Theoretical foundations

Microeconomic consumer theory

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Introduction to choice models

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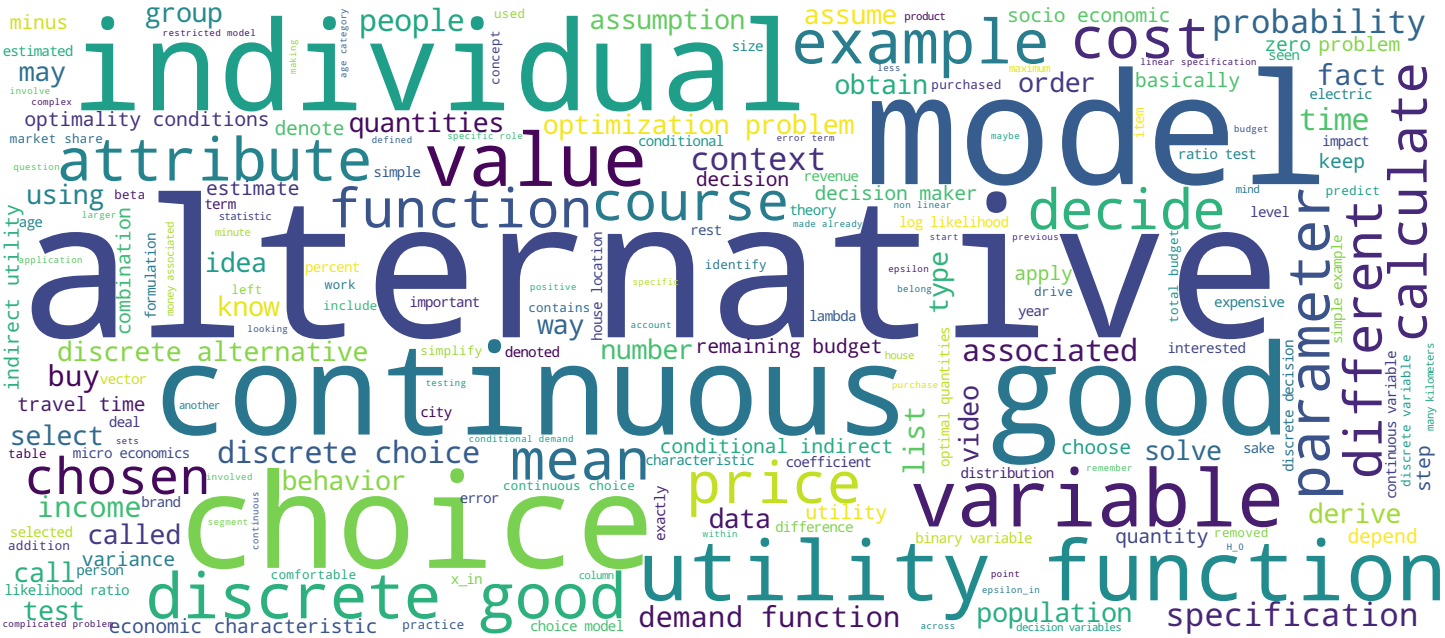

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Video

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The case of discrete goods

So we have seen how micro-economics models the behavior of economic agents in the context of continuous goods. They solve an optimization problem and by calculating the optimality conditions of this optimization problem, we can derive a demand function. But we are interested in discrete goods. So let's see how this theory extents to the case discrete goods (actually of both continuous and discrete goods), and see why it becomes more complex and also how we will deal with this complexity. This is the objective of this video.

Notes

Summary

0m 05s

Microeconomic theory of discrete goods

The consumer

- selects the quantities of continuous goods: $Q = (q_1, \ldots, q_L)$
- chooses an alternative in a discrete choice set $i = 1, \ldots, j, \ldots, J$
- discrete decision vector: $(y_1, \ldots, y_J)$, $y_j \in \{0, 1\}$, $\sum_j y_j = 1$.

