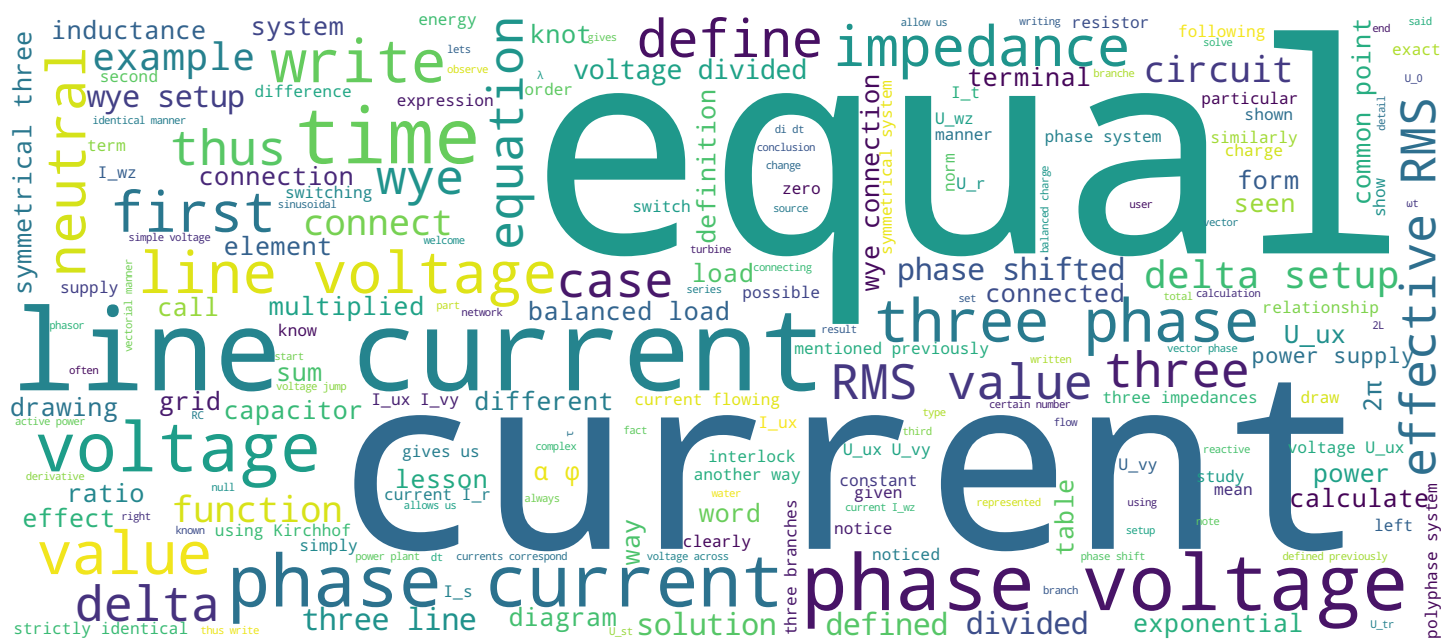


## LECON 4

Yves PERRIARD & Paolo GERMANO  
Laboratoire d'Actionneurs Intégrés





- Introduction
- Charge triphasée équilibrée
- Connexion en étoile
- Connexion en triangle
- Conclusion

Electrotechnique II

Hello and welcome to this lesson dedicated to symmetrical three-phase systems, and in particular to the definition of the load in a wye or delta circuit. In this lesson, after a short introduction we will define what the symmetrical three-phase load is and then study the different wye or delta assemblies and finally, end on a conclusion.

