Diagrammes de Bode

Annexe

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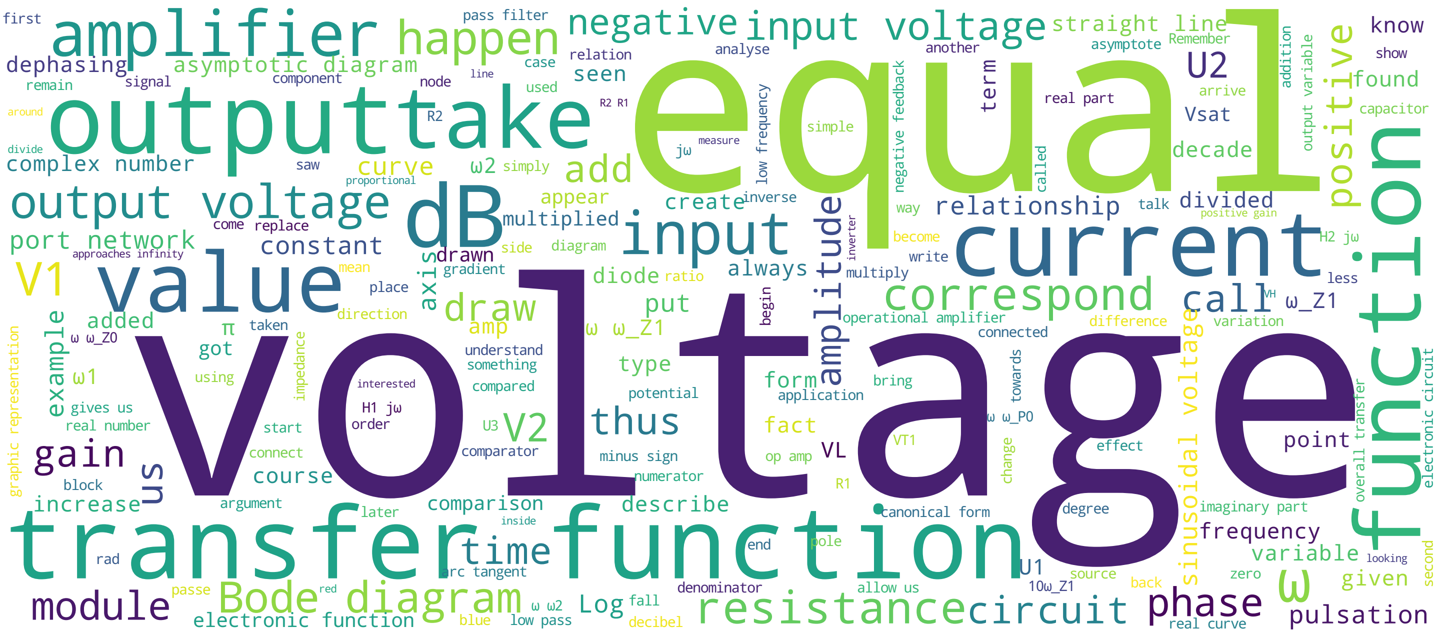
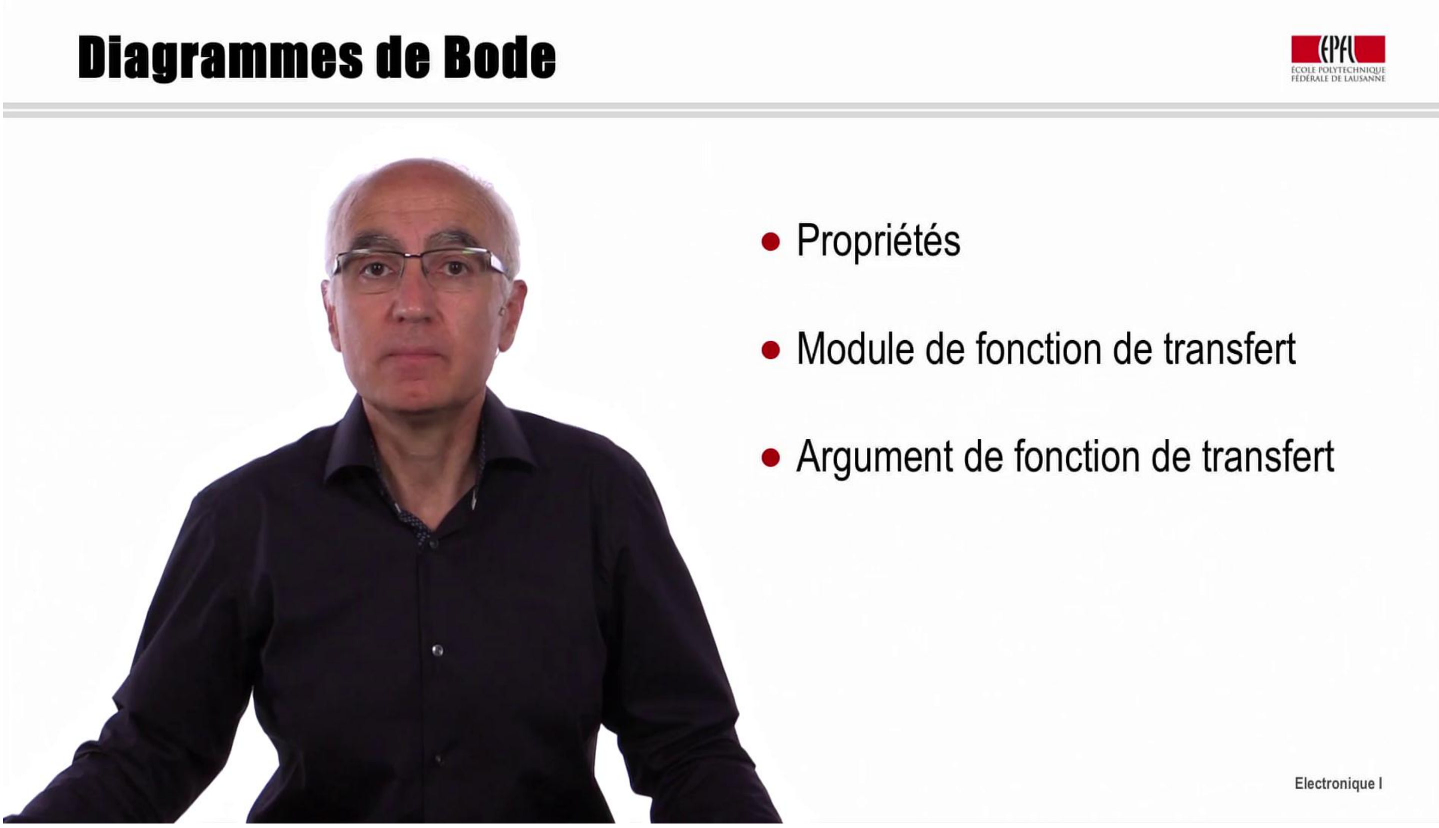


Video

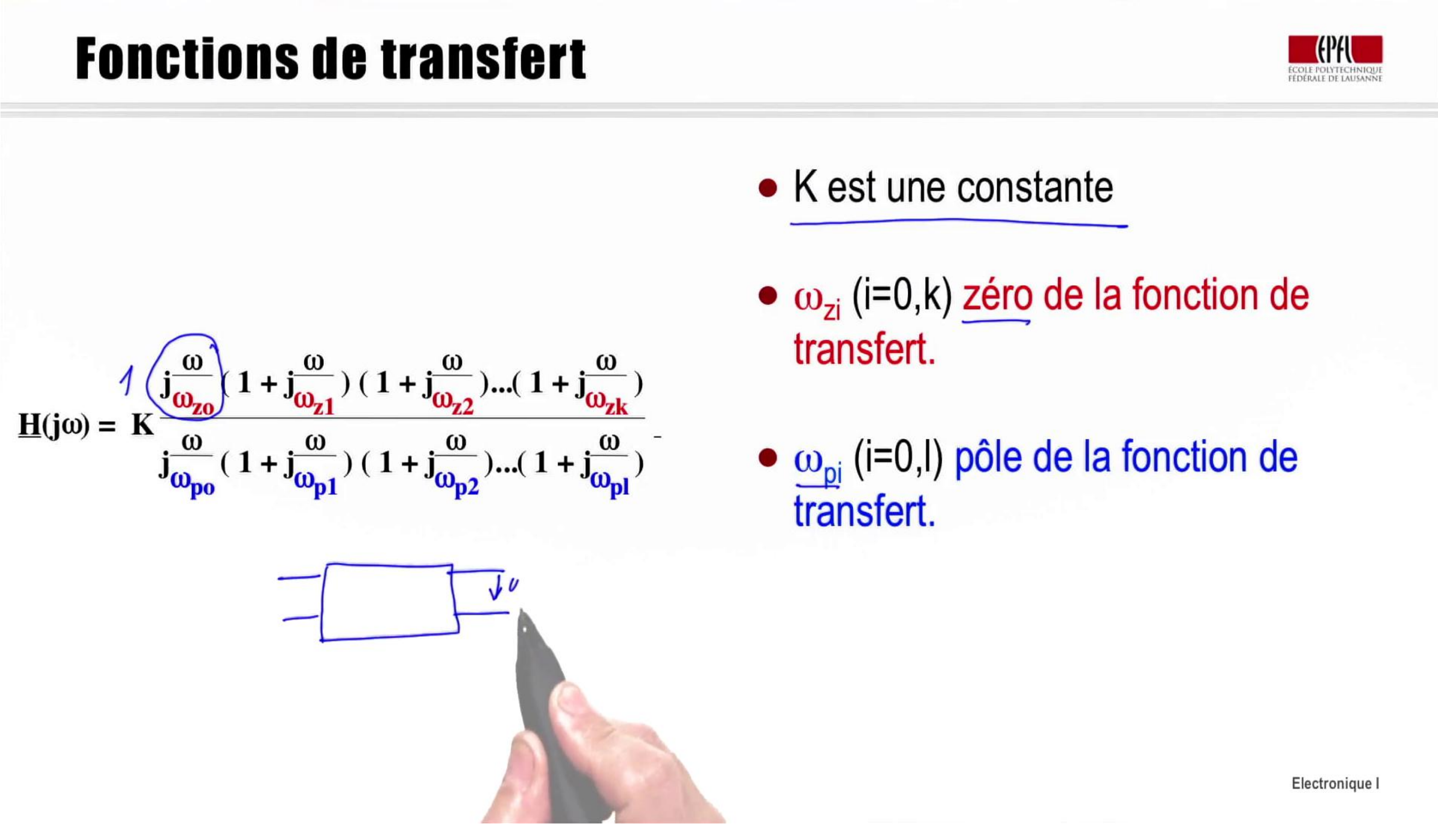


Today we are going to look at the Bode diagram. So, what is a Bode diagram? Well, a Bode diagram is just a way of representing what we call a transfer function. A transfer function is a relationship between an output and an input variable of a two-port network in electronics or other fields. So, when we have an output variable - for example, an output or input voltage, so that we have a relationship between the input and output, and we speak of a Bode diagram, this in fact refers to a vibration in sinusoidal form. This will help us greatly in analysing electronic functions. I think that it is easier to understand when we are really talking about electronic functions - then we see that all these functions are described by the Bode Diagram module and phase. So, by way of introduction, I am going to start off by discussing what a diagram is intended to show, that is, a mathematical formulation. Next, we will see that there are two ways of presenting a Bode diagram, module and phase. So, we are going to learn to carry out the module transfer function and next, we are going to look at the phase transfer function. And I think that the best way to understand this is by pushing through and getting to the chapters in which we will look at real electronic functions.

Notes

Summary

0m 05s

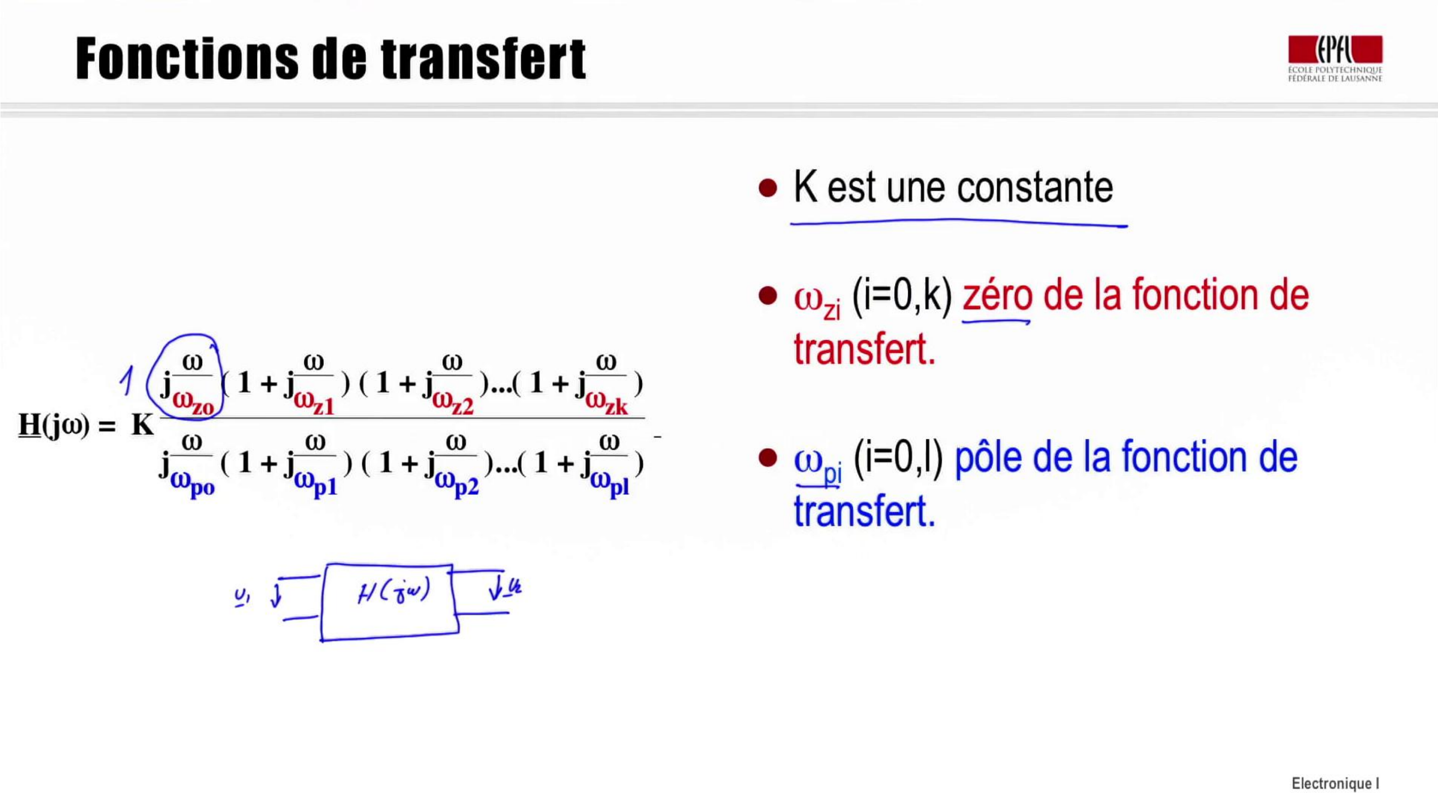


Let us therefore begin with what is called a transfer function. This is a transfer function. In a transfer function, you can see that there is a numerator, a denominator, and a value, *K*, which is a constant. As for the numerator, we refer to it with a *ω_Z*. *Z* is for zero transfer function. As for the denominator, we have the subscript *P* because this is the pole of the transfer function. You will note that we have the form *j(ω/ω_Z0)* and that the same form recurs with the denominator. Thus we also have *1/j(ω/ω_P0)*. So the form at the numerator is also found at the denominator although it is inverted, 1 over. It is the same case with *1+j(ω/ω_Z1))* and *1/(1+j(ω/ω_P1))*. So, when we create these two forms, this one, that one, and their inverses, as well as the constant *K*, we are going to see later that this so-called canonical representation is going to correspond with a block. In this block, we have placed electronics. At the output, there is an output variable. This output variable could be voltage. So, let us consider the example where it is voltage. There is a sinusoidal vibration at the output due to sinusoidal vibration at the input.

