

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

EPFL, Lausanne, Switzerland

1.5. Quality of Integrate-and-Fire Models



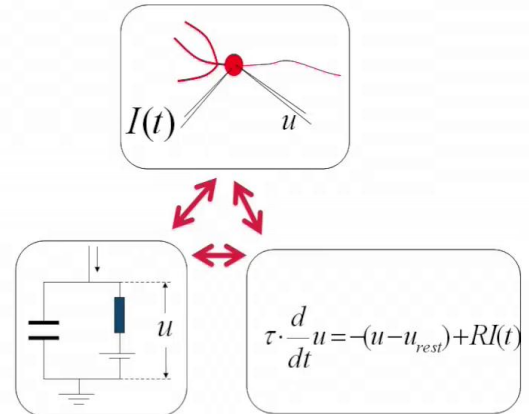
EPFL



Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



Welcome back. In the previous part, I've talked about the passive membrane. The passive membrane corresponds to a RC circuit, and is described by a linear differential equation. The passive membrane is a reasonable description for some neuronal properties in the subthreshold regime. However, it cannot account for spike firing. In order to generate spikes, we need to introduce the notion of a threshold.

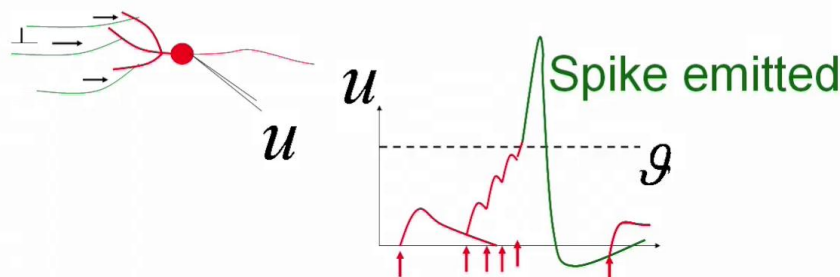
Notes

Summary



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Neuronal Dynamics – Integrate-and-Fire type Models



Input spike causes an EPSP
= excitatory postsynaptic potential

Simple Integrate-and-Fire Model:

passive membrane
+ threshold

- output spikes are events
- generated at threshold
- after spike: reset/refractoriness

Leaky Integrate-and-Fire Model

So we have seen a threshold already in a first symbol model that I introduced at the beginning of this week, there I said each incoming pulse causes a response, causes a post-synaptic potential. If several pulses arrive, one after the other, then these potentials are added. And at some point, if a threshold is reached, an outward spike is generated. Now, a simple integrate-and-fire model is our model of a passive membrane combined with a threshold. So we take our model of a positive membrane between a linear differential equation, combine it with a threshold, and then we have the Leaky Integrate-and-Fire Model.