Notes

Summary



0m 04s

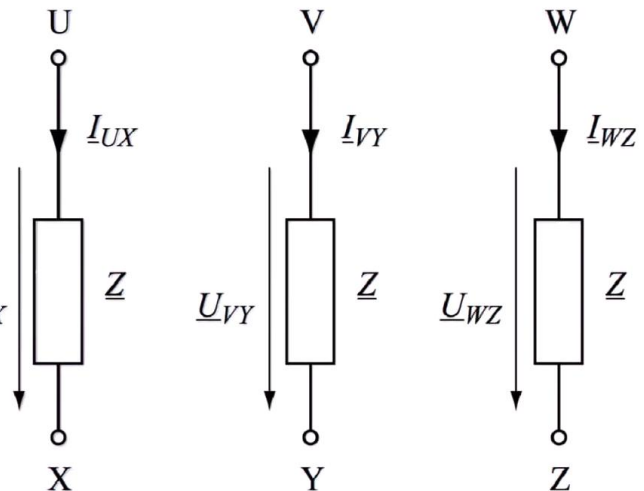
# CHARGE TRIPHASÉE ÉQUILIBRÉE

$$\underline{Z} = Z e^{j\varphi} \quad \text{phase de l'utilisateur.}$$

Tension de phase:  $\underline{U}_{ux}, \underline{U}_{vy}, \underline{U}_{wz}$

Courant de phase:  $\underline{I}_{ux}, \underline{I}_{vy}, \underline{I}_{wz}$

Pour un système symétrique:  $I_{ph} = \frac{U_{ph}}{Z}$



Electrotechnique II

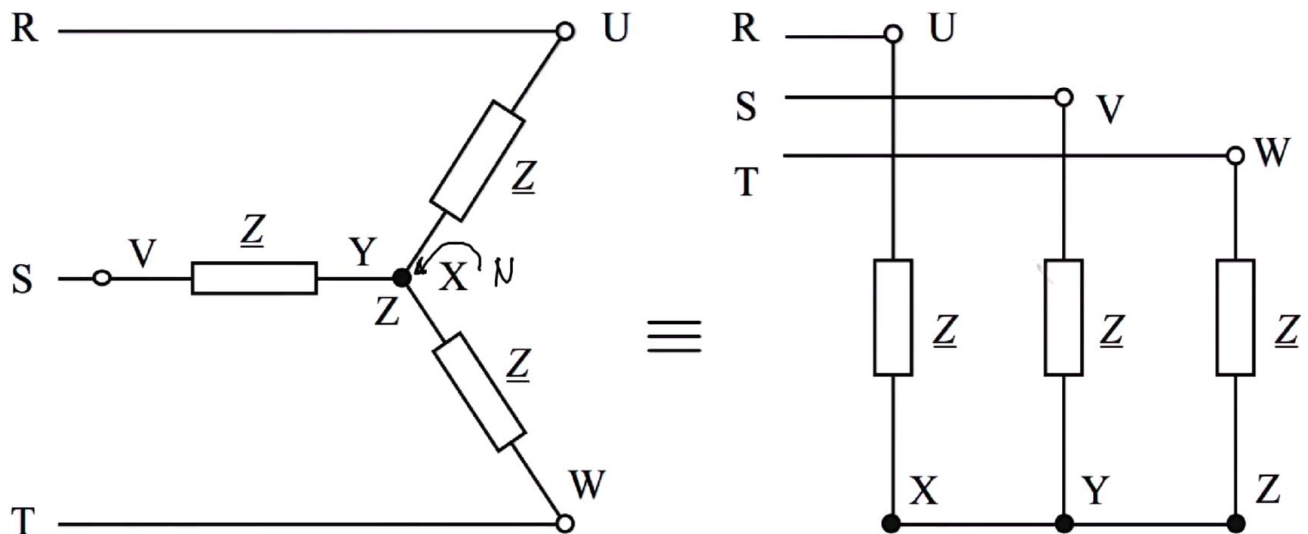
By definition, a balanced load is a set of three impedances strictly identical that we have named here  $Z$ . Three identical impedances through which three currents flow  $I_{ux}, I_{vy}, I_{wz}$  three defined voltages  $U_{ux}, U_{vy}$  et  $U_{wz}$ . The impedance  $Z$  common to the three branches is defined as  $Ze^{j\varphi}$  and is commonly called the phase of the user. We can then define in this diagram what we call the phase voltage and what we call the phase current. First of all, the phase voltage, well, this phase voltage is the voltage across each of the branches of the  $Z$  impedance, we have three branches so we have  $U_{ux}, U_{vy}$  and  $U_{wz}$ . Similarly, we can define a phase current and these phase currents are the currents flowing through these impedances. namely:  $I_{ux}, I_{vy}$  and  $I_{wz}$ . Since all three  $Z$  impedances are strictly equal the effective (RMS) value of the three phase voltages, as well as that of the three phase currents, will always be equal. Only the phase shift between these values can be different thus defining which branch we are talking about. For a symmetrical system we can then define what is known as the phase current in effective (RMS) value which is worth the phase voltage divided by the norm of the  $Z$  impedance.

