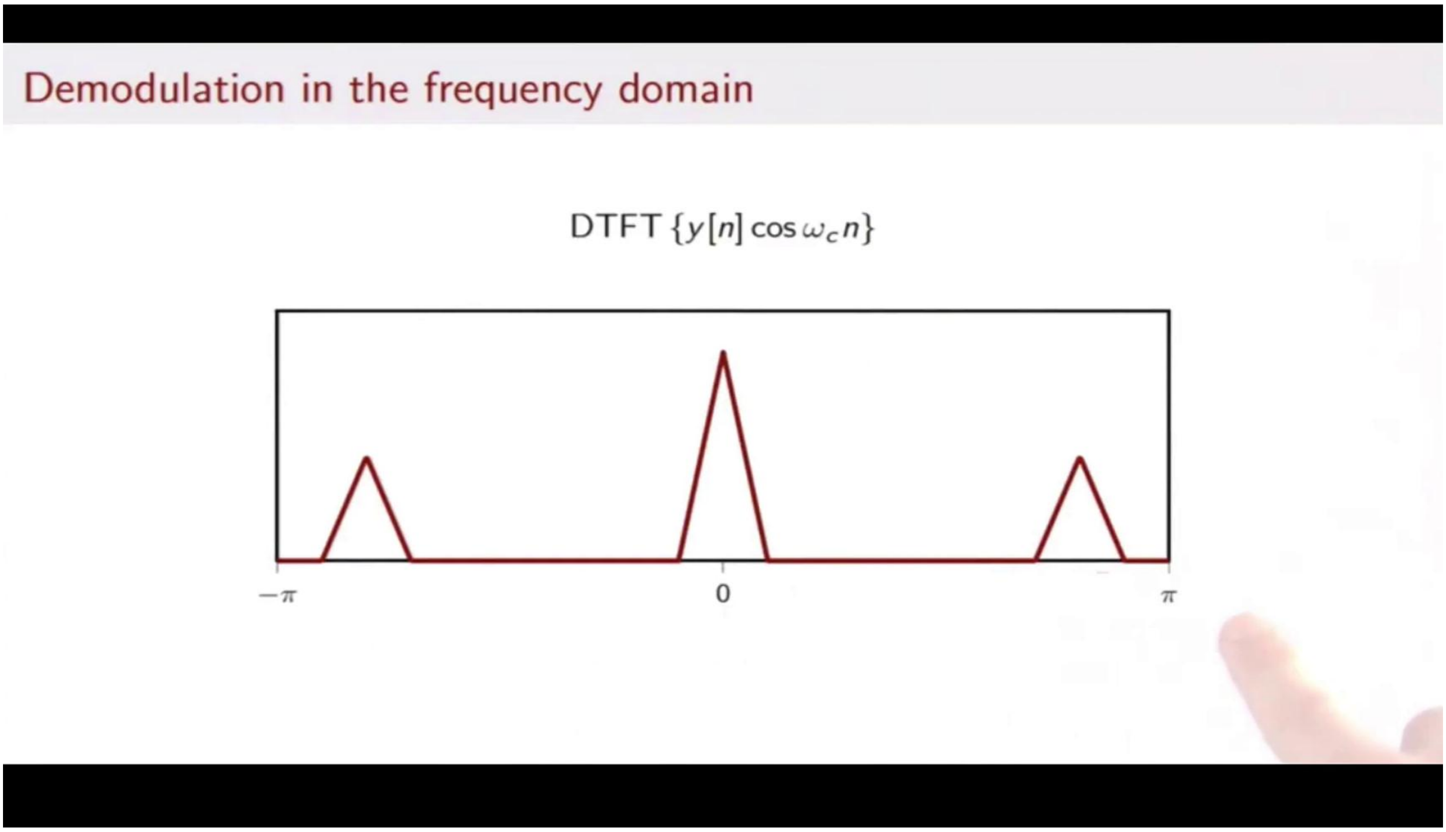


The total is the sum of these two, which has a peak around the origin, which is of height two and the two other peaks, which are around pi.

Notes

Summary

6m 24s



We have now the picture of the DTFT of the demodulated version. It looks like the original spectrum around the origin, but it has these two peaks closer to minus pi and pi, which were not present in the original signal.

Notes

Summary

6m 37s

Demodulation in the frequency domain

- we recovered the baseband signal exactly...
- but we have some spurious high-frequency components
- in the next Module we will learn how to get rid of them!

We have the baseband, but we have these two spurious high-frequency components and we will have to learn how to actually get rid of them. This will be the topic of the next module.

Notes

Summary

6m 54s