Notes

Summary

1m 33s

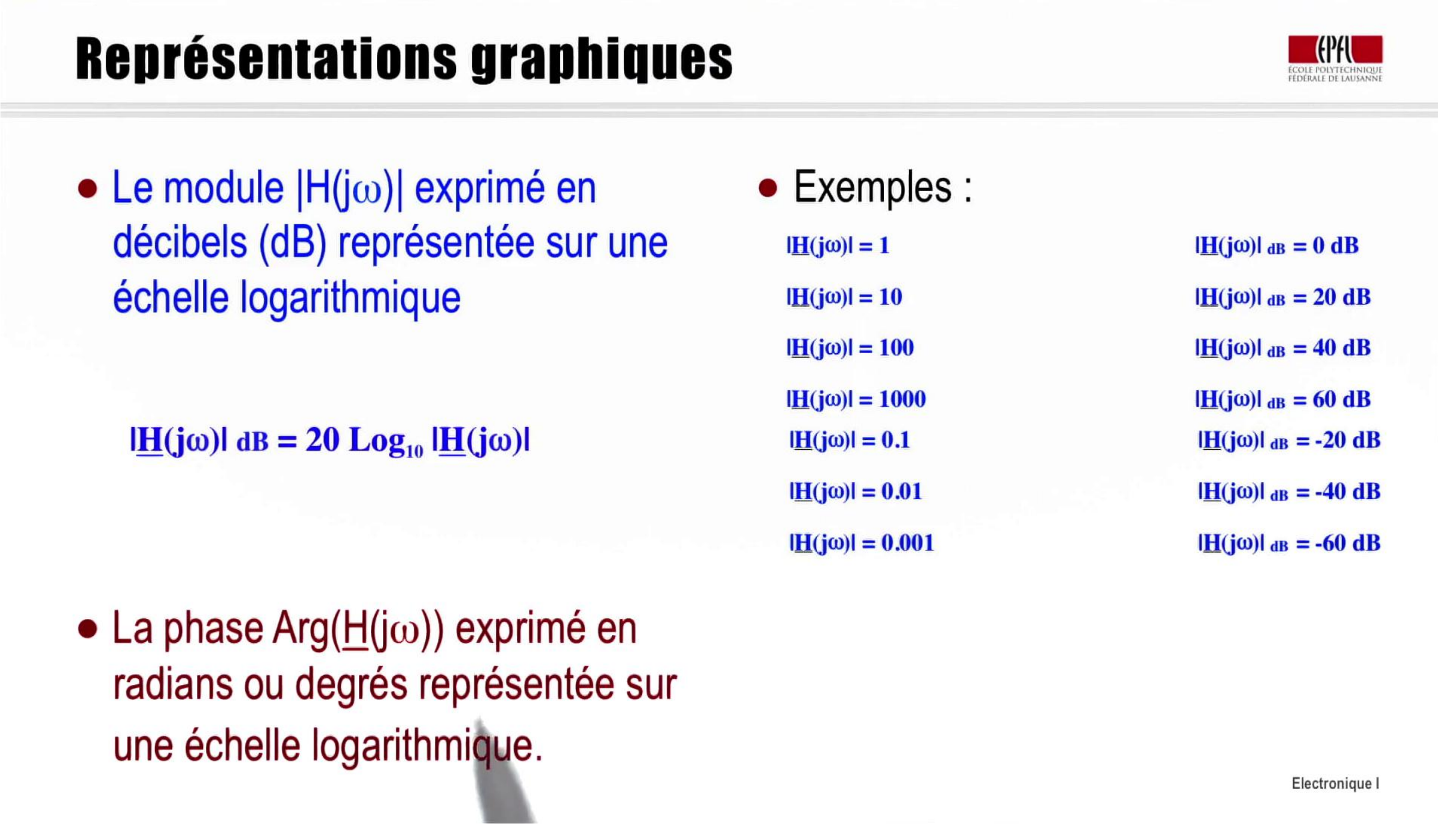


And that which is inside this, we call the transfer function. And all the canonical forms that you see represented here, this one repeated for a second time, and a third time, and same with the denominator, we can create these using electronics. So, when you continue the Electronics course, you will see that the function *1/(1+j(ω/ω_))* corresponds with a low-pass filter. That means that the output voltage, as compared with the input voltage, has a relationship that gives us a function in which the low frequency passes through this filter and will be cut at a point that we call the pole of the transfer function, which is linked to this *ω_P1* and which corresponds with a well-known value. The variable is thus *ω*. As for the unit *H(jω)*, we are going to see right after this that it is the relationship between one voltage and another, and there is thus no unit. And we are going to talk about decibels because we are going to look for the 20 Log of the output voltage divided by the input voltage.

Notes

Summary

3m 16s

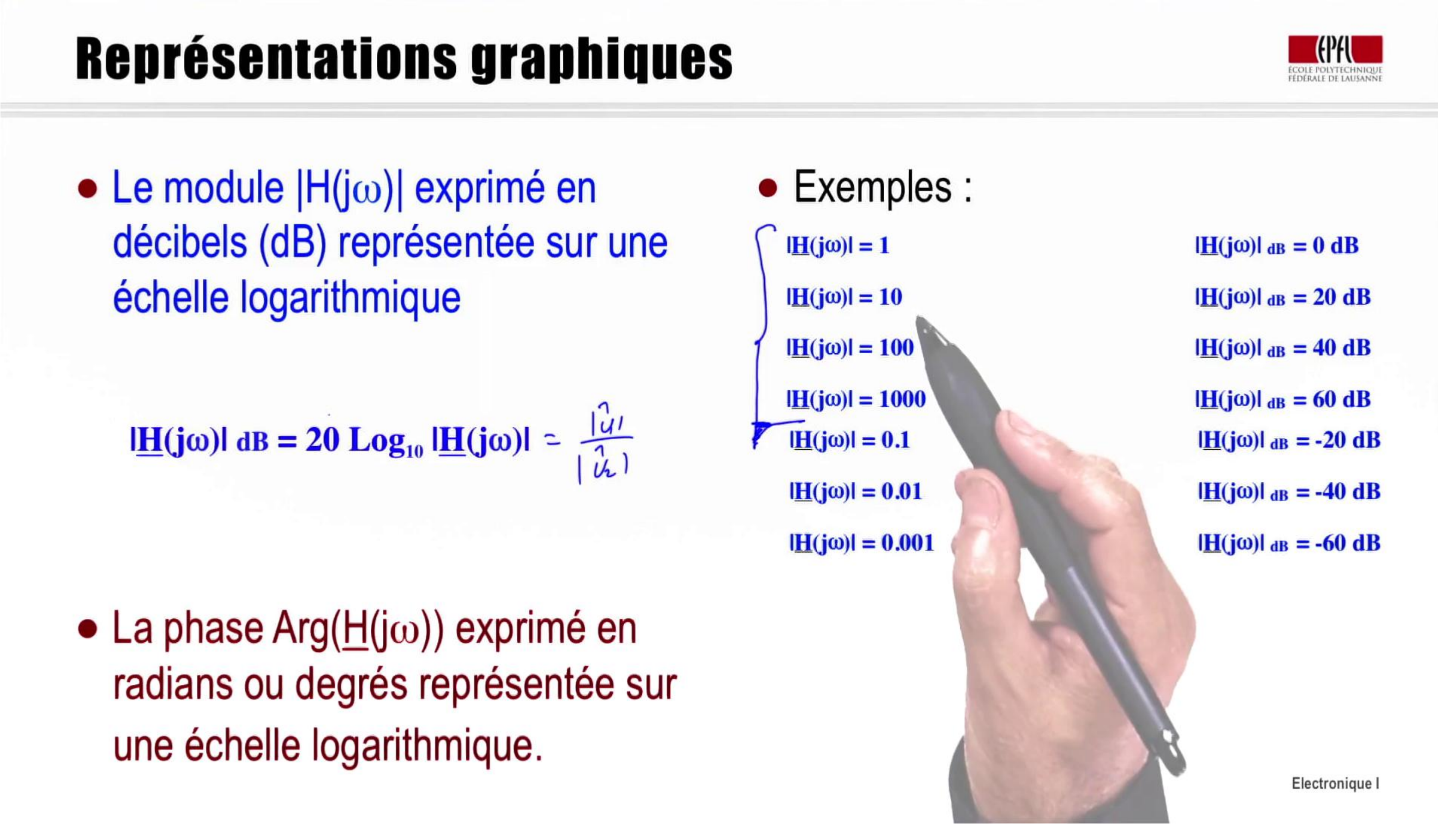


We are going to use a graphic representation. This graphic representation is there to show us, for a set of sinusoidal voltages, what happens with the transfer function module, which will tell us what happens with the amplitudes of these different voltages that are going to cross, therefore I am talking about the sinusoidal voltage which is going to traverse these two-port networks we have just seen. The phase also describes what happens with the phase of these sinusoidal voltages at the output compared to the input, with the variableω. Therefore we are going to change the ω, the voltage pulses, and we are going to see their amplitude values, the relationship with the amplitude values that we call the module of the transfer function, and which we describe as being a module of *H(jω)*, for which we use this logarithmic function, so we are going to look for the 20 Log base 10 of *H(jω)*. Now why would we want to do it like that? It is simple, because the phase will describe the signs. So, voltage which is positive at the input to a two-port network becomes negative. And, even if the amplitude is the same, one sees here, that there has been no change in amplitude, it's Log 20 of something that is equal to one.

Notes

Summary

4m 28s

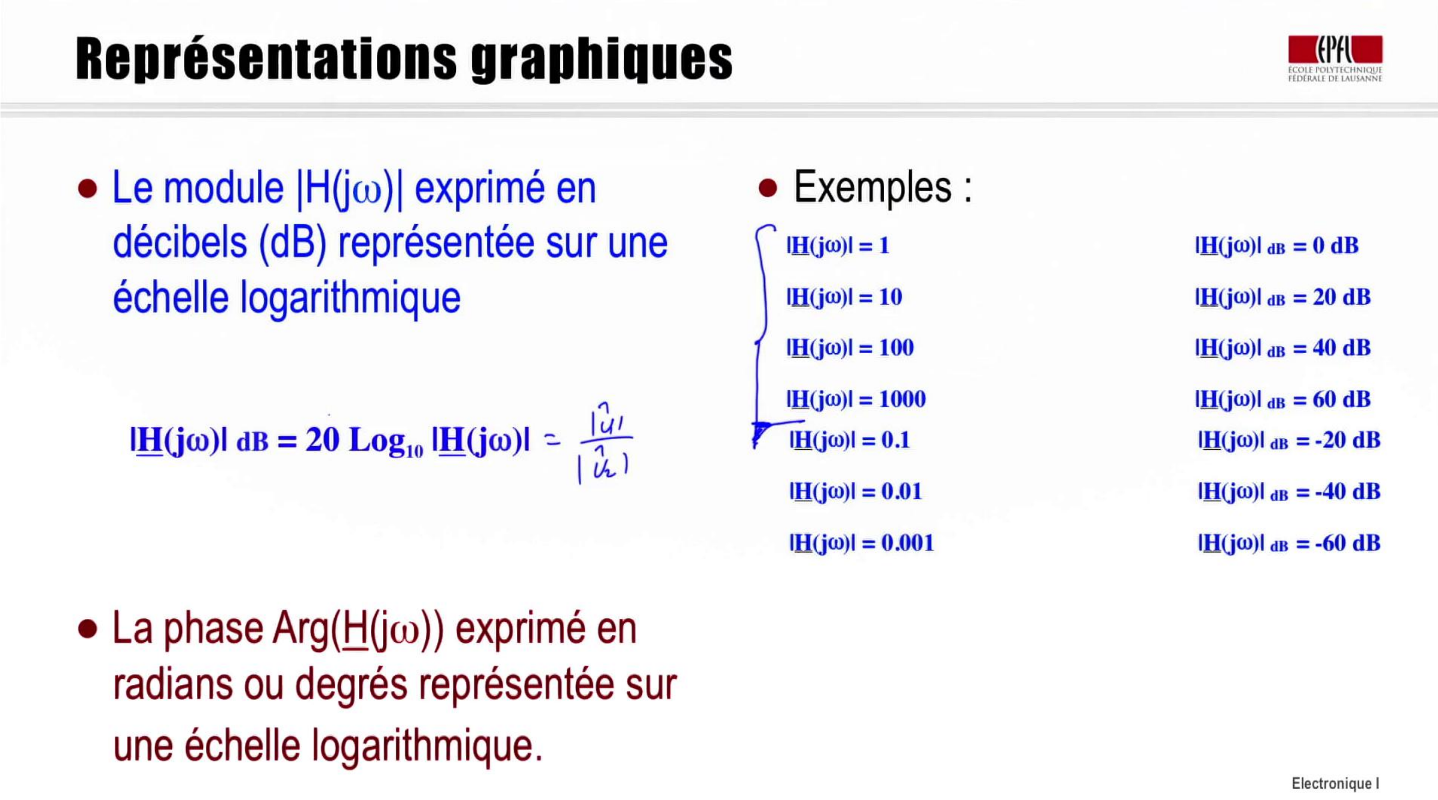


So 20 Log of 1 means the output voltage, i.e. the amplitude divided by; if the peak value is the same, there has been no change. If this is equal to 1, then that will give us the Log of 1 equal to 0, and then we are back with 0 *dB* again. It is the same if the output voltage if 10 times greater than the input voltage; in that case, we will find 20 *dB*. Every time we multiply by 10, we add 20 *dB* due to this gradient of 20 Log base 10 of the *U1/U2* relationship. So you can see it here. Each time we multiply by 0, we add 20 *dB*. However, each time we divide by 10, the output is 10 times less than the input voltage, and we will find the minus sign appearing, it is -20 *dB*, -40 *dB*. It's the same every time we divide by 10 - we see reduction of 20 *dB*. When this happens, we talk of amplification. Up to here. So, from there to there, if one excludes this - because there, we have a *follower*. If we talk about voltage, we call that a voltage *follower*, which is to say that the output voltage is equal to the input voltage. And from there, the output voltage is amplified in comparison with the input voltage.

