Theoretical foundations

Ingredients of choice theory

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Introduction to choice models

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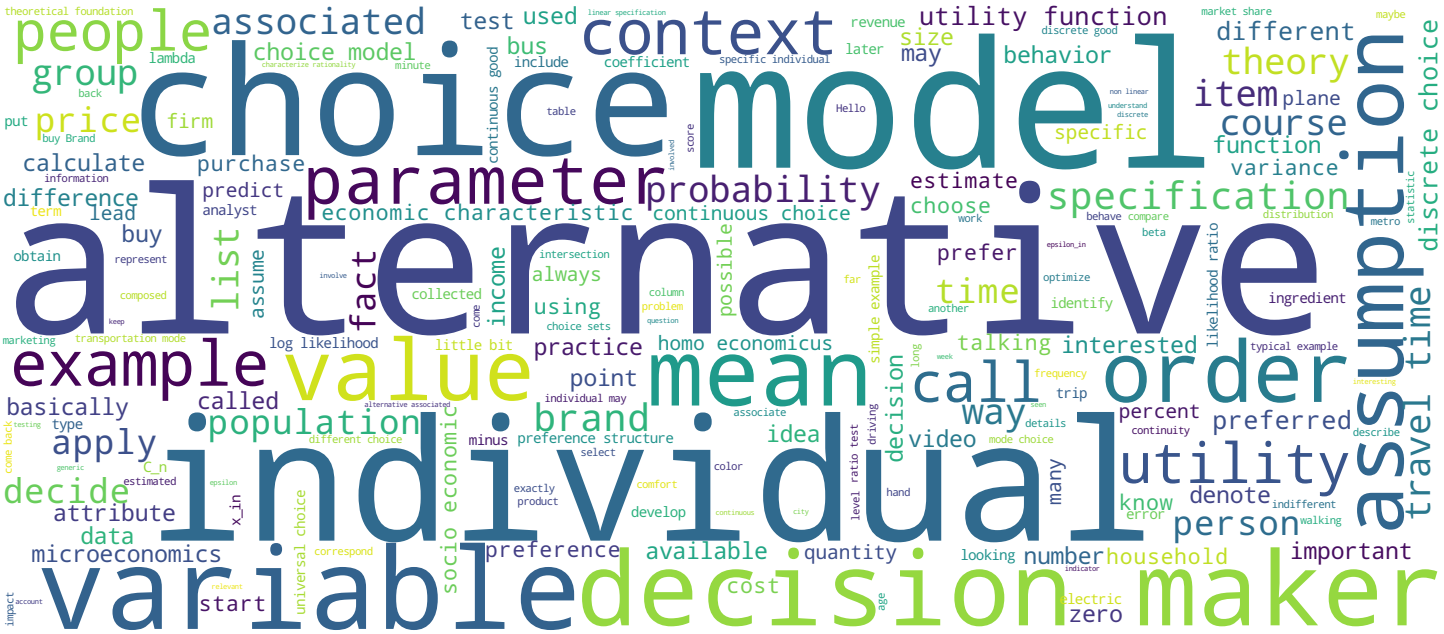

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Video

Ingredients of choice theory

Hello, this week will be dedicated to the theoretical foundations of choice models. In this video, we will start to look at the different ingredients that we need to develop a choice theory. And afterwards we will go in more details through these ingredients.

Notes

Summary

0m 04s

Choice theory

Theory of behavior that is

- descriptive: how people behave and not how they should
- abstract: not too specific
- operational: can be used in practice for forecasting

What we need is a theory of choice behavior which is descriptive: we want to predict how people behave and not how they should behave. It has to be abstract, meaning that we want to be able to apply it in different contexts: in transportation demand analysis, in marketing, in business, finance, what have you.. but also and most importantly it has to be operational. The theory must lead to operational models that can be used in practice for forecasting.

Notes

Summary

0m 21s

Building the theory

Define

1. who (or what) is the decision maker,
2. what are the characteristics of the decision maker,
3. what are the alternatives available for the choice,
4. what are the attributes of the alternatives, and
5. what is the decision rule that the decision maker uses to make a choice.