Notes

Summary



# CONNEXION EN ÉTOILE



Electrotechnique II

The wye connection is a way to connect this balanced load, these three strictly identical  $Z$  impedances in such a way as to form a Y, or wye as shown in the drawing, here, that you have on the left. the three impedances are connected here in a common point X, Y, Z often called the neutral. it is possible to connect this neutral to an additional terminal generally, when the load is perfectly symmetrical the three currents that flow in the common knot here at the centre, is null. It is thus unnecessary to connect this neutral. Another way to draw this wye schema is shown on the right figure it is strictly identical, it's simply another way to draw this wye connection.

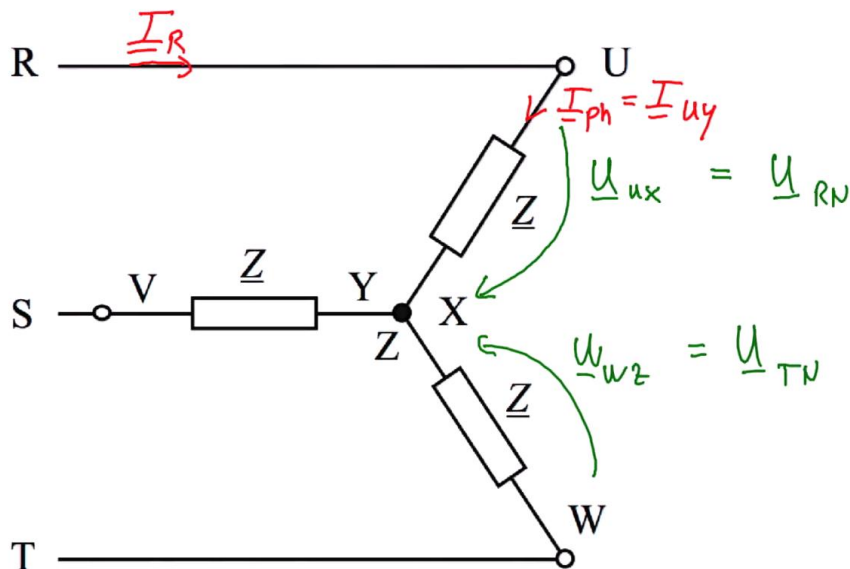
Notes

Summary



2m 26s

# CONNEXION EN ÉTOILE



$$U_{vy} = U_{SN}$$

$$\Rightarrow U_{ph} = U$$

$$I_R = I_{uy}$$

Electrotechnique II

A few remarks now concerning the power supply R, S and T that define the symmetrical three-phase power supply on which we connect our balanced symmetrical load. What we can say is that we have by definition three phase voltages, just like we have defined previously, so here  $U_{ux}$  for example. This  $U_{ux}$  corresponds to the voltage  $U$  between the R terminal of the line and the neutral point. We can write for the three other voltages namely  $U_{wz}$  and this  $U_{wz}$  is equal to  $U$  between T and the neutral. And finally for the third namely  $U_{vy}$  and this  $U_{vy}$  is equal to the voltage between S and N. In other words the phase voltages of our three impedances presented here, are in fact equal to simply  $U$ . In effect the effective (RMS) value that we find on the grid. Three other important elements : the currents. We have here the current flowing through the phase and this phase, we can also specify : the current between U and Y. If we compare this current to the one defined previously in other lessons with the current that we have called "line current"  $I_R$  we can note that this line current  $I_R$  is equal to the phase current  $I_{uy}$ , why? Because at this knot, here there is no possibility for the disappearance of part of the current, thus all the current that enters the line,  $I_R$  must go through the phase, present here, Z. And we will find the exact same configuration for the two other line currents.