Notes

Summary

5m 51s

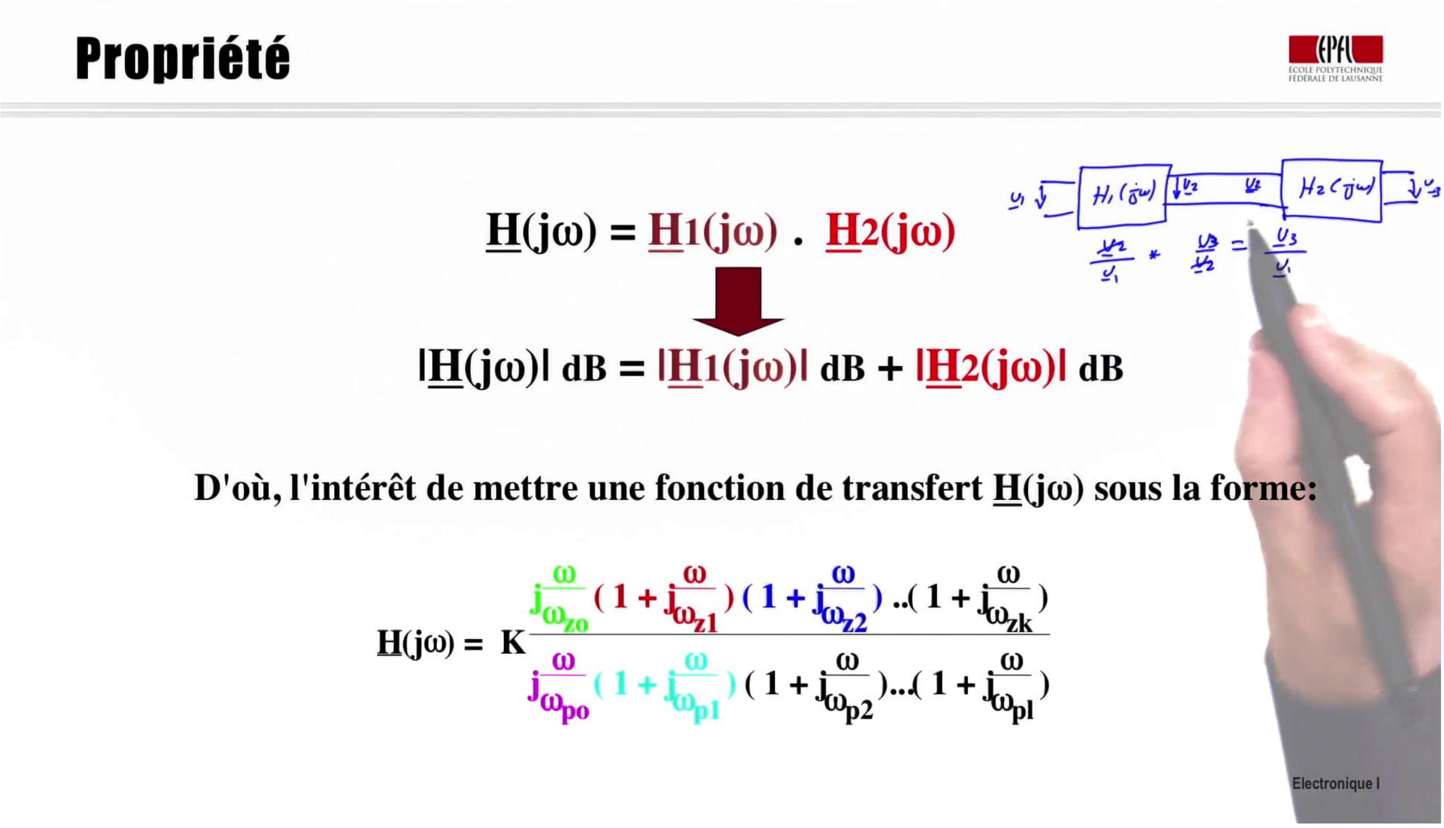


And there, we are going to talk about an amplifier, and we see it with a positive value which appears on that side. When we have a reduction, which is to say that the output voltage is reduced in comparison with the input voltage, we see that it's 0,1; 0,001, a division by a factor of 10. And we see the value is negative. And this must be so, because the logarithm is going to give us the minus sign. Thus when one has a minus sign, this does not describe what is happening with the voltage, knowing that the voltage is a sinusoidal voltage that is going to fluctuate between positive and negative. But when on comes to the phase diagram, we can easily distinguish the phase. Thus if the voltage is out of phase by 180 degrees, and so the positive part has become negative, and it is in a phase diagram that we are going to understand it. And so, this explanation describes the base of two graphic representations, one graphic representation that shows the amplitude, and one graphic representation that shows the phase.

Notes

Summary

7m 16s

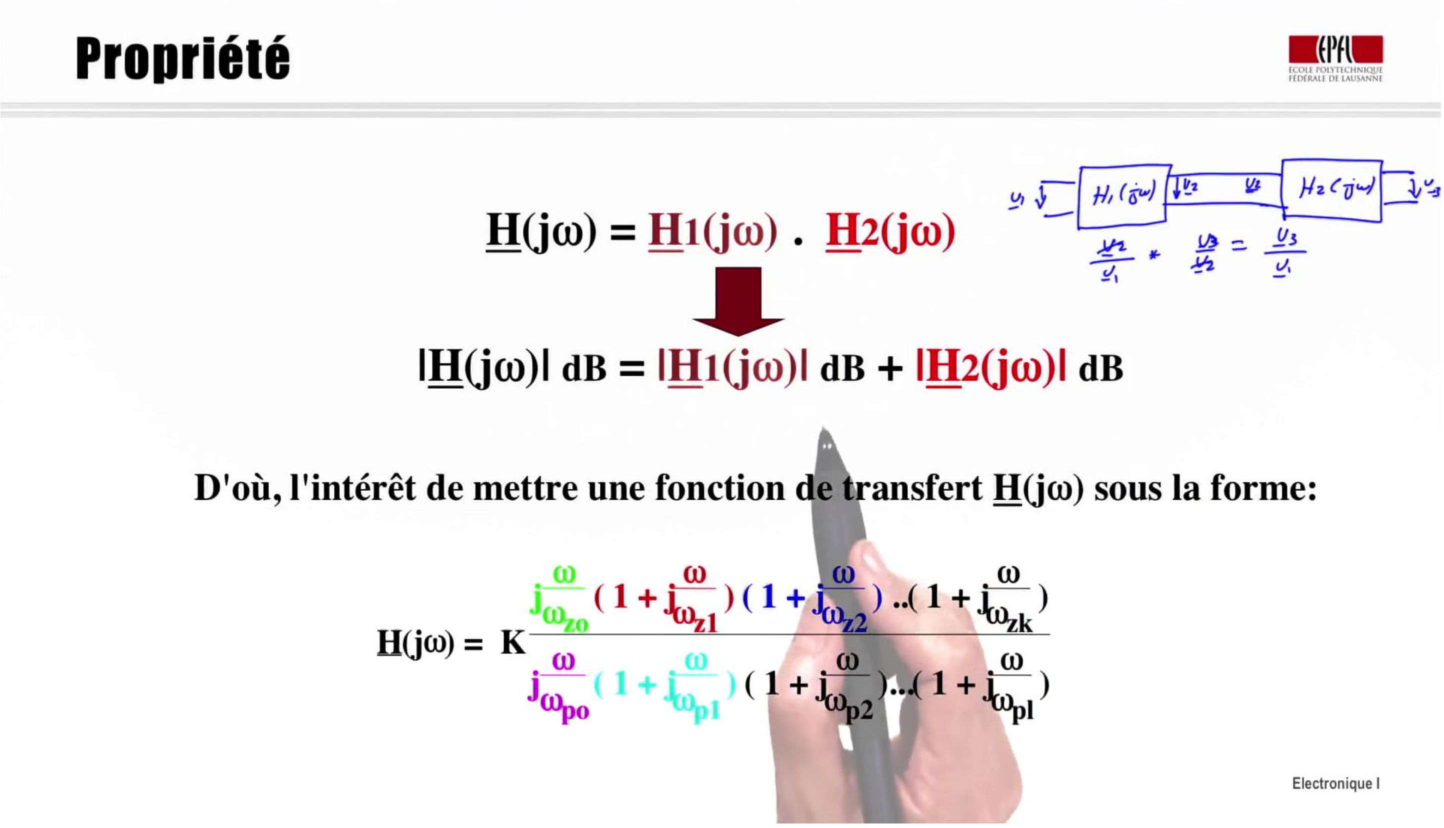


When we take two two-port networks, one of which I will call *H1(jω)* and another which follows, and we place them like that, one after the other, and so we call it *H2(jω)*. The overall transfer function, which is to say that if we put one voltage at the input, sinusoidal voltage, I put a line below it to show that it is a complex representation, and we look at what happens at the output and I call it *U2*. This same voltage is read by the second, and so I have *U2* which appears here and I am going to output voltage, *U3*, which is here. Thus if I describe the module of the transfer function as being *U2/U1*, the output voltage over the input voltage, followed by *U2*, which is the same, divided by the voltage. Sorry, there I should write *U3* divided by the voltage *U2*. And so, we are going to see that by simplifying *U2* and *U2*, it is in fact the overall transfer function which is *U3* divided by *U1*. Therefore this is a transfer function, and we are interested in the output as a ratio of the input. The intermediate voltage *U2* can be simplified and does not appear.

Notes

Summary

8m 22s



Thus if you take *H(jω)* and you say that it's *H1(jω)* and you say that it's *U2/U1* and the function *H2(jω)* which is equal to *U3/U2*, well then, their product, which is what we have just made here, corresponds with the overall transfer function of the connection of one behind the other which is going to give us *U3* divided by *U1*. That is, in fact, what happens. We take an audio application, and we imagine that we have a preamplifier and an amplifier. The preamplifier can receive - whether from a microphone, tuner, or MP3 player - a *U1* signal, convert it into a *U2* voltage, amplifying the voltage, enter into a power amplifier and pass through with increased power or a voltage at the output *U3*. You will see that the overall transfer function is the product of one compared to the other. What interests us is the input of a microphone and the output to a speaker that we are going to put here. So, we are interested in the product of the transfer functions *H1(jω)* by *H2(jω)*. All this transforms itself into an addition. Because when one expresses it in decibels, the fact of using a logarithm changes multiplication into addition, and that will give us the sum.

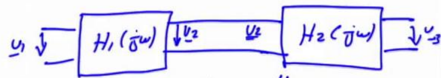
Notes

Summary

10m 02s

Propriété

$$\underline{H}(j\omega) = \underline{H}1(j\omega)\ .\ \underline{H}2(j\omega)$$



$$\frac{v_2}{v_1} * \frac{v_3}{v_2} = \frac{v_3}{v_1}$$

$$|\underline{H}(j\omega)|\ dB = |\underline{H}1(j\omega)|\ dB + |\underline{H}2(j\omega)|\ dB$$

D'où, l'intérêt de mettre une fonction de transfert $\underline{H}(j\omega)$ sous la forme:

$$\underline{H}(j\omega) = K\frac{j\frac{\omega}{\omega_{zo}}(1 + j\frac{\omega}{\omega_{z1}})(1 + j\frac{\omega}{\omega_{z2}})..(1 + j\frac{\omega}{\omega_{zk}})}{j\frac{\omega}{\omega_{po}}(1 + j\frac{\omega}{\omega_{p1}})(1 + j\frac{\omega}{\omega_{p2}})...(1 + j\frac{\omega}{\omega_{pl}})}$$

Electronique I

If this is the *H1(jω)* module of the transfer function, it is sufficient to add the *H2(jω)* module to the *H1(jω)* module. So we succeed in transforming a multiplication through addition. If we are capable of representing a diagram in which the module describes what happens with that block as a function of the variation of a variable - here, pulsation - and which in other terms is the frequency, because ω is equal to $2\pi f$, so if one has to draw the Bode diagram of *H1(jω)*, and the Bode diagram of *H2(jω)*, we have only to add the two to obtain the type of circuit this could produce. So, the use of putting a transfer function into this format, I have put them in colour to show the different types of functions that we are going to study, and, well, it is this that is going to change into addition.

Notes

Summary

11m 20s

But : faciliter le tracé du diagramme de Bode

$$\underline{H}(j\omega) = K \frac{j\frac{\omega}{\omega_{zo}}\left(1 + j\frac{\omega}{\omega_{z1}}\right)\left(1 + j\frac{\omega}{\omega_{z2}}\right)..\left(1 + j\frac{\omega}{\omega_{zk}}\right)}{j\frac{\omega}{\omega_{po}}\left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)...\left(1 + j\frac{\omega}{\omega_{pl}}\right)}$$

$$|\underline{H}(j\omega)|_{dB} = |K|_{dB} +$$

$$\left|j\frac{\omega}{\omega_{zo}}\right|_{dB} + \left|1 + j\frac{\omega}{\omega_{z1}}\right|_{dB} + \left|1 + j\frac{\omega}{\omega_{z2}}\right|_{dB} + ... + \left|1 + j\frac{\omega}{\omega_{zk}}\right|_{dB} +$$

$$\left|\frac{1}{j\frac{\omega}{\omega_{po}}}\right|_{dB} + \left|\frac{1}{1 + j\frac{\omega}{\omega_{p1}}}\right|_{dB} + \left|\frac{1}{1 + j\frac{\omega}{\omega_{p2}}}\right|_{dB} + ... + \left|\frac{1}{1 + j\frac{\omega}{\omega_{pl}}}\right|_{dB}$$

Electronique I

Let us now take this same function and watch what will happen when we begin to add these canonical formats, that is, the constant, in decibels, is added to everything that is a canonical function of each block. Because here, I have a series of multiplications, as if I had taken electronic blocks and aligned them one behind the other. And I have a function which seems to be very complex but if each block corresponds with an electronic project that we control, then we have only to add the Bode diagrams that we are going to draw, and learn to draw for each of these functions. This, then, appears to be something that is rather abstract but someone working in electronics will utilise it regularly because he or she will realise that with each of these forms, there is a clear and established function that we perform, and we have acted so that these canonical forms are transformed into a circuit which will be presented within the scope of this course.

Notes

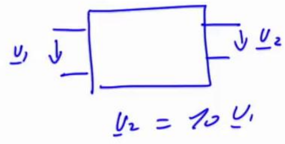
Summary

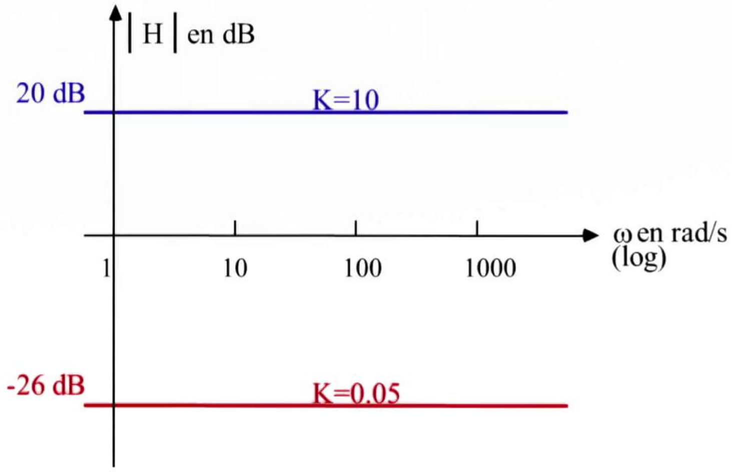
12m 19s

Module constant

Electronique I

Si $|K| > 1$, alors $|K|_{dB} > 0$





I am now going to go through each term in turn, I will begin with the constant module of a transfer function. Suppose that you have a transfer function, and you place a signal at the input. So you have a two-port network. In this two-port network, you have a sinusoidal voltage, *U1*, and you expect to find a voltage *U2* at the output and you find repeatedly that your voltage *U2* is equal to 10 times *U1*. So we have a voltage of 1V, and you will find 10V at the output as this is a sinusoidal function. And suppose that you change the frequency of this voltage *U1* at the input and you see that *U2* remains at the same frequency at the input, of course, because it's the same excitation signal, but despite all the variation in frequency to which you can subject *U1*, there is no effect on the relationship between the two. Thus we have an amplifier and whatever passes through it at the input, it multiplies by factor 10 without changing the amplitude of the output voltage with respect to the input voltage. Therefore we will say that where the constant *K* is positive, - and I give the following example - we always have 20 Log of 10, and that will give me 20 *dB*.

