Testing

Non nested hypotheses

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Introduction to choice models

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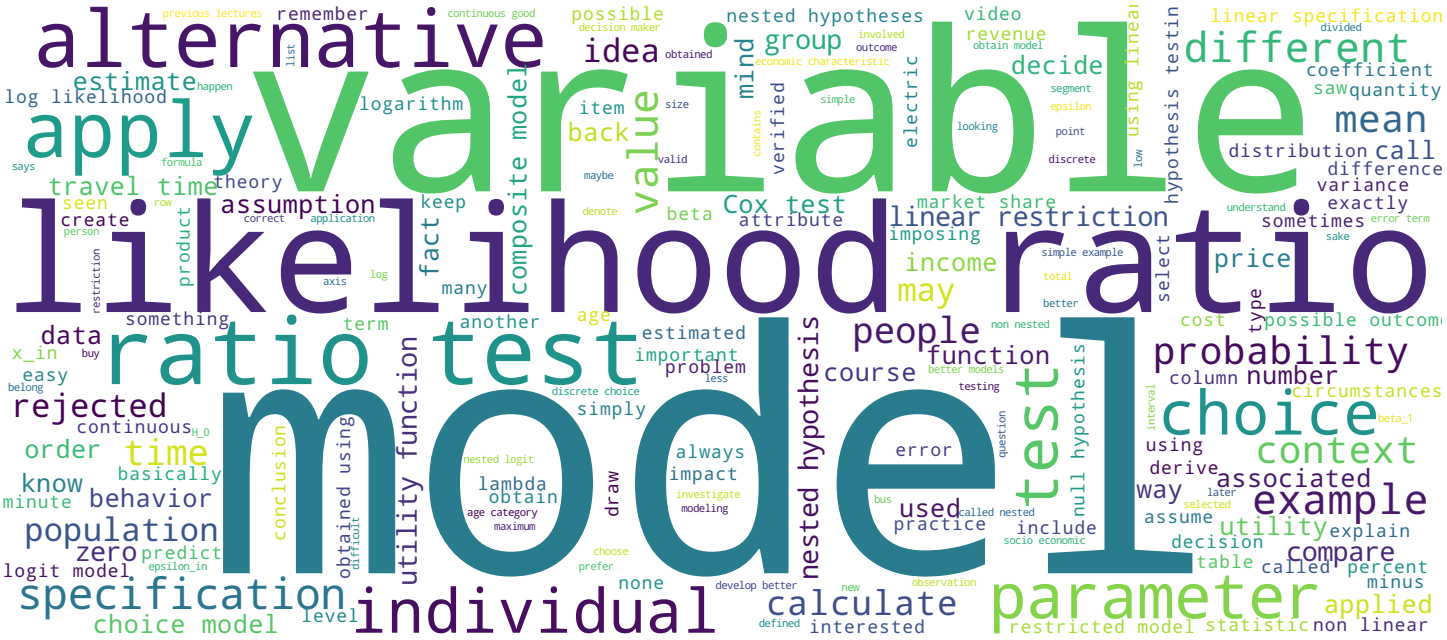

The Cox test

We have seen before that the likelihood ratio test is very powerful. It can be applied in many circumstances. But we have to keep in mind that it cannot always be applied. There are circumstances where it does not apply, where it is not valid to apply it. In this case, we need to consider other tests. One of them is the Cox test. So let's see first what are the circumstances where the likelihood ratio test cannot be applied.

Notes

Summary

0m 04s

Non nested hypotheses

Nested hypotheses

- Restricted and unrestricted models
- Linear restrictions
- H_0: restricted model is correct
- Test: likelihood ratio test

Non nested hypotheses

- Need to compare two models
- None of them is a restriction of the other
- Likelihood ratio test cannot be used

The likelihood ratio test is applied when we want to compare two models, a restricted and an unrestricted one. And when the restrictions are linear. We saw several examples in previous lectures. In this case the null hypothesis is that the restricted model is correct, and we can apply the likelihood ratio test. In the jargon of hypothesis testing, this is usually called "nested hypothesis". But sometimes, we want to compare two models and none of them is a linear restriction of the other. In this case you cannot apply the likelihood ratio test. So we say that we are in a context of non-nested hypotheses. As an aside, I would like to emphasize the fact that some choice models are called "nested logit models". But they have absolutely nothing to do with the nested hypothesis. So the term "nested" in our context here has nothing to do with the nested logit model.

Notes

Summary

0m 29s

Example

Model 1

$$V_{in} = \beta_1 x_{ink} + \cdots$$
$$V_{jn} = \beta_2 x_{jnk} + \cdots$$
$$\vdots$$

Model 2

$$V_{in} = \beta_1 \log(x_{ink}) + \cdots$$
$$V_{jn} = \beta_2 \log(x_{jnk}) + \cdots$$
$$\vdots$$

Let's take an example. Let's say that I have two models. And in the first model, I include my variables in a linear way in the utility function. But then, I may be interested in non linear specifications. And I consider that this variable will be involved as a log, as we saw in previous lectures. But in this case we cannot apply the likelihood ratio test, because no model is a linear restriction of the other one. These are simply two different specifications. We cannot derive one by restricting some parameters of the other one.

