



- Introduction
- Systèmes triphasés symétriques
- Définition des tensions de ligne et courant de ligne
- Calcul du rapport des tensions
- Conclusion

Electrotechnique II

Hello After a general view of polyphase systems we will now focus and more particularly study symmetrical three-phase systems. In this lesson after a short introduction we will define what is a symmetrical three-phase system define what a line voltage is a phase-to-phase voltage, and similarly for currents, and compute the relationship between these different values.

Notes

Summary



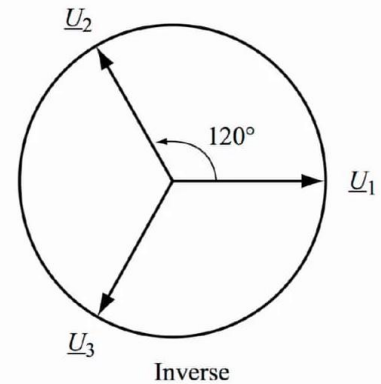
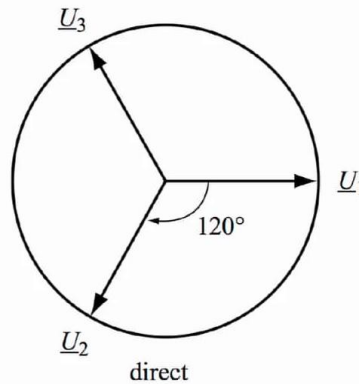
0m 04s

SYSTÈMES TRIPHASÉS SYMÉTRIQUES

$$k = 1 \quad u_1 = u_2 = u_3$$

$$m = 3$$

$$\underline{u}_1 = u e^{j\alpha}$$



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And finishing on a conclusion We have here a classical case of a three-phase symmetrical system that we have carried over on a phasor diagram with over here the U_1 voltage U_2 U_3 , in a forward system with U_1 , U_2 and U_3 in the reverse system that we have seen in the previous lesson. So we clearly have here for the order of succession of the phases, a k equal to 1. The number of phases equal to 3 and a symmetrical system because the three values U_1 , U_2 , and U_3 are equal. We can particularly see that the order of succession of the phases or phase-shift between the different phases, is 120 degrees, or $2\pi/3$. We will now be able to characterise these 3 phasors and write the equation for the 3 phasors in this symmetrical three-phase system. First of all, the first phasor U_1 is equal to $Ue^{j\alpha}$. I'm leaving the angle α on purpose to allow us complete freedom later on the position of U_1 in the complex plane. While on the drawing that we have here, in both drawings for the forward or reverse system we have U_1 with a null angle. But let's leave, in a general manner, α thus the other two phasors U_2 and U_3 become for the former $Ue^{j(\alpha - 2\pi/3)}$, we have a phase shift of $2\pi/3$ and the last one as well also of 120 degrees.

Notes

Summary



SYSTÈMES TRIPHASÉS SYMÉTRIQUES

$$k=1 \quad u_1 = u_2 = u_3$$

$$m=3$$

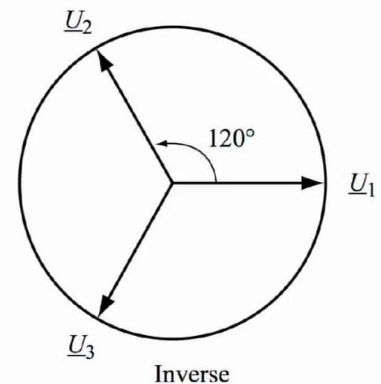
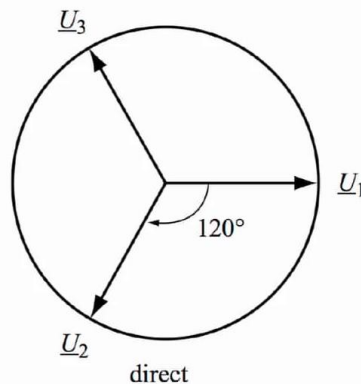
$$\underline{u}_1 = u e^{j\alpha}$$