Notes

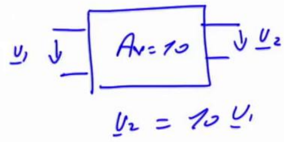
Summary

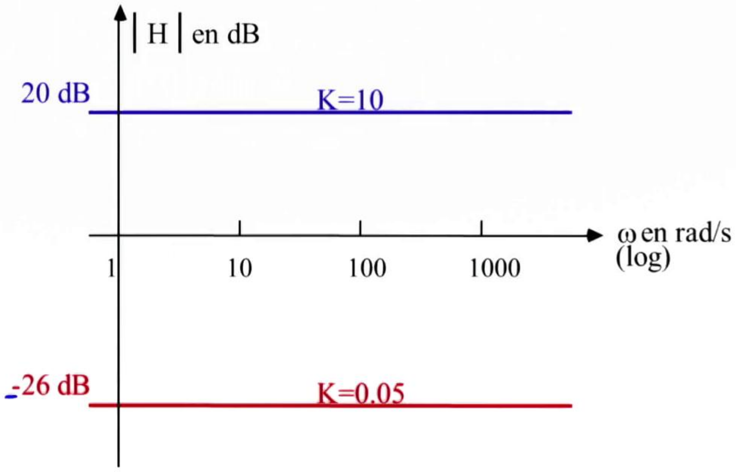
13m 18s

Module constant

Electronique I

Si $|K| > 1$, alors $|K|_{dB} > 0$





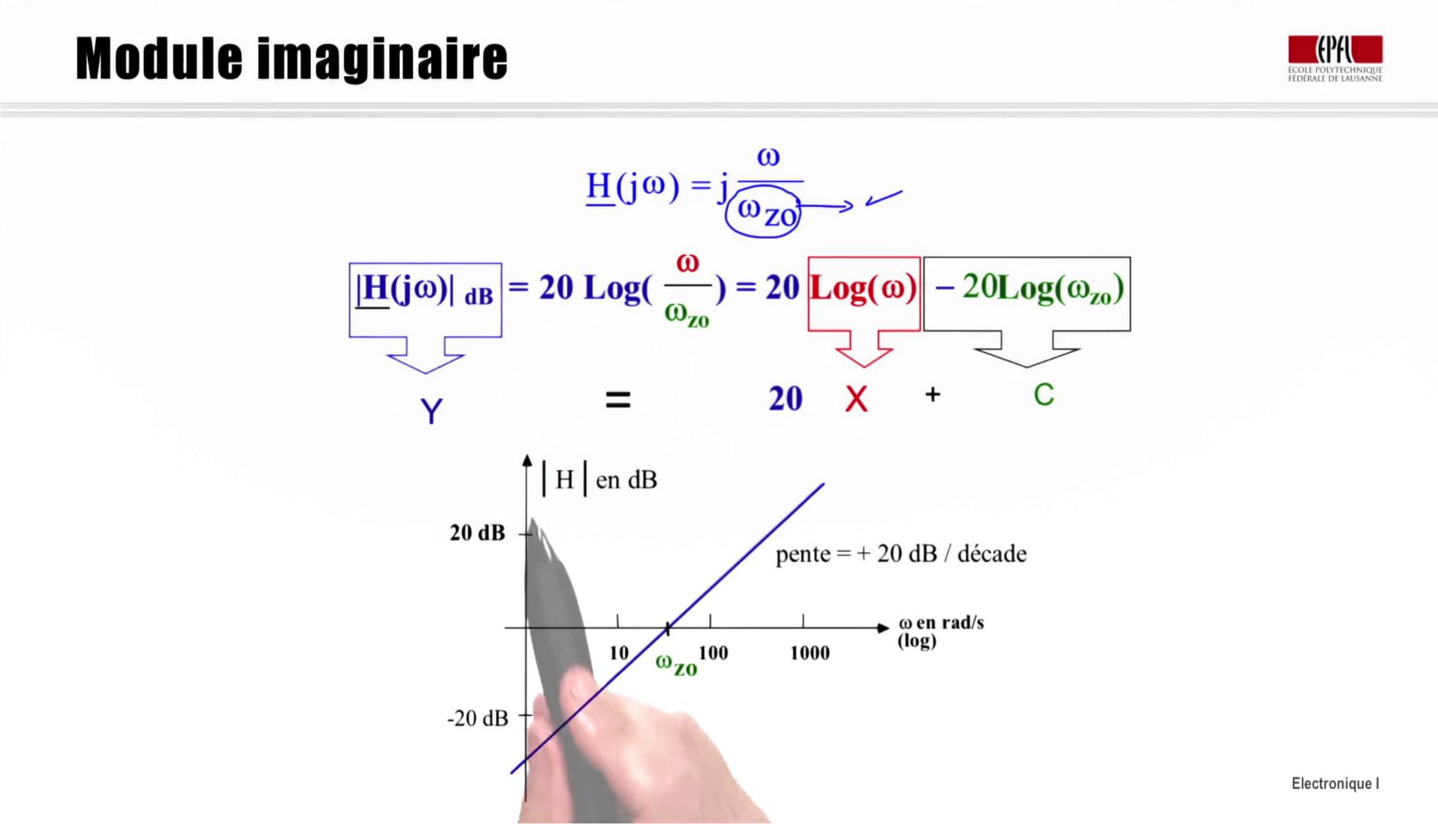
Electronique I

Now, if we look at what happens if I have an attenuator, which is to say, if I put a function here and if for that function, whenever I put a voltage at the input, I find the 0,05 amplitude of this voltage at the output. Thus I have attenuated it by a certain degree, and the negative sign will appear because it is 20 Log of 0,05 that will give me -26. And as you see, I use the decibel unit because *U2/U1* has no unit. Representation in decibels allows us to describe a non-unit and by using this representation with the 20 Log, then, we see that multiplication by 10 makes it easier for us to understand that each increase by a decade, will correspond with an addition of 20 *dB* to this one on the right. If I now add a second amplifier that gives me a gain equal to 10 and I add another one behind it that is the same, then in fact, I will see that I have 40 *dB*, so I have multiplied by 100.

Notes

Summary

14m 40s



I am going to take another canonical function, the one containing a complex number. There, I have a *j(ω/ω)_Z0)* and I am going to describe it in decibels. Thus the module of *H(jω)*, is 20 Log of the module of a complex number. Therefore the module of a complex number *j(ω/ω_Z0)* will give me *20 Log(ω/ω_Z0)*, so that I can write *20 Log(ω) - 20 Log(ω_Z0)*. Practically, *ω_Z0* will correspond to a given value, that we know; it is a numerical value that will depend, in my electronic circuit, on the values - perhaps on a resistance or a capacity, or a residence and an inductance. So there are things that I know, the values that correspond with the values of the components. In contrast, the variable is *ω*. So, given that we are going to draw the Bode diagram on that axis and return the Log of *ω*, as if I was saying that the Log of *ω* is *X* and that is a constant. So that makes *20X* plus a constant. If I want to bring that back to a Bode diagram, there I have drawn the axe containing *Log(ω)* and those are decibels, it is a *20X+C* line, it is a line with a slope equal to 20 *dB* that will pass through *ω_Z0*, where *y* is equal to 0, I will find that *x* is equal to *ω_Z0*.

Notes

Summary

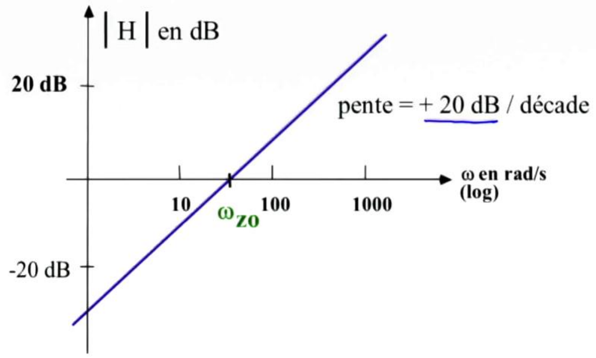
15m 46s

Module imaginaire

$$\underline{H}(j\omega) = j\frac{\omega}{\omega_{ZO}}$$

$$\boxed{|\underline{H}(j\omega)|_{dB}} = 20\,Log\left(\frac{\omega}{\omega_{zo}}\right) = 20\,\boxed{Log(\omega)}\;\boxed{-\,20Log(\omega_{zo})}$$

$$Y \quad = \quad 20 \quad X \quad + \quad C$$



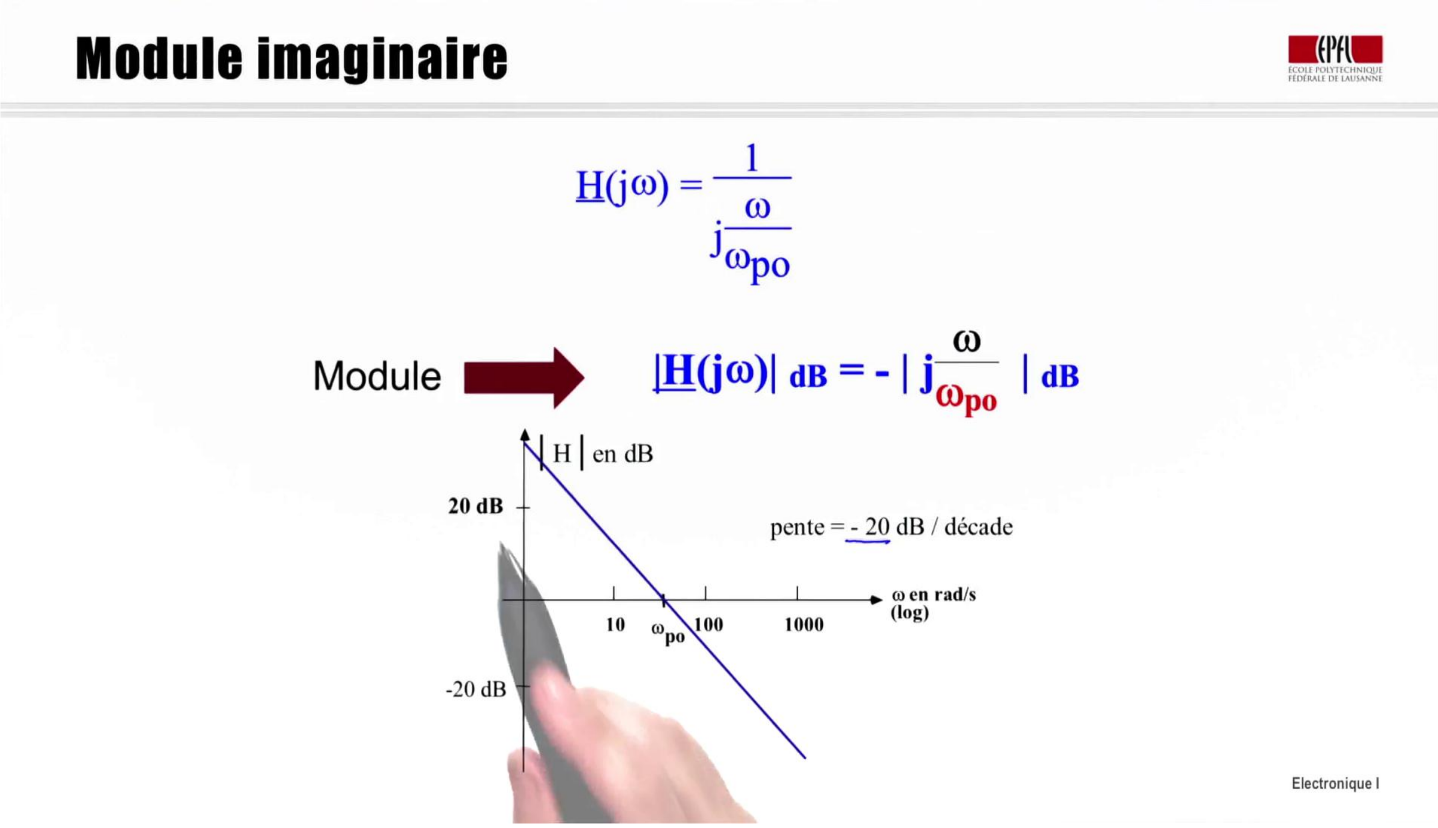
Electronique I

So that is what we refer to as the transfer function module of the canonical function which is equal to *j(ω/ω_Z0)* and which corresponds with a relationship between a voltage, a voltage and a current, between two currents, at the output of a two-port network. So the nature of the variable is independent of this function, and it is simply going to describe what happens when we transfer an input variable to the output variable.

Notes

Summary

17m 28s



I would like to look at the inverse of what we have just seen. So, instead of taking *j(ω/ω_Z0)*, I will take its inverse, *1/j(ω)*, and given that this is going to happen to the denominator, I will call that the pole of the transfer function and I will write *ω_P0*. I am also going to draw the Bode diagram of this module, diagram of the Bode module. And there, you can see that it is actually very simple - there is no need to spend a lot of time analysing it. *20Log(1)* is 0. For *-j(ω/ω_P0)*, its module is *ω/ω_P0*. And 20 Log of this is going to give me the minus sign behind, so that what we have just seen will happen again but with a negative slope. There, we will end up with -20 *dB* per decade. So, that will give me a transfer function with a slope equal to -20 *dB*/decade which passes through *ω_P0* when the *y* axis gives me zero. Later this will correspond, as we will see, with an integrator function in the temporal domain and, thanks to that, there is an electronic circuit that we are going to learn which is made by an operational amplifier, for example, and which will allow us to obtain this function.

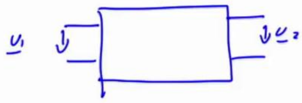
Notes

Summary

17m 58s

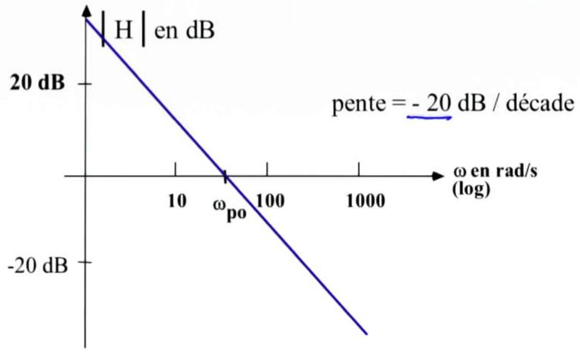
Module imaginaire

$$\underline{H}(j\omega) = \frac{1}{j\dfrac{\omega}{\omega_{po}}}$$



Module

$$|\underline{H}(j\omega)|_{dB} = -\left| j\frac{\omega}{\omega_{po}} \right|_{dB}$$



Electronique I

That function will thus correspond with a two-port network. In this two-port network, we are going to add some electronics. And the electronics, when you apply an input voltage of *U1* to it, will give you *U2* at the output. This voltage should be sinusoidal. When you vary the pulsation of this sinusoidal voltage, you then increase or decrease the frequency and you will observe *U2* and see that this voltage *U2* will cross this curve. If you increase the frequency, and thereby also the pulsation, you will see that the amplitude of the voltage *U2* will fall and fall and fall and fall following this curve and so it will fall in increments of 20 *dB*/decade. Thus each time we go from 10 to 100, we lose factor 10 on the amplitude of the output voltage. And this diagram describes this clearly. Therefore later, when one knows that we have added a transfer function to it, and when you will see that it involves an integrator, and then, you can directly draw this line without hesitating and you will see that you bring through what is in the denominator, the value that we talked about, which is going to correspond with a constant value given by the values of the components that we are going to use.

