

# LE PONT DE WHEATSTONE

## LEÇON 11

## Électrotechnique I

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## Généralités



- Analogie
- Description du circuit
- Equilibre - Condition d'équilibre
- Exemples d'application
- Conclusion

Electrotechnique I

Hello, today, we will discuss about a particular circuit much used in metrology: it is the Wheatstone Bridge. This is the content of today's lesson: first of all, we will do an analogy with another measuring device that is the beam balance. Then, we will describe the circuit that builds the Wheatstone bridge. Then we will establish a condition for the bridge to be at equilibrium, and calculate it's equilibrium condition. Finally we will give two application examples and we will end with a conclusion.

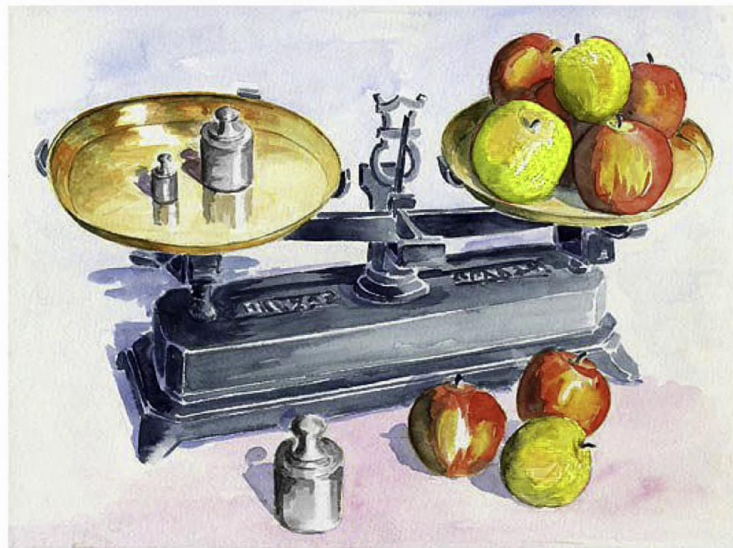
Notes

Summary



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## Analogie



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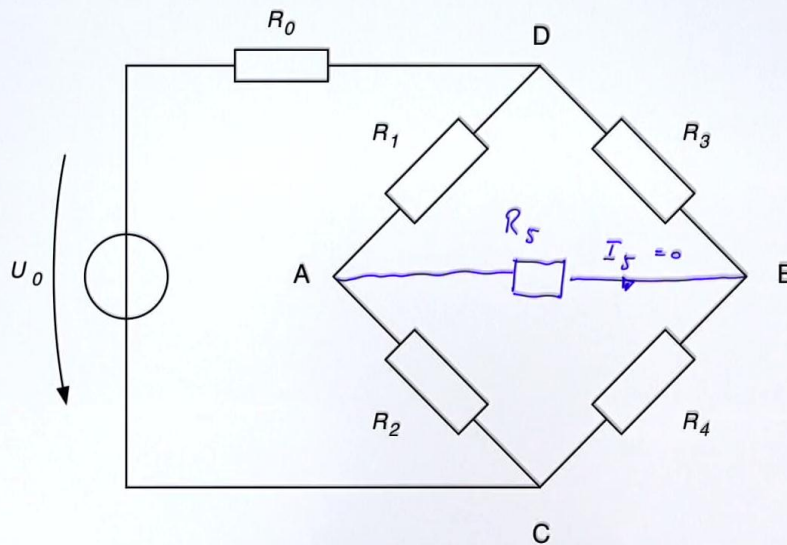
The Wheatstone bridge is an interesting circuit example capable of satisfying, a particular equilibrium condition, a little bit like balance beams.

Notes

Summary



## Description du circuit - Equilibre



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So, here is the Wheatstone bridge. It is a circuit that is essentially made of two branches of two resistors in series. And, the condition for the bridge to be at equilibrium is that the voltage between the points A and B has to be equal to zero. So if we connect a resistor between these two points, resistor that we can name  $R_5$  well, the current that crosses this resistor, that we name  $I_5$ , must be equal to zero, at equilibrium. So if we connect a galvanometer, which is a device allowing the measure of the current. If we connect a galvanometer in series with this resistor her, this last one will indicate a null value. This condition is independent of the value of  $R_0$  and of  $U_0$ . It should be noted that such a principle can be used as continuous current. This is what we are going to work on today, to perform the measurement of an unknown resistor, or generalize an alternative state to define the unknown impedances.