$$\underline{u}_2 = u e^{j(\alpha - \frac{2\pi}{3})}$$

$$\underline{u}_3 = u e^{j(\alpha - \frac{4\pi}{3})}$$

$$\underline{u}_1 + \underline{u}_2 + \underline{u}_3 = 0$$

$$u_1 + u_2 + u_3 = 0$$



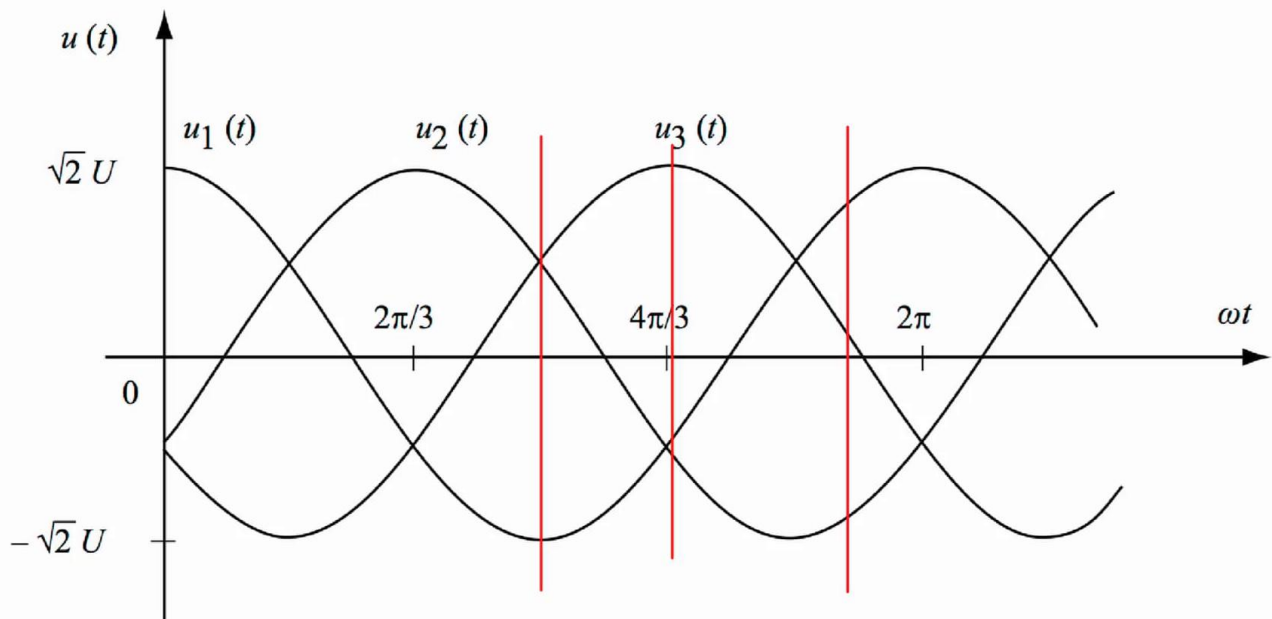
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So here are the three phasors that will be the basis of our symmetrical three-phase system. Now if we observe in a temporal manner what these 3 values are worth, we will notice one thing. Already on this phasor diagram, you can see it here: if I combine these three vectors, I will return to zero. This means that, at every instant, every moment, these three vectors are equal to zero when combined. In other words, we can write at all times: phasor 1 of the voltage, plus the second voltage, and the third voltage are always equal to zero. Even more surprising, we can also do this for the temporal part: at all times, the three instantaneous voltages are also equal to zero.

