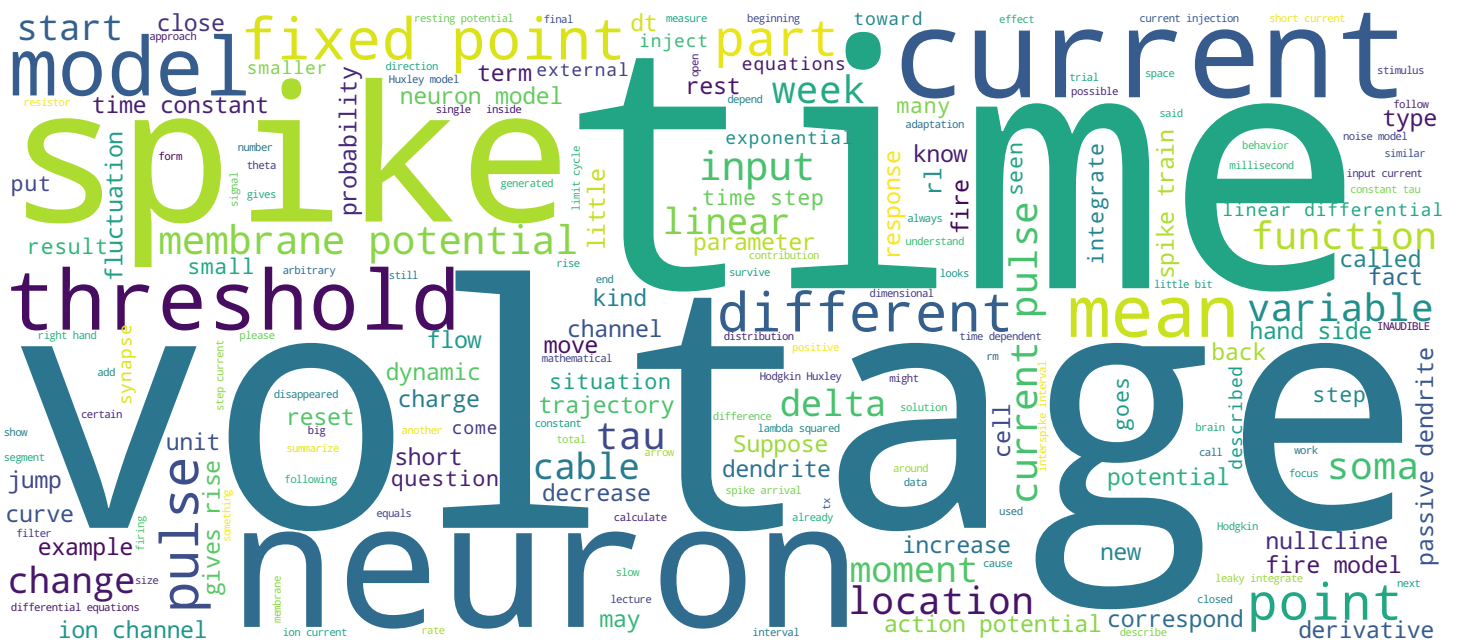


ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE



EPFL



Week 3 – part 4: Cable equation



✓ 3.1 Synapses

✓ 3.2 Short-term plasticity

✓ 3.3 Dendrite as a Cable

3.4 Cable equation

3.5 Compartmental Models
- active dendrites

Welcome to part four of the third week. In the previous part, I have derived a cable equation. And I said that, depending on what type of ion channels you put into the equations, you can get different behaviors.

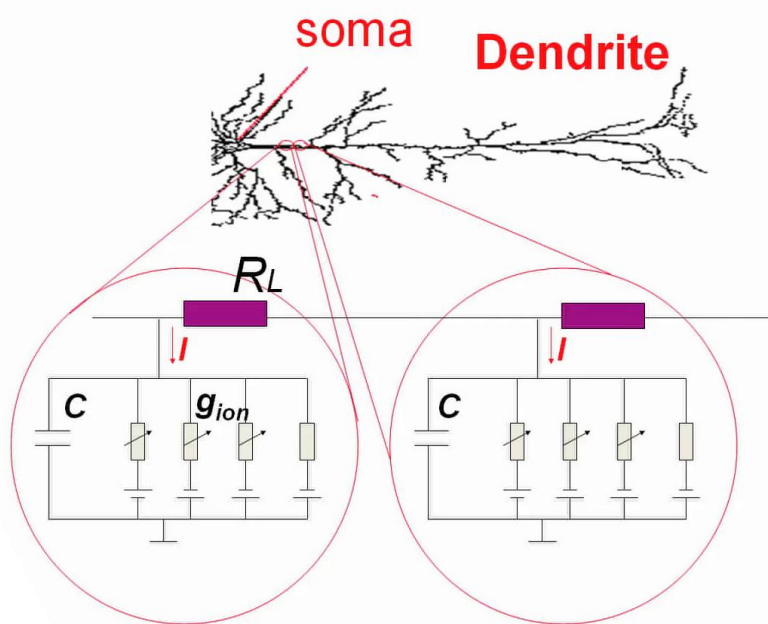
Notes

Summary



0m 04s

Neuronal Dynamics – 3.4. Cable equation



In the following, I would like to focus on the case of a passive dendrite.