Notes

Summary

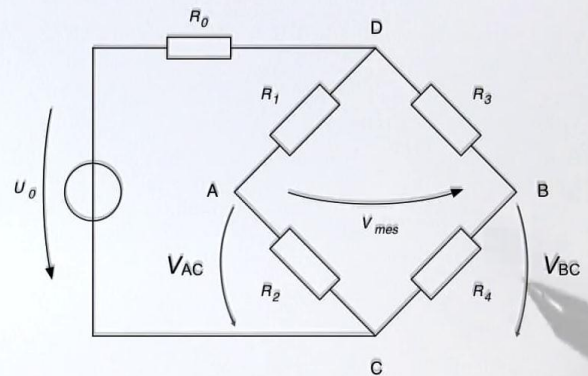


## Condition d'équilibre

$$V_{AC} = V_{BC} \quad I_5 = 0 \quad U_5 = 0$$

$$U_5 = V_{AC} - V_{BC} = 0$$

$$V_{AC} = V_{DC} \cdot \frac{R_2}{R_1 + R_2} \quad V_{BC} = V_{DC} \cdot \frac{R_4}{R_3 + R_4}$$



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The purpose is to cancel the voltage between the point A and the point B, this voltage  $U_5$ , whatever the values of  $U_0$ ,  $R_0$  and  $R_5$  are. To get the equilibrium criteria, we will first of all calculate, the voltage that there is between the terminal A and the terminal C, and secondly, the voltage that is between the terminal B and the terminal C and by making these two voltages equal, we will get a voltage  $U_5$  that is equal to zero. The equilibrium condition is that  $V_{AC}$  must be equal to  $V_{BC}$  which will give a current  $I_5$  equal to zero and a voltage  $U_5$  also equal to zero. We can also define this voltage  $U_5$  between the points A and B as being:  $U_5 = V_{AC} - V_{BC}$ . This must be equal to zero. The voltage  $V_{AC}$  is given by the following relationship: It is the voltage  $V_{DC}$  between the point D and the point C, the voltage  $V_{DC}$  multiplied by the resistor  $R_2$  divided by the sum  $R_1 + R_2$ . We recognize the voltage divider. We do the same thing for the voltage at the terminals of B and C. So we have that  $V_{BC}$  is equal to  $V_{DC}$  multiplied by  $R_4$ , divided by  $R_3 + R_4$ . And again, this expression of the voltage divider. For the equilibrium condition, the voltage  $V_{AC}$  needs to be equal to the voltage  $V_{BC}$ .

Notes

Summary



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## Condition d'équilibre

$$V_{AC} = V_{BC} \quad I_s = 0 \quad U_s = 0$$

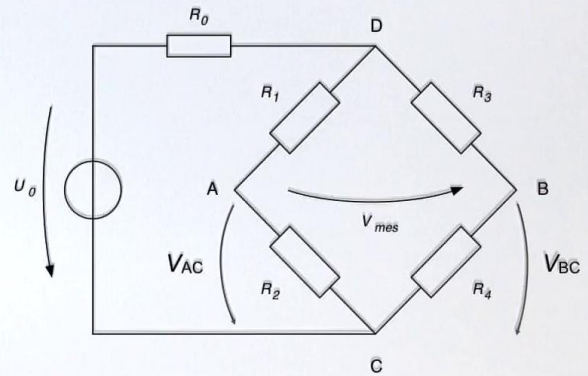
$$U_s = V_{AC} - V_{BC} = 0$$

$$V_{AC} = V_{DC} \cdot \frac{R_2}{R_1 + R_2} \quad V_{BC} = V_{DC} \cdot \frac{R_4}{R_3 + R_4}$$

$$\frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4} \Rightarrow R_3 R_2 + \cancel{R_2 R_4} = R_4 R_1 + \cancel{R_4 R_2}$$

$$\Rightarrow \underline{\underline{R_2 R_3 = R_1 R_4}}$$

La condition pour que l'équilibre soit atteint:  
"Les produits croisés des résistances doivent être égaux"