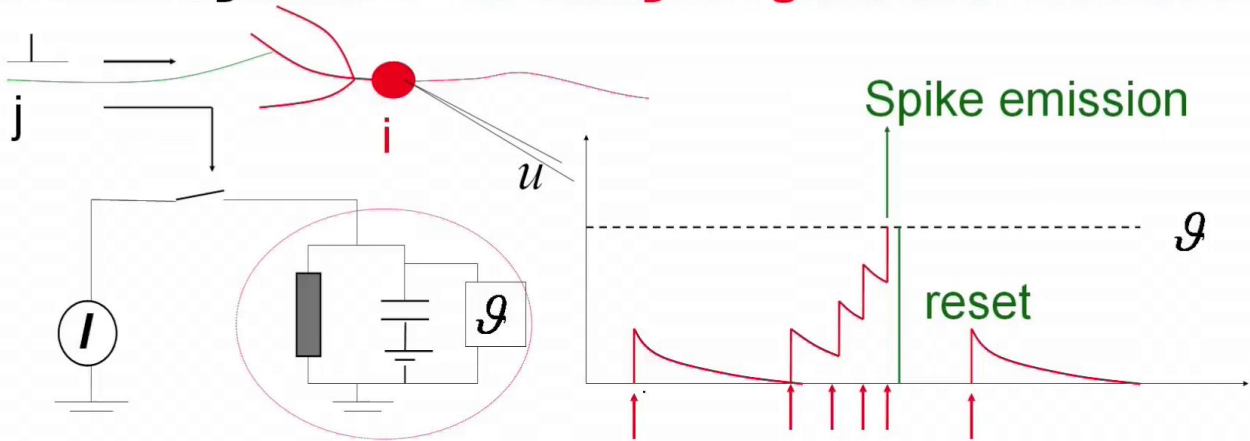
Notes

Summary



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Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

linear

$$u(t) = g \Rightarrow \text{Fire+reset } u \rightarrow u_r$$

threshold

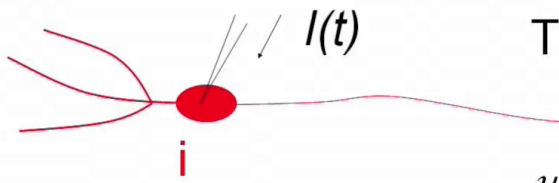
So here's our RC circuit. Suppose a pulse comes in. A pulse corresponds to a short current pulse onto the RC circuit. If several pulses come in, then we would just add up these individual contributions until a threshold is reached. At this moment, a spike is generated and the membrane potential is reset to a new value. This is for the case of spike input. If spikes correspond to a short current pulse, then each of these linear response functions are just exponential pulses. Now more generally, I can drive my neuron model also with time-dependent input current.

Notes

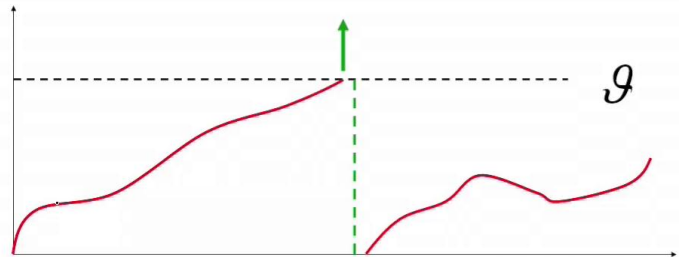
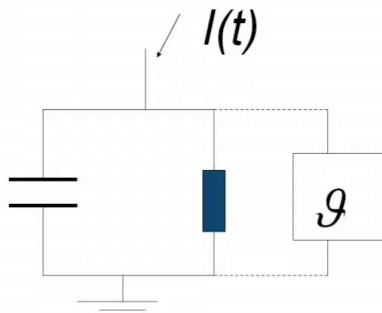
Summary



Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



Time-dependent input



- spikes are events
- triggered at threshold
- spike/reset/refractoriness

So let's suppose we have some arbitrary time-dependent input current. It would integrate this until the threshold is reached. Afterwards, the membrane potential is reset and the whole process starts again.

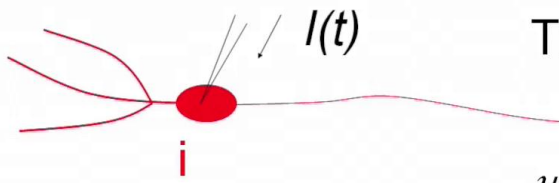
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Summary

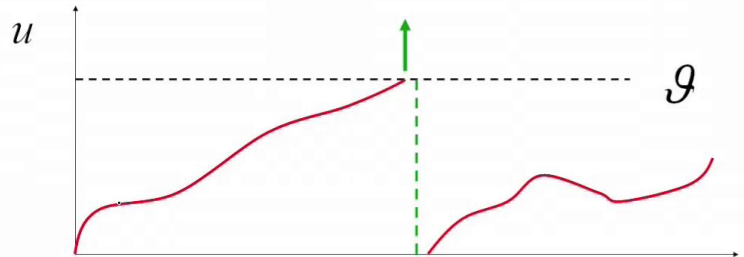
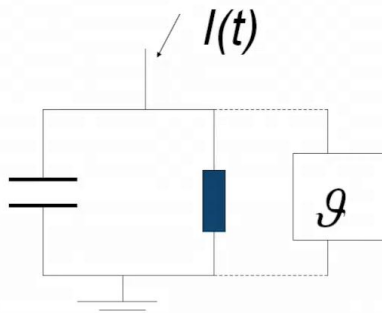


