ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Fourier Filtering

Image Processing & Analysis for Life Scientists

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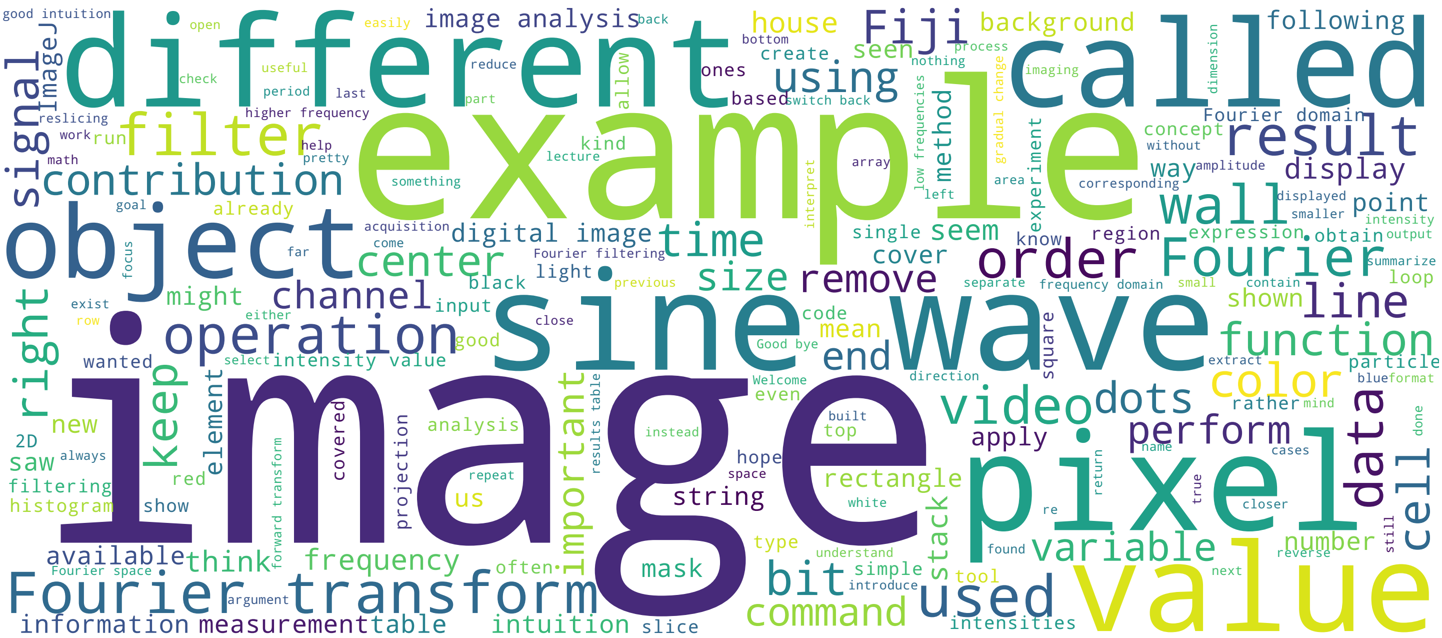