Notes

Summary



$$\underline{I}_{ux} = \frac{\underline{U}_{ux}}{\underline{Z}} = \frac{\underline{U}_{RN}}{\underline{Z}} = \frac{U}{Z} e^{j(\alpha - \varphi)} = \underline{I}_R$$

$$\underline{I}_{vy} = \frac{U}{Z} e^{j(\alpha - \varphi - \frac{2\pi}{3})} = \underline{I}_S$$

$$\underline{I}_{wz} = \frac{U}{Z} e^{j(\alpha - \varphi - \frac{4\pi}{3})} = \underline{I}_T$$

Electrotechnique II

Notes

We can then write for these three line currents firstly that  $\underline{I}_{ux}$  the first branch and  $\underline{U}_{ux}$  the phase voltage divided by  $Z$ , the impedance. We've just written that this  $\underline{U}_{ux}$ , the voltage between  $U$  and  $X$  is the voltage between  $R$  and the neutral always divided by the impedance  $Z$ . We can write that this is simply  $U$  divided by  $Z$  effective (RMS) value and the norm of  $Z$  multiplied by  $e^{j(\alpha - \varphi)}$ . This for the first current the current that we will also call line current since it is equal to we've said before, to  $\underline{I}_R$ . We can write the exact same thing for the two other currents. First,  $\underline{I}_{vy}$ . We immediately simplify, we write the final result:  $U/Z e^{j\alpha}$  and we have here  $(\alpha - \varphi)$  and this is phase-shifted by  $2\pi/3$ . And the same result for the third current  $\underline{I}_{wz} = U/Z (\alpha - \varphi - 4\pi/3)$ . We can also write that this is equal to  $\underline{I}_S$ , and that this is equal to  $\underline{I}_T$  namely the three line currents. We can thus see that in a wye setup our line currents correspond to the phase currents and that the line voltages are not equal to the phase voltages.

Summary



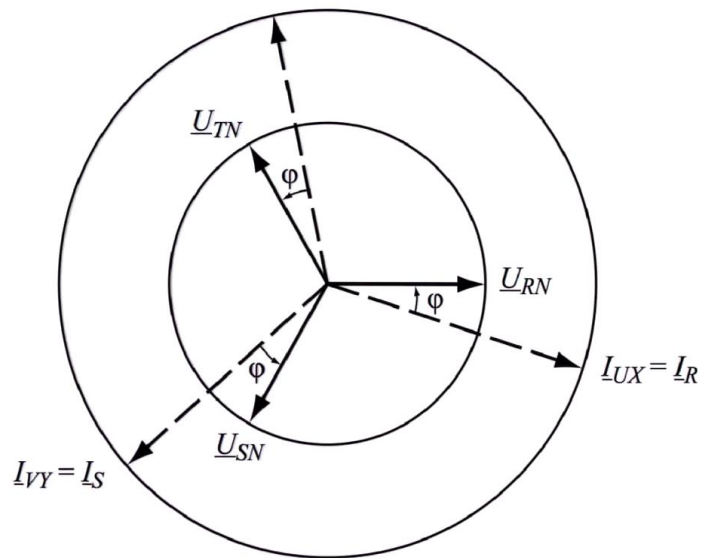
5m 58s

$$\underline{I}_{ux} = \underline{I}_R$$

$$\underline{I}_{vy} = \underline{I}_S$$

$$\underline{I}_{wz} = \underline{I}_T$$

$$I_\ell = I_{ph} = \frac{U}{Z}$$



Electrotechnique II

This can be seen in this diagram where we have represented the three phase voltages  $\underline{U}_{rn}$ ,  $\underline{U}_{tn}$ ,  $\underline{U}_{sn}$  and we discover here the current vectors that are generally bigger than the voltage vectors since these currents correspond to the line currents. We can thus write that  $\underline{I}_{ux}$ , as mentioned previously is the line current,  $\underline{I}_r$   $\underline{I}_{vy}$  is the line current  $\underline{I}_s$  and  $\underline{I}_{wz}$  is the current  $\underline{I}_t$ . In short the line current, in effective (RMS) value is equal to the current flowing through the phase in effective (RMS) value, and that is equal to the voltage divided by the impedance  $Z$ .

