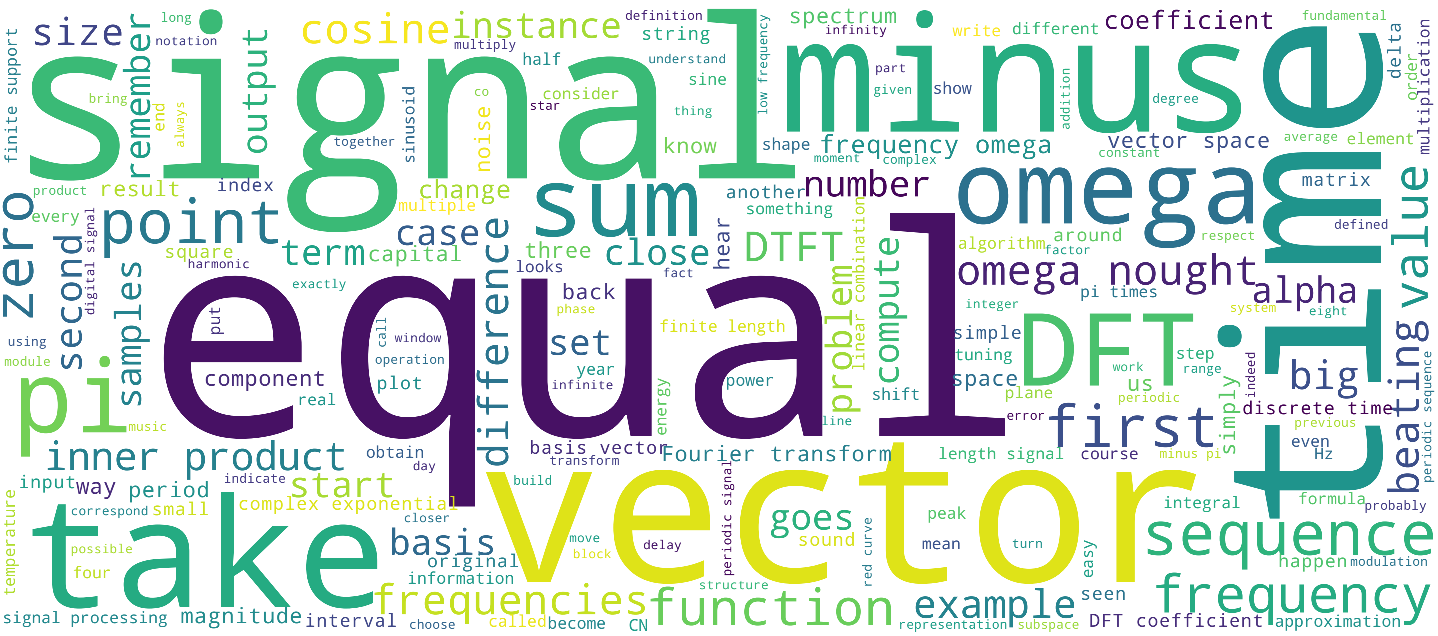
Another application: tuning a guitar

Problem (abstraction):

- reference sinusoid at frequency ω_0
- tunable sinusoid of frequency ω
- make $\omega = \omega_0$ "by ear"



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Video

The procedure

1. bring ω close to ω_0 (easy)
2. when $\omega \approx \omega_0$ play both sinusoids together
3. trigonometry comes to the rescue:

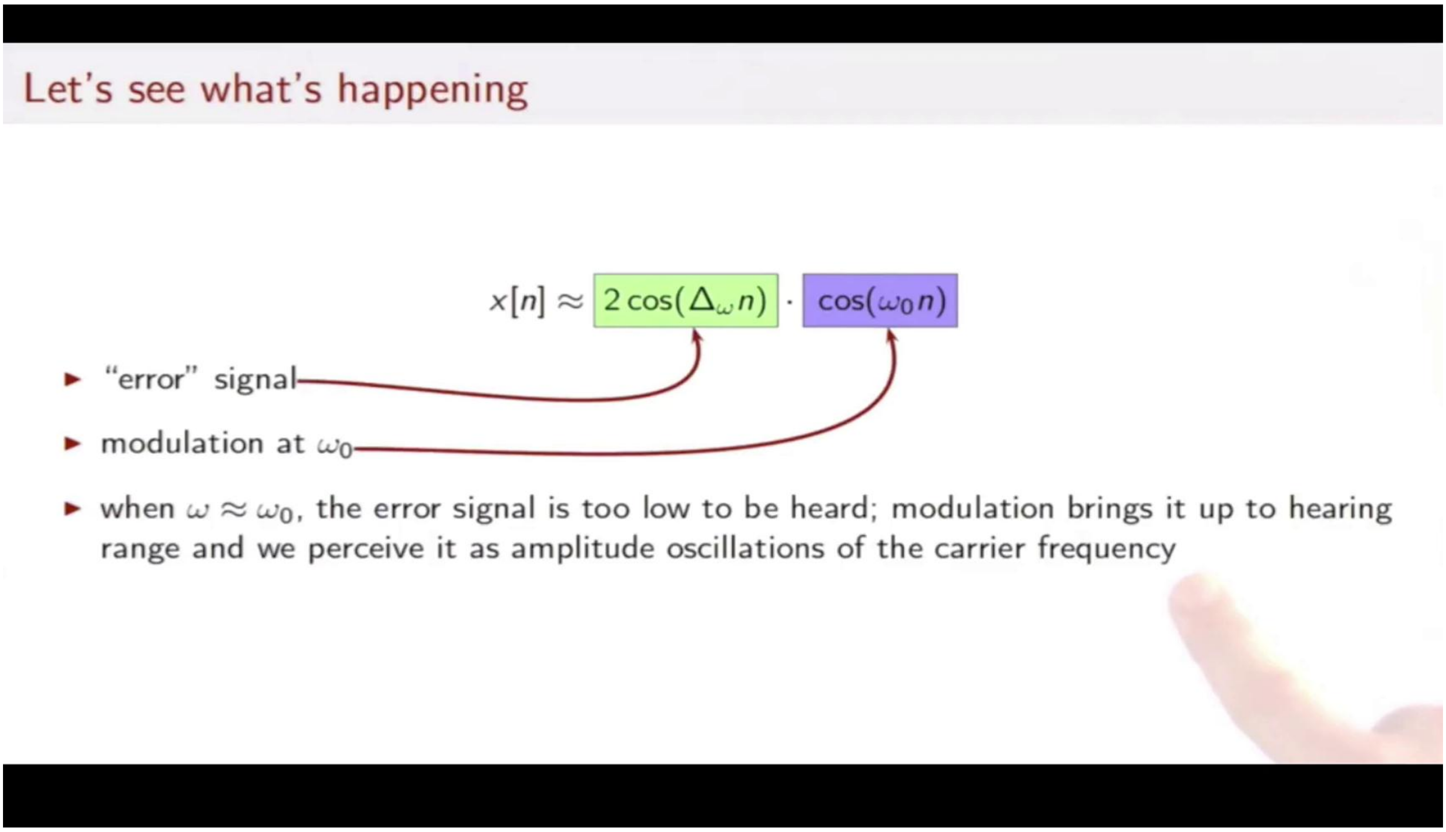
$$\begin{aligned} x[n] &= \cos(\omega_0 n) + \cos(\omega n) \\ &= 2\cos\left(\frac{\omega_0 + \omega}{2} n\right)\cos\left(\frac{\omega_0 - \omega}{2} n\right) \\ &\approx 2\cos(\Delta_\omega n)\cos(\omega_0 n) \end{aligned}$$

Finally, we're going to see a real application, a really useful application that is it is tuning your guitar. The abstraction of the problem is that you have reference sinusoid at some frequency omega nought, you have a tunable sinusoid of frequency omega, and we would like to make omega and omega nought as close as possible actually equal, and this only by listening to it. What we are going to hear is a beating between these two frequencies when they are close enough. Then by tuning, we can bring this beating to essentially frequency zero at what point omega is equal to omega nought. We have tuned our guitar string with respect to a reference frequency. How are we going to go about this? Well, first, we bring omega close to omega nought. That's easy if you have a minimum of musical ear. When these two frequencies are close, we play both sinusoids together. Then we have to remember trigonometry and we write x n, which is a sum of cos omega nought n plus cos of omega n in terms of a sum and a difference of these two frequencies, which finally can be written approximately as two times the cos of the difference, delta omega times then and the cosine of the base frequency omega nought.