Notes

Summary



0m 19s

Neuronal Dynamics – 3.4 Cable equation

Mathematical derivation

$$\frac{d^2}{dx^2}u(t,x) = c r_L \frac{d}{dt}u(t,x) + r_L \sum_{ion} i_{ion}(t,x) - r_L i^{ext}(t,x)$$

$$\sum_{ion} i_{ion}(t,x) = leak \quad \text{passive dendrite}$$

$$\sum_{ion} i_{ion}(t,x) = Ca, Na, \dots \quad \text{active dendrite}$$

$$\sum_{ion} i_{ion}(t,x) = Na, K, \dots \quad \text{axon}$$

✎ ✏ ☐ ➡

A passive dendrite means that I only have an ohmic current, a current that corresponds to Ohm's law, a current that does not give rise to action potentials. That would be an active current-- the currents that we have in the soma or in [INAUDIBLE].

Notes

Summary



0m 26s

Neuronal Dynamics – 3.4 Derivation for passive cable

$$\frac{d^2}{dx^2} u(t, x) = c r_L \frac{d}{dt} u(t, x) + r_L \sum_{ion} i_{ion}(t, x) - r_L i^{ext}(t, x) \quad (*)$$

$$\sum_{ion} i_{ion}(t, x) = leak \quad \text{passive dendrite}$$

$$\frac{d^2}{dx^2} u(t, x) = c \cdot r_L \frac{d}{dt} u(t, x) + r_L \frac{u(t, x)}{r_m} - r_L i^{ext}(t, x)$$

$$r_m \frac{d^2}{dx^2} u(t, x) = c \cdot r_m \frac{d}{dt} u(t, x) + u(t, x) - r_m \cdot i^{ext}$$

$$I_{ion} = i_{ion} dx$$

$$I^{ext} = i^{ext} dx$$

$$\sum_{ion} i_{ion}(t, x) = \frac{u(t, x)}{r_m}$$



So let's look on this passive dendrite. And let's derive the cable equation for the case of a passive dendrite right now. So up here, we have the equation that I derived in part two. This is the cable equation. The cable equation for arbitrary choices of ion current. But now, let's focus on a passive dendrite. Remember that the little i is a specific current per unit length of cable. Now, Ohm's law is that these ion currents can be described by a linear relation between the voltage at location x and the membrane resistance. And the little r indicates that, again, this is something per unit length. So let's plug this in here. So I insert u . So I have r_L , and then I have u of t and x divided by the little r_m , the membrane resistance. I copy the other terms, minus $r_L i^{ext}$. I have a little $r_L d/dt u$ of t and x . And these different terms, if the right hand side of the equation, on the left hand side, I have the second derivative of the voltage with respect to the spatial coordinate, x . So let's now multiply this equation by r_m . So I move this term to the other side-- $r_m d^2 dx^2 u$ of tx $c r_m d/dt u$ of tx plus u of tx minus $r_m i^{ext}$. And you may have wondered, what happened to this r_L ?

Notes

Summary



0m 50s

Neuronal Dynamics – 3.4 Derivation for passive cable

$$\frac{d^2}{dx^2} u(t, x) = c r_L \frac{d}{dt} u(t, x) + r_L \sum_{ion} i_{ion}(t, x) - r_L i^{ext}(t, x) \quad (*)$$

$$\sum_{ion} i_{ion}(t, x) = leak \quad \text{passive dendrite}$$

$$\frac{d^2}{dx^2} u(t, x) = c \cdot r_L \frac{d}{dt} u(t, x) + r_L \frac{u(t, x)}{r_m} - r_L i^{ext}(t, x)$$

$$I_{ion} = i_{ion} dx$$

$$I^{ext} = i^{ext} dx$$

$$\left(\frac{r_m}{r_L} \right) \frac{d^2}{dx^2} u(t, x) = c \cdot r_m \frac{d}{dt} u(t, x) + u(t, x) - r_m \cdot i^{ext}(t, x)$$

