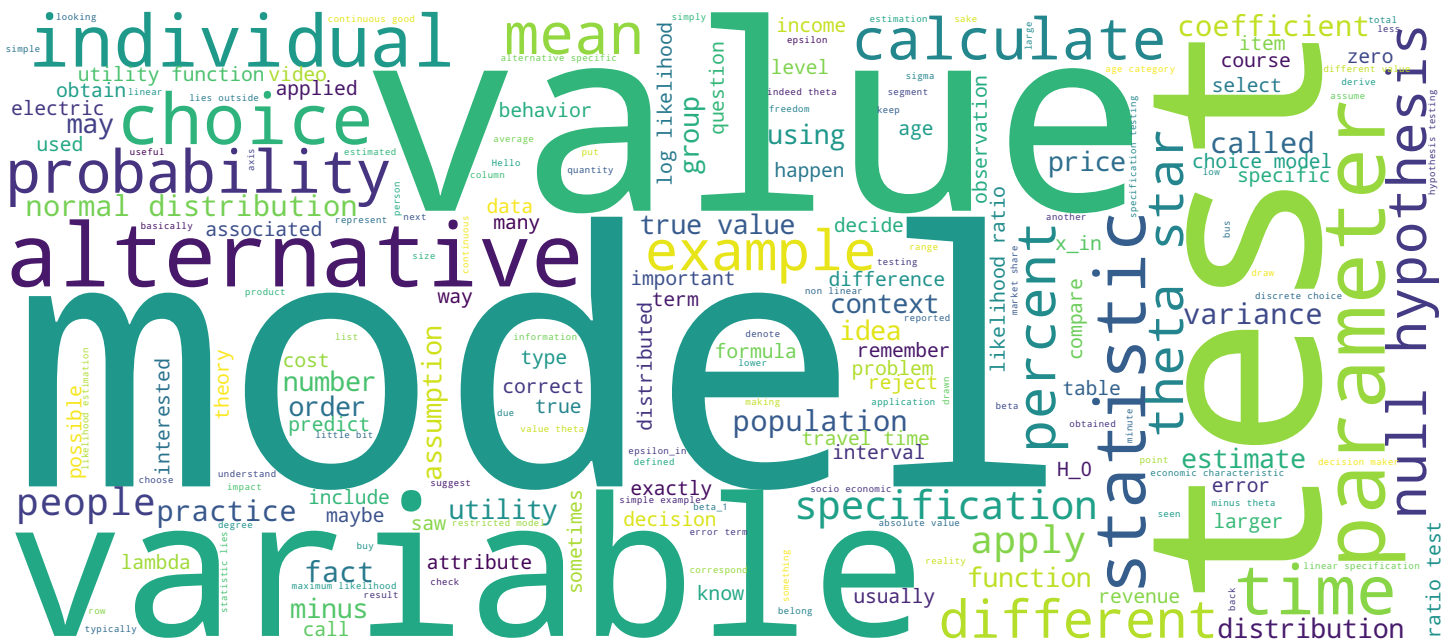


t-tests

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Introduction to choice models



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Video



EPFL

Usage of the t -tests



Hello. In this video, we will look at one of the most important statistical tests that is applied on choice models.

Notes

Summary



0m 04s

t-test

Question

Is the parameter θ significantly different from a given value θ^* ?

- ▶ $H_0 : \theta = \theta^*$
- ▶ $H_1 : \theta \neq \theta^*$

Statistic (assuming maximum likelihood estimator)

Under H_0 , if $\hat{\theta}$ is normally distributed with known variance σ^2

$$\frac{\hat{\theta} - \theta^*}{\sigma} \sim N(0, 1).$$

It's the t-test. So the objective of the t-test is to analyze the question that the parameter, a parameter of the model, let's say theta, is equal to a given target value theta star. Actually, we would like to see if the value that we obtained from estimation is actually significantly different from this value theta star. Basically, the null hypothesis is that the true value of the parameter is indeed theta star. And the fact that we have a different value from the estimation, theta hat, is a result of the fact that we have drawn a sample from the population and that we have used inference. So this is purely inference errors. That's what we mean by "significant". Now, if H_0 is true, meaning that the true value of theta is equal to theta star, what we can show is that the statistic that is represented here has a given distribution. So this statistic, the distribution of theta hat, the estimated value, is distributed as a normal distribution. And more specifically, theta hat minus theta star, which in average should be equal to zero if theta star is the true value of the parameter, divided by sigma, which is the standard error of the estimate, is a normal distribution with mean zero and variance one.