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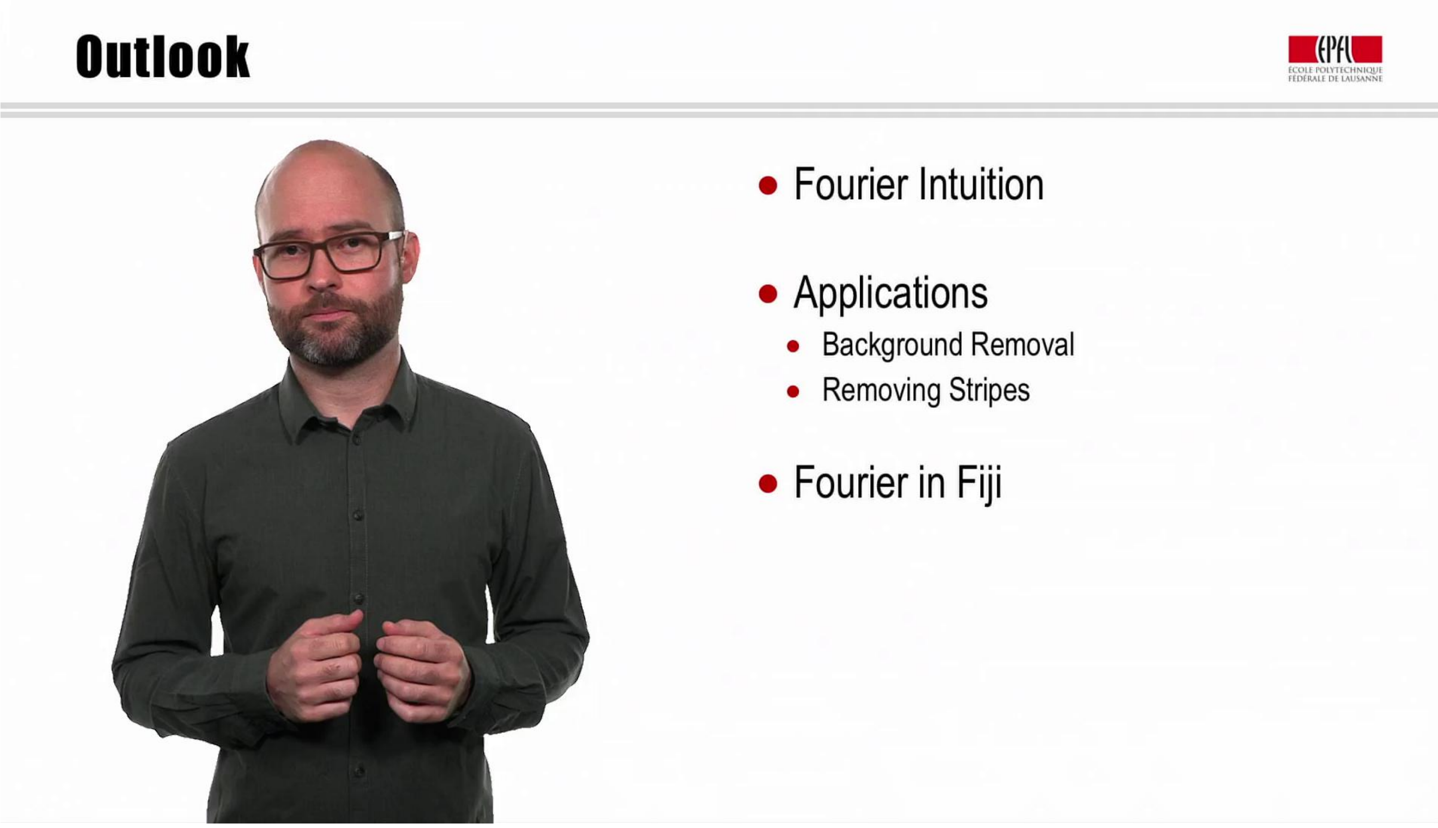
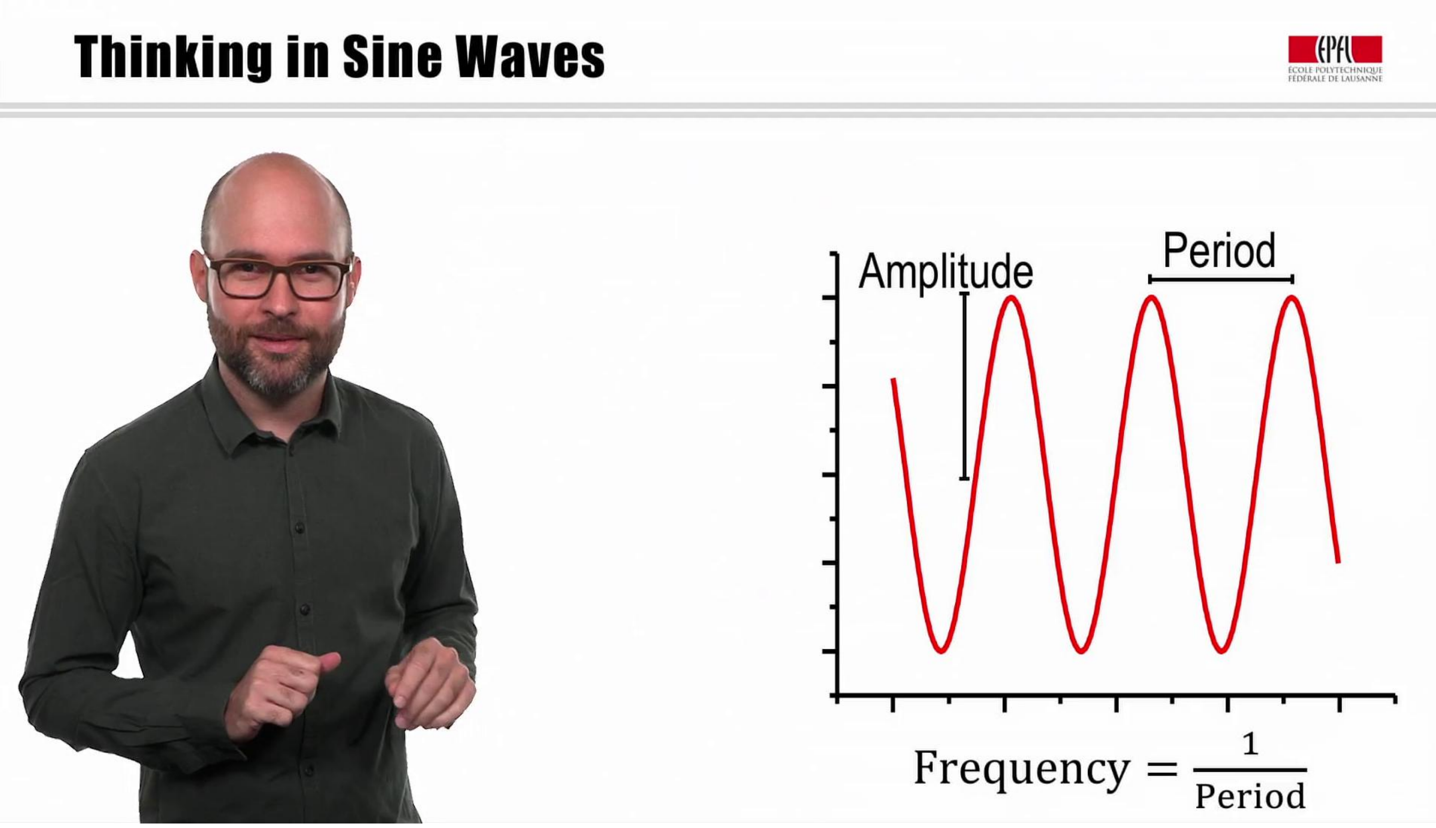