λ^2 τ_m

$$\left\| \lambda^2 \frac{d^2}{dx^2} u(t, x) = \tau_m \frac{d}{dt} u(t, x) + u - r_m i^{ext}(t, x) \right\|$$

time

$$\sum_{ion} i_{ion}(t, x) = \frac{u(t, x)}{r_m}$$

Well, let's divide by r_L at the same time. So I have here r_L , the longitudinal resistance. It just disappeared here, has disappeared here. It has disappeared here. Now, let's introduce some shorthand notation. C times r_m -- I call this $[\tau_m]$. This term here, r_m over r_L , I call λ^2 squared. This gives me the final equation, λ^2 squared, second derivative with respect to space of the voltage $[\tau_m]$, little m , first derivative of the voltage with respect to time, plus u of t, x $[r_m]$ external of t and x . This situation here is the cable equation for the passive dendrite, or the passive cable equation. u is units of voltage. r times i is unit of voltage. This is $[\tau_m]$ with respect to time. Therefore, $[\tau_m]$ or τ_m is a time constant. This is the second derivative with respect to space of the voltage. The right hand side has units of voltage. Therefore, this λ^2 squared has units of space squared. λ is a length. λ^2 squared is the length squared.

Notes

Summary

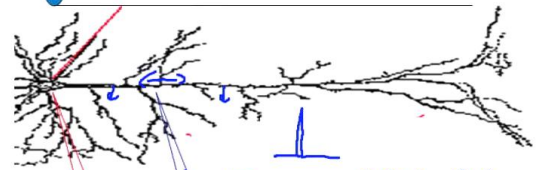


Neuronal Dynamics – 3.4 dendritic stimulation

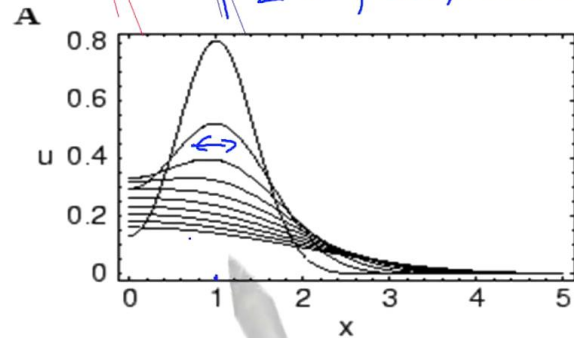
passive dendrite/passive cable

$$\lambda^2 \frac{d^2}{dx^2} u(t, x) = \tau_m \frac{d}{dt} u(t, x) + u - r_m i^{\text{ext}}(t, x)$$

soma



$$I = q \delta(t) \cdot \delta(x - x_0)$$



So now, we have a partial differential equation, a differential equation with two different derivatives, one with respect to space, one with respect to time-- a partial differential equation, as before. But it's now completely linear. This is linear in the voltage. This is linear in the voltage. This is linear in the voltage. And for these kinds of linear differential equations, it's possible to find solutions analytically. So let's consider the following paradigm. We inject a current pulse-- a very short current pulse, very short. So inject a current pulse at time t at location x minus x_0 . Now, this current pulse will put a lot of charge at this location. The charge will spread out in both directions. The charge will also leave the dendrite through ion channels. As a result, the peak that was at location one-- this is where we injected the current-- this peak becomes broader and broader. Initially, it's sharp. And it becomes broader and broader. And because of these transversal currents, the surface under peak will also decrease over time. For this solution to this differential equation, the dendrite was approximated just as one linear cable, which is very long in this direction [INAUDIBLE] infinitely long.