Note

- In theory, one alternative of the discrete choice combines all possible choices made by an individual.
- In practice, the choice set will be restricted for tractability

Now, in this context the consumer, in addition to selecting the quantities of each of the continuous goods, has also the opportunity to select discrete goods. Indeed, there are many discrete decisions in life. You can decide to go to the university or not. You can decide to get married or not. You can decide to buy a car. And if you decide to buy a car which brand, which size. You can decide many things that can be represented by discrete choice set. So here, we will assume that this choice set contains J alternatives whatever they are. And it can be the combination of a lot of different discrete alternatives. Therefore, we have to associate decision variables to this choice set. These decision variables are denoted by y. And the idea is that it's a binary variable. Binary variable means that it can take only values 0 and 1. 0 being associated with the fact that you don't select this alternative and 1 being associated with the fact that you choose this alternative. And for the sake of simplicity, we will assume that each individual can select only one among the J alternatives. So the sum over J of these y_j is equal to 1. Of course, as I said, although the theory says that one alternative can combine many possible choices made by an individual, when you do a concrete and an operational study, you need to restrict the choice set for the sake of tractability.

Notes

Summary

0m 39s

Example

Choices

- House location: discrete choice
- Car type: discrete choice
- Number of kilometers driven per year: continuous choice

Discrete choice set

Each combination of a house location and a car is an alternative

So let's take an example where we have both continuous choices and discrete choices. The example is an example where a family has to decide where to live. What would be the location of the house. And what car to buy. And also, how many kilometers to drive during the year. The house location is typically a discrete choice. You need to decide which city and within the city which one of the houses should be chosen and should be selected. Car type is also a discrete choice. It depends on the brands and the type of cars. But the number of kilometers driven per year is a continuous choice. And of course all these decisions are interrelated. Because if I decide to drive a low number of kilometers, basically I will choose a small car, while if I drive many kilometers, I may decide to buy a more expensive car or a larger car or more comfortable car. In this example the discrete choice set is composed of all combinations of a house location and a car. So the combination of these two decisions is one alternative in the context of the theory that we are describing now.

Notes

Summary

2m 08s

Optimization problem

$$\max_{Q,y} \widetilde{U}(Q, y, \tilde{z}^T y)$$

subject to

$$p^T Q + c^T y \leq I$$
$$\sum_j y_j = 1$$
$$y_j \in \{0, 1\}, \forall j.$$

where $c^T = (c_1, \ldots, c_i, \ldots, c_J)$ is the cost of each alternative

Solving the problem

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly

Of course, the rest is the same. So we have a utility function. But now, the utility function depends not only on the quantities of the continuous goods that the decision maker is purchasing, but also as a function of the discrete choice decisions that he's making. And, of the attributes of the discrete alternatives. So we denote by z tilde the list of attributes of the J alternatives. And when you multiply by y, basically what you obtain is the list of attributes of the chosen alternative. And so again we have an optimization problem to solve. But now, this optimization problem involves two sets of variables. The Q variables are the continuous variables, the quantity of each of the continuous goods to be purchased or to be chosen. And now we have, in addition, the discrete variables, the binary variables that identify the chosen discrete alternative. The budget constraint is the same. But we have of course to take into account the cost of selecting alternative i. So now the total money which is spent is the money associated with the choice of continuous good p^T Q, and the money associated with the purchase of the discrete good which is c transposed y, where c is the vector of the cost of each alternative.