In order to build the theory, we need to go through these steps. First we need to identify who is the decision maker, who is the person we want to model the behavior. Two, we need to know information about this individual, this decision maker. We need to know his characteristics. We will call these socio-economic characteristics. Then, we need to identify the choice problem and in particular what are the alternatives that are available to this individual to make a choice. The next step will be to be able to identify variables that describe these alternatives. We will call these variables 'attributes'. And finally, once we have collected all this data about the person, the decision maker, and the alternatives, we need to identify somehow the algorithm, the decision rule, that the decision maker is actually applying in order to come up with a choice. OK.

Notes

Summary

0m 59s

Decision maker

Individual

- a person
- a group of persons (internal interactions are ignored)
  - household, family
  - firm
  - government agency
- notation: n

So let's start with the decision maker. In this course, we will assume that the decision maker is an individual, is a physical person. Well actually, this can be a little bit generalized. If you want to apply the methodology to a group of persons such as a household, a family or maybe a firm or an agency, this is fine. But in this case, it means that you ignore all internal interactions that are involved in the decisions of this group of individuals. So the individual, the person, is the atomic representation of the decision maker. And throughout the course, we will denote this individual, this decision maker by the letter n. And so n will be used as an index characterizing the individual.

Notes

Summary

1m 57s

Characteristics of the decision maker

Disaggregate models

Individuals

- face different choice situations
- have different tastes

Characteristics

- income
- sex
- age
- level of education
- household/firm size
- etc.

We are developing disaggregate models. Therefore we want to be able to model the fact that different individuals are facing different choice situations, but also that they have different tastes and different behaviors. In order to do that, we need to collect some information about them. We will call this information, the variables characterizing the decision maker, we will call them "socioeconomic characteristics". And they can involve many of the things that are usually available from census data, or other survey data, like income, sex, age, level of education,... If you are talking about the household or a firm, the size of this household or the size of this firm etc. All these variables about the decision maker are collected and are called "characteristics".

Notes

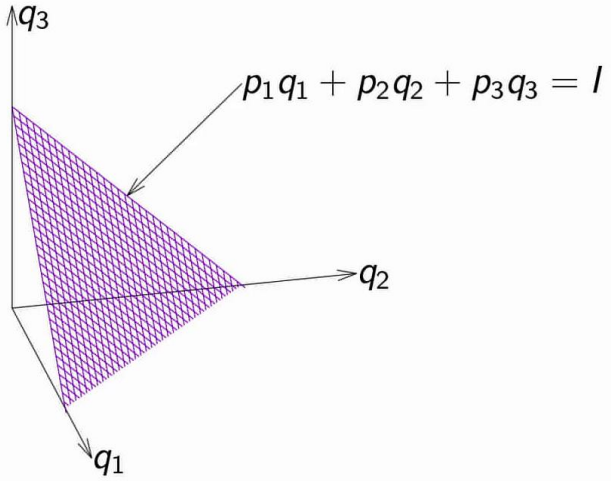
Summary

2m 48s

Alternatives: continuous choice set

Commodity bundle

- q_1: quantity of milk
- q_2: quantity of bread
- q_3: quantity of butter
- Unit price: p_i
- Budget: I



Now the alternatives. We will look at discrete choice, as opposed to continuous choice. So in order to make the difference, let me define what a continuous choice set is first, and then we'll move to the discrete choice afterwards. So a typical example is when you want to purchase different items. In this example, I use milk, bread and butter. And what you need to do is to decide which quantity of each of these items you will purchase given the price of these items, and given the total budget that you have available. This is a typical example of microeconomics. And we'll come back to this latter on. In this case, the choice set, so all the possible decisions that you can make, can be characterized by the intersection of this plane (the plane with equation p1 q1 + p2 q2 + p3 q3 = I, so basically all possible combinations that achieve the feasible budget), and the intersection of this plane with the positive orthant, because of course you can buy only positive quantities of each of these items. In this case, the choice set is continuous. It is represented actually by this triangle.