Hello! This video will cover filtering in the Fourier domain or frequency domain. Don't be afraid! The goal is not to cover the math of Fourier filtering, but rather to understand its underlying principle. We are going to establish some intuition about the Fourier domain and what it represents, with some examples. We will explore some useful applications of Fourier filtering to biological images. And finish by seeing where Fourier methods can be found in Fiji.

Notes

Summary

0m 05s



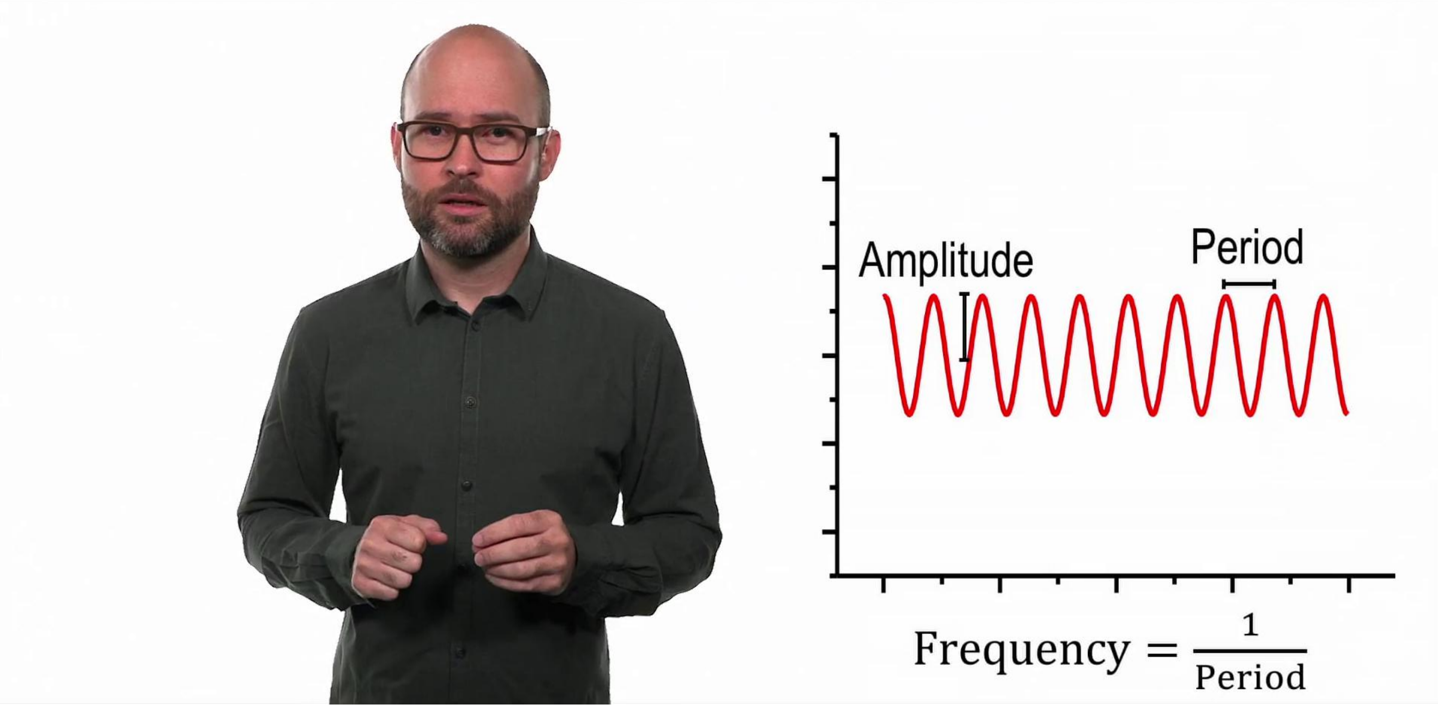
So what is the Fourier domain? Imagine I give you the challenge to use nothing but sine waves to represent signals. Any kind of signal, like sound or images. Sine waves are represented by their amplitude (how strong they are) and their period, or frequency (how often they repeat). Just a reminder that the frequency is the inverse of the period. This is pretty much the only mathematical formula in this video, in case you were worried.