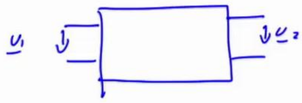
Notes

Summary

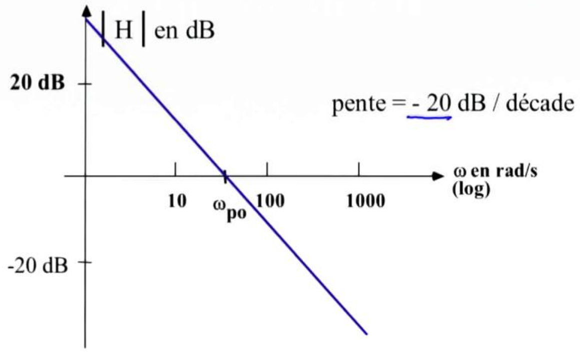
19m 20s

Module imaginaire

$$\underline{H}(j\omega) = \frac{1}{j\dfrac{\omega}{\omega_{po}}} \rightarrow cte.$$



Module ➡ $|\underline{H}(j\omega)|_{dB} = -\left| j\dfrac{\omega}{\omega_{po}} \right|_{dB}$



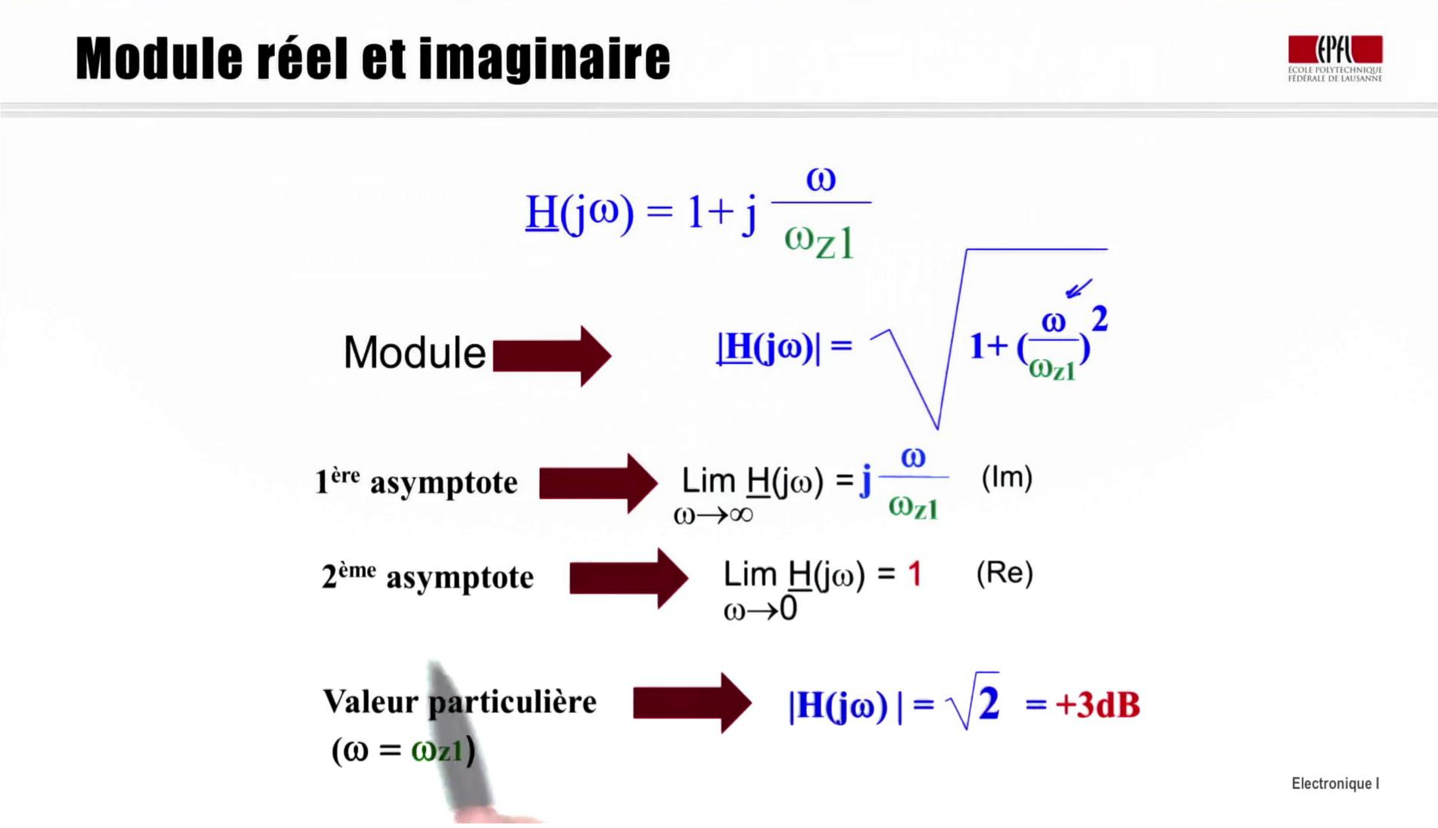
Electronique I

So, if we bring that here, and we make a gradient of -20 dB, we are going to see clearly the behaviour of our transfer function, which is inside this block. So we will see that the ratio between the output and input voltages is perfectly described by this.

Notes

Summary

20m 41s

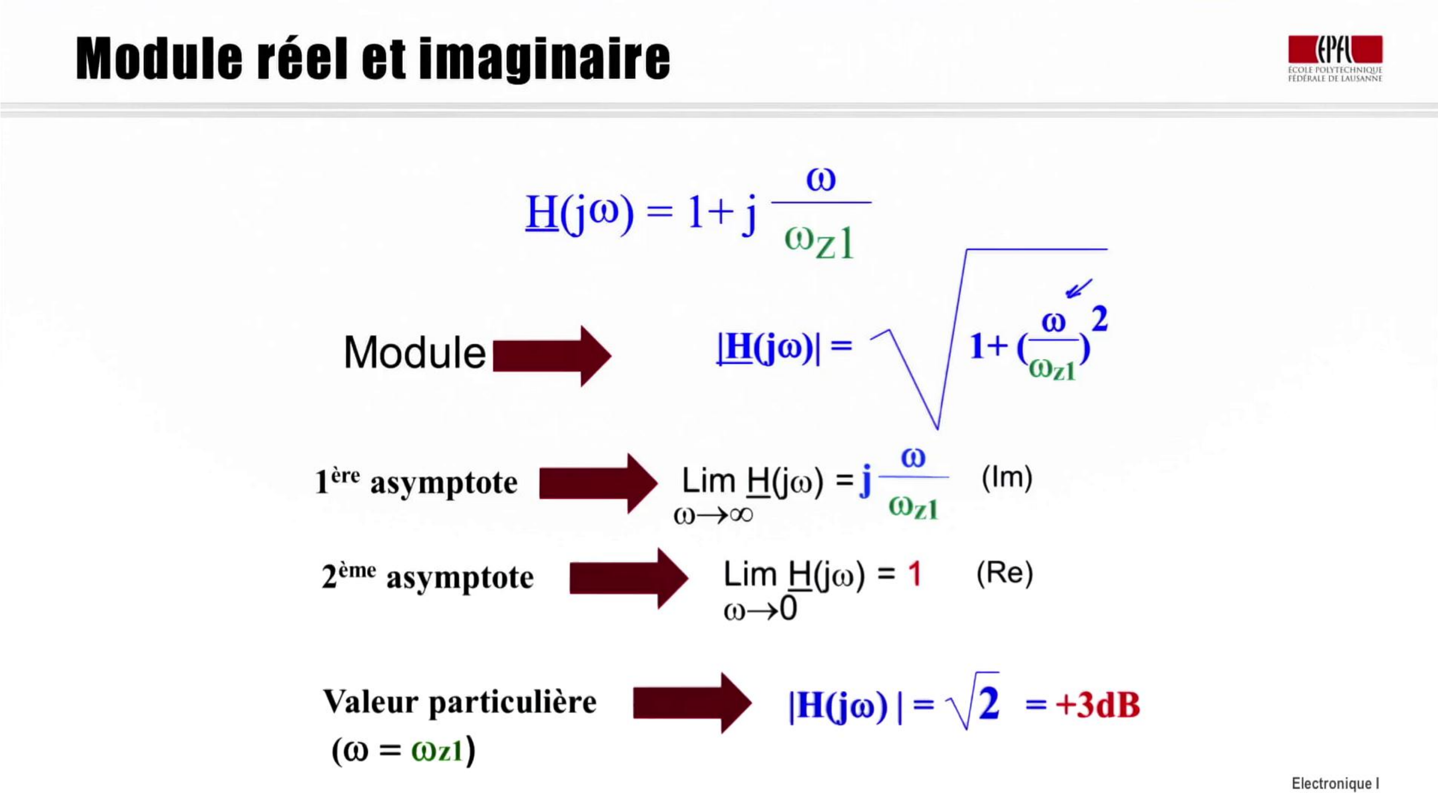


Let us now take *1+j(ω/ω_Z1)*. There, contrary to what we saw before, we have drawn a pure real number, then drawn a pure imaginary number. Here, I am taking a function that contains both a real and an imaginary part. Next I must of course write the module of this complex number which is the square root of the real part of the square plus the imaginary part of the square. And now, I must draw the module in a Bode diagram where the variable is *ω*. The *ω_Z1* is known and I must add that. Thus we can draw that as we draw any analytical function. We begin by looking at the asymptotes. Thus we make *ω* approach infinity. If *ω* approaches infinity we have, in this function, an imaginary part which becomes much more dominant in comparison with the real part. Thus we can say that it is a pure quasi-imaginary number and ignore the real part. So I will find that, when *ω* approaches infinity, the function is similar to that which we have just drawn, which we called *j(ω/ω_Z1)* and I ignore the real part, which happens to be equal to 1. So we have *1+j* times infinity. There is a second asymptote, which is when I make *ω* approach 0.

Notes

Summary

21m 00s



And, while ω approaches 0, I end up with a pure real number and the pure real number will be equal to 1, and the imaginary part is going to disappear. I am going to look for a particular point that belongs to this module. I then replace ω by ω_{Z1} and I will find $\sqrt{2}$. $20Log(\sqrt{2})$ is nearly equal to 3 dB. So I have a first asymptote, a second asymptote and a particular value which belongs, which is equal to +3 *dB*. And I am going to draw the asymptotic diagram that I have just had here and afterwards, draw the real diagram that I am will find for a function of $1+j(\omega/\omega_{Z1})$.

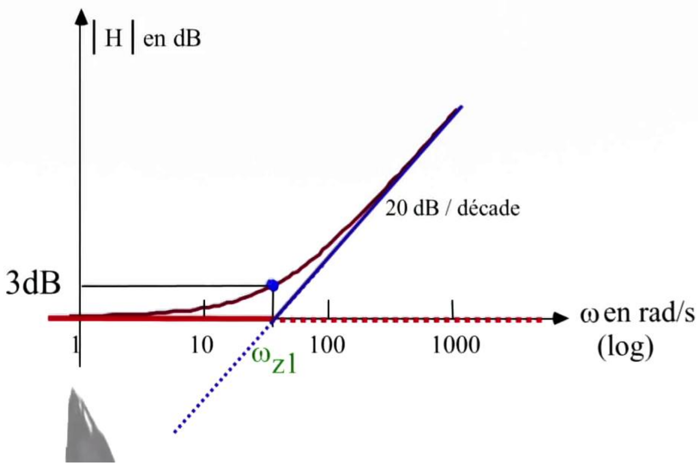
Notes

Summary

22m 24s

Module réel et imaginaire

$$\underline{H}(j\omega) = 1 + j\,\frac{\omega}{\omega_{z1}}$$



Electronique I

We are going to find that. We are going to find that we have this axis, which will be an asymptote, the *x* axis. And we have this second axis which will be the second asymptote. In fact, it will correspond where ω approaches infinity and we have seen that we can ignore the 1. So, that gave us $j(\omega/\omega\_1)$ and we saw that it's a straight line that passes through ω_Z1 and that the real curve that we will measure at the electronic output, where the electronic element represents a transfer function which is this one, and which will correspond to this straight line, or this red curve, which runs asymptotically around the... around a low frequency which runs asymptotically on the *x* axis and which will then move asymptotically towards this straight line with a gradient of +20 dB/decade. And, just as we had when ω was equal to ω_Z1, we now have $1+j$, making $+3\ dB$. Thus there that point, and this point corresponds with a particular point. So we have only to sketch this curve, which is this one.

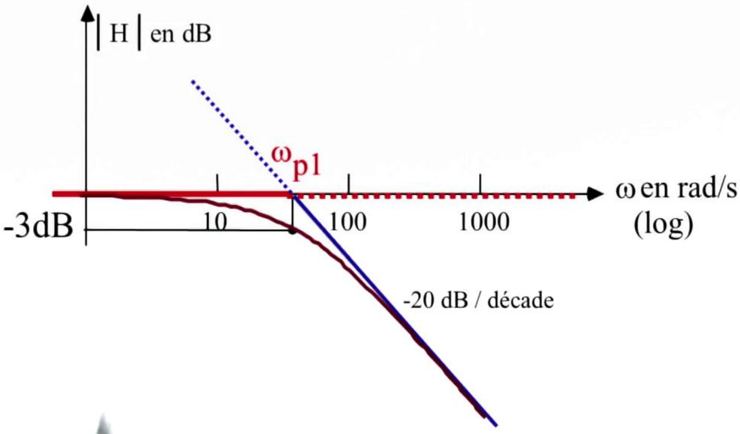
Notes

Summary

23m 12s

Module réel et imaginaire

$$\underline{H}(j\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega p1}}$$