Notes

Summary





Electrotechnique II

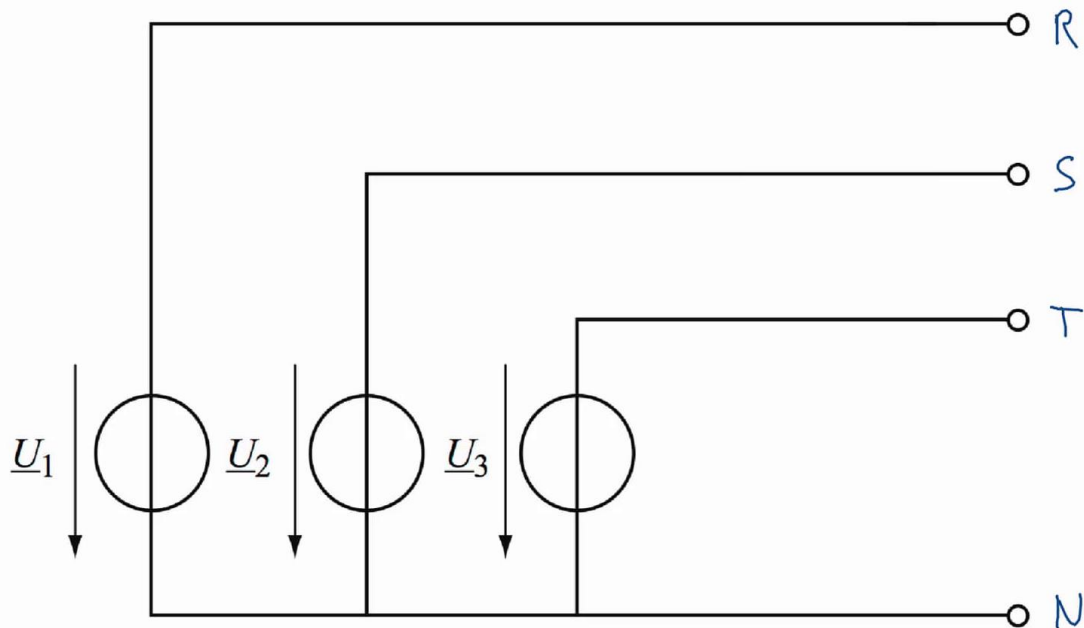
And this we shall now see in the following graph where we have represented the three voltages U_1 , U_2 and U_3 in a temporal manner. What do we see? We can see that at any point of time that I choose, here, for example on this curve, if I add at the instant I've marked with the red line if I add U_2 and U_3 we can see that the 2 values are exactly equal to the value of U_1 . The sum of these three voltages is zero, as expected. Let's take another instant and once again, here, the sum of these three voltages of U_1 , U_2 and U_3 we find zero. We can again do a last one, by really taking a random line the sum of this voltage this voltage and this voltage will give us exactly zero. This system thus has an undeniable advantage being symmetrical we will have some form of equalization of the energy due to the fact that the sum of these voltages is always equal to zero.

Notes

Summary



3m 25s



Electrotechnique II

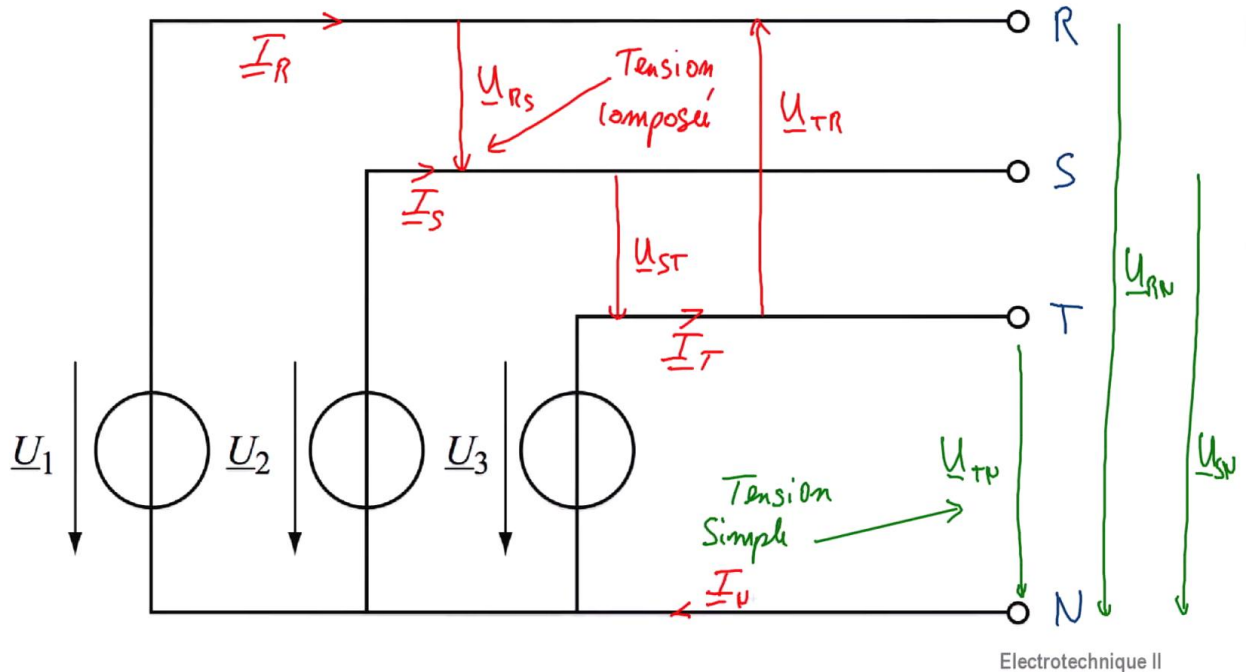
We must now model this source and we will define some number of values that are linked to a symmetrical three-phase source that I present to you first of all according to a model that we give here in a wye setup. We can see that our three voltages presented previously, are represented by sources, here, U_1 , U_2 and U_3 that they are all connected to each other through a common point and that we have three outputs, so we can see, of the voltages that we call lines. We will define, on these lines different values. First of all, we name these lines by letters that allow us to navigate later on with a three-phase symmetrical source. First of all, here the R line the S line, and the T line. with the common point that we name N, and that symbolizes the neutral point. About these different values we can say one thing, firstly, U_1 , U_2 , U_3 will have the same effective value (RMS value) will have the same frequency, and the three will be phase-shifted by $2\pi/3$. We can still characterize a certain number of elements. First of all, we can see that there is a voltage between the R line and the neutral, the S and the neutral, the T and the neutral and that we can compute or measure the voltages between R and S directly, S and T or T and R. Well, with all these different values we will need to name them in order to navigate and characterize these different elements.