Notes

Summary

0m 31s

Thinking in Sine Waves



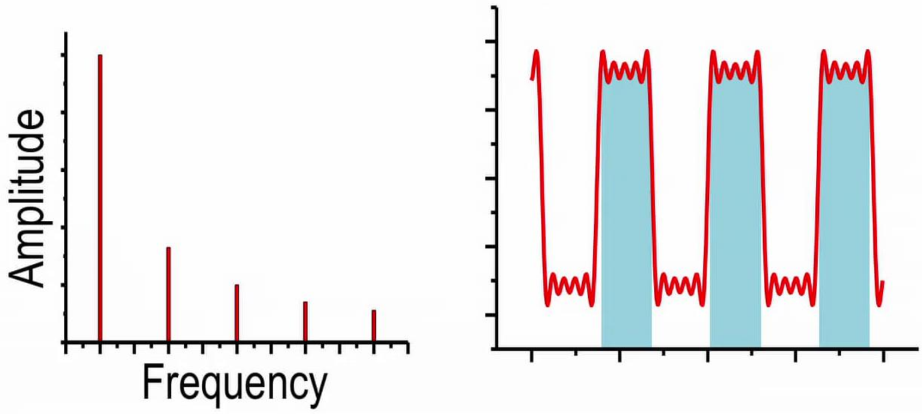
This sine wave has a higher frequency than the previous one, and a lower amplitude, for example.