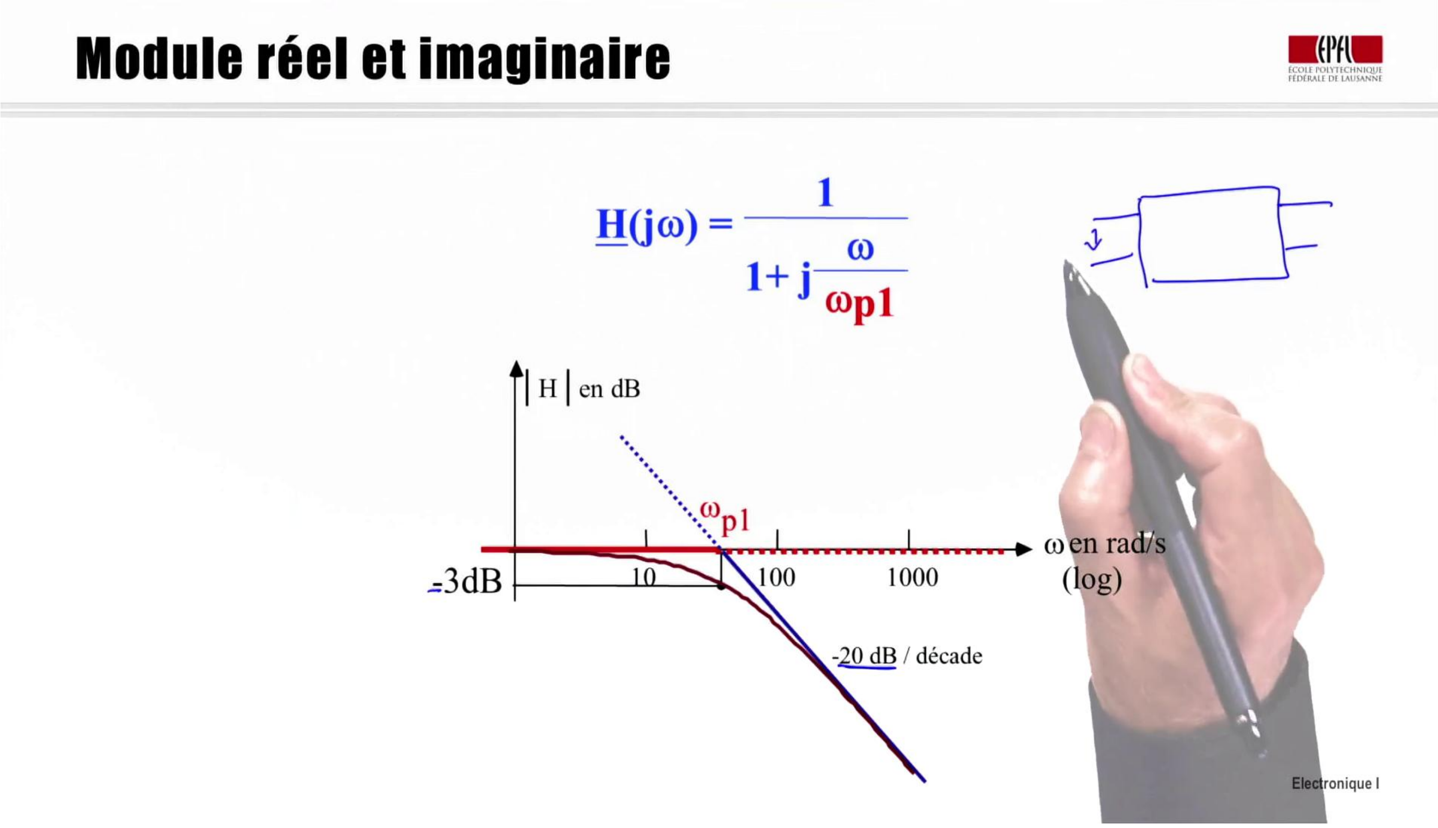
Electronique I

So we see that for this electronic function that, later, we are going to create with electronic circuitry, when ω is sufficiently close to 0, and thus has a DC voltage, we will find ourselves with a function which moves directly in an asymptotic direction where, for ω is equal to 0, the amplitude of the output will be almost zero. And the more we increase the pulsation, the more we will see that the transfer function is going to give us a variable at the output which will increase and which will move asymptotically on the asymptote $j(\omega/\omega\_Z1)$. I would like to take the inverse of this and I believe that now those following the development of the Bode diagram have properly understood that one takes $1+j(\omega/\omega\_Z1)$ and places 1 above it, and one replaces the ω_Z with a pole of the transfer function, and for the inverse of that, it is enough to take what we saw before, which is to say, this same curve was symmetrical in that direction, and we now find it towards the bottom because it's 20 Log 1, which is equal to 0, less than minus what we have just analysed, i.e. $20 \, Log(1+j(\omega/\omega\_P1))$. So, we have only to add the minus sign and we find the same thing.

Notes

Summary

24m 28s

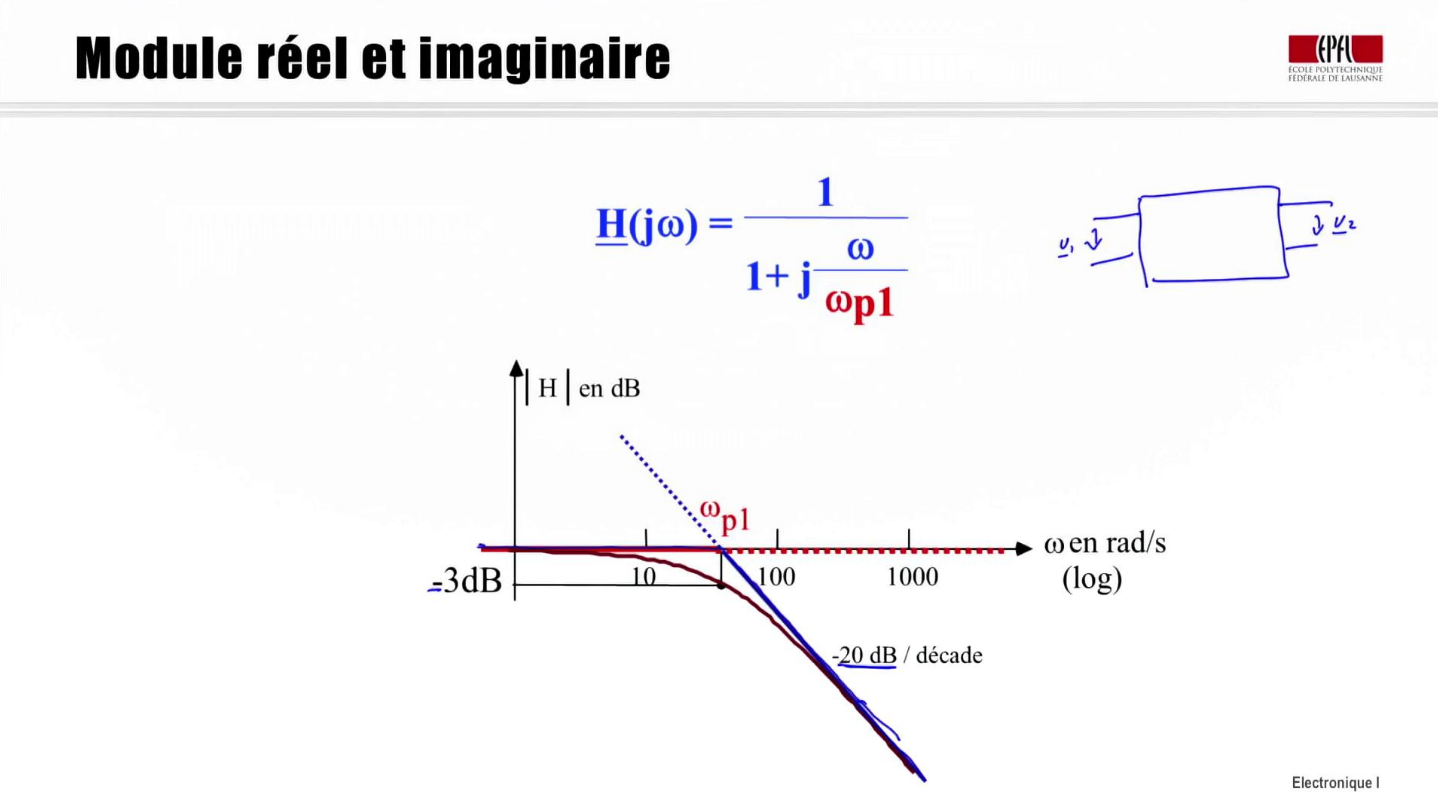


We find that the *x* axis, which remains an asymptote, but this straight line now has a gradient equal to -20 *dB*. The fact of finding -20 *dB* is now going to allow us to draw the curve which passes through -3 *dB*, which is when *ω* is equal to *ω_P1*, thus when we have the variable *ω* here. We have a transfer function which becomes *1/(1+j)* and the module of this transfer function, is the module of *1/1√(1+1)*, and that is going to give us -3 *dB*. And here we find the rather well-known characteristic of what we have a low-pass filter, which will also be created, and which corresponds with a function where low frequencies pass through, and are multiplied by 1, and one finds them, the output voltage, it's a low-frequency two-port network, what is at the input appears at the output. The sinusoidal voltage *U1* appears as it is, as the output with its amplitude.

Notes

Summary

25m 48s



And when we increase the pulsation, we begin to see a reduction in the output voltage and afterwards, we continue to reduce it, and the asymptotic diagram that you see here, without the curve that passes through - the real curve - we speak very often of an asymptotic representation, i.e., we are often satisfied with doing what I am going to mark in blue, that's that part there - and, of course, this straight line, which is already in blue, and we speak of this asymptote or asymptotic diagram of the function *1/(1+j(ω/ω_P1))*, which is typically a function of a low-pass filter with a pole at *ωP1*.

Notes

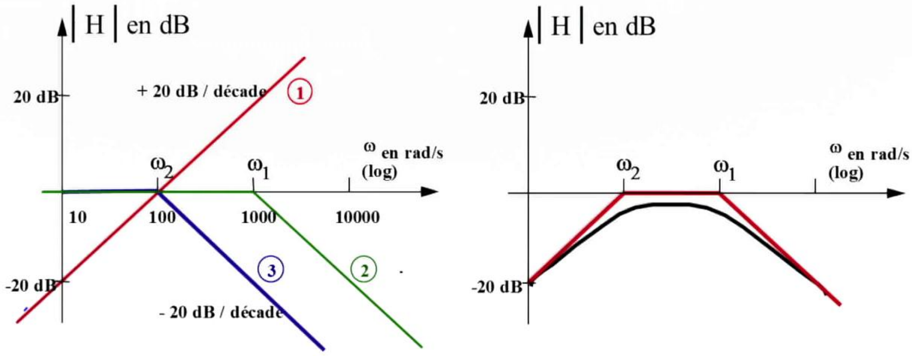
Summary

26m 54s

Exemple

$$\underline{H}(j\omega) = \frac{1 \quad j\frac{\omega}{\omega_2}}{\left(1 + j\frac{\omega}{\omega_1}\right)\left(1 + j\frac{\omega}{\omega_2}\right)}$$

$\omega 2 = 100$ rad/s
$\omega 1 = 1000$ rad/s



Electronique I

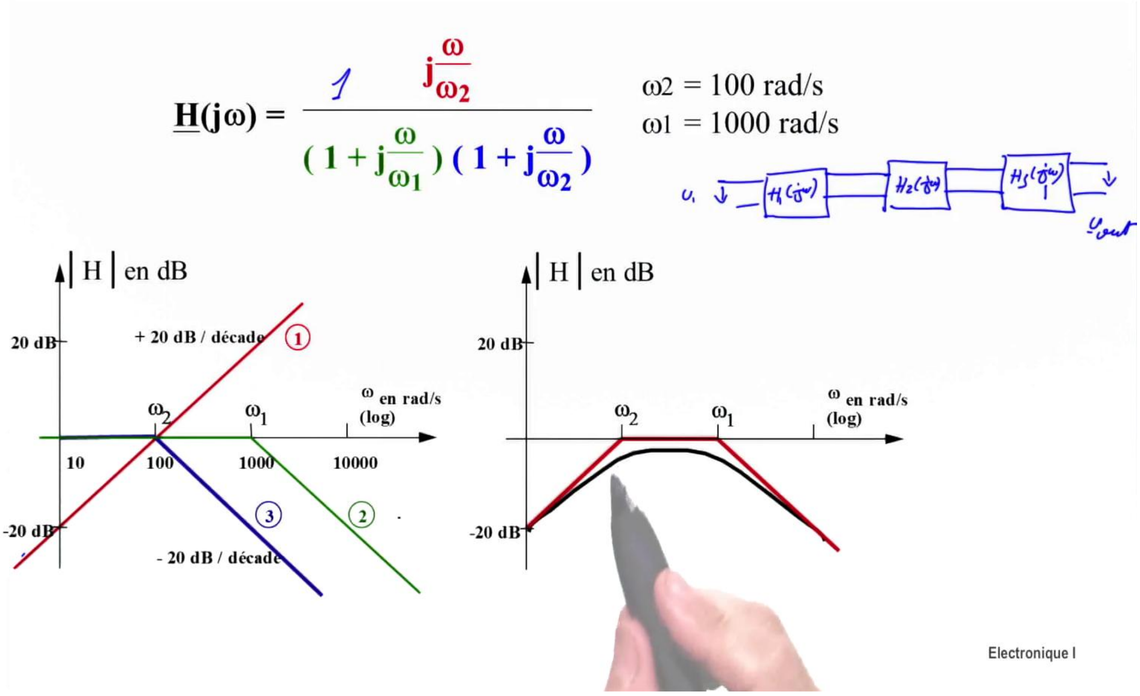
I am going to deal with a full example which contains a Bode function or a transfer function *H(jω)*, which contains three parts. I have a part *j(ω/ω2)*, which is found at the numerator. So that is a 0 of the transfer function. I have *1/(1+j(ω/ω1))* and I have *1/(1+j(ω/ω2))*. And I have given the values for *ω2* et *ω1*, saying that *ω2* is found at 100 rad/s, and *ω1* is found at 1000 rad/s. And I have added the three functions to a single Bode diagram. One function independent of the other, now in the same colours. So, the function of *j(ω/ω2)* is a straight line, we saw that, with a gradient equal to +20 dB/decade. Each time we multiply by 10, a decade, we gain 20 *dB* in amplitude and as we increase the pulsation, the more we are going to travel to the output of this two-port network; I have *ω/ω2* an amplitude which is proportional at the input, according to this straight line. After, if you take the three transfer functions, it is as if we had taken a first two-port network followed by a second, followed by a third, and we put placed one after the other and we imposed an input voltage and we watched an output voltage, which I will call *U_out*, which is a sinusoidal voltage, it is going to be a sinusoidal voltage.

Notes

Summary

27m 40s

Exemple



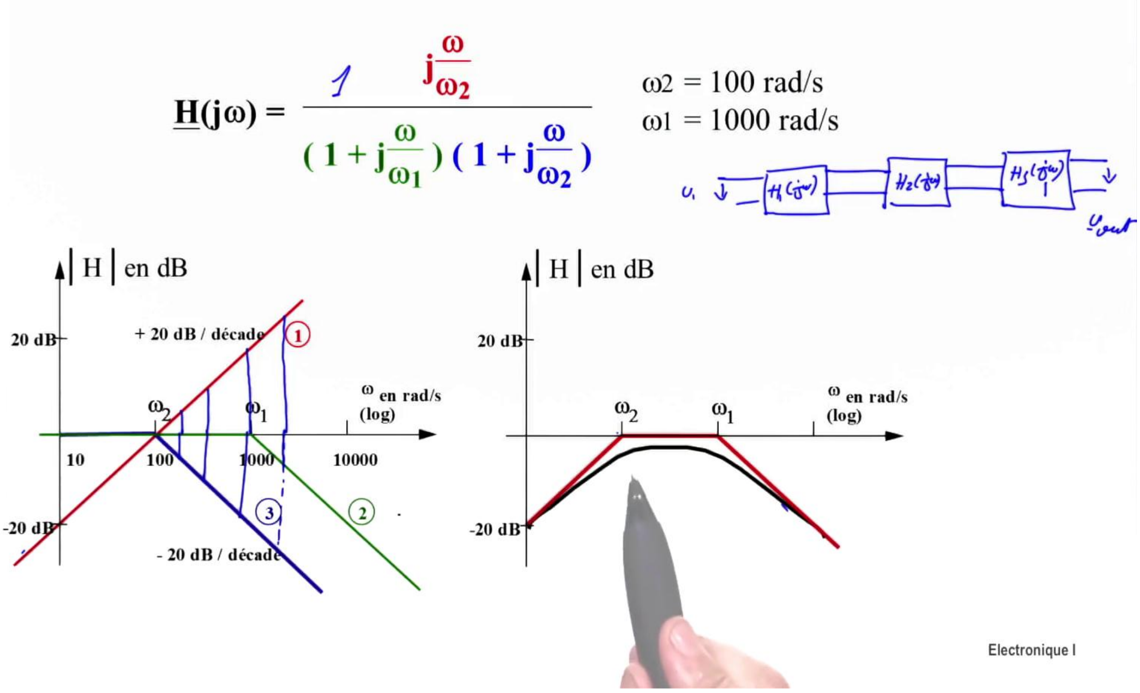
I am going to vary the pulsation of this voltage. I then observe the pulsation at the output. And I will see all of the Bode diagram that is going to influence the output voltage. Therefore, if you draw the red function individually, this one here. It is thus *H1(jω)*. There, it is *H2(jω)*. And there, it is *H3(jω)*. And this is *H1(jω), H2(jω), H3(jω)*. If you take the three functions and you add them together, we have thus drawn the first one. We are going to draw the second. So I only have to take *1/(1+j(ω/ω2))* which corresponds with this asymptotic diagram. There, I have only drawn the real diagram; I have managed to keep the asymptotic diagram. And I have done the same for the second, which has the same form, *1/(1+j(ω/ω1))*, except that I have *ω2*, *ω1*. The *ω1* is at 1000 rad/s, *ω2* is at 100 rad/s. And I have only to add these three curves together. I will thus take +20 dB/decade which will always be added to 0. I will find it here. Up to *ω2*, there is only the curve 1 that dominates. The two others have given me an asymptotic diagram equal to 0. So, we see no effect here, and I draw this gradient.