Notes

Summary



7m 46s

Le Neutre ?

$$\begin{aligned}\underline{I}_N &= \underline{I}_R + \underline{I}_S + \underline{I}_T \\ &= \frac{U}{Z} \left[ e^{j(\alpha-\varphi)} + e^{j(\alpha-\varphi-\frac{2\pi}{3})} + e^{j(\alpha-\varphi-\frac{4\pi}{3})} \right] \\ &= 0\end{aligned}$$

Electrotechnique II

Notes

What about the neutral ? An important question about this common point that we have noticed between the X, Y and Z connection. To find out, we just need to add together the three currents that converge at this common point at this knot and, using Kirchhof, we can very obviously write that it is the sum of the three line currents listed. We know and we have defined what were, or are, these line currents. We can then simply add them up in a vectorial manner and we have for the first  $I_R e^{j(\alpha-\varphi)}$  another vector phase-shifted by  $2\pi/3$  and the third phasor, a vector phase-shifted by  $4\pi/3$ . and this gives us 3 vectors perfectly phase-shifted by  $2\pi/3$  and the total is equal to 0. And thus, as I was saying previously does not necessitate the connection to the neutral, since anyway, in this neutral, or common point, there normally isn't any current flowing. This is then true only if the load is a balanced load and if the grid is symmetrical Let's move on to another form of connection.

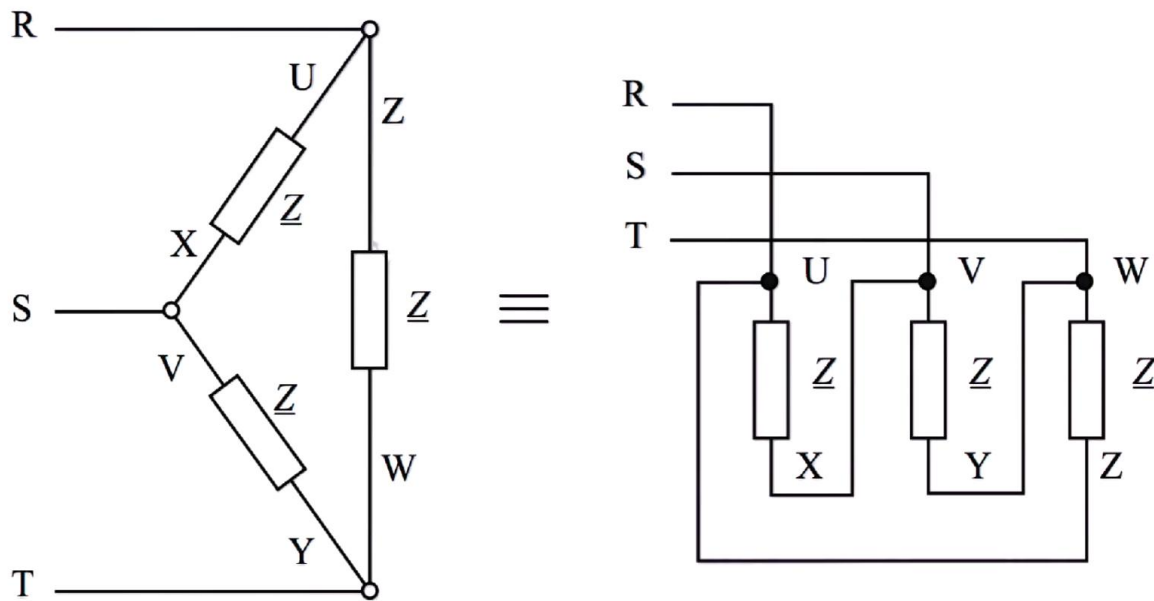
Summary



8m 46s



# CONNEXION EN TRIANGLE



Electrotechnique II

It is possible to connect this balanced charge, not in a wye setup, but in a delta. We can clearly see the difference the connection mode looks like a triangle, or delta, a triangular shape, drawn like this that can, in several cases, be drawn according to this figure on the right with a slightly more complex way of drawing, but that often when we have only straight lines, either vertical or horizontal allows us to solve these drawing problems. So sometimes we can see this drawing in this shape and in a slightly clearer manner in the shape on the left.