Notes

Summary

0m 00s

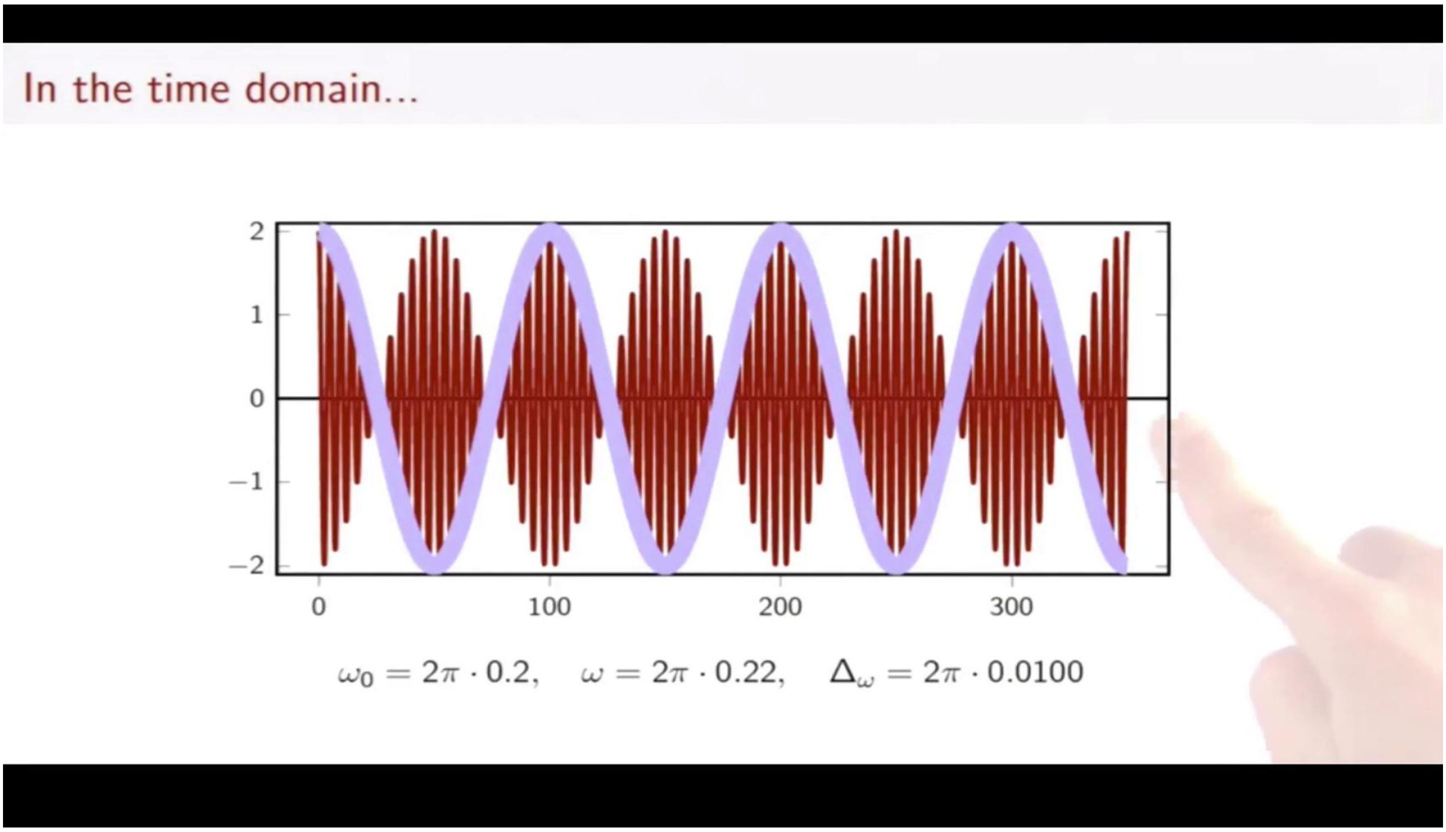


From this formula, we see there are two components. There is the error signal, the cosine of delta omega n and there is a modulation signal, the cosine at omega nought. When omega is close to omega nought, the error signal is very low frequency, so we cannot really hear it because it's such a low frequency. The modulation will bring it up to the hearing range and we are going to actually hear it as an oscillation of the carrier frequency. We're going to see this pictorially in just a moment.