Notes

Summary

29m 30s

Exemple



From there, I am going to add that quantity there to that quantity, and each time it's the same quantity. I have +20 *dB*/decade - 20 *dB*/decade, which will give me 0 *dB*/decade. So that gives us a flat Bode diagram until I get here, *ω1*. For the asymptote here, I have +20 *dB* on one side, and on the other I have -20 *dB*, but after that, I have +20 *dB* - 20 *dB*/decade and also a second -20 *dB*/decade, which is going to give me -20 *dB*/decade here. And when I add all that together, you see in red the asymptotic diagram of the entire function and you see above it the values which would have been measured if you had taken the output voltage and measured it in comparison with the input voltage; you would have seen that at low frequency, the output voltage is low in comparison with the input voltage and then it is going to be amplified, amplified, amplified, and arrive at point *ω2* where we have an attenuation of -3 *dB*. Which is the same as at *ω1*. And we find that the real curve passes just below the asymptotic diagram.

Notes

Summary

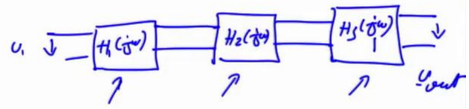
31m 15s

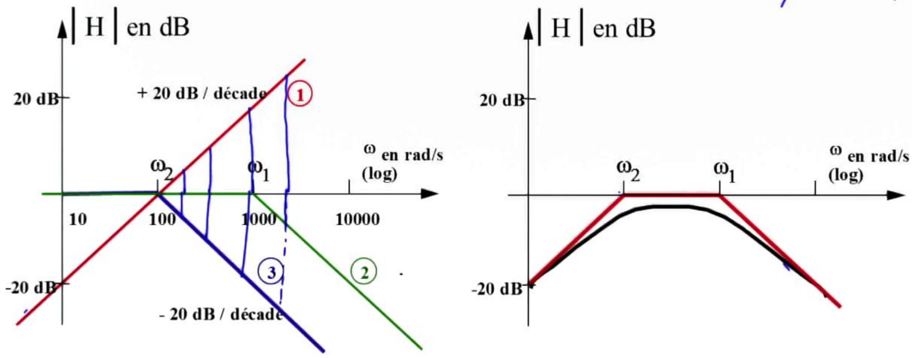
Exemple

$$\underline{H}(j\omega) = \frac{1 \qquad j\frac{\omega}{\omega_2}}{\left(1 + j\frac{\omega}{\omega_1}\right)\left(1 + j\frac{\omega}{\omega_2}\right)}$$

$\omega 2 = 100$ rad/s

$\omega 1 = 1000$ rad/s





Electronique I

And afterwards, it continues asymptotically on -20 *dB*/decade such that the frequencies increase when we arrive at *ω2*, then remain more or less constant, and afterwards, start to be attenuated. And there, the pulsation between *ω2* and *ω1* creates a section where we more or less see the greatest gain, we call that the band-pass filter, where I allow through a frequential frequency-band between the two pulsations *ω2* et *ω1*, otherwise we would be attenuating on both sides. So we see the usefulness of this representation and we will see the usefulness when each function that you see there becomes an electronic function that we are going to create with electronic circuits. And when we place them one after another, we have no need to analyse the entire circuit, it is sufficient to analyse one independently of the other, draw the Bode diagram and see all of it simply by graphically adding what happens.

Notes

Summary

32m 38s

Diagramme de Bode - argument ou phase

$$\underline{H}(j\omega) = K \frac{j\frac{\omega}{\omega_{zo}}\left(1+j\frac{\omega}{\omega_{z1}}\right)\left(1+j\frac{\omega}{\omega_{z2}}\right)\ldots\left(1+j\frac{\omega}{\omega_{zk}}\right)}{j\frac{\omega}{\omega_{po}}\left(1+j\frac{\omega}{\omega_{p1}}\right)\left(1+j\frac{\omega}{\omega_{p2}}\right)\ldots\left(1+j\frac{\omega}{\omega_{pl}}\right)}$$

$$\mathrm{Arg}(\underline{H}(j\omega)) = \mathrm{Arg}(K) +$$

$$\mathrm{Arg}\left(j\frac{\omega}{\omega_{zo}}\right) + \mathrm{Arg}\left(1+j\frac{\omega}{\omega_{z1}}\right) + \mathrm{Arg}\left(1+j\frac{\omega}{\omega_{z2}}\right) + \ldots + \mathrm{Arg}\left(1+j\frac{\omega}{\omega_{zk}}\right) +$$

$$\mathrm{Arg}\left(\frac{1}{j\frac{\omega}{\omega_{po}}}\right) + \mathrm{Arg}\left(\frac{1}{1+j\frac{\omega}{\omega_{p1}}}\right) + \mathrm{Arg}\left(\frac{1}{1+j\frac{\omega}{\omega_{p2}}}\right) + \ldots + \mathrm{Arg}\left(\frac{1}{1+j\frac{\omega}{\omega_{pl}}}\right)$$

Electronique I

Now that we have fully analysed what happens with the function of the Bode module, and we have thus seen that, for a transfer function, in a Bode diagram module, it is going to describe for us what happens with the output voltage in terms of amplitude in comparison with the input voltage in terms of amplitude. We will take the same form again, or the same canonical forms. I will represent them for a second time and this time I am going to be more interested in the phase. So, we have a complex number. If I have a real, complex number, the *K* that you see there, I am going to find the argument for a real number, meaning the arc tangent of *K*. Otherwise, if we are working with a complex number *j(ω/ω_Z0)*, I am going to look for the argument of a transfer function which is pure and imaginary and I will do the same for all the rest and I know that the arguments are going to add up. The same goes as for the modules, by paying attention to the argument, we will find that the arguments are going to give us diagrams where, if we draw independently from one to another, and if we add graphically, we will see that happens with the whole of the function once it has been created by separate electronic functions for which we have mastered the canonical forms in both mathematical form, with a complex number, and in terms of implementation as electronic elements - e.g.

Notes

Summary

33m 42s

Diagramme de Bode - argument ou phase

$$\underline{H}(j\omega) = K \frac{j\frac{\omega}{\omega_{zo}}(1+j\frac{\omega}{\omega_{z1}})(1+j\frac{\omega}{\omega_{z2}})\ldots(1+j\frac{\omega}{\omega_{zk}})}{j\frac{\omega}{\omega_{po}}(1+j\frac{\omega}{\omega_{p1}})(1+j\frac{\omega}{\omega_{p2}})\ldots(1+j\frac{\omega}{\omega_{pl}})}$$

$$\mathrm{Arg}(\underline{H}(j\omega)) = \mathrm{Arg}(K) +$$

$$\mathrm{Arg}(j\frac{\omega}{\omega_{zo}}) + \mathrm{Arg}(1+j\frac{\omega}{\omega_{z1}}) + \mathrm{Arg}(1+j\frac{\omega}{\omega_{z2}}) + \ldots + \mathrm{Arg}(1+j\frac{\omega}{\omega_{zk}}) +$$

$$\mathrm{Arg}(\frac{1}{j\frac{\omega}{\omega_{po}}}) + \mathrm{Arg}(\frac{1}{1+j\frac{\omega}{\omega_{p1}}}) + \mathrm{Arg}(\frac{1}{1+j\frac{\omega}{\omega_{p2}}}) + \ldots + \mathrm{Arg}(\frac{1}{1+j\frac{\omega}{\omega_{pl}}})$$

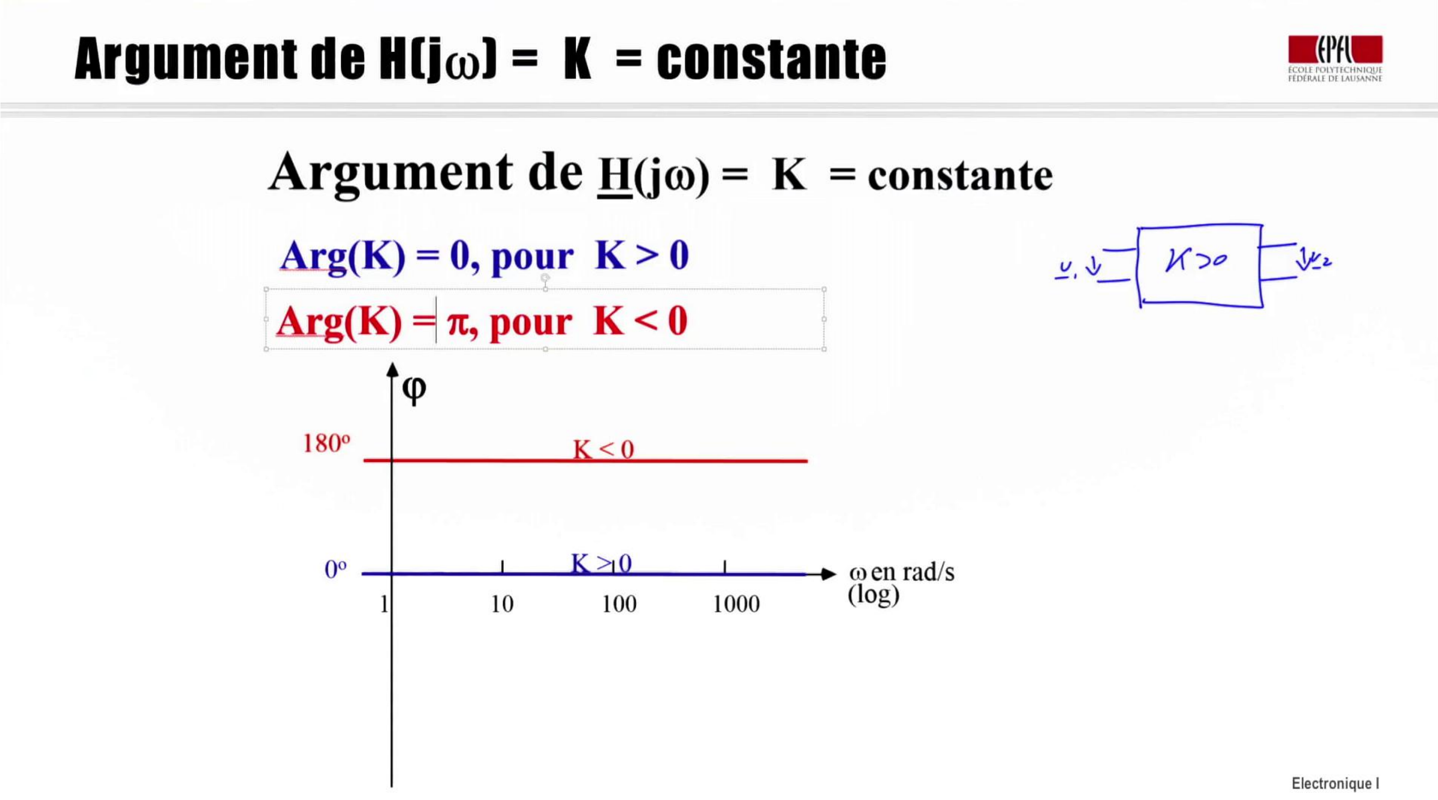
Electronique I

in terms of which type of circuit to build, which will be studied in the electronics course.

Notes

Summary

35m 07s



I am going to begin with a transfer function for real, pure numbers. So we take the *arc tangent* of a real, pure function and I know that if the argument is equal to 0, I am on the real axis, and then, I can say that the value, the 0 axis, this axis here, I have ω and there, I have the phase. And if the function K is negative, and if I then place an amplifier there with a positive gain. So, let us take a two-port network and added a positive gain, for example, one that is equal to 10. If it is a real pure number equal to 10, for example, and it is thus strictly positive, we will find ourselves with voltage $U2$ and voltage $U1$ which are in phase. The voltages will not be out of phase. However, if this constant K is negative, we will be out of phase by 180 degrees, so we will have an inverter. We are thus going to talk about positive or negative gain. And positive gain gives us the x axis, and there is now dephasing. And on this axis, where we have the phases, if the gain is negative, we are going to have a dephasing of 180 degrees, of π, which means that I have inverted the output voltage by comparison with the input voltage. We call this an *inverter*.

Notes

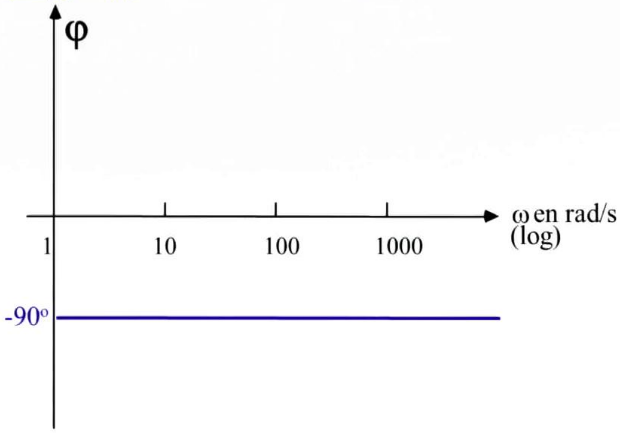
Summary

35m 12s

Fonction imaginaire

$$\underline{H}(j\omega) = \frac{1}{j\frac{\omega}{\omega_{po}}}$$

$$\mathbf{Arg(\underline{H}(j\omega)) = -\pi/2 \quad ou \ -90^o}$$



Electronique I

The order that we used for performing the analysis for the modules, is the same one we will use for the phases. So here, we will take the canonical form *j(ω/ω_Z0)*, which is pure and imaginary, and we are going to analyse what happens to it. And there, we will see, it is very simple. We know that one is on the imaginary axis and we will have a dephasing equal to *π/2*. So, for the pure imaginary, the imaginary axis will give us a dephasing of *π/2*. So, for a function such as this one, when we create it with electronic components, each time I change the pulsation, I will find at the output a dephasing equal to a constant which is equal to 90 or *π/2* as a phase. We will now take its inverse, i.e. *1/j(ω/ω_P0)*. Here again, it is the *arc tangent* of 1 minus the *arc tangent* of the section which is found at the denominator and which will surely give us a dephasing of *-π/2*. So we find ourselves with a dephasing of *-π/2*, which means that the output voltage compared to the input voltage in my two-port network, has a phase delay equal to *π/2*.