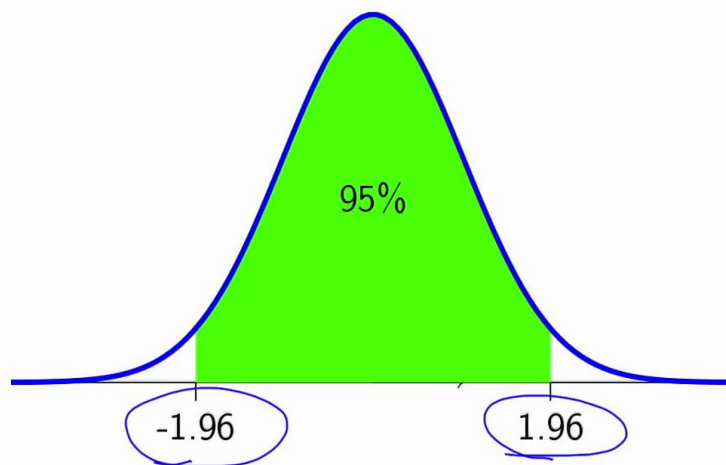
Notes

Summary



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t -test: under H_0



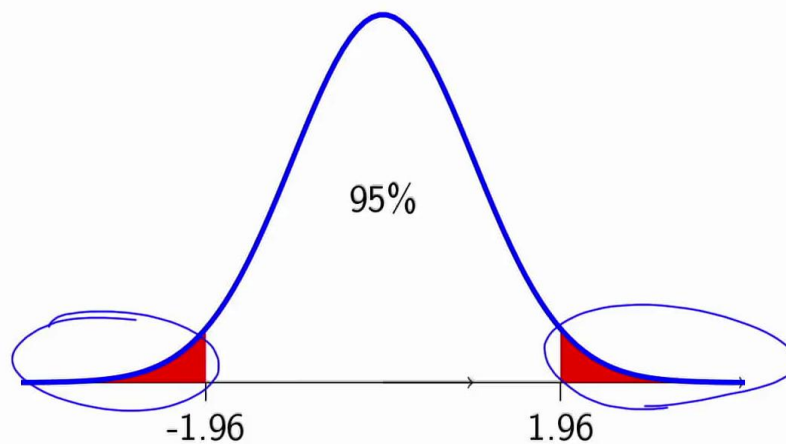
So it means that, if H_0 is true, so if the true value of the parameter is indeed θ^* , 95 percent of the time, when we draw a sample and we estimate the value of θ from the sample, the value that we will obtain will be between -1.96 and 1.96, which correspond to the quantiles of the normal distribution.

Notes

Summary



t-test: if the statistic lies outside



H_0 is rejected at the 5% level.

Now, if we look at this logical relationship in the other direction, it means that if the statistic lies outside this range (so, if we happen to calculate the statistic, and its value is below -1.96 or above 1.96), in that case, we can actually safely reject the null hypothesis. We know that we will make a mistake 5 percent of the time, but this is what we are willing to accept as a margin of error. So, this is the idea of the t-test. When the statistic lies outside the interval -1.96 and 1.96, we say OK, maybe it's due because we have drawn a wrong sample or very specific sample. Maybe... It will happen only 5% of the times. But we believe more that it is due because of the fact that H_0 is not correct. So we want to reject it.