Electrotechnique I

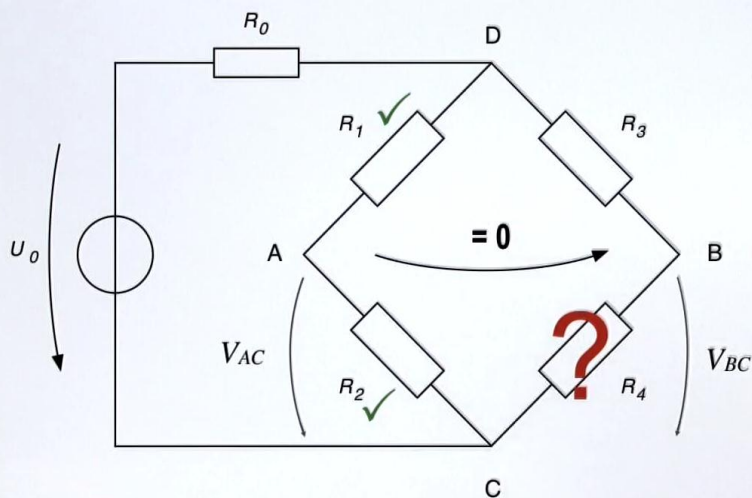
We can rewrite this equation the following way: [Silence] that we can expand and that gives: [Silence] We can simplify this equation by deleting these two products. Which gives us the following relationship: The product  $R_2 * R_3$  must be equal to the product of  $R_1 * R_4$  so that the bridge is in equilibrium. Rather than embarrass ourselves with indices which are, moreover, not from a conventionality we can clearly see here that the indices  $R_1, R_2, R_3, R_4$  can be arbitrarily chosen we will then remember the following rule, that says that: the equilibrium condition of the bridge is reached when the cross products of the resistors,  $R_1 * R_4$  is equal to the product of the resistors  $R_3 * R_2$  [Silence] Now that we have defined the equilibrium condition we can see that we can use this circuit, or this bridge, in two different ways: either by adjusting the resistors  $R_1, R_2, R_3, R_4$  so that the voltage  $V_{ab}$  is equal to zero, or by the equilibrium of the circuit, that satisfies this equation her, we look at the voltage that appears at the terminals  $V_{ab}$  when we change one or several of the four resistors.

Notes

Summary



## Exemple d'application 1 – Mesure de résistance



$$R_1 \cdot R_4 = R_2 \cdot R_3$$

$$V_{AC} = V_{DC} \frac{R_2}{R_1 + R_2}$$

$$V_{BC} = V_{DC} \frac{R_4}{R_3 + R_4}$$

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So the first mode of use, is the resistance measuring. If we start with a circuit where a first resistor is known and a second resistor, we can get the result, the computation of a fourth resistor simply by adjusting the resistor, here,  $R_3$ , in a way that the voltage  $V_{ab}$ , that appears between these points, becomes equal to zero, namely that we satisfy this equation here.

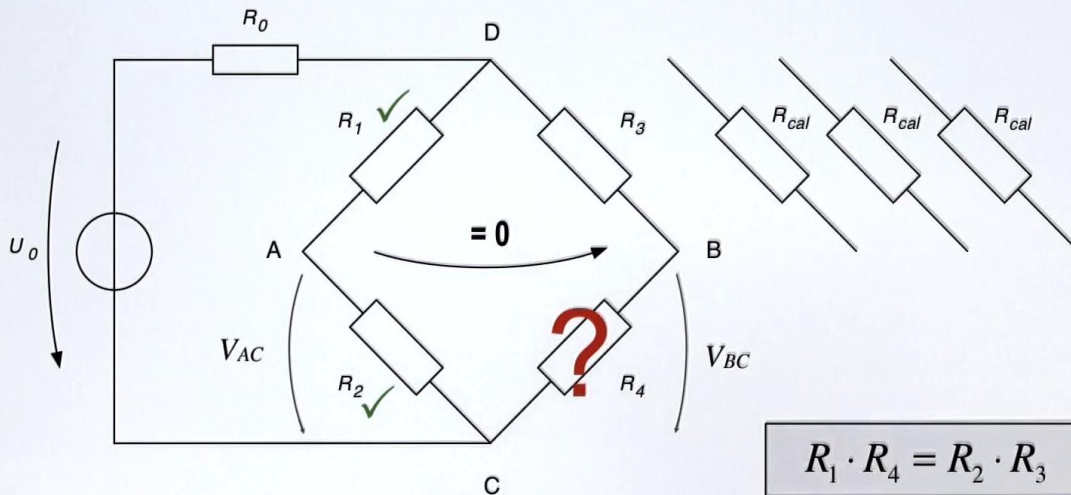
Notes

Summary



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## Exemple d'application 1 – Résistances calibrées