Notes

Summary

36m 44s

Fonction Réelle et Imaginaire

$$\underline{H}(j\omega) = 1 + j\,\frac{\omega}{\omega_{Z1}}$$

$$\mathrm{Arg}(\underline{H}(j\omega)) = \mathrm{Arctg}\left(\frac{\omega}{\omega_{z1}}\right)$$

1ère asymptote → $\lim_{\omega\to\infty} \underline{H}(j\omega) = j\,\frac{\omega}{\omega_{Z1}}$ (Im)

2ème asymptote → $\lim_{\omega\to 0} \underline{H}(j\omega) = 1$ (Re)

Valeur particulière ($\omega = \omega_{z1}$) → $\mathrm{Arg}(\underline{H}(j\omega)) = 45^{o} = \pi/4$

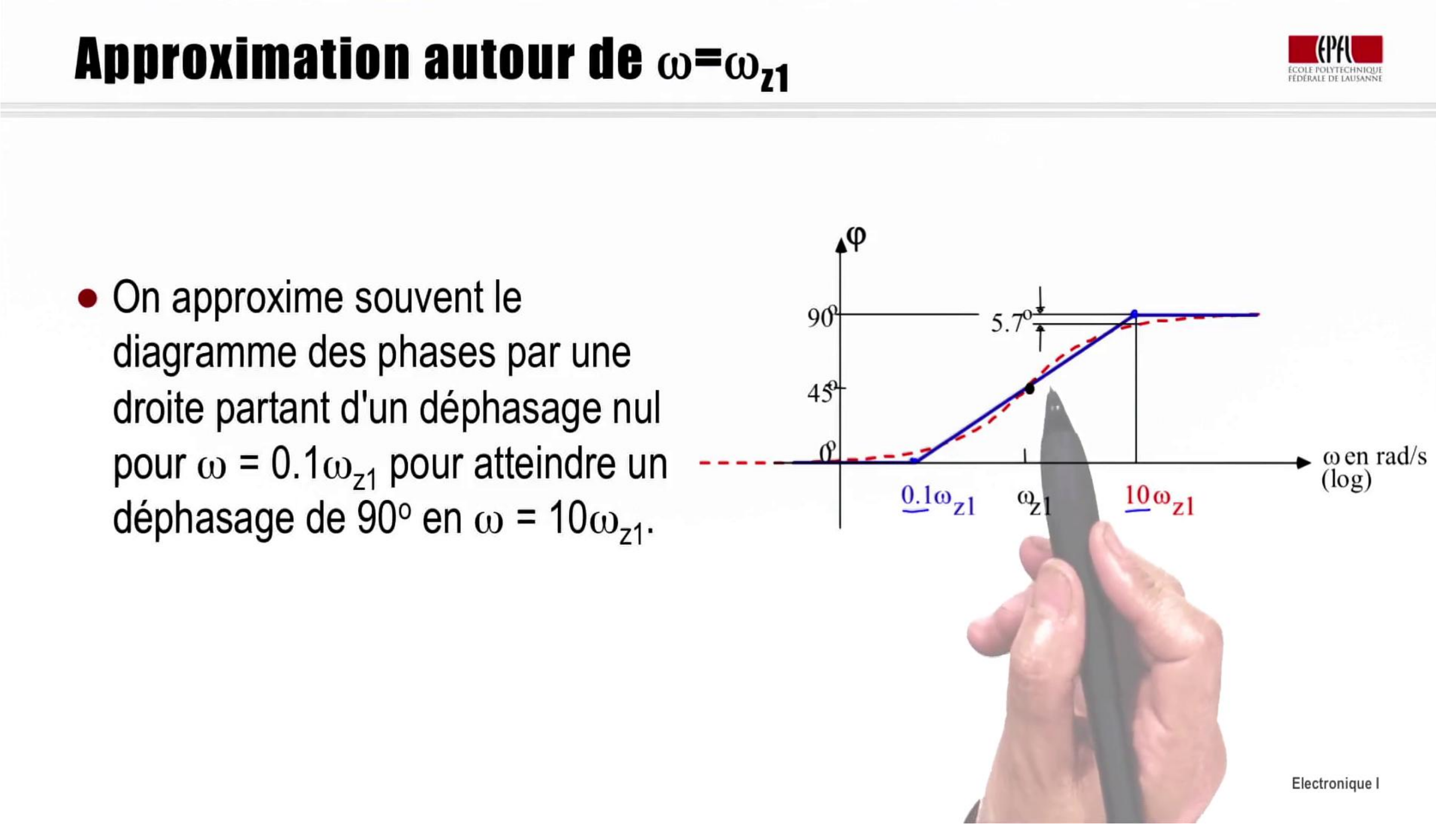
Electronique I

A function which is a little more developed, is the one where we have a real and an imaginary part. So the *arc tangent* of this will of course be *Arctg (ω/ω_Z1)*. So, once more, we are going to draw the function *arctg(ω/ω_Z1)*, which we know well, and which has an asymptote when *ω* approaches infinity. So if this term is equal to infinity, the imaginary part is much more determinant than the real part which is equal to 1, and I can thus simplify this 1 and find myself with a complex number *j(ω/ω_Z1)* of which the tangent arc is what we have just drawn a moment ago, when we drew the first asymptote of *j(ω/ω_Z1)*, the pure imaginary. It is the same where *ω* approaches 0; this term, or this transfer function, becomes real and pure and is equal to 1. So we have two asymptotes and we do the same again, as in the module, by making *ω* equal to *ω_Z1* and we will find the complex number *1+j*, which corresponds with a dephasing of 45 degrees, which is of course equal to *π/4*.

Notes

Summary

38m 02s



And here is the diagram of the Bode phase of such a function. So, asymptotically, I have this axis and that axis. We have found that there is the *x* axis, thus the *ω* axis, and we have a dephasing of 90 degrees, of *π/2*, which will be the second axis. Meanwhile, we have a particular point at which pulsation *ω* is equal to *ω_Z1* which corresponds with this point. To draw the asymptotic diagram that corresponds with a real diagram that you see in a dotted line just below, we multiply the particular value *ω* equal to *ω_Z1* by 10 and we divide it by 10. And we have this point and that point and we draw this straight blue line between *10ω_Z1* and *1/10ω_Z1* and that gives us, in blue, the asymptotic phase diagram of the function *1+j(ω/ω_Z1)*. And the real curve, when we draw it with a calculator or a computer, or whether we measure the values because we have created electronic circuits capable of doing so, we will see that it develops asymptotically on the *x* axis. We will have the curve passing from the other side of this straight line up to *ω_Z1*. It will pass again above this straight line, to then return and develop asymptotically above it.

Notes

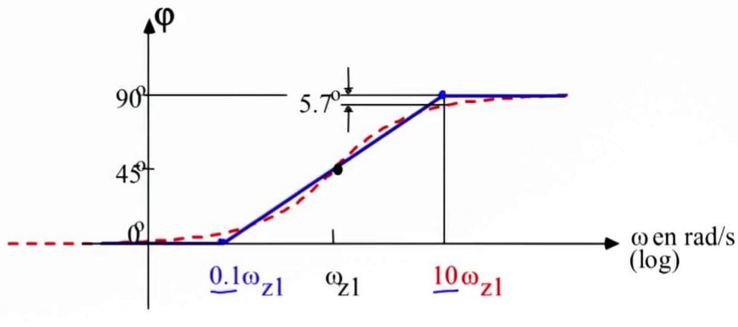
Summary

39m 13s

Approximation autour de $\omega=\omega_{z1}$

Electronique I

- On approxime souvent le diagramme des phases par une droite partant d'un déphasage nul pour $\omega = 0.1\omega_{z1}$ pour atteindre un déphasage de 90° en $\omega = 10\omega_{z1}$.



And the error is 5.7 degrees when one is at *10ω_Z1* and *0,1ω_Z1*. So, in practice, we are satisfied, when it comes to manual sketches, with the asymptotic diagram and we know exactly what happens with the real curve, with this real error that we have for this point *10ω_Z1*, which corresponds with the order of magnitude of six degrees greater or less than, according to where we are. And otherwise, in relation to the asymptotic straight line, we will find ourselves on one side or the other according to where we are in *ω_Z1*. But, as for the sketches, we draw the curve in blue. And for the measures, we find this curve which is approximated by the curve in blue.

Notes

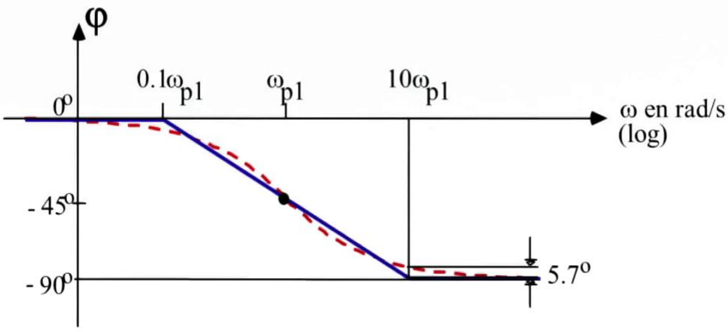
Summary

40m 56s

Fonction Réelle et Imaginaire

$$\underline{H}(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega p1}}$$

$$\mathrm{Arg}(\underline{H}(j\omega)) = -\,\mathrm{Arg}(1 + j\frac{\omega}{\omega_{p1}})$$



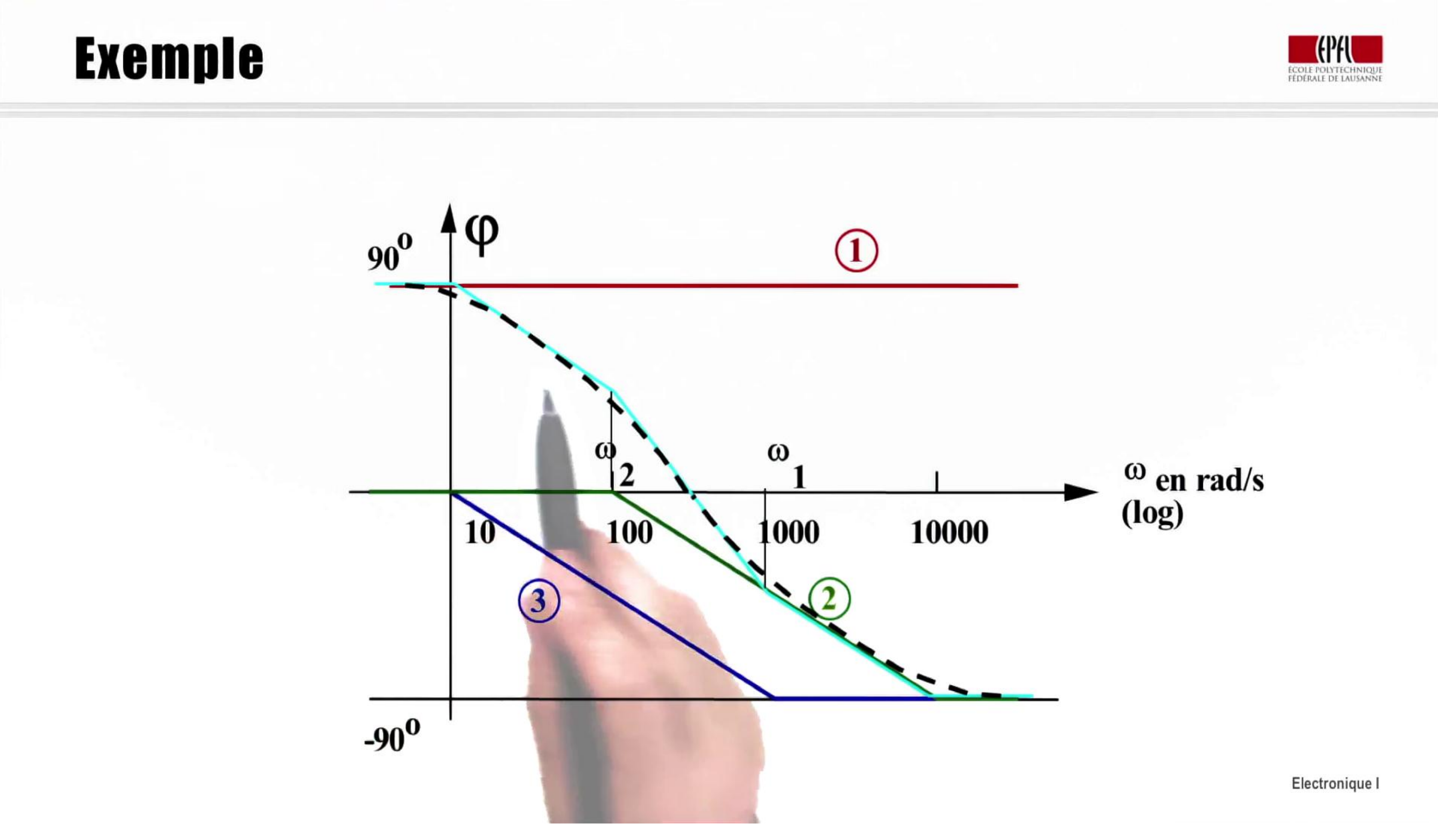
Electronique I

And you have also really understood that if you take the 1 on the function that we have just drawn, we then only have to take the same curve, place a minus sign before the argument, and we will have axial symmetry, and we thus find that this same explanation reappears with axial symmetry in relation to the *x* axes.

Notes

Summary

41m 43s



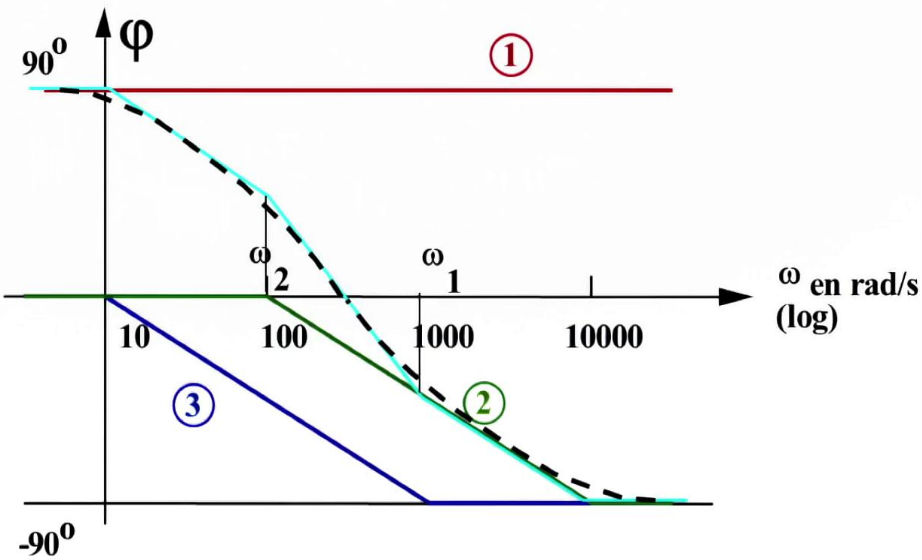
And, lastly, the example which has been given and drawn for the transfer function module, I will use it again to draw the argument of this transfer function. So we will see our three canonical forms again. We will see the examples for *ω1* and *ω2* again. We will find a dephasing of 90 degrees for what is marked in red. This counts for both *1/(1+j(ω/ω1))* and *1/(1+j(ω/ω2))*, and we find the two asymptotic diagrams which have the same form, but with a delay. For *ω1*, we are at 45 degrees here. Sorry, I meant for *ω2*. And for *ω1*, we are always at -45 degrees for the second curve, otherwise the form of the curve is the same and here we have only to add graphically. So, I am going to add graphically these three curves and I am going to find the Bode phase function of this transfer function which will be this one. There it is. Here I have added the curve which is in light blue, or cyan. You see the asymptotic diagram. And there, the dotted lines correspond with the real function of the phase. We start at low frequencies and a phase equal to *π/2*. And when the frequency increases and the pulsation is beyond by an order of magnitude of 10 000, we begin to see a dephasing of *-π/2*.

Notes

Summary

42m 07s

Exemple



Electronique I

And between the two, the phase changes according to the addition of three curves which, in reality, correspond with three electronic functions places one after the other. We have just seen, in this annex which has been added to this MOOC, all of the canonical Bode diagrams which might be used in this MOOC. So, these Bode diagrams allow us to find the electronic functions. And when we create them, they will allow us to understand the frequential behaviour easily, just by analysing the module and phase Bode diagrams of each electronic function, whether they are derivators, integrators, or a simple gain, i.e. the linear functions which are given as examples in this course. Those who have not taken a course on Bode functions will thus be able to understand this. And this is thus an annex to this MOOC.

Notes

Summary

43m 49s