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Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



Time-dependent input



*Math development:
Response to step current*

Now let's focus on one particularly simple case, the situation of a step current input.

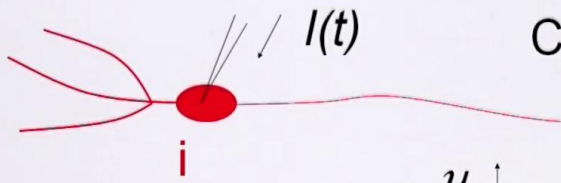
Notes

Summary

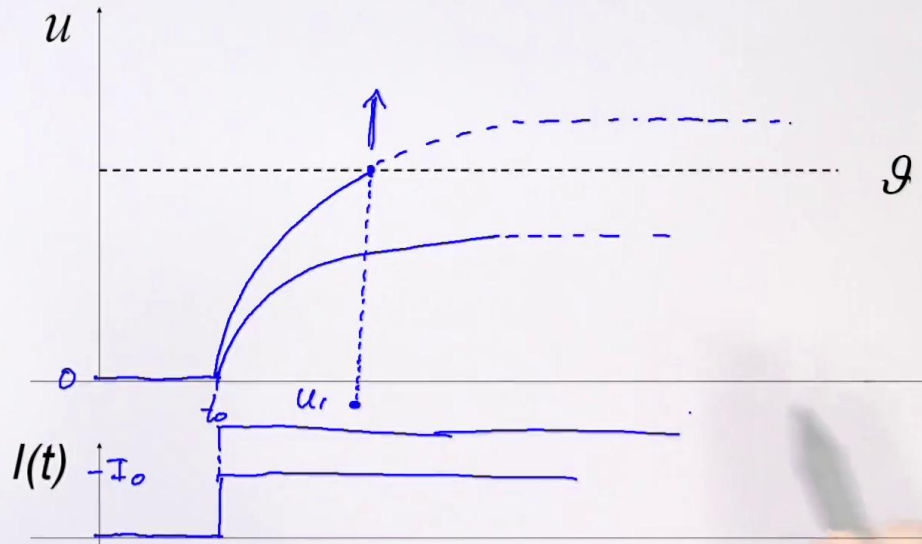
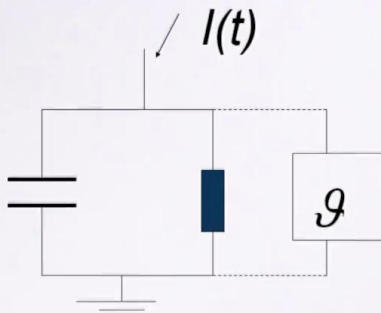


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Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



CONSTANT input/step input



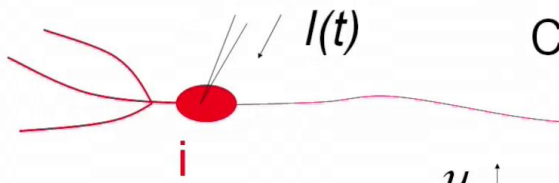
So we have an input current, and this input current is a step. So it's zero for some time, and then it switches to a new value which I call i_0 . Let's suppose that by suitable mathematical transformation, I've removed the resting potential and set it to zero. So up to time t_0 we would be a value of zero. Now we know that a step current input will cause a convergence to a new value-- for example, this value here-- with some time constant τ that corresponds to the time constant of the RC circuit. So for this value of i_0 , the voltage, u , never reaches the threshold value θ . Now let's take a bigger value of the input current, in which case the trajectory may look like this. And then it would converge to a new stationary value. However, because of the threshold, it cannot convert to its new stationary value. But the neuron will fire at this point here. This is the moment of the outward spike. And in this moment in time, we reset the membrane potential to a new value, which I called u_r . u_r can be at the resting value of 0. It can be below. It could also be above. And now the whole process starts again. It's the same current. The current continues. So the current again tries to approach its stationary value.