Notes

Summary

1m 26s

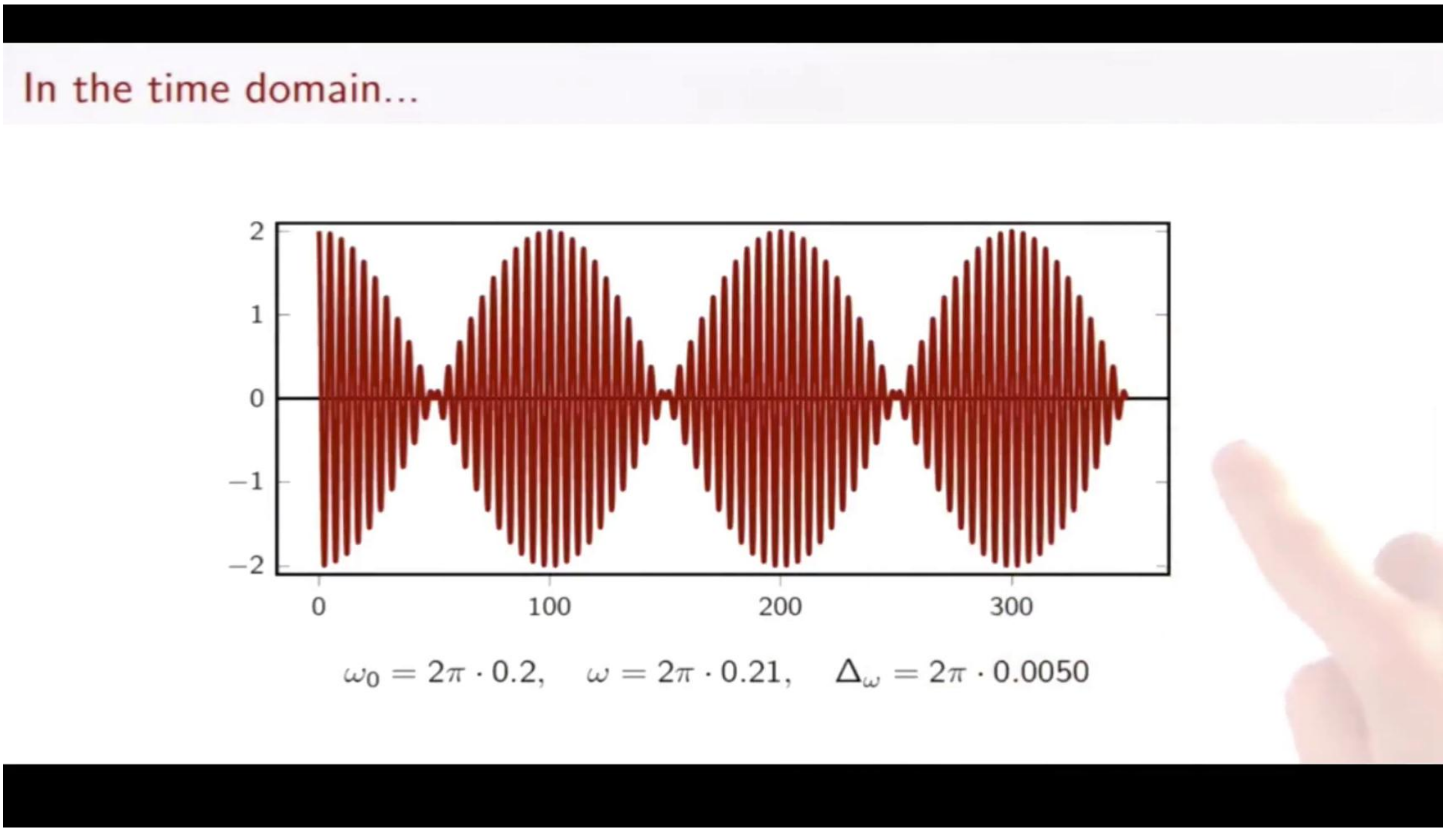


Let's look at the pictorial demonstration here. We start omega zero is 2 pi times 0.2, omega is 2 pi times 0.22. The difference, which is actually half of the difference between the two is 2 pi times 0.01. We see now, interestingly, we have the carrier frequency, which is a red curve modulated by the difference