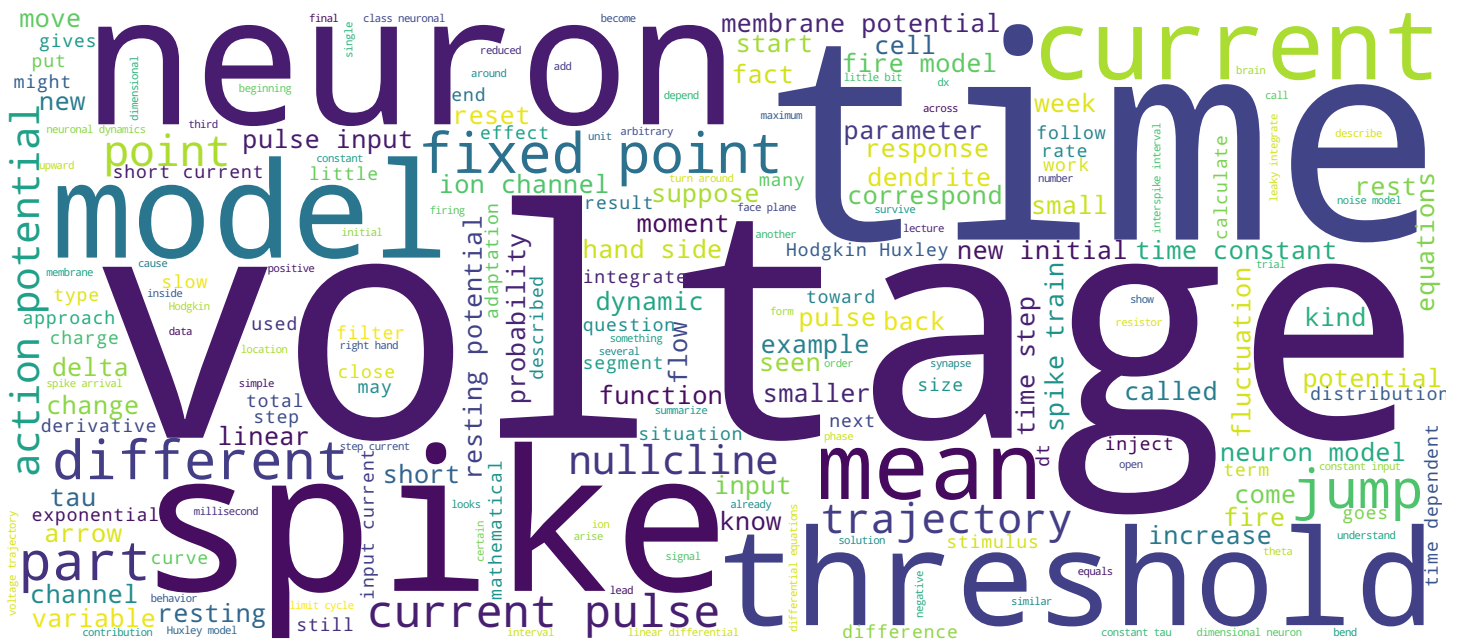


ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

EPFL, Lausanne, Switzerland



Video



Week 4 – part 3: Analysis of a 2D neuron model



✓ 4.1 From Hodgkin-Huxley to 2D

✓ 4.2 Phase Plane Analysis

- Role of nullcline

4.3 Analysis of a 2D Neuron Model

- MathDetour 3: Stability of fixed points

4.4 Type I and II Neuron Models

- where is the firing threshold?

- MathDetour 4: separation of time scales

4.5. Nonlinear Integrate-and-fire

- from two to one dimension

Welcome to part three of week four of the class neuronal dynamics.

Notes

Summary



0m 04s

Neuronal Dynamics – 4.3. Analysis of a 2D neuron model



2-dimensional equation

$$\tau \frac{du}{dt} = F(u, w) + RI(t)$$

stimulus



$$\tau_w \frac{dw}{dt} = G(u, w)$$

Enables graphical analysis!

- Pulse input
- Constant input

I promised that once we have reduced the system to two equations, we can do a lot of things. We can do phase plane analysis. And phase plane analysis can be used to understand spike generation. For example, doing pulse input or doing constant input. And this is the topic of this part.

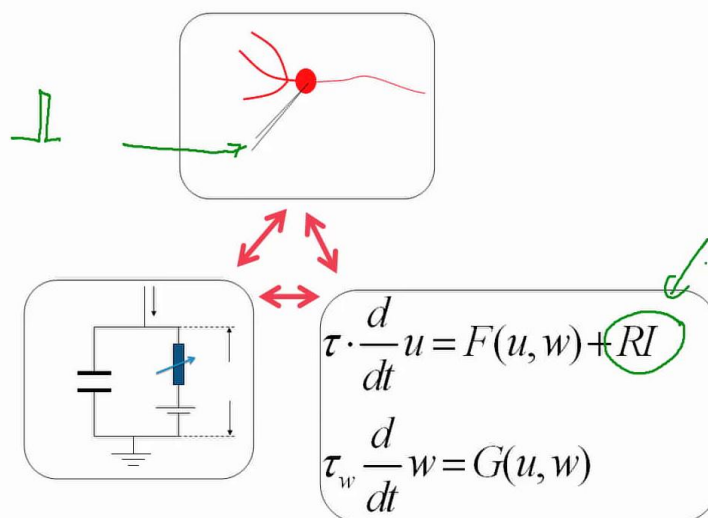
Notes

Summary



0m 11s

Neuronal Dynamics – 4.3. 2D neuron model : Pulse input



pulse input



⌂ / ☰ ➡

So let's consider pulse input. An experimentalist may inject a very short current pulse into the neuron. The effect of this can be described in a Hodgkin-Huxley type model. Now we have reduced this to a two-dimensional neuron model. The pulse input would come in here, in the voltage equation.