Notes

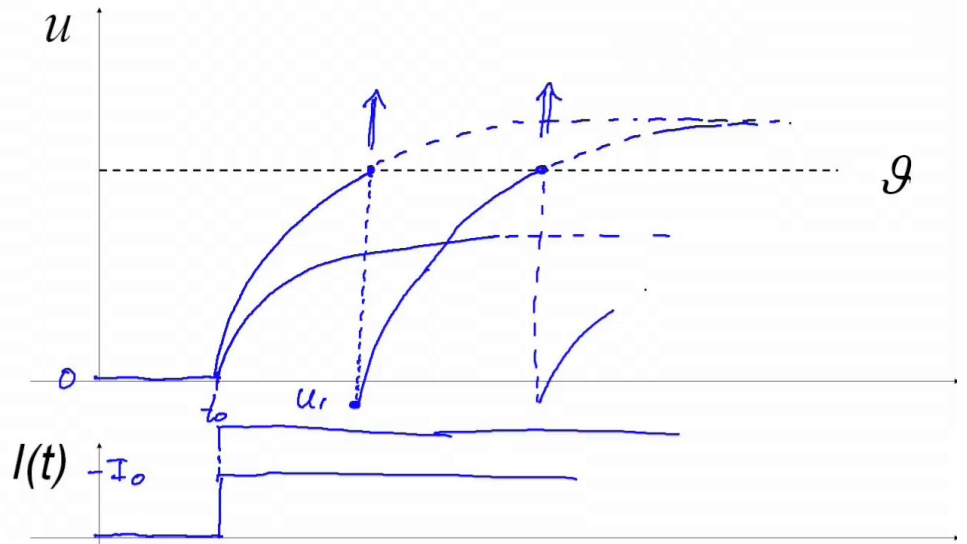
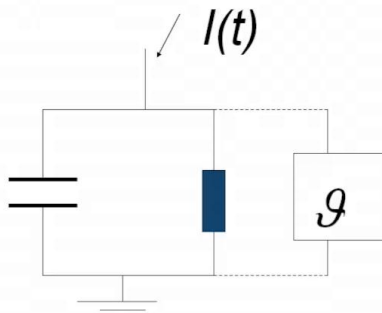
Summary



Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



CONSTANT input/step input



But again it hits the threshold and it's reset. And again, the whole process continues. Thus, for a constant input current, we can expect that this Leaky Integrate-and-Fire Model will fire regularly.