Notes

Summary

0m 56s

Thinking in Sine Waves

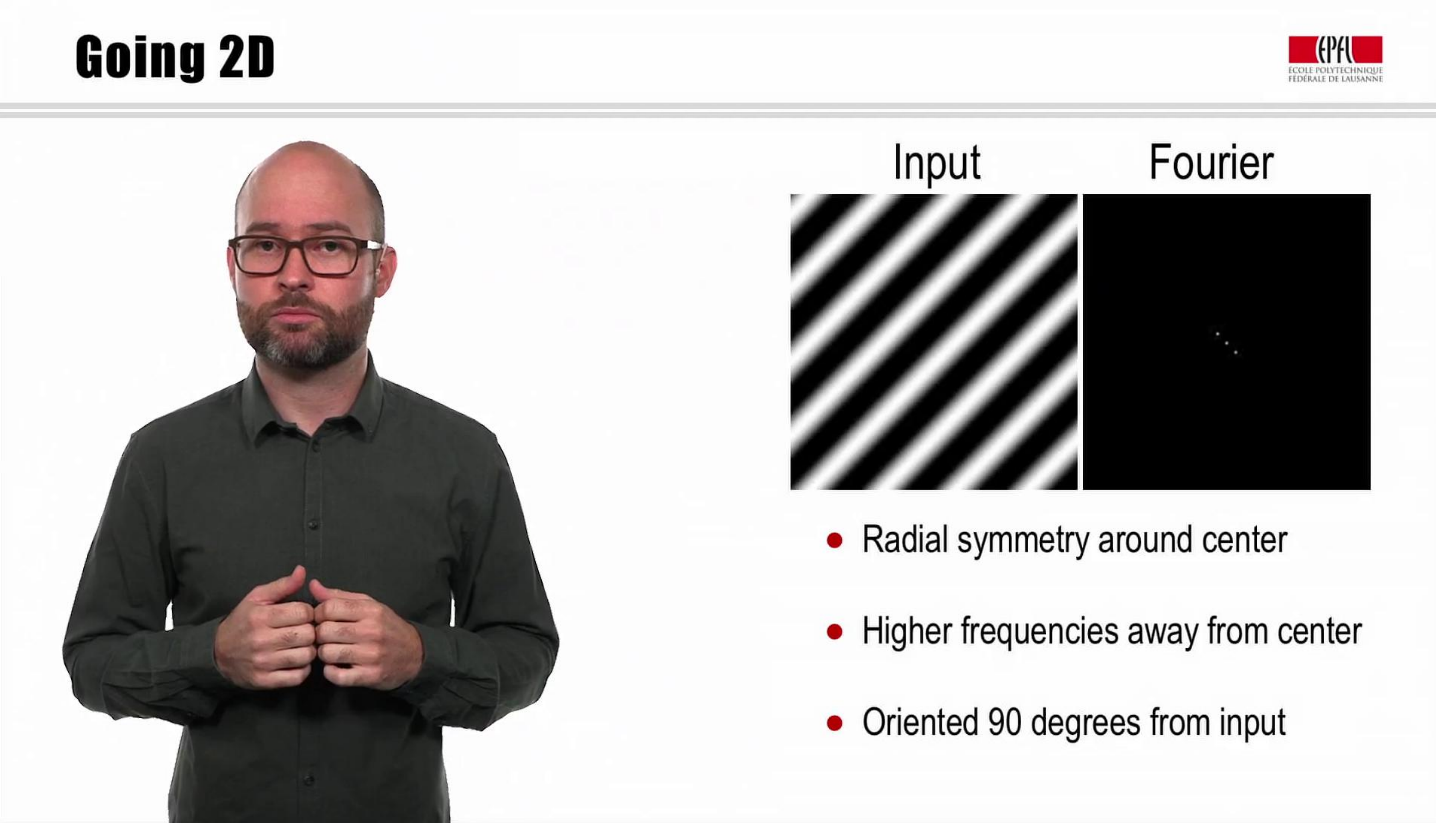


So we have these sine waves, and now I'm asking you to represent these rectangles with your sine tools. Ok 2D is a bit rough, so let's focus in the horizontal direction only. Our first intuition would be to put a sine wave on top like this. Not bad, the peaks match with the top and the empty spaces when the squares are gone, but we lost the sharp edges… All we've got are sine waves, so let's try and add another sine wave that's got twice the frequency of the first one, just to try it out and let's sum them all All right! here it seems we are starting to get a flatter top and bottom of the rectangle, which is good, so let's keep adding more sine waves We can keep going like this, but at some point I'm hoping you get a good intuition for it: we can represent things by just summing up different sine waves together. Another way to see what we've done is instead of plotting the amplitude over time or over space, we will plot it as a function of the frequency. So here we can see the different components that make up the signal we've just built. We see the contribution of the five sine waves we used to try and model that rectangle. Now, what does that look like in 2D?

Notes

Summary

1m 02s



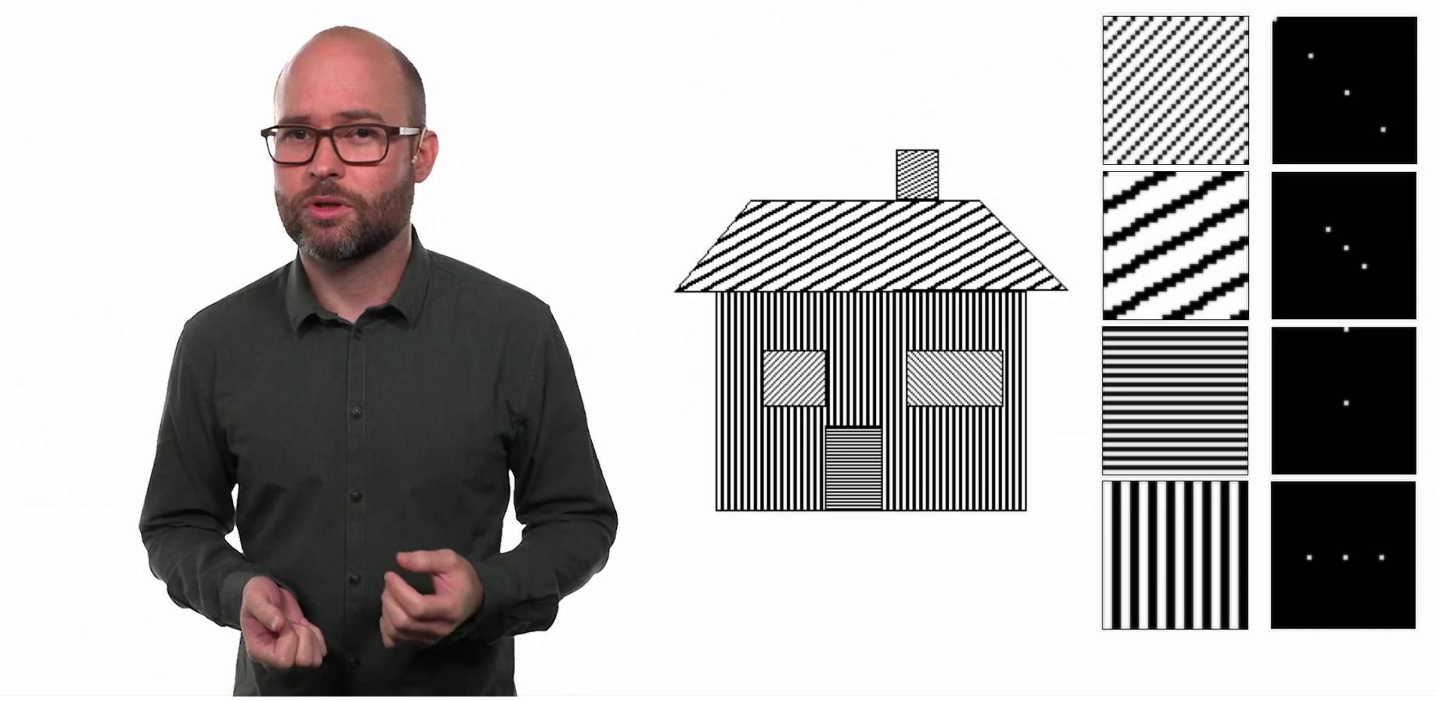
Here we see an animation of a nice ‘pure’ sinusoidal signal and the corresponding Fourier amplitudes on the right. Let’s interpret what we see. The lines are vertical and the dots in Fourier are symmetric around the center apparently. Also, the bigger our sine wave, the closer the dots are to the center, and the smaller the sine wave, or higher frequency, they seem to have dots that are farther away from the center. Let's look at the same thing, but now let's tilt the signal by 45 degrees. We notice that the dots in Fourier are following a direction that seems perpendicular to the direction of the sine lines So let's tentatively conclude the following: we can say that the Fourier transform of an image seems to be symmetric, with respect to the center We can think that higher frequencies tend to be further away from the center. And that the components in Fourier are oriented perpendicular to the main direction of the image.

