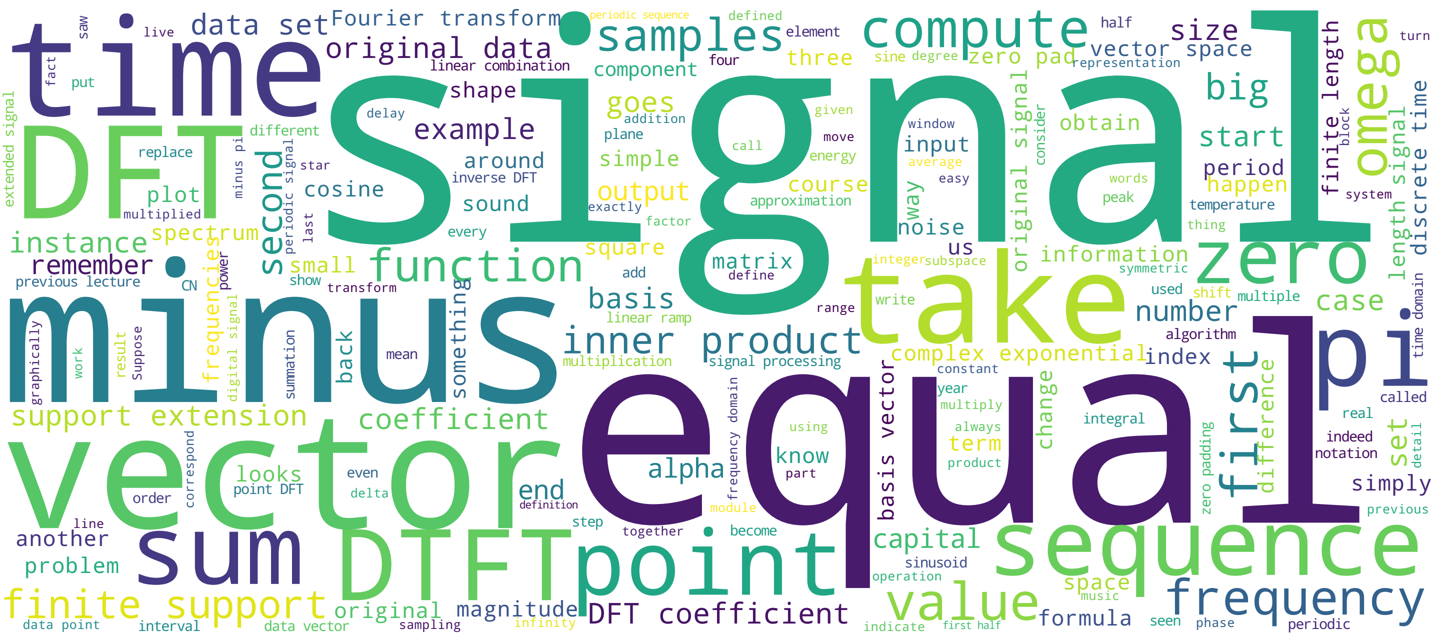
About zero-padding

When computing the DFT numerically
one may "pad" the data vector with zeros to obtain "nicer" plots



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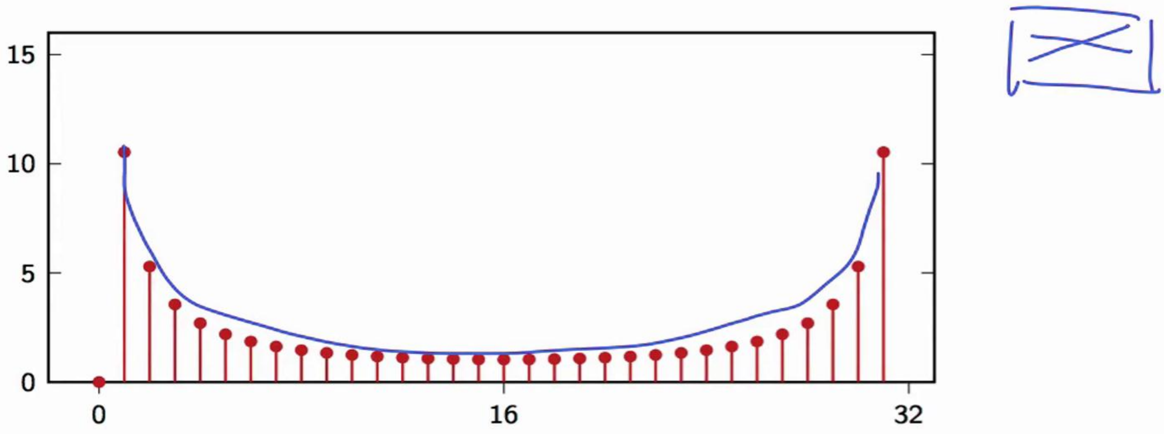
Video