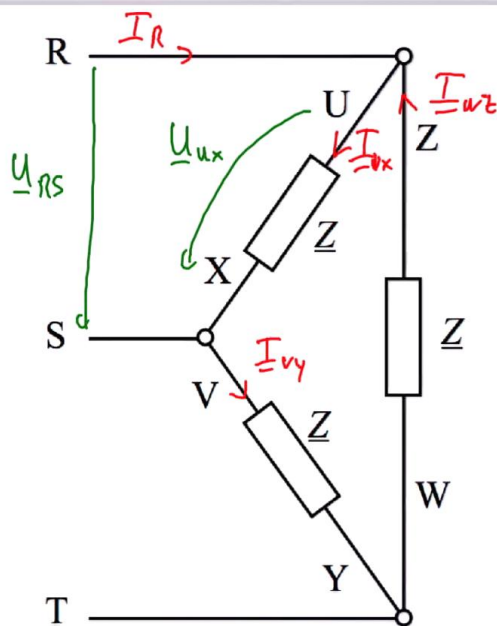
Notes

Summary



10m 13s

# CONNEXION EN TRIANGLE



Tensions de phases se confondent avec les tensions de ligne

Electrotechnique II

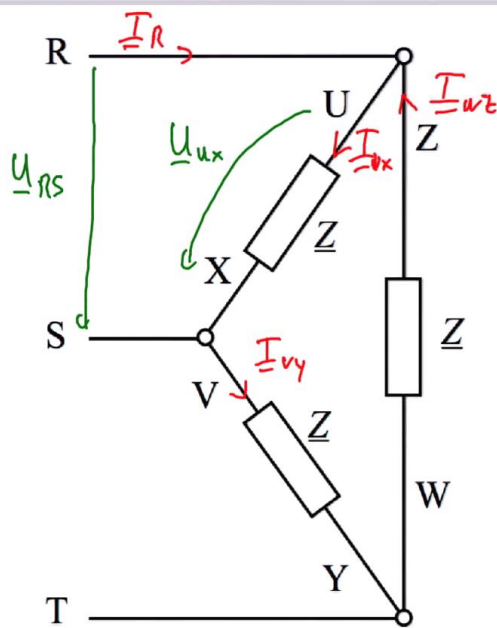
By studying now more in detail this new diagram, or this new way of connecting, we can observe a certain number of elements. The first concerning the currents. The line currents, which is here can no longer simply cross the current of the first phase  $I_{ux}$  it will share itself here between  $I_{ux}$  and the other current  $I_{wz}$ . In effect, we have here a knot that makes it so that the current is not equal but is split. We will have, similarly the current here  $I_{vy}$  and the current, the third  $I_{wz}$ . On the other hand, for the voltages we will notice a surprising thing compared to a wye connection. Here is the voltage  $U_{ux}$  we can see that this voltage is measured between the terminals R and S and thus we notice that this voltage  $U_{ux}$  is actually the same voltage as  $U_{rs}$ . And this in an identical manner for all three phases. We can then write that the phase voltages are confused with the line voltages. In a more strict manner, we can say that  $U_{ux}$  as mentioned previously is U between the R and S terminals, namely the line voltage similarly, for the other two,  $U_{vy}$  is equal to  $U_{st}$  and for the third  $U_{wz}$  to  $U_{tr}$ . In other words this phase voltage in effective (RMS) value, is equal to the line voltage of grid, and thus  $\sqrt{3} \cdot U$ .