Notes

Summary

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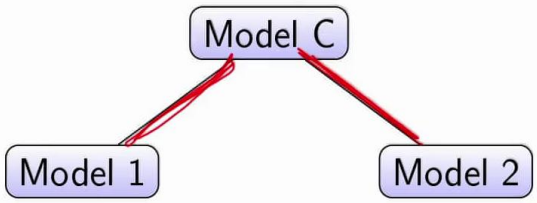
Cox test

Back to nested hypotheses

- We want to test model 1 against model 2
- We generate a composite model C such that both models 1 and 2 are restricted cases of model C.



The idea of the Cox test is that, because we know how to use likelihood ratio tests, let's try to go back to a context where the nested hypotheses are verified, and then we can use the likelihood ratio test. So the idea is to create from model 1 and model 2, the two models that we would like to compare, we create a composite model. I call it model C. the idea is that the first model can be obtained using linear restriction of model C. And, also, the second model can be obtained using linear restrictions of model C. What we have in mind by doing that is that it will be possible to apply a likelihood ratio test between model 1 and model C, because the nested hypothesis is verified in this context. And it will also be possible to apply a likelihood ratio test between model 2 and model C. And we know how to apply likelihood ratio tests.

Notes

Summary

1m 59s

Example

Model 1

$$V_{in} = \beta_1 x_{ink} + \cdots$$
$$V_{jn} = \beta_2 x_{jnk} + \cdots$$
$$\vdots$$

Model 2

$$V_{in} = \beta_1 \log(x_{ink}) + \cdots$$
$$V_{jn} = \beta_2 \log(x_{jnk}) + \cdots$$
$$\vdots$$

Model C

$$V_{in} = \beta_{11} x_{ink} + \beta_{12} \log(x_{ink}) + \cdots$$
$$V_{jn} = \beta_{21} x_{jnk} + \beta_{22} \log(x_{jnk}) + \cdots$$
$$\vdots$$

Let's go back to our example to have a complete feeling of what this composite model would be. So remember, we have to compare the first model with a linear specification of the variable x, with the second model, where the variable x was involved using the logarithm. Now we create a composite model, where we basically put these two specifications together. And we have this model C, where we have both the linear terms and the logarithm terms. So as you can see, we can obtain model one by imposing this two parameters to be equal to zero. This is a linear restriction. So, therefore, the nested hypothesis is verified, and we can apply the likelihood ratio test. Similarly, we can obtain model 2 by imposing these two parameter to be equal to zero. It's again a linear restriction. We have the nested hypothesis, and the likelihood ratio test can be used.

Notes

Summary

3m 00s

Cox test

Testing

- We test 1 against C using the likelihood ratio test
- We test 2 against C using the likelihood ratio test

Conclusions

C against 1	C against 2	Conclusion
1 is not rejected	2 is rejected	Prefer 1
1 is rejected	2 is not rejected	Prefer 2
1 is rejected	2 is rejected	Develop better models
1 is not rejected	2 is not rejected	Use another test

So that's what we do. To apply the Cox test, we test model one against model C. and we can use the likely equation test, because we have the nested hypothesis. Then we test model two against model C, using again the likelihood ratio test. So what conclusions can we draw? Well, there are four possible outcomes of this comparison. The first two outcomes is when one model is rejected and the other one is not rejected. Well in this case, the conclusion is easy. We will prefer the model which has not been rejected. Now what happens if both models are rejected. So basically, it says that the model C is better than model one and the model C is better than Model 2. Well be careful in this case, it doesn't mean that modern C should be accepted but it definitely means that you should develop better models. Model C is very artificial. It has been just obtained by pulling together the two specifications for the sake of the test. OK so don't use it necessarily for specification. But develop better models. And the last possible outcome is that none of the models is rejected. So in this case the Cox test does not help us to decide if model 1 is better than model 2. Then we need to use another test.

Notes

Summary

3m 58s

Cox test

Testing

- We test 1 against C using the likelihood ratio test
- We test 2 against C using the likelihood ratio test

Conclusions

C against 1	C against 2	Conclusion
1 is not rejected	2 is rejected	Prefer 1
1 is rejected	2 is not rejected	Prefer 2
1 is rejected	2 is rejected	Develop better models
1 is not rejected	2 is not rejected	Use another test

And we will see later in the course a test called the Davidson and MacKinnon test that can be used, and we will also explain how to calculate a so-called adjusted likelihood ratio index, that is usually denoted by "rho bar squared", that can also be used to compare these two models.

Notes

Summary

5m 15s

The advantage of the Cox test is that it's very easy. Because you go back to something that you know, meaning calculating likelihood ratio tests, in a very simple way. But there are two difficulties that we need to keep in mind. Is the two models are very different, the composite model may be extremely complicated and difficult to estimate. Actually, sometimes the estimation may even fail. In this case, you cannot apply it. And the second difficulty is that, as we saw, one of the possible outcomes of the test may be that none of the two models is rejected. And this is not conclusive. This is why we would like to investigate other types of tests in order to test non-nested hypotheses.

Notes

Summary

5m 32s