Notes

Summary



2m 07s

Applying the test

Statistic

$$P(-1.96 \leq \frac{\hat{\theta} - \theta^*}{\sigma} \leq 1.96) = 0.95 = 1 - 0.05$$

Decision

H_0 can be rejected at the 5% level ($\alpha = 0.05$) if

$$\left| \frac{\hat{\theta} - \theta^*}{\sigma} \right| \geq 1.96.$$

OK, so the statistic that we calculate (theta hat minus theta star divide by sigma) the probability that it lies between minus 1.96 and 1.96 is equal to 95 percents, as we saw in the picture. And, actually, 95 percents is one minus five percents. Five percents is the level at which we would like to apply the test. So now, the decision that we apply in practice is that, we consider rejecting H_0 at the five percents level if the statistic, in absolute value, is larger or equal to 1.96. But, remember, I use here five percents for the sake of the example, but in the context of specification testing, there is absolutely no problem to use higher values for the level, meaning lower values for the t-test.

Notes

Summary



2m 59s

Comments

- ▶ If $\hat{\theta}$ **asymptotically** normal
- ▶ If variance unknown
- ▶ A t test should be used with N degrees of freedom.
- ▶ When $N \geq 30$, the Student t distribution is well approximated by a $N(0, 1)$

So why is it called the t-test? In theory, if θ hat is asymptotically normal, which is the case with maximum likelihood estimation, and if the variance is unknown (which is usually the case: we use the estimated variance and not the true one) then a t-test should be used with N degrees of freedom. But in practice, you can see that, when N , the number of observations in the sample, is thirty or larger, then this t distribution, the so-called Student t distribution, is well approximated by a normal distribution. So, therefore, in practice, because we most of the time have more than thirty observations, we can use the normal distribution to apply this statistic. Even if it is called a t-test, it is applied with a normal distribution.

Notes

Summary



3m 47s

p value

- ▶ probability to get a t statistic at least as large (in absolute value) as the one reported, under the null hypothesis
- ▶ it is calculated as

$$p = 2(1 - \Phi(t))$$

where $\Phi(\cdot)$ is the CDF of the standard normal.

- ▶ the null hypothesis is rejected when the p -value is lower than the significance level (typically 0.05)

Now, sometimes, it's useful to represent the same information using a different value, which is called a "p-value". The p-value will basically convey exactly the same information as the t-test, but in a different format. What is the p-value? The p-value is the probability to get a t-statistic which is at least as large, in absolute value, as the one which has been reported, if the null hypothesis is correct. If you calculate a t-statistic, you can actually derive the p-value using this formula: two times one minus the CDF of the normal distribution calculated at t . And now you can use the p-value exactly like you use the t-test. Now, in practice, you would reject the null hypothesis when the p-value is lower than the level you are targeting. So, typically, five percents. If p is lower than five percents, you would reject the null hypothesis.

Notes

Summary



4m 36s

Comparing two coefficients

Hypothesis

$$H_0 : \beta_1 = \beta_2.$$

Statistic

$$\frac{\hat{\beta}_1 - \hat{\beta}_2}{\sqrt{\text{Var}(\hat{\beta}_1 - \hat{\beta}_2)}}$$

where

$$\text{Var}(\hat{\beta}_1 - \hat{\beta}_2) = \text{Var}(\hat{\beta}_1) + \text{Var}(\hat{\beta}_2) - 2 \text{Cov}(\hat{\beta}_1, \hat{\beta}_2)$$

Distribution

Under H_0 , distributed as $N(0, 1)$.

The t-test can be also used to compare two coefficients. In this case, the null hypothesis is that two coefficients are equal. This applies in particular when you are making the assumption that a variable is alternative-specific, so you assign a different coefficient for the variables in different alternatives. But then you may want to test if it is reasonable or not. So you test if the two coefficients that you have put are actually equal, in reality. So this is your null hypothesis. The idea of the t-test is exactly the same. You calculate a statistic. This statistic is given as the difference of the two coefficients divided by the square root of the variance of this difference (and you can apply the formula for the variance in order to obtain this calculation here), and this statistic again is distributed as a normal zero, one if the null hypothesis is true.

Notes

Summary



5m 35s



So now I suggest that you practice a little bit with these t-statistics for testing a parameter against a specific value, calculate the p-value, and also use the t-statistic to compare two coefficients, to check the hypothesis that two coefficients are equal.

Notes

Summary



6m 34s