EPFL

DFT of 32-tap sawtooth



$\mathbf{x} = [x_0\, x_1\, \ldots\, x_{31}]$

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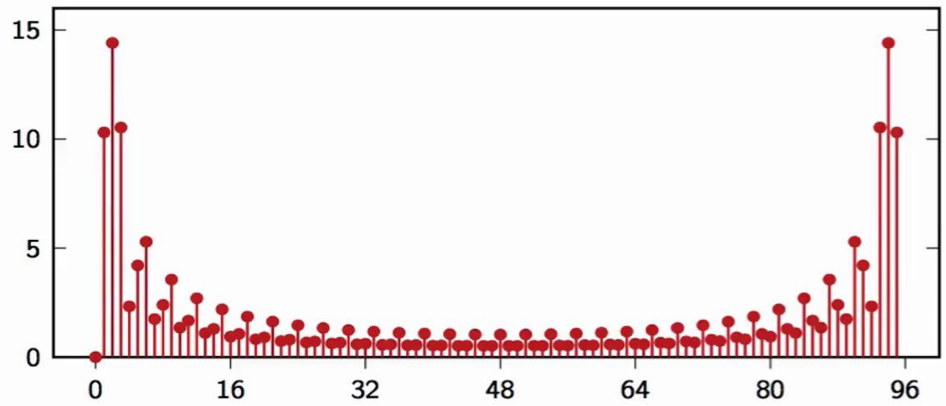
Suppose you have a set of capital and data points and you want to compute the DFT numerically as in Python or MatLab. The DFT algorithms that you find in numerical packages, allow you to specify the length of the DFT, in other words, you can compute the DFT over a length that is longer than the length of the data set, and this is achieved by zero padding the original data vector, namely by appending the proper number of zeros to the end of the data set. Now, when you do that, when you zero pad vector and you compute the DFT, it looks like the set of DFT coefficients that you obtain after zero padding is richer. Shows more detail about the spectral characteristics of the signal. But this is really not the case. It's just an illusion and we will show why that is so. Let's start with a simple example. We take the linear ramp that we have used in the previous lecture going from minus one to one over 32 points. If we compute the DFT of the signal, we go from 32 samples in the time domain to 32 DFT samples in the frequency domain and we have the shape that we saw in the previous lecture, this hyperbolic shape here. Now, suppose we tried to zero pad this data vector and we extended to 96 points.

Notes

Summary

0m 00s

DFT of 32-tap sawtooth, zero-padded to 96 points



$$\mathbf{x} = [\underbrace{x_0\, x_1\, \ldots\, x_{31}}\ \underbrace{0\, \ldots\, 0}_{64}]$$

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Namely, we take the linear ramp for the first 32 points and then we add 64 zeros at the end of the vector. We now take the DFT. It's a 9-6 point DFT, and indeed it looks like we have more data. It looks like the plot is richer and something's happening. But this is just an illusion and we will show mathematically why.

Notes

Summary

1m 18s

About zero-padding

$\mathbb{C}^N$

$$x_M[n] = \begin{cases} x[n] & \text{for } 0 \leq n < N \\ 0 & \text{for } N \leq n < M \end{cases}$$

$\mathbb{C}^M$

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Let's define the signal X underscore M as the extension of the original data set. This is again, a finite length signal that lives in CM which is obtained from the original signal that lives in CN, and we extend the original signal with M minus N zeros.