$$V_{AC} = V_{DC} \frac{R_2}{R_1 + R_2}$$

$$V_{BC} = V_{DC} \frac{R_4}{R_3 + R_4}$$

$$R_1 \cdot R_4 = R_2 \cdot R_3$$

$$\Rightarrow R_4 = \frac{R_2 \cdot R_{cal \text{ eq}}}{R_1}$$

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So the analogy with the beam balance is total. We have here an unknown that we will like to know the mass, we will simply add here calibrated weights such that the beam balance comes back to equilibrium. This is what we do here with the bridge. If we know the resistor  $R_1$  and the resistor  $R_2$  The resistor  $R_4$  can be obtained by adding or replacing  $R_3$  by known calibrated resistor values, so that the voltage  $AB$  becomes equal to zero. Which makes that the resistor  $R_4$  will be equal to the product of the resistor  $R_2$  multiplied by the equivalent calibrated resistor, divided by  $R_1$ .  $R_1$  and  $R_2$  are known and also the calibrated resistors.

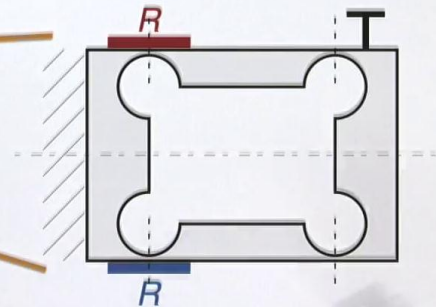
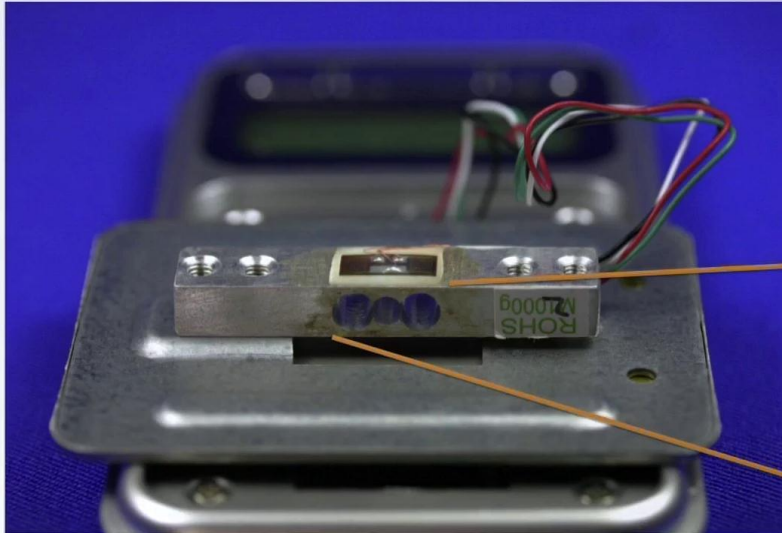
Notes

Summary





## Exemple d'application 2 – Balance électronique



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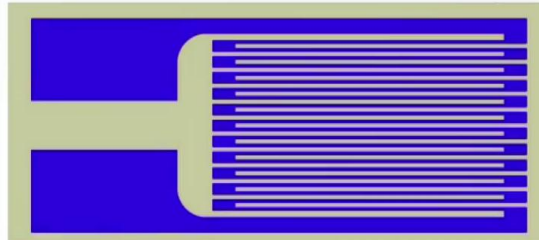
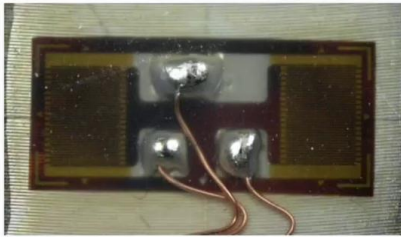
The second application example is an electronic scale, a scale such as the ones in trade, that allows to measure a force, or a weight, by using strain gauges. This scale, here disassembled, is made of a deformable mechanical structure where a part is linked to the stationary part of the balance, the table, and another part that is connected to the plate. Schematically, here, this element here, is what we call a bearing neck, that is a deformable structure with the four necks that are here very thin, and so we put a resistance here on the part that deforms itself, and when we will push here on the plate all the structure will deform itself and change the resistance.