Notes

Summary



4m 48s



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The first elements that we will give are the voltages between the lines and the neutral. We will have here a voltage that we will call \underline{U}_{rn} and then we will have \underline{U}_{sn} and finally \underline{U}_{tn} . These are complex values thus, we underline the values. Another possibility, as I have mentioned it earlier, is to take values directly here. We will thus have \underline{U}_{rs} we will have here \underline{U}_{st} and finally the value \underline{U}_{tr} . We will name the voltages that are directly taken between the lines composed voltages. By extension, we shall also call them line voltages. And for the voltages between R, S, T and the neutral we will call these voltages simple voltages. Also by extension, we will call these voltages either simple voltages, or phase voltages. Three elements must still be mentioned once the voltages are defined we can still define the currents that flow in the different lines. First of all, we will define the three currents that are really flowing in the lines \underline{I}_r , \underline{I}_s and \underline{I}_t and a fourth element can still be mentioned it's the return of current on the neutral that we will name \underline{I}_n but we will quickly see that this current \underline{I}_n is often null if the entire system is balanced and symmetrical the current will always be null sometimes even, the neutral will not be connected.

Notes

Summary



Tension Simple: \underline{U}_{RN} , \underline{U}_{SN} , \underline{U}_{TN}

Tension Composée: \underline{U}_{RS} , \underline{U}_{ST} , \underline{U}_{TR}

$$\underline{U}_{RS} = \underline{U}_{RN} - \underline{U}_{SN}$$

$$\underline{U}_{ST} = \underline{U}_{SN} - \underline{U}_{TN}$$

$$\underline{U}_{TR} = \underline{U}_{TN} - \underline{U}_{RN}$$

$$\underline{U}_{RN} = U e^{j\alpha}$$

$$\underline{U}_{SN} = U e^{j(\alpha - \frac{2\pi}{3})}$$

$$\underline{U}_{RS} =$$



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We will once again mention here what the simple voltage is or what the line voltage is according to the diagram that we've just seen about the three-phase source. First of all, the simple voltage that we define as being the measure of the voltage or the definition of the voltage between the R line and the neutral, likewise for every line. In the same manner, the composed voltage is defined this time directly between the lines namely \underline{U}_{rs} , \underline{U}_{st} and \underline{U}_{tr} . Now, if we can calculate the relationship between the simple voltage and the composed voltage we will simply apply Kirchhoff on the previous diagram. We then find 3 relatively simple loop equations that are $\underline{U}_{rs} = \underline{U}_{rn} - \underline{U}_{sn}$ and similarly for the other two equations. We will take this first equation and write, for this first equation what is the result of the subtraction of these two vectors or these two phasors. By definition, we have, \underline{U}_{rn} equal to $U e^{j\alpha}$. and we have \underline{U}_{sn} equal to $U e^{j(\alpha - 2\pi/3)}$. We can then write and bring out $U e^{j\alpha}$ for both elements and finally write $\underline{U}_{rs} = \dots$. We bring out $U e^{j\alpha}$ firstly, we have \underline{U}_{rn} , from where the 1 is issued minus and then the second part so, e^{j} and simply $-2\pi/3$.

Notes

Summary



8m 42s

Tension Simple: \underline{U}_{RN} , \underline{U}_{SN} , \underline{U}_{TN}

Tension Composée: \underline{U}_{RS} , \underline{U}_{ST} , \underline{U}_{TR}

$$\underline{U}_{RS} = \underline{U}_{RN} - \underline{U}_{SN}$$

$$\underline{U}_{ST} = \underline{U}_{SN} - \underline{U}_{TN}$$

$$\underline{U}_{TR} = \underline{U}_{TN} - \underline{U}_{RN}$$

$$\underline{U}_{RN} = U e^{j\alpha} \quad \underline{U}_{SN} = U e^{j(\alpha - \frac{2\pi}{3})}$$

$$\underline{U}_{RS} = U e^{j\alpha} \left(1 - e^{j(-\frac{2\pi}{3})} \right)$$

$$\underline{U}_{RS} = U e^{j\alpha} \left(\frac{3}{2} + j \frac{\sqrt{3}}{2} \right)$$

$$\underline{U}_{RS} = \sqrt{3} U e^{j(\alpha + \frac{\pi}{6})} = \sqrt{3} \underline{U}_{RN} e^{j\frac{\pi}{6}}$$

$$U_p = \text{Tension de ligne} = \sqrt{3} U$$

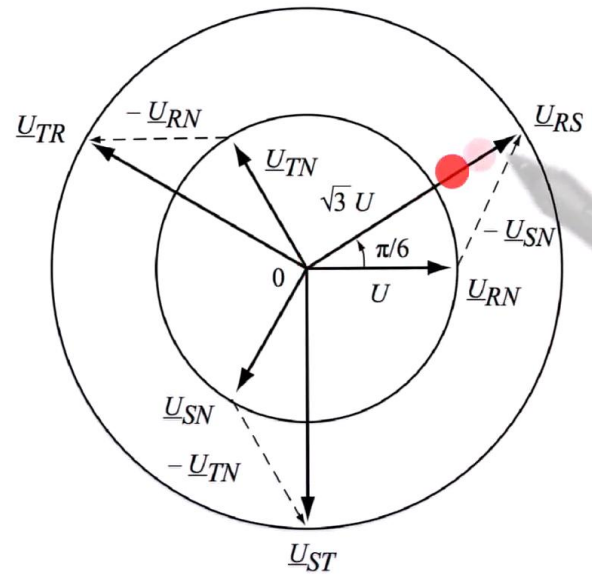
Electrotechnique II

This being said we can transform this equation, now, into a a+bj relationship, that will gives us the following result \underline{U}_{rs} will be equal to $Ue^{j\alpha}$ and that multiplies $3/2$ plus $j * (\sqrt{3}/2)$. By reconverting everything in the form of a Euler type phasor we then have finally, \underline{U}_{rs} equal to $\sqrt{3}U * e^{j(\alpha + \pi/6)}$. We recognize here $Ue^{j\alpha}$ which is the simple voltage, and thus the composed voltage is actually $\sqrt{3}$ times the simple voltage \underline{U}_{rn} that underlines it with a phase-shift of $\pi/6$. We shall see that for the other three, for other two line voltages, it's exactly the same, thus everytime all of these values are simply raised by a $\sqrt{3}$ factor compared to the simple voltage and phase-shifted by a factor of $\pi/6$.