Notes

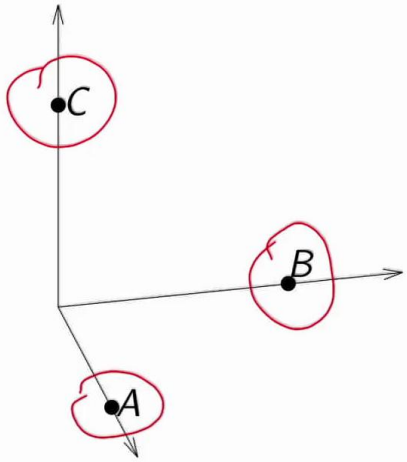
Summary

3m 39s

Alternatives: discrete choice set

List of alternatives

- Brand *A*
- Brand *B*
- Brand *C*



In our context, we are interested in discrete choice. So typically when the choice is made among a list of alternatives. Suppose that you want to purchase some item and you have to decide which brand you will purchase and you can decide between brand A, brand B, brand C. In this case if I use the same presentation of the choice set as I did in the previous slide, each of the axes is associated with one of the brands. Now the only decision that I can make is 0, I don't buy this brand, or 1, I buy this brand. So this point here which is labeled A, corresponds to the fact that I decide to buy Brand A. And in this case I cannot buy Brand B or Brand C because I can choose only one of these items. So the choice set, in this representation, is composed of the three alternatives which are represented here. This is a discrete set. Interestingly, discrete choice sets have more complex mathematical properties than continuous choice sets, as we will see in the derivation of the model.

Notes

Summary

4m 58s

Alternatives: discrete choice set

Choice set

- Non empty finite and countable set of alternatives
- Universal: $\mathcal{C}$
- Individual specific: $\mathcal{C}_n \subseteq \mathcal{C}$
- Availability, awareness

Example

Choice of a transportation mode

- $\mathcal{C}$ ={car, bus, metro, walking }
- If decision maker n has no driver license, and the trip is 12km long

$$\mathcal{C}_n = \{\text{bus}, \text{metro}\}$$

So the assumption that we will make is that the choice set is a list of alternatives, which is non-empty, finite, and countable. And, we will discuss it later on, but we will make the difference between the universal choice set which is the set of all alternatives which are relevant in the context of the model from the point of view of the analyst. But also each individual may be faced with a different choice set. So we will usually denote by C the universal choice set and we will add an index n when we mean that this choice set C_n is associated with a specific individual. In order to develop the individual choice set C_n we will need to take into account the fact that some alternatives may not be available to everybody or that a given individual may not be aware that an alternative is on the market. Let's take a simple example of a transportation mode choice model. In this case, the analyst can be interested in understanding, in a given city, if people take the car, take the bus, the metro, or walk, in order to go to work in the morning peak hour. But now, if you take a specific individual decision maker n, maybe this person has no driver's license and the trip that he has to do to reach his workplace is 12 kilometers long.

Notes

Summary

6m 07s

Alternative attributes

Characterize each alternative *i* for each individual *n*

- price
- travel time
- frequency
- comfort
- color
- size
- etc.

Nature of the variables

- Discrete and continuous
- Generic and specific

Well in this case driving a car is not an option because he's not allowed to do so and walking is not an option either because it's far too long a trip to make it walking. Therefore, this individual has a choice set which is composed of bus and metro only. Once we have identified the choice set, the list of alternatives for each individual in the population, we need to characterize them. We need to find the variables that describe these alternatives. And they can be of many types. It can be the price of the alternative, the travel time, if we're talking about transportation mode choice, the frequency of the bus, the comfort of the bus. If we are talking about marketing and you're interested in packaging, the color of the package maybe important, the size of the package may be important, and so on. The list is very long and is really application dependent. The point is that we would like to be able to involve as many variables as possible but of different nature as well. We have discrete and continuous variables. Typically travel time would be a continuous valuable while color or comfort would be a discrete variable. We will have generic and specific variables. Generic: for example, travel time.

Notes

Summary

7m 28s

Alternative attributes

Characterize each alternative *i* for each individual *n*

- price
- travel time
- frequency
- comfort
- color
- size
- etc.

Nature of the variables

- Discrete and continuous
- Generic and specific

Whatever the mode of transportation you use, you will have to face a travel time. But frequency, frequency is a variable that does not apply to driving a car. So this frequency is specific to alternatives which are related to public transportation.

Notes

Summary

8m 43s

Decision rule

Homo economicus

Rational and narrowly self-interested economic actor who is optimizing her outcome

Preferences

- $i \succ j$: i is preferred to j,
- $i \sim j$: indifference between i and j,
- $i \succsim j$: i is at least as preferred as j.