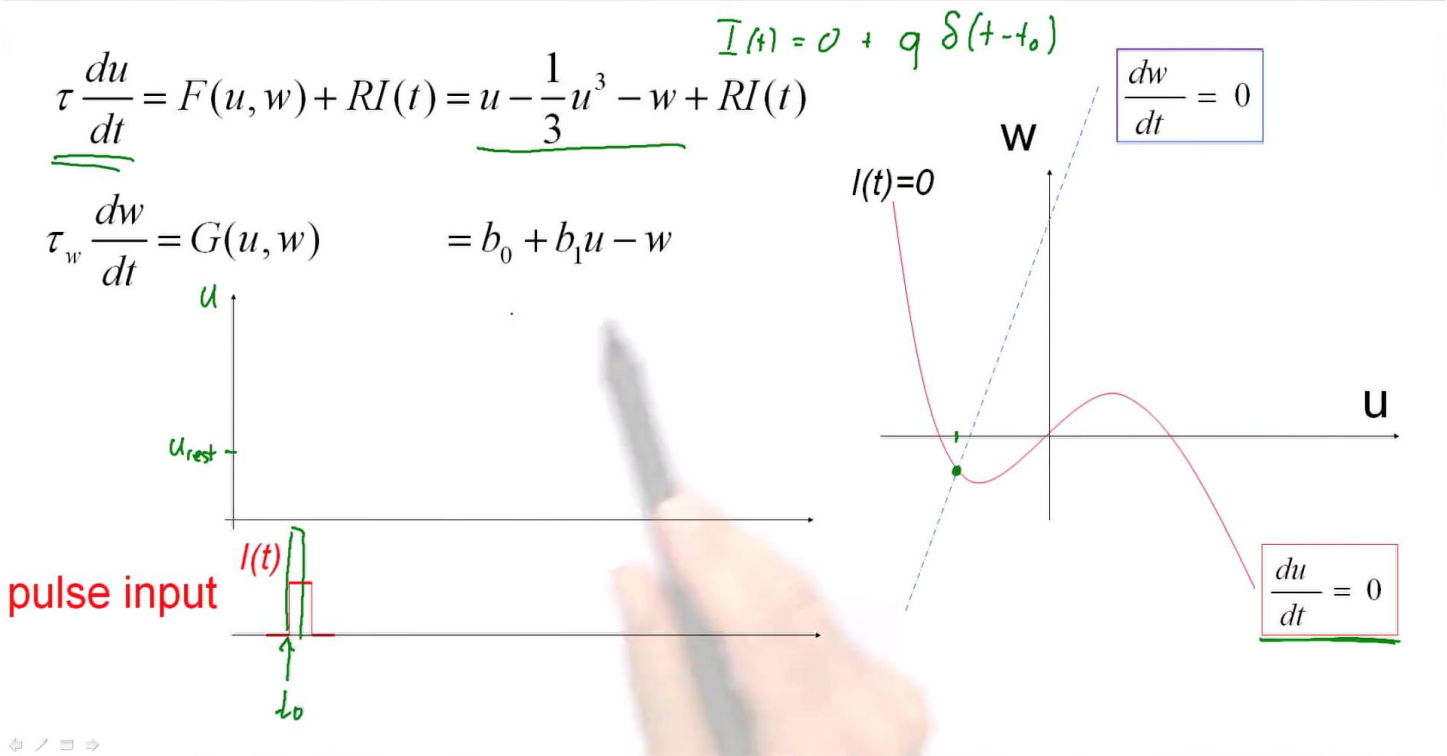
Notes

Summary



0m 36s

Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input



Let's see how this works. So here I've written, again, the equations of the FitzHugh-Nagumo model. There is a u -nullcline, defined by du/dt equals 0, which is given by a cubic function. That's the red curve here. And then we have the w -nullcline, which is given by a linear function indicated by the blue line here. The input is zero. So I of t is zero, except at this very short moment when we inject a current pulse. Now in the limit, we can say we inject a charge Q at the moment t_0 . So this might be my moment t_0 . This is still a finite pulse, but as we have seen, I can make the pulse shorter and of larger amplitude. And then this current will go into the voltage equation. So what's going to happen with the voltage? Well, at the beginning, we are at the fixed point at the resting potential. This will be some negative value of the voltage. And if I bought the voltage here, say, this is my resting value. Now the short current pulse is inserted into the voltage equation. On the left hand side, I have the derivative of the voltage. If I put in the Dirac current pulse, I put in this charge q in very short moment of time, then the voltage will jump from its previous value to a new value.