Notes

Summary

2m 08s

The Fourier House



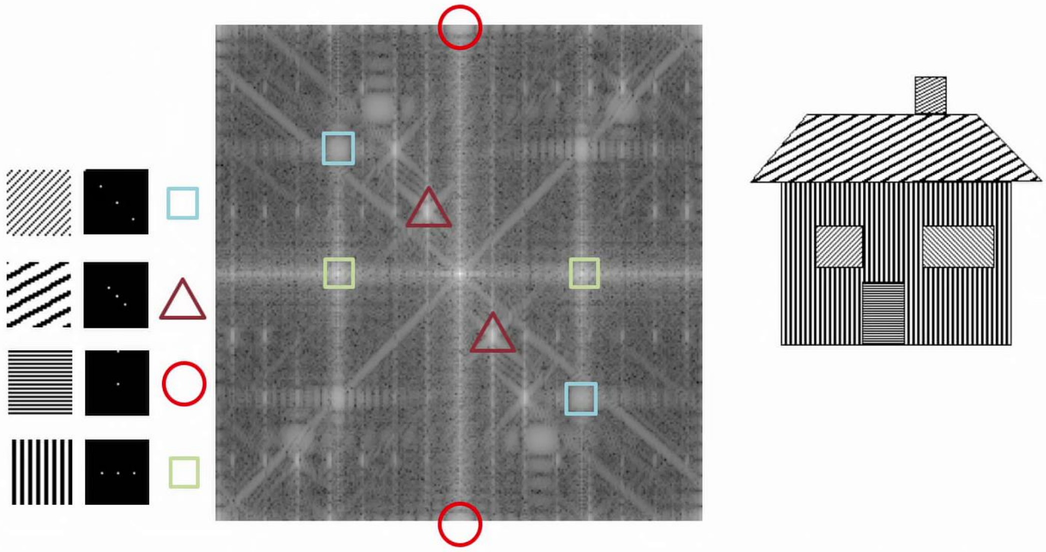
All right! That's it for the intuitions: let’s go to a bit more complicated example. We saw that different ‘pure’ waves represent something like dots on a Fourier plot like the examples here. So what about a more complex shape like this one? Suppose we have a house where each element shows a periodic signal in a different direction and of a different size. Now we are asked to extract the walls of the house and separate them somehow from the rest of these elements. We’ve seen in the previous videos how the idea of filtering is to enhance the contribution of elements we want to keep and to reduce the contribution of elements we want to remove. Here we’re going to be doing the same thing, but in Fourier space.

Notes

Summary

3m 07s

Finding Dominant Frequencies



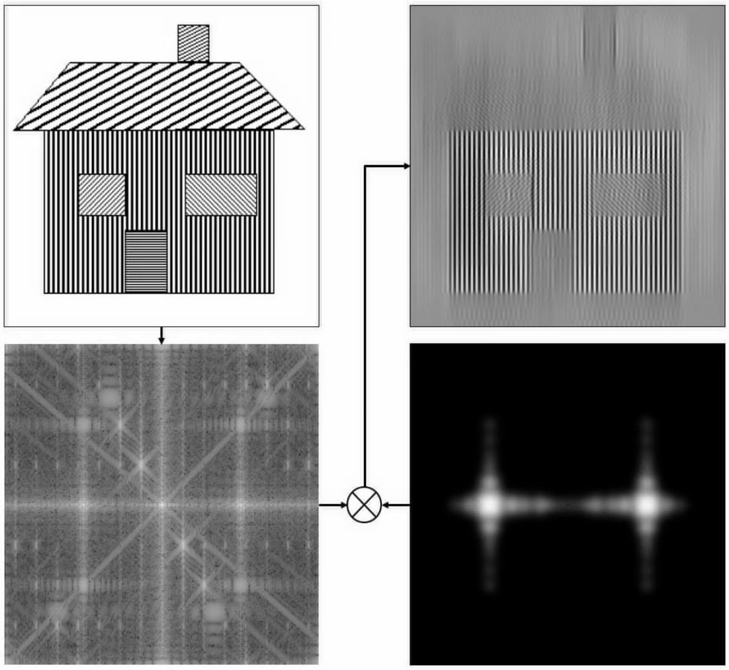
So if I take what is called the Fourier transform of the house we see it's a bit less clean than what we had before, but we can still find the main components of our house. Here I'm showing you the left window, the roof, the door and the wall.