Notes

Summary

3m 22s

Optimization problem

$$\max_{Q,y} \widetilde{U}(Q, y, \tilde{z}^T y)$$

subject to

$$p^T Q + c^T y \leq I$$
$$\sum_j y_j = 1$$
$$y_j \in \{0, 1\}, \forall j.$$

$Q \geq 0$

where $c^T = (c_1, \ldots, c_i, \ldots, c_J)$ is the cost of each alternative

Solving the problem

- Mixed integer optimization problem
- No optimality condition
- Impossible to derive demand functions directly

And then we need to add the constraints that only one alternative can be chosen, and the discrete variables can take the values only 0 and 1. And as before there is the constraint then that Q must be nonnegative. This problem is more complicated than the previous one that we had for the continuous goods. It's called a mixed integer optimization problem. Integer because it involves integer variables (these are the y variables), and we call it mixed because it also includes continuous variables (variables Q). Such an optimization problem has no optimality condition. So we cannot do what we did in the case of the continuous goods. We cannot derive a demand function based on the optimality conditions because there is no such optimality conditions. So we need to deal with it in a different way.

Notes

Summary

4m 46s

Solving the problem

Step 1: condition on the choice of the discrete good

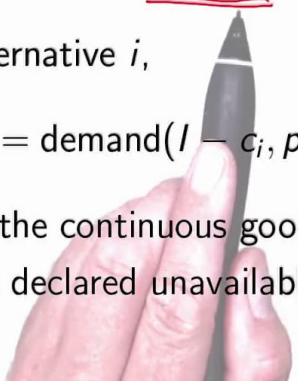
- Fix the discrete good, that is select a feasible y.
- The problem becomes a continuous problem in Q.
- Conditional demand functions can be derived:

$$q_{\ell|y} = \text{demand}(I - c^T y, p, \tilde{z}^T y),$$

or, equivalently, for each alternative i,

$$q_{\ell|i} = \text{demand}(I - c_i, p, \tilde{z}_i).$$

- $I - c_i$ is the income left for the continuous goods, if alternative i is chosen.
- If $I - c_i < 0$, alternative i is declared unavailable and removed from the choice set.



As often when we have to solve a complicated problem, we try to reduce it to a simpler problem that we are able to solve. This is what we will do here. We will try to transform this complicated problem to a sequence of problems which involve only continuous goods. Let's see how it works. We will solve the problem in two steps. The first step is that we will assume that the decision based on the discrete goods has been made already. We fix the discrete good. We say, OK, this person has made already the discrete decision, so the value of y is set. Now the only remaining variables are the Q, the continuous variables. Therefore, we can do what we did before. We can calculate the optimality conditions and we can derive a demand function. We call this a conditional demand function because it's conditional on the choice of the discrete good. So Q_l given y is the demand (so, the continuous quantities that will be purchased) assuming that y is the discrete decision. And it's a function of the remaining budget, which is the total budget I minus the cost of the discrete alternative. It's also a function of the prices as before and the attributes of the discrete goods.

Notes

Summary

5m 37s

Solving the problem

Step 1: condition on the choice of the discrete good

- Fix the discrete good, that is select a feasible y.
- The problem becomes a continuous problem in Q.
- Conditional demand functions can be derived:

$$q_{\ell|y} = \text{demand}(I - c^T y, p, \tilde{z}^T y),$$

or, equivalently, for each alternative i,

$$q_{\ell|i} = \text{demand}(I - c_i, p, \tilde{z}_i).$$

- $I - c_i$ is the income left for the continuous goods, if alternative i is chosen.
- If $I - c_i < 0$, alternative i is declared unavailable and removed from the choice set.

Equivalently, you can write this from each alternative i. So here we write the quantity of the continuous good, given that alternative i has been chosen. It's a function of the remaining budget, the total budget minus the cost of alternative i, the price of the continuous goods, and the attributes of alternative i. We have to realize that I minus c_i, so the income which is left after the alternative i has been purchased, must be positive. If it is negative, it means that you cannot afford to buy alternative i. In this case, this alternative will be declared unavailable because it's too expensive for your budget and will be removed from the choice set.