Notes

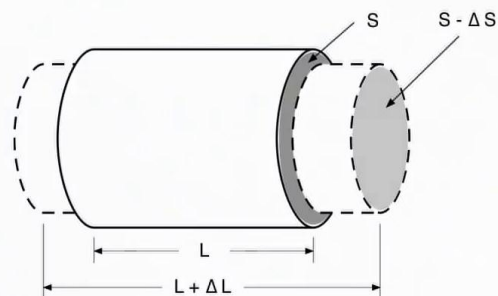
Summary



## Exemple d'application 2 – Jauge d'extension



$$R = \rho \cdot \frac{L}{S}$$



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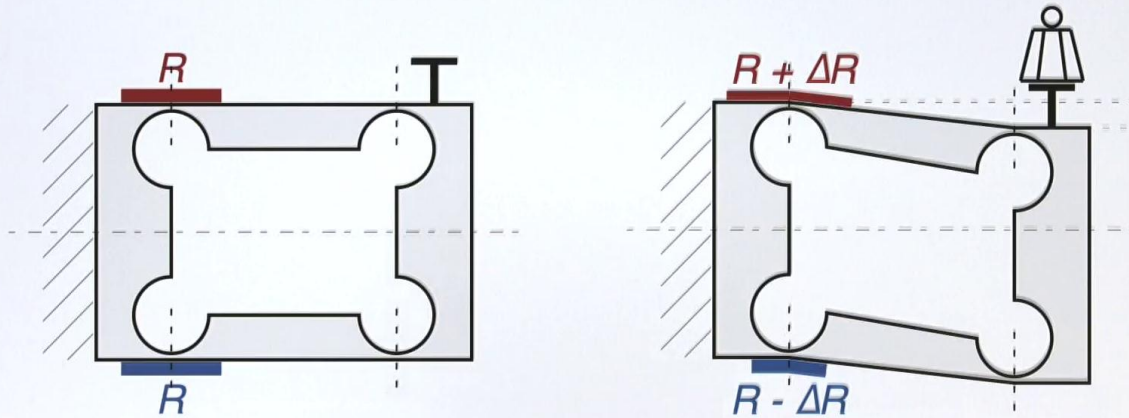
We can see here a picture a little bit more detailed of such strain gauges. Here, schematically, to picture the element in a better way, we see that it is a coil with  $L$  times the length of a copper cable and if we pull on this gauge, each of these strands will lengthen and the section will decrease so the volume remains the same. So we have a resistance that will change. If we pull on the strain gauge, the resistance will increase and if we compress the strain gauge, the resistance will decrease.

Notes

Summary



## Exemple d'application 2 – Déformation



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We see here on these two sketches the bearing neck in its equilibrium position and when we apply a force on the plate so we deform this extremity here the other one remaining stationary and so the resistance, or the strain gauge, that is positioned at this place will elongate, so we will increase its resistance, while this one will be compressed. We will decrease its resistance of a value  $\Delta R$ .

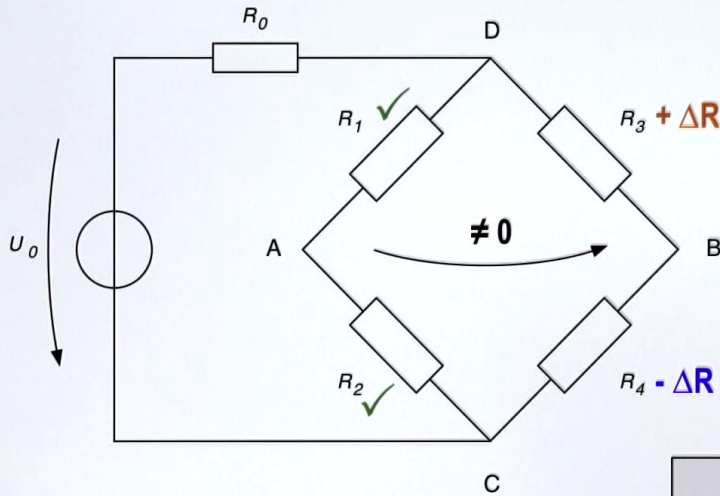
Notes

Summary



9m 14s

## Exemple d'application 2 – Mesure du déséquilibre



$$V_{mes} = V_{DC} \left( \frac{R_4 - \Delta R}{(R_3 + \cancel{\Delta R}) + (R_4 - \cancel{\Delta R})} - \frac{R_2}{R_1 + R_2} \right)$$