Notes

Summary

3m 52s

Fourier Filtering



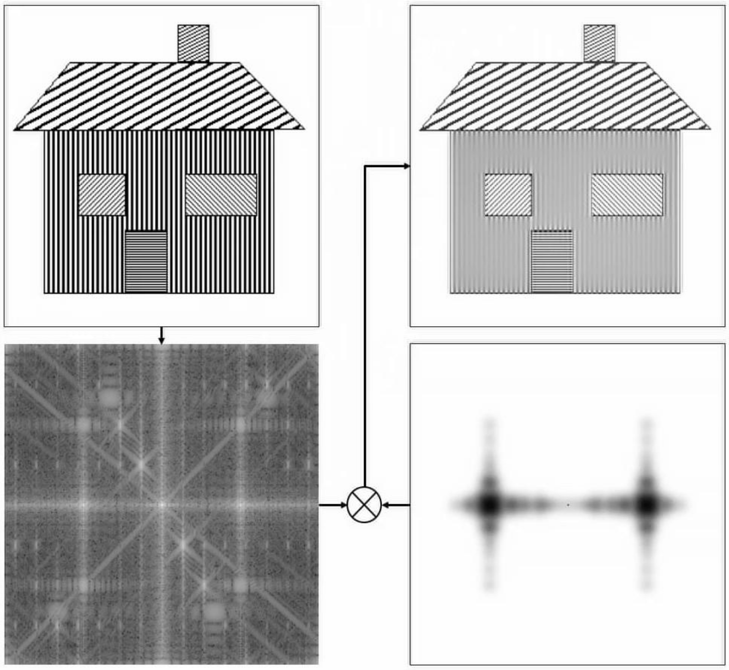
All right! Let's try something. If I wanted to keep the wall, I should hide all the other frequencies on that image except the ones we know represent the wall. So let's take the Fourier transform and make a mask out of the frequencies we believe come from the wall. Then we multiply the two images together in Fourier and then switch back into the spatial domain. And we see that we've managed to remove the contributions from most of the other elements, except the walls. This is the other really cool thing about Fourier which is called the forward transform. Which means you can go from a regular image to Fourier, then do some operations in the Fourier space and then just switch back and see the result. This is called the inverse Fourier transform.

Notes

Summary

4m 08s

Fourier Filtering



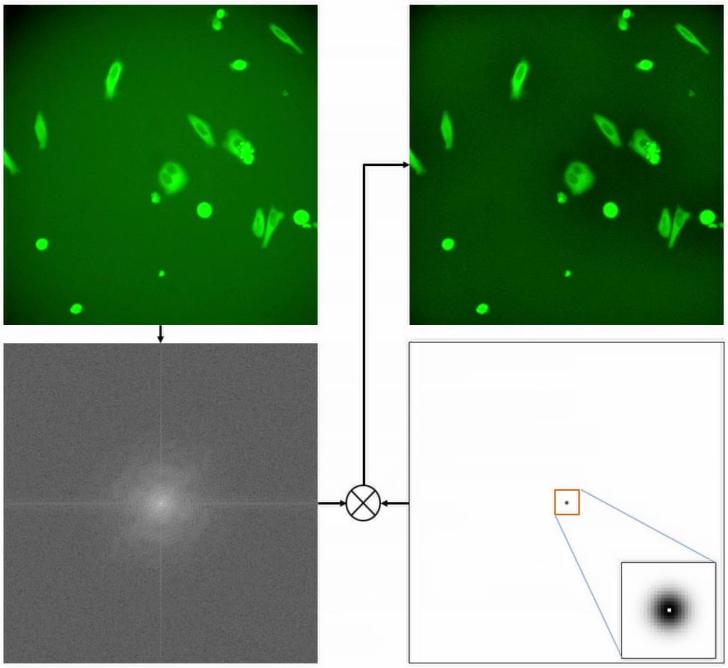
Now we can do it the other way Suppose we wanted to keep everything except the wall I would just reverse the filter And then perform the same multiplication again And go back from Fourier into real space. And here I get the walls that are torn down.

Notes

Summary

4m 53s

Background Removal



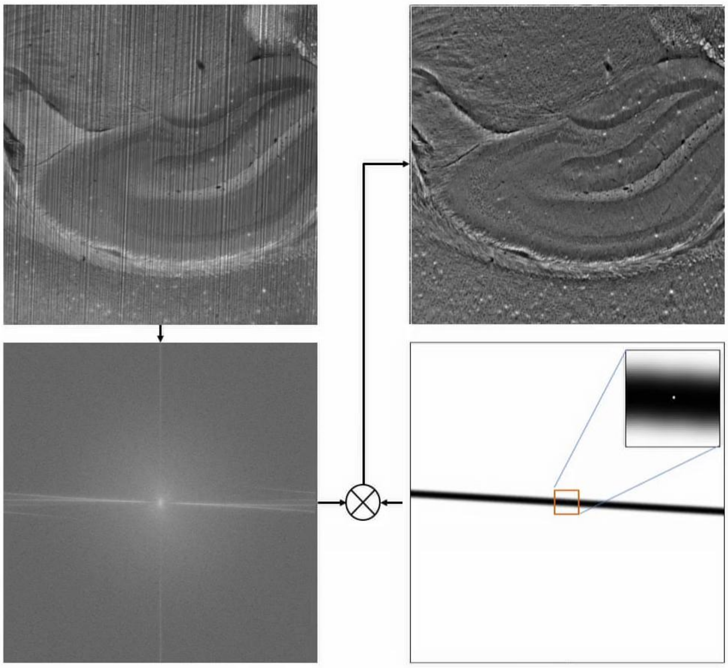
Now this was a rather simplistic example. Let’s look at some real life applications. You've seen that sometimes samples suffer from uneven illuminations. That is, a gradual change in the intensities of the background. If we think of them in the frequency domain, large gradual changes means low frequencies (those close to the center of the image). If we apply a Fourier filter that keeps all the high frequencies and removes the low frequencies (shown in black here), we are able to remove this background.