Notes

Summary

7m 03s

Solving the problem

Conditional demand functions

$$\text{demand}(I - c_i, p, \tilde{z}_i),\ i = 1, \ldots, J$$

Conditional indirect utility functions

Substitute the demand functions into the utility:

$$U_i = \widetilde{U}(\text{demand}(I - c_i, p, \tilde{z}_i), \tilde{z}_i) = U(I - c_i, p, \tilde{z}_i),\ i = 1, \ldots, J$$

So we have this conditional demand function. We know for each values of the cost of alternative i, the budget, the price of the continuous goods and the attributes of alternative i, we can calculate the optimal quantity of the continuous goods. But this can be substituted into the utility function. So the utility function is a function of the quantities of the continuous good. But now, we know the optimal quantities. So for each alternative i, assuming that it is chosen, we can calculate the optimal quantities of the continuous goods and plug them into the utility function. So these are called the conditional indirect utility functions. Conditional because it's conditional on the fact that alternative i has been chosen. And indirect because they don't depend directly on the quantities but they depend on the optimal quantities derived from the demand function. So, some of this can be denoted by the function of the remaining budget, the price, and the attributes. These conditional indirect utility functions is what we will call in the rest of this course the utility functions. This is what will be involved to decide which of the discrete goods will be selected.

Notes

Summary

7m 49s

Model for individual n

$$\max_y U(I_n - c_n^T y, p_n, \tilde{z}_n^T y)$$

Simplifications

- S_n: set of characteristics of n, including income I_n.
- Prices of the continuous goods (p_n) are neglected.
- c_{in} is considered as another attribute and merged into $\tilde{z}_n$

$$z_n = \{\tilde{z}_n, c_n\}.$$

$$\max_i U_{in} = U(z_{in}, S_n)$$

Well let's see the step two now: the choice of the discrete goods. So now for each alternative i, we have been able to derive this conditional indirect utility function which basically calculates the utility that you will get if you alternative i, and if you make an optimal decision on the continuous goods. So basically what we want to do is to find the alternative which is associated with the maximum utility. And to do that we will simply enumerate all alternatives, calculate this utility for each of them (this conditional indirect utility function), and then select the alternative with the highest U_i. And here, actually, it's simple because we have no income constraint anymore, because the income is already in the utility function. We know already that this alternative is affordable because if it was not it would have been removed in the previous step from the choice set. It means that the model for individual n is simply calculating the maximum of a list of utility functions. And each of them depends on the remaining budget, the price of the continuous goods, and the attributes of the alternatives. Now in order to use it in the context of discrete choice, that will be the focus of this course, we'll do some simplifications.

Notes

Summary

9m 15s

Model for individual n

$$\max_y U(I_n - c_n^T y, p_n, \tilde{z}_n^T y)$$

Simplifications

- S_n: set of characteristics of n, including income I_n.
- Prices of the continuous goods (p_n) are neglected.
- c_{in} is considered as another attribute and merged into $\tilde{z}_n$

$$z_n = \{\tilde{z}_n, c_n\}.$$

$$\max_i U_{in} = U(z_{in}, S_n)$$

We will denote by S_n the set of characteristics of individual n. And, actually, although income plays a very specific role in this formulation (as you could see), we will simply add income into the set of characteristics as any other characteristic, to simplify the presentation of the problem. We will also neglect the prices of the continuous goods. We will keep them out as they don't impact the utility of the discrete goods. And also the cost itself, the cost of the alternative which plays also a specific role in this formulation, is considered as another attribute and will be merged into the vector z of attributes. So it means that to simplify, we will obtain the utility function for each alternative i, for each individual n, and this utility function will be a function of the attributes of the alternative and the socio economic characteristics of the decision maker. And we keep in mind that this list of attributes includes the cost and this list of socio economic characteristics includes the income.

Notes

Summary

10m 44s

So the good news is that we have a strong theoretical foundation. We start from micro-economics, which is basically derived for continuous goods, we extend it to discrete goods so we can use all the machinery which is associated with micro-economics. Actually, in the next step, you should look at the simple example, in order to understand better how this concept of utility works in a practical case.

Notes

Summary

11m 50s