Notes

Summary

1m 40s

About zero-padding

$$
\begin{aligned}
X_M[h] &= \sum_{n=0}^{M-1} x'[n]e^{-j\frac{2\pi}{M}nh} = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{M}nh} \\
&= \sum_{n=0}^{N-1}\left(\frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}nk}\right)e^{-j\frac{2\pi}{M}nh} \\
&= \frac{1}{N}\sum_{k=0}^{N-1} X[k]\left(\sum_{n=0}^{N-1} e^{-j\left(\frac{2\pi}{M}h-\frac{2\pi}{N}k\right)n}\right) \\
&= \bar{X}(e^{j\omega})\big|_{\omega=\frac{2\pi}{M}h}
\end{aligned}
$$

DTFT

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When we take the DFT of this extended signal, well, we can write out the DFT formula and we see that the sum here goes from zero to M minus one. This is the new extended signal. What happens is that the last M minus N samples of this extended signal are equal to zero. So we can just replace the upper limit of the summation by N minus one. Here we have N, here we have M. Okay, now we use the same trick that we've used in the previous lecture. We replace the value of XN by the inverse DFT formula. So we compute X of N as the inverse DFT in N. Here we have the sum of the inverse DFT of the original data vector and then we interchange the summation and we obtain a formula that once again links the original DFT coefficients to something here that is a sum of complex exponentials. Now, if you compare this formula to the formula for the DTFT of finite support extension of a finite data set, you will see that the value of the zero pad at the DFT is equal to the value of the DTFT of the finite support extension of the original data set, computed in omega, which is equal to 2 pi over M multiplied by H. The coefficient of the zero-padded DFT in H is equal to the sample of the DTFT of the finite support extension computed in omega equal to two pi over M times H.

Notes

Summary

2m 02s

About zero-padding

- zero padding does not add information
- a zero-padded DFT is simply a sampled DTFT of the finite-support extension

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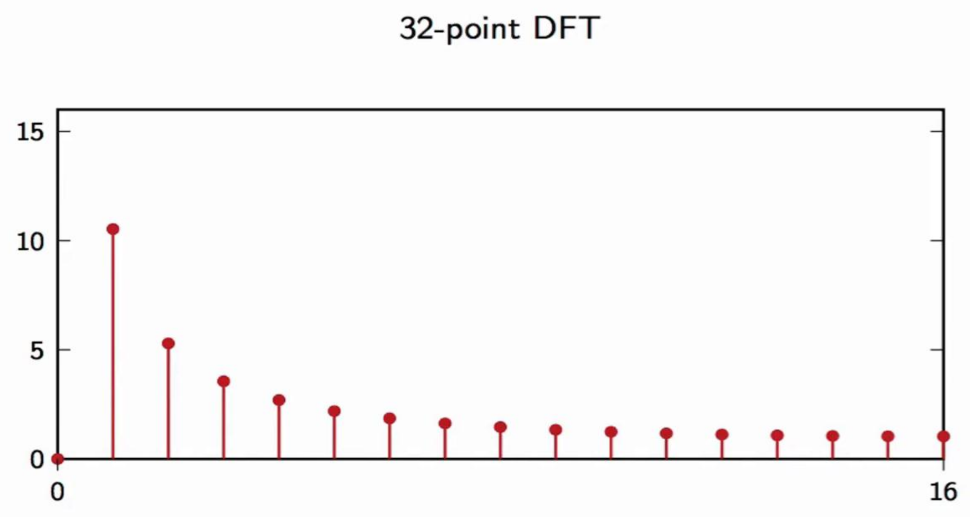
Zero padding therefore does not add any information. If we zero pad a signal and compute the DFT, we're just sampling the DTFT of the finite support extension of that signal more finally over the frequency axis.