by cosine of delta of omega. We see this the beating in blue overlaid to the red curve, which is modulation.

Notes

Summary

1m 58s



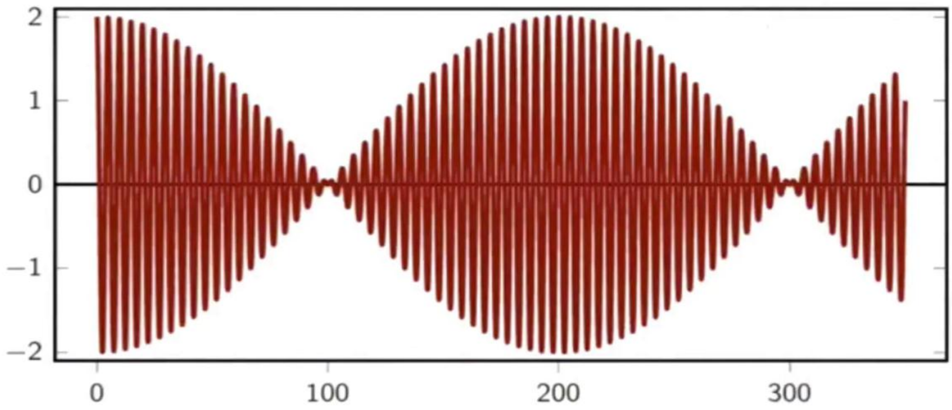
We can change the frequency omega to 0.21 times 2 pi. The difference now is 0.005. The beating is slower. We pick omega is equal to 2 pi times 0.205.

Notes

Summary

2m 36s

In the time domain...



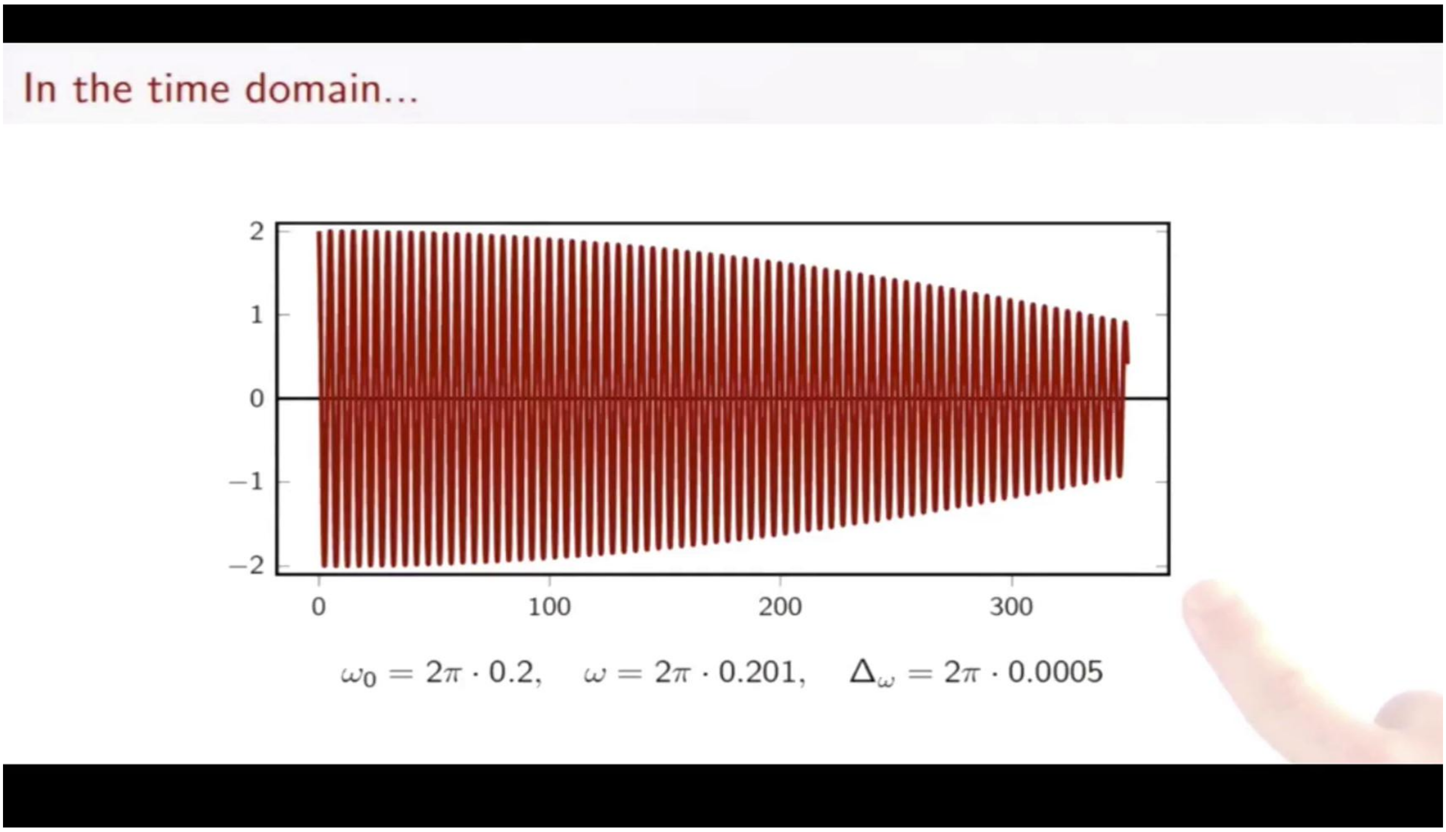
$$\omega_0 = 2\pi \cdot 0.2, \quad \omega = 2\pi \cdot 0.205, \quad \Delta_\omega = 2\pi \cdot 0.0025$$

The beating is even slower.

Notes

Summary

2m 51s



Here we take an example where omega is very close to omega nought. The beating is extremely slow and we almost see only the modulating frequency omega nought and the very slow variation due to the beating by cosine of delta of omega.

Notes

Summary

2m 54s

It's time to see a video demonstration how to tune a guitar using this very simple principle. You probably have seen musicians do it on stage. Now, you understand the math behind it.

Notes

Summary

3m 13s

Okay, after all these math, let's try to do something useful, like tuning an electric base. An electric base has an E string, which is a frequency of 41.2 Hz, an A string which has a frequency of 55 Hz. To use the result we have just seen, we want to find two frequencies that we want to make equal. As long as they are nought, there will be a beating that we can adjust. We are going to hear this in just a moment. The first harmonic of the E string here is at 83.4 Hz. The third harmonic is at 164.8. For the A string, the first harmonic is at 110 and the second harmonic is at 165. We're going to use these two harmonics to do the tuning. You can hear when they are out of tune. There is a beating.

Notes

Summary

3m 26s

As you get them closer and closer, the beating slows until it's actually zero beating and then the two frequencies are similar.

Notes

Summary

4m 24s

Then we can start playing something like or something else.

Notes

Summary

4m 34s