Notes

Summary

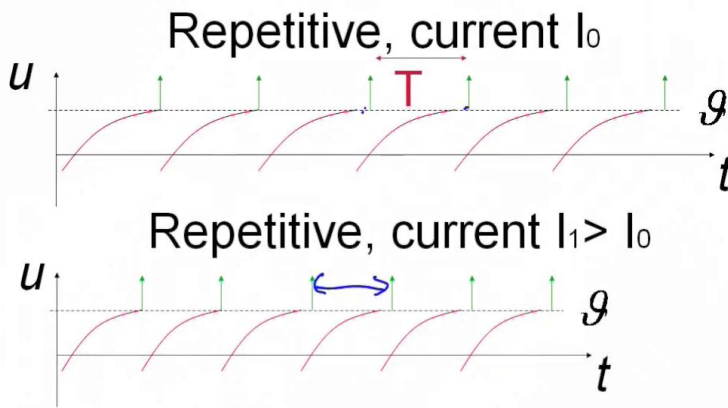


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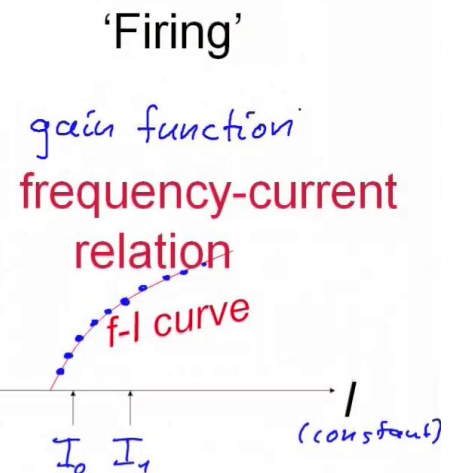
Leaky Integrate-and-Fire Model (LIF)

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI_0$$

LIF
If $u(t) = \vartheta \Rightarrow u \rightarrow u_r$



$$f = 1/T$$



Its trajectory approaches the threshold, it's reset. It approaches the threshold, it's reset. Approaches the threshold, is reset, and so forth. If I increase the current, then the approach to the threshold is faster. The slope is higher, therefore threshold is approached more quickly. And therefore, if I look at the distance at the interval between two spikes, here I have a large interval. For a bigger current, I have a shorter interval. Instead of plotting the interval, I plot $1/T$. For periodic fires, the neuron enters into a regime of periodic firing. It has a fixed firing frequency you find as one of the period. Now for a low current, i_0 , this is my firing frequency. For larger current i_1 , it's a different firing frequency. If I repeat this kind of argument for other values of the current, I would construct a plot which gives the frequency as a function of the constant input current. Now this is the frequency-current relation, or the F-I curve, also called the gain function of the neuron.

Notes

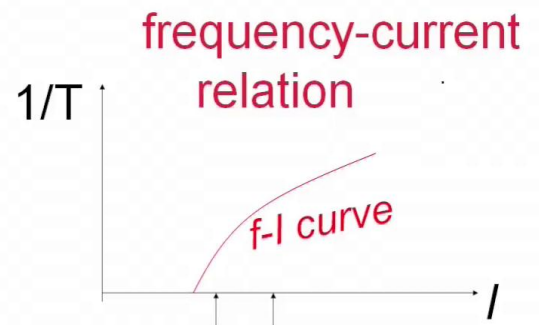
Summary



Neuronal Dynamics – Homework 1.2



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



take 10 min!

So we have seen in part 1.2 that for a passive membrane, we can calculate mathematically the response to a step current. We can now use this situation to calculate the period of firing or the frequency of firing of a periodically firing Leaky Integrate-and-Fire model.

Notes

Summary



7m 12s

Neuronal Dynamics – Homework 1.2. Leaky Integrate-and-fire Model

LIF $\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI_0$ If firing: $u \rightarrow u_r$

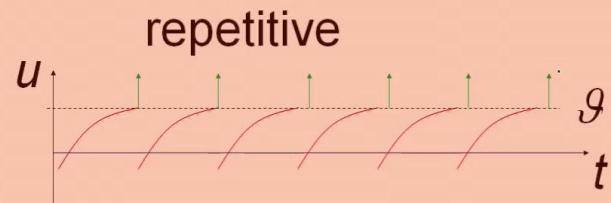
Exercise!

Calculate the interspike interval T for constant input I .

Firing rate is $f=1/T$.

Write f as a function of I .

What is the frequency-current curve $f=g(I)$ of the LIF?



And this is what I would like you to do next. Take the solution of the step current. Calculate the time between two firings, the interval capital T . Look at the frequency you find as $1/T$, and write down this frequency as a function of the constant input. In short, the question is what is the frequency current curve of the Leaky Integrate-and-Fire model?

Notes

Summary



7m 37s