Notes

Summary



# CONNEXION EN TRIANGLE



Tensions de phases se confondent avec les tensions de ligne

$$\underline{U}_{ux} = \underline{U}_{RS}$$

$$\underline{U}_{vy} = \underline{U}_{ST} \Rightarrow U_{ph} = U_l = \sqrt{3} U$$

$$\underline{U}_{wz} = \underline{U}_{TR}$$

Par Kirchhoff :  $I_l = \sqrt{3} I_{ph}$

Electrotechnique II

For the currents using Kirchhoff by writing the different equations of the three line currents and of the three phase currents, we can write in a general manner that the line current is equal to  $\sqrt{3}$  times the phase current. Furthermore, these line currents form a symmetrical system that will be delayed by  $\pi/6$  compared to the phase currents.

Notes

Summary



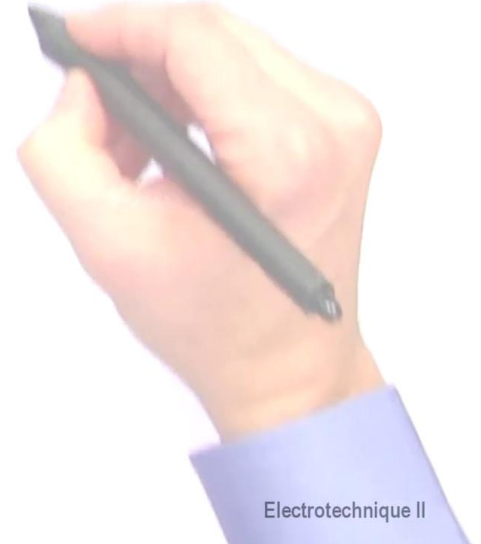
12m 51s

Résumé :

Couplage

Etoile ( $\gamma$ )

| Triangle ( $\Delta$ )



Electrotechnique II

We have here the final diagram that shows the line currents in a delta setup  $I_r$   $I_s$  and you have  $I_t$ , here, in the diagram and you have, in a vectorial manner this demonstration with  $I_{wz}$  minus  $I_{wy}$ , namely, in effect the manner of doing, using Kirchhof, by adding on the knot of the three lines, the currents that meet there. We can thus demonstrate that the line current is equal to  $\sqrt{3}$  times the phase current. In other words for example for  $I_r$  and the difference with  $I_{ux}$ , thus the phase the first phase Z,  $I_{ux}$  we can clearly see a ratio of  $\sqrt{3}$  between the lengths of these two vectors. And all of this phase shifted, as mentioned previously, by  $\pi/6$  creating a new symmetrical phase-sifted system. We can, to sum up give a table of the setups, in wye or in delta and what this gives for the phase voltage and the phase current. We will thus write in this table the setup namely, is the setup in a wye configuration often written as Y to look like the letter Y or a delta that we often write as a delta triangle like this. Thus we work, here, a table, that will allow us to give this configuration for the two values that we've seen previously, either the phase voltage or the phase current.

Notes

Summary



13m 27s

# CONNEXION EN TRIANGLE

Résumé :

	Couplage	Etoile (Y)	Triangle (Δ)
Tension de phase		$\frac{U_l}{\sqrt{3}}$	$U_l$
Courant de phase		$I_l$	$\frac{I_l}{\sqrt{3}}$

Electrotechnique II

first, in the case of a wye, we have noticed that the phase current is equal to the line current in a wye setup. However, the phase voltage has a  $\sqrt{3}$  ratio, namely the line voltage divided by  $\sqrt{3}$  for the phase voltage. It is in fact the exact opposite for a delta setup, we have a phase voltage equal to the line voltage, in this delta setup and for the current a ratio of  $\sqrt{3}$ .

Notes

Summary



15m 15s



- Définition de la charge équilibrée
- Dans un montage en étoile, les courants de ligne se confondent avec les courants de phase
- Dans un montage en triangle, les tensions de ligne se confondent avec les tensions de phase

Electrotechnique II

To finish here is a little conclusion to say that we have defined a symmetrical balanced load. We have seen the wye setup, the delta setup to define the phase voltages , phase currents and their relationship with line currents and line voltages. And finally, that these relationships are often crossed thus we often have one of the currents equal to the other, or with a ratio of  $\sqrt{3}$  and in an identical manner for the voltage. Thank you and until next time.

Notes

Summary



15m 52s