Notes

Summary

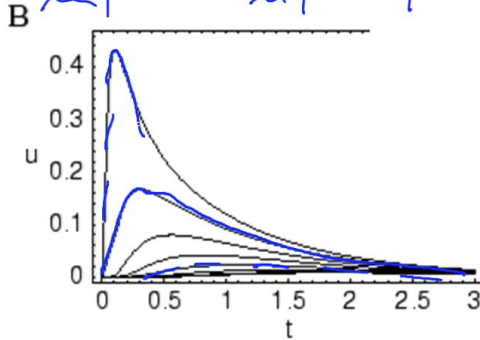


5m 27s

Neuronal Dynamics – 3.4 dendritic stimulation

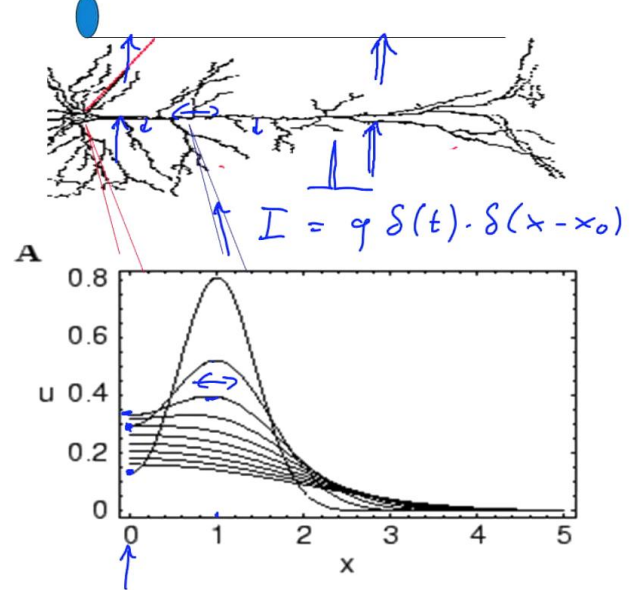
passive dendrite/passive cable

$$\lambda^2 \frac{d^2 u(t, x)}{dx^2} = \tau_m \frac{du(t, x)}{dt} + u - r_m i^{\text{ext}}(t, x)$$



Stimulate dendrite, measure at soma

soma



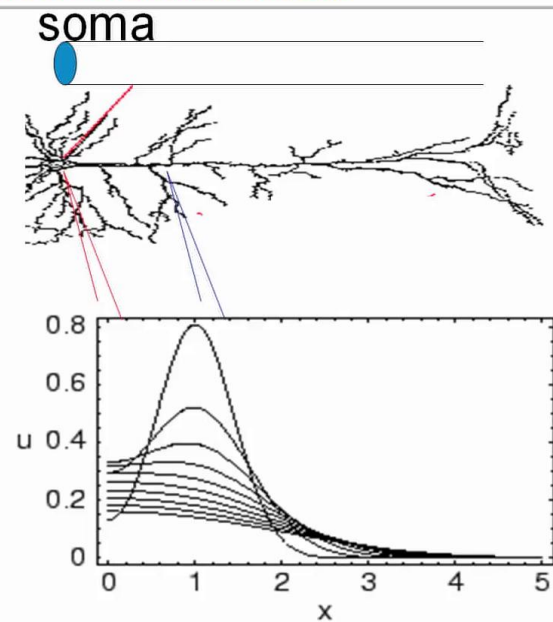
But at the soma, it's closed. This will be where action potentials are generated. Here, we are interested just in the dendritic stimulation. Therefore, we just closed here. So my pulse injected here will give rise to a very sharp peak at the beginning. But then, it will decrease and disappear over time. We can follow the effect of the current injection here, not just at this location, but also at many other locations. For example, we can follow it at the soma. So what do you see at the beginning? The voltage was very little. Then the voltage increased. This would be the second curve. This is the third curve. The voltage increases. And then it decreases again. Therefore, as a function of time, the voltage first increases and then decreases again. So this is for dendritic stimulation. We stimulate at location one. And we measure the time course at the soma. If we stimulate further out in the dendrite-- if there's stimulation here-- [INAUDIBLE] in the cable, then the total effect at the soma is much less. If you stimulate close to the soma, then there's an initial sharp rise and a sharp peak measured at the soma.

Notes

Summary



Neuronal Dynamics – 3.4 dendritic stimulation



To summarize, the cable equation allows us to understand how charge gives rise-- through synapses or through current injection on the dendrite-- how charge spreads out over the dendrite and how it gives rise to a post-synaptic potential, how it gives rise to a time course, how it gives rise to a potential at the soma.

Notes

Summary



9m 15s

Neuronal Dynamics – Quiz 3.4

Multiple answers possible!

The space constant of a passive cable is

☐ $\lambda = \frac{r_m}{r_L}$

☐ $\lambda = \frac{r_L}{r_m}$

☐ $\lambda = \sqrt{\frac{r_L}{r_m}}$

☐ $\lambda = \sqrt{\frac{r_m}{r_L}}$

Dendritic current injection.

If a short current pulse is injected into the dendrite

☐ the voltage at the injection site is maximal immediately after the end of the injection

☐ the voltage at the dendritic injection site is maximal a few milliseconds after the end of the injection

☐ the voltage at the soma is maximal immediately after the end of the injection.

☐ the voltage at the soma is maximal a few milliseconds after the end of the injection

It follows from the cable equation that

☐ the shape of an EPSP depends on the dendritic location of the synapse.

☐ the shape of an EPSP depends only on the synaptic time constant, but not on dendritic location.

With this, I would like to close this part. And please, take a couple of minutes for the quiz before you go on to the final part of this week, where we discuss mold neuron models with dendrites.

Notes

Summary



9m 48s