So we have two ingredients so far. On the one hand, we have the decision maker with a list of variables that we call socio-economic characteristics and on the other hand we have the list of alternatives that form the choice set. And each of them is associated with a list of variables that we call the attributes. Now let's see how do we put all of this together in order to come up with a decision, with a choice. The theoretical foundations of choice modeling are rooted in microeconomics. And, therefore, the first assumption that we will make is that individuals are actually "homo economicus". An "homo economicus" is an abstract individual who is supposed to behave in a way which is rational and narrowly self-interested. And the objective of this homo economicus is to optimize her outcome. This is the first assumption. The second assumption is that each individual is able to identify preferences. And this preference I denote them by this indicator. So the first notation here means that alternative i is preferred to alternative j. This one represents indifference between the two alternatives. And when we combine the two we have that i is at least as preferred as j. But they can be indifferent as well. So there is a way to represent the preferences.

Notes

Summary

8m 59s

Decision rule

Rationality

- Completeness: for all alternatives i and j,

$$i \succ j \text{ or } i \prec j \text{ or } i \sim j.$$

- Transitivity: for all bundles i, j and k,

$$\text{if } i \succsim j \text{ and } j \succsim k \text{ then } i \succsim k.$$

- "Continuity": if i is preferred to j and k is arbitrarily "close" to i, then k is preferred to j.

When we put these two assumptions together, we basically need to characterize rationality using assumptions about the preferences. And the main three assumptions that characterize rationality are the following. The first is completeness. It means that you can give me any pair of alternatives, I will always be able to tell you if individual n prefers i, prefers j, or is indifferent between the two. The second assumption, which is probably the most questionable in practice (but we'll come back to this later on), is transitivity. For any three alternatives i, j and k, if i is preferred to j, and j is prefer to k, than automatically i will be preferred to k. And the last one, let me call it "continuity" (with some quotation marks too, because it's not very formal) but the idea is that if i is preferred to j, and we have another alternative k which is very close to i, very similar to i, in this case k would also be preferred to j. That's what we mean by continuity here.

Notes

Summary

10m 29s

Utility

$$U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : i \rightsquigarrow U_n(i)$$

Consistent with the preferences

$$U_n(i) \geq U_n(j) \Longleftrightarrow i \succsim j.$$

- Unique up to an order-preserving transformation.
- Captures the attractiveness of an alternative.
- Measure that the decision maker wants to optimize.

The interesting thing is that if you use these assumptions, it can be shown that there exists a mathematical formula, a mathematical function, called the utility function, that associates each alternative i in a choice set with a score, a value of the utility that we denote by U_n(i). And the point is that this score, this utility, is completely consistent with the preference structure of the individual. It means that if alternative i is prefered to alternative j, it is equivalent to say that the utility of i is greater or equal to the utility of j. And this is extremely useful because now it means that if this preference structure exists, there is always a way to translate it into a mathematical formulation that we can compare and which is quantitative. So we can manipulate it, and code it in a computer. If you look at the microeconomics textbooks, you can find a lot of results about this theory of utility. You can show that this function, under some simple assumptions, is unique up to a order-preserving transformation. Of course, if you multiply all the utilities by 2, the order will be the same, it will not change anything. So the idea is that this utility captures the attractiveness of an alternative.

Notes

Summary

11m 46s

Utility

$$U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : i \rightsquigarrow U_n(i)$$

Consistent with the preferences

$$U_n(i) \geq U_n(j) \iff i \succsim j.$$

- Unique up to an order-preserving transformation.
- Captures the attractiveness of an alternative.
- Measure that the decision maker wants to optimize.

And it's a measure that the decision maker wants to optimize. So basically this leads to the assumption that each decision maker is actually a maximizer, is a person who solves an optimization problem which is: identifying the alternative associated with the maximum utility.

Notes

Summary

13m 14s

Behavioral assumptions

- the preference structure of the decision maker is fully characterized by a utility associated with each alternative
- the decision maker is a perfect optimizer
- the alternative with the highest utility is chosen

So, in summary, the behavioral assumptions that we will use in our context, is that the preference structure of the decision maker is completely characterized by a utility function which is associated with each alternative. And the decision maker is assumed to be an optimizer. And a perfect one, who is able to choose the alternative associated with the highest utility.

Notes

Summary

13m 32s

Behavioral assumptions

Now, we will start to go into a little bit more details and to do that we will go back to the basics of microeconomics. So I will provide just a quick reminder of microeconomics for the context of continuous goods. But then we will see how this theory can be extended for the case of discrete goods, that will lead to the discrete choice context that we are looking for.

Notes

Summary

14m 01s