Notes

Summary





Electrotechnique II

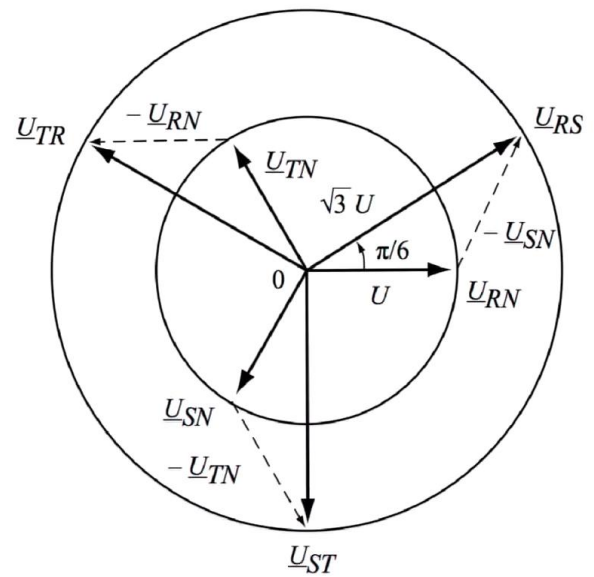
Essentially we can write in a completely general manner that the line voltage is equal to $\sqrt{3}$ times the effective (RMS) value of the simple voltage. By taking now a diagram in which we have put all of these different phasors one after the other in a complex diagram we will observe a certain number of elements: first of all, we can recover our three simple voltages \underline{U}_{rn} , \underline{U}_{tn} and \underline{U}_{sn} . We've aligned here the simple voltage \underline{U}_{rn} with zero. You have here a circle that defines the value of the RMS of the U grid. If we apply, in this case in a completely graphical manner what we have done earlier, meaning $\underline{U}_{rn} - \underline{U}_{sn}$, we can observe and obtain here a new phasor that is the composed voltage \underline{U}_{rs} that we have just calculated. We discover that this value is $\sqrt{3}$ times bigger than the effective (RMS) value of the grid, or than the previous voltage, the simple voltage, and that it is phase-shifted by $\pi/6$. Similarly, the three composed voltages will now form a new symmetrical three-phase system where the amplitude is $\sqrt{3}$ times greater than the simple voltages and all of them phase-shifted by $\pi/6$. It is the foundation of every sinusoidal three-phase symmetrical system to have this relationship between simple voltages and composed voltages.

Notes

Summary



$$U_l = \sqrt{3} U$$



Electrotechnique II

I remind here the extremely important expression that the line voltage will always be $\sqrt{3}$ times the simple voltage. In the most common type of domestic power supply grid in Europe, the modulus of the simple voltages is generally normalized to 220 volts.

Notes

Summary



14m 23s

Tension simple à 220 V (220 - 240 V)

Tension composée à 380 V ($220 \times \sqrt{3} = 380$) (380 - 420)

Electrotechnique II

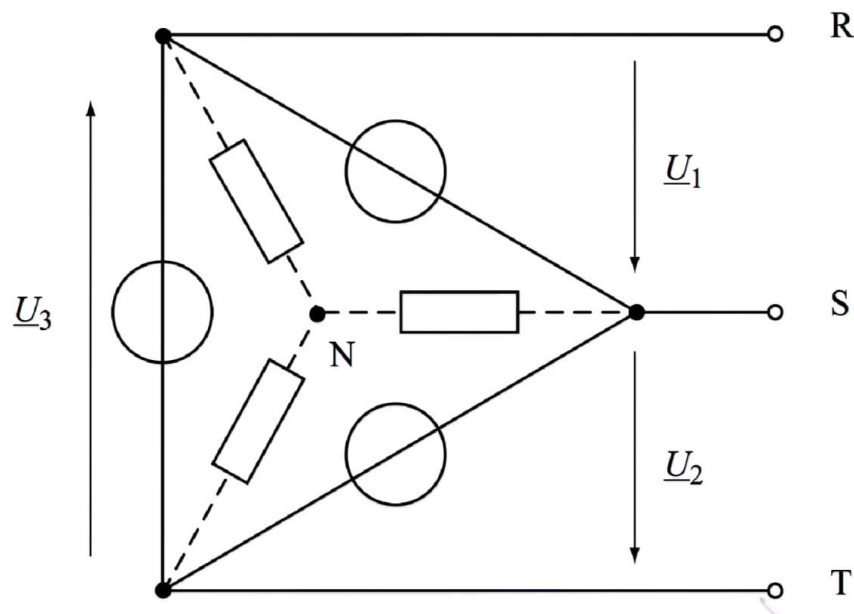
So, simple voltage at 220 volts generally, this 220 volts is more often included between 220 and 240 volts and can oscillate throughout the year generally. Thus if we compute now the composed voltages linked to these simple voltages we can observe that if we multiply the composed voltage by $\sqrt{3}$ it is now at 380 volts since it is 220 multiplied by $\sqrt{3}$ since we can have 220 volts version and a 240 volts version the composed tensions will oscillate between 380 and 420 volts. Thus, in a three-phase system, we always have available either a simple voltage around 220 volts or a composed voltage of 380 volts.