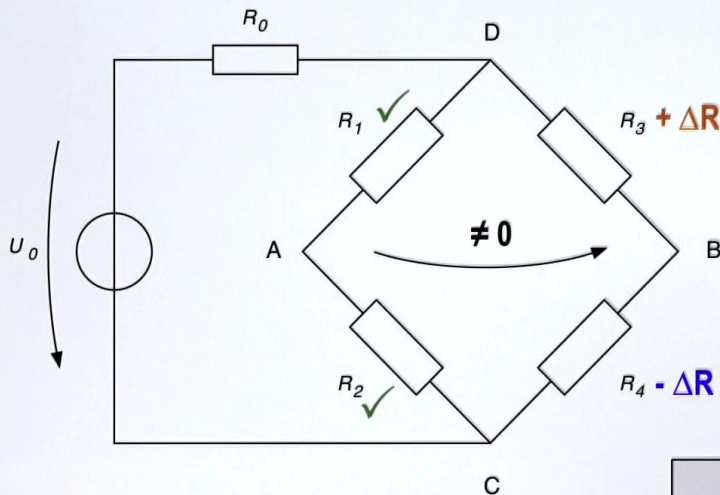
Here is the circuit used within the electronic balance. We have four resistors where two of them are known and fixed. The two others are the strain gauge resistors, and that are capable to fluctuate. At equilibrium, these resistors are equal and so the bridge is at equilibrium the voltage  $V_{ab}$  is equal to 0. Under stress, the bearing neck will change. The resistor  $R_3$  will increase, the resistor  $R_4$  decreases, and so at equilibrium  $V_{ab}$  will be broken and a voltage between these two points will appear. We find the equation that we have expanded before, with the crossed products that must be equal, the resistor  $R_4$  will be changed of a value  $-\Delta R$ , the resistor  $R_3$  will be modified of a value  $+\Delta R$ . We see in this equation that  $\Delta R$  can simply be changed here, compensate itself. And the voltage that appears at the terminals, between the terminals A and B namely given by this equation that depends on the fluctuations of the resistor and therefore the deformation that has been done on the scale. There will be a voltage between the points A and B that will appear, it is a little bit like the needle of the balance, here that will deviate from the equilibrium position if we continue our analogy.

Notes

Summary



## Exemple d'application 2 – Mesure du déséquilibre



$$V_{mes} = V_{DC} \left( \frac{R_4 - \Delta R}{(R_3 + \cancel{\Delta R}) + (R_4 - \cancel{\Delta R})} - \frac{R_2}{R_1 + R_2} \right)$$

Note that we can arrange the resistors in different ways to increase the sensibility of the circuit, eventually go from two gauges to four gauges. But anyway, the big advantage of this set up, is that it is indifferent to external disturbances, for example the temperature. At equilibrium, if the circuit's ambient temperature increases, then the bridge stays at equilibrium because all the resistors will increase because of the temperature. And therefore, the equilibrium will be guaranteed.

Notes

Summary







- Circuit particulier – Associé à la métrologie
- En régime continu et alternatif
- Peut être utilisé de deux manières
- Présente des avantages

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So we have seen a particular circuit linked to metrology, it is the Wheatstone bridge. We have done the expansion of the functioning in DC mode but it is also valid for the alternative mode. We have seen that we could use this bridge in two different ways either by searching for the equilibrium either by starting at equilibrium, and moving away from this equilibrium and calculate the resistance fluctuations. This circuit has a lot of advantages, as being indifferent to the temperature. It is clear now that, there are many electronic devices that allow to measure the resistances, and that make the circuit a little bit obsolete, but the structure of the bridge remains used in many assemblies, even in embedded electronics, when it is not possible to add another device.

Notes

Summary



11m 50s