Notes

Summary

5m 08s

Stripe Removal



More interestingly, in light sheet microscopy, sometimes shadows can appear on the images that can cause long stripes that deteriorate the quality of our signal. If we can identify these contributions in Fourier, which are represented by an almost horizontal line here, we can filter these out by creating the appropriate filter. And now we have the signal and everything else is intact.

Notes

Summary

5m 38s

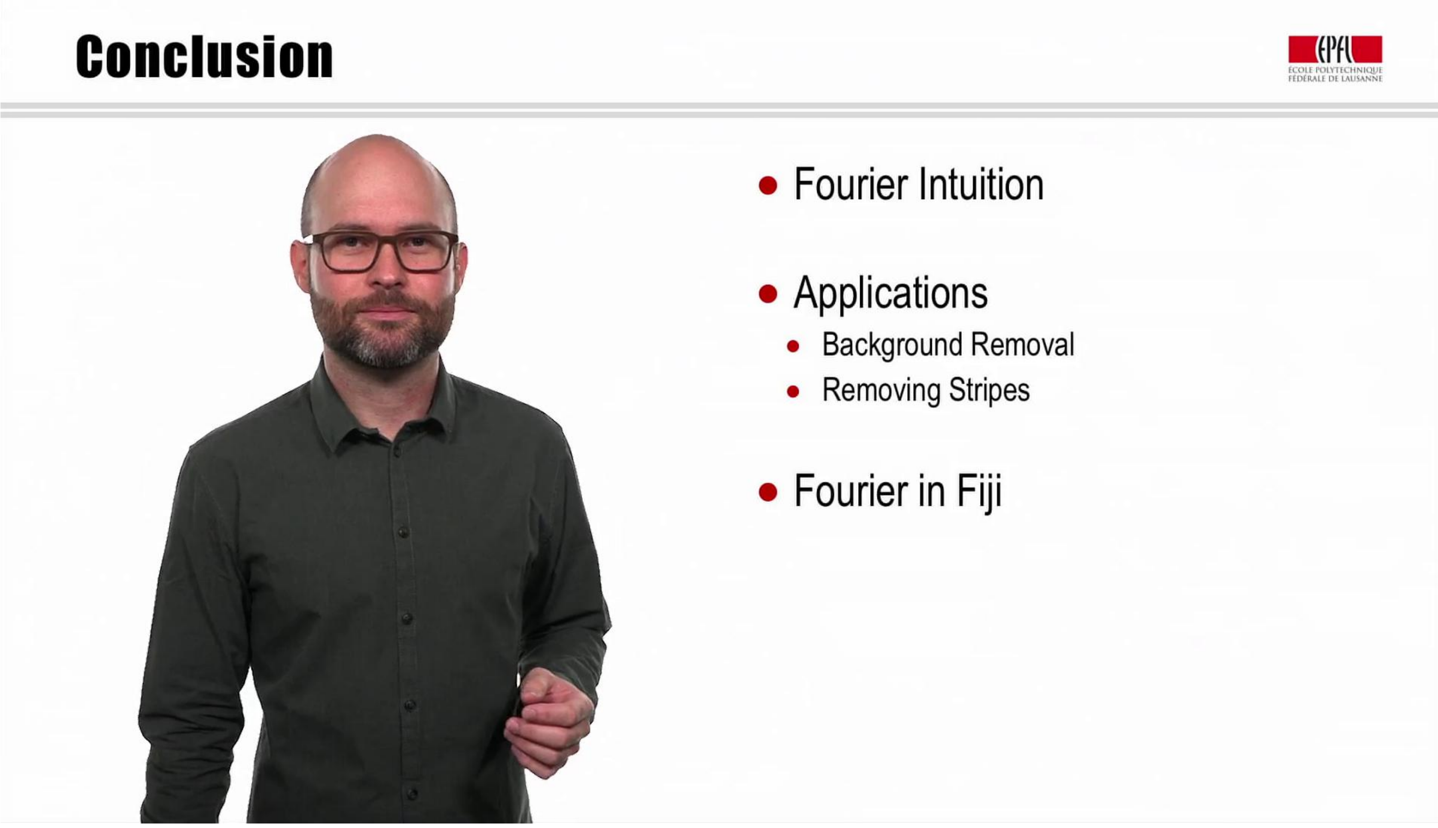
Fourier In Fiji

In Fiji, the entire menu for Fourier transforms is in Process FFT (Fast Fourier Transform). FFT stands for Fast Fourier Transform, which is a very smart implementation of the Fourier algorithm that is suited for digital images. And it is, as you might have guessed, fast! What we called the forward transform is the FFT, and the reverse is the inverse FFT. The examples we used in these slides make use of the Custom Filter command here. This brings us to the end of this video on the Fourier transform.

Notes

Summary

6m 01s



While in most cases you don’t need to use Fourier Transform directly yourselves, I hope that this video has given you a good intuition for understanding what it is. Our goal was not to be exhaustive or have a mathematical approach to Fourier which is a very rich domain and without it we wouldn’t have most of the technology we use nowadays. We invite you to check out the macros that perform the examples I’ve shown here so you can experiment for yourselves. Good bye.

Notes

Summary

6m 34s