Notes

Summary



14m 42s



Electrotechnique II

There is another way of showing or modelling a symmetrical three-phase source. Instead of using a wye circuit like we've done previously it is possible to show that by connecting the source, here U_1 , U_2 and U_3 in a delta we also get R, S and T at whose terminals we find our three line voltages and the line currents that are flowing in an identical manner compared to our presentation in a wye circuit. The big difference compared to a wye connection is that here, in this schema there is no common point, thus no neutral and to bring up the neutral, it is necessary to construct a system built using three very high impedance resistors connected in a mid-point that recreates an artificial neutral. In the end, the only important element is to create three lines : R, S, T that will be useful later for connecting loads and thus, in the end, that our source is connected either in a wye connection or in a delta connection this will make or will have only little difference. Thus, I advise you for the comprehension of the next part of this class to keep a mental picture of this wye connection that allows us to more simply bring up the common point of the neutral.

Notes

Summary



15m 43s

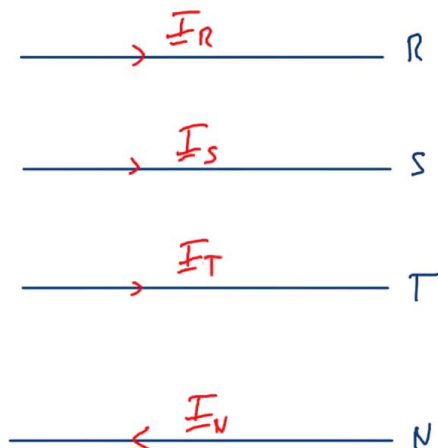
We still need to mention a word about the current of the neutral that is this connector that recieves the three lines so as to recover the energy coming from the load connected on R, S and T. I will re-draw here, for a better understanding the three lines R, S, and T that exit our symmetrical three-phase grid.

Notes

Summary



17m 14s



$$\underline{I}_N = \underline{I}_R + \underline{I}_S + \underline{I}_T$$

Electrotechnique II

Notes

We have R, we have S, we have T we also know that there is a return the neutral that will allow us us to bring back this energy If now we write here the line current I_R the line current I_S and the line current I_T we can write that once we have connected a R, S, T load and a neutral current using Kirchoff, it is possible to write this relatively simple equation it's that the sum of the currents I_R , I_S and I_T is equal to this neutral current We will see and we will demonstrate thereafter that when these currents are perfectly symmetrical, thus phase-shifted by 120 degrees and of equal amplitude the neutral current, as we've mentioned it previously, will be null To conclude we have seen in this lesson that a symmetrical three-phase system is relatively simple to study since we only need to concern ourselves with a single element everytime and the others are simply phase-shifted by $2\pi/3$ we have defined the phase voltage, the composed voltage we have seen what the line current is, that the sum of these line currents gives us the neutral current and that the ratio between the composed voltage and simple voltage is $\sqrt{3}$.

Summary



17m 40s

CONCLUSION



- Systèmes symétriques simples à étudier
- Définition des tensions de ligne et de la phase
- Définition des courants de ligne
- La tension de ligne vaut $\sqrt{3}$ la tension de phase
- La somme des courants de ligne définit le courant de Neutre, retour commun

Electrotechnique II

We will thus, in the next lessons study the loads connected to this symmetrical three-phase source

Notes

Summary



19m 13s