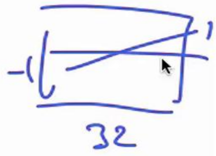
Notes

Summary

3m 47s

DFT of 32-tap sawtooth, zero-padded





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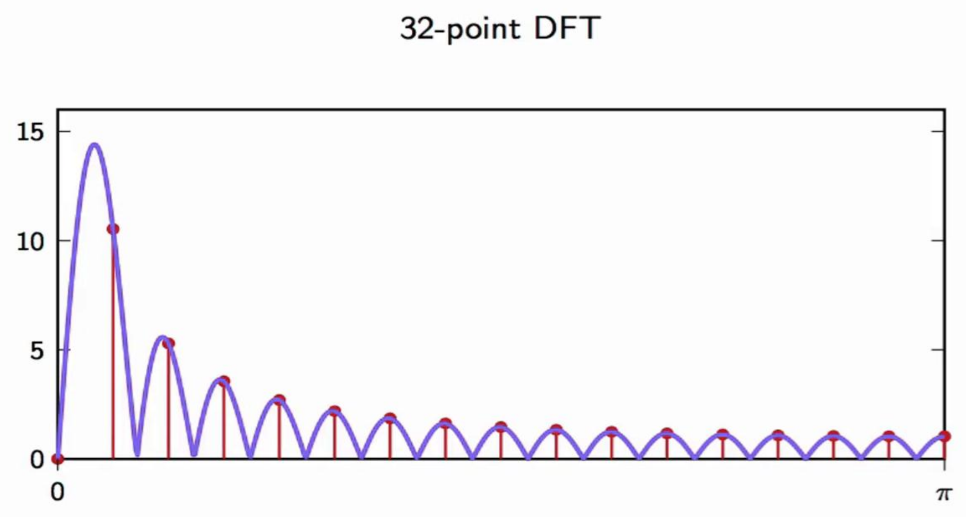
Let's see this graphically. This is the 32-point DFT of the linear ramp from minus one to one. We're just showing the first half of the DFT coefficient the rest is just symmetric.

Notes

Summary

4m 04s

DFT of 32-tap sawtooth, zero-padded



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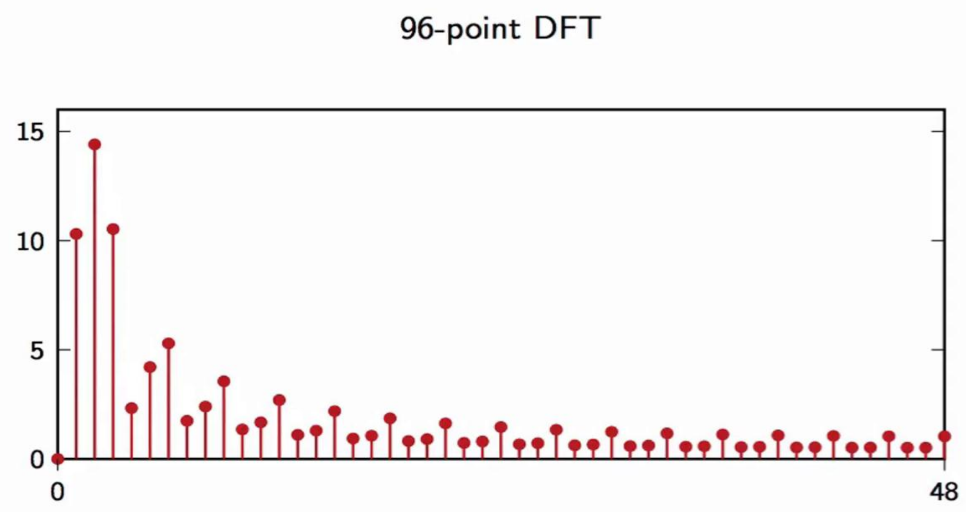
If we compute the DTFT of the finite support extension of this 32-point signal, we obtain this interpolated curve here, we saw that in the previous lecture, we just interpolating the DFT points with this sync-like interpolator, which is just the DTFT of the rectangular signal. Now if we zero pad, the original signal and compute the DFT, we're going to be sampling this blue curve more finally.

Notes

Summary

4m 19s

DFT of 32-tap sawtooth, zero-padded



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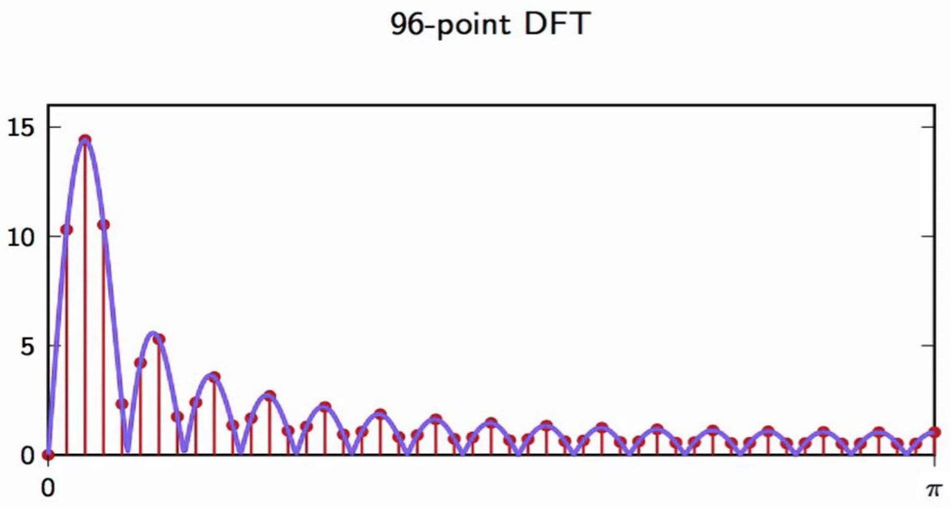
A 9-6 six point DFT that look like this. Well, if we superimpose the DTFT of the finite support extension of the original data, that's what you have.

Notes

Summary

4m 48s

DFT of 32-tap sawtooth, zero-padded



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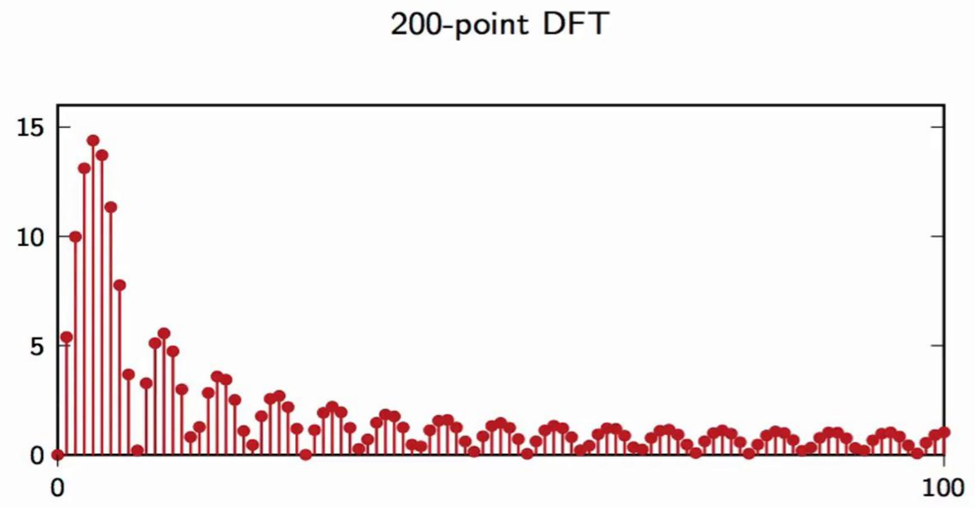
The samples are just finer samples of the original data set.

Notes

Summary

4m 58s

DFT of 32-tap sawtooth, zero-padded



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The zero padded DFT does not add more information. It just eventually converges to the DTFT of the finite support extension of the signal, which of course once again, only uses the original data points.

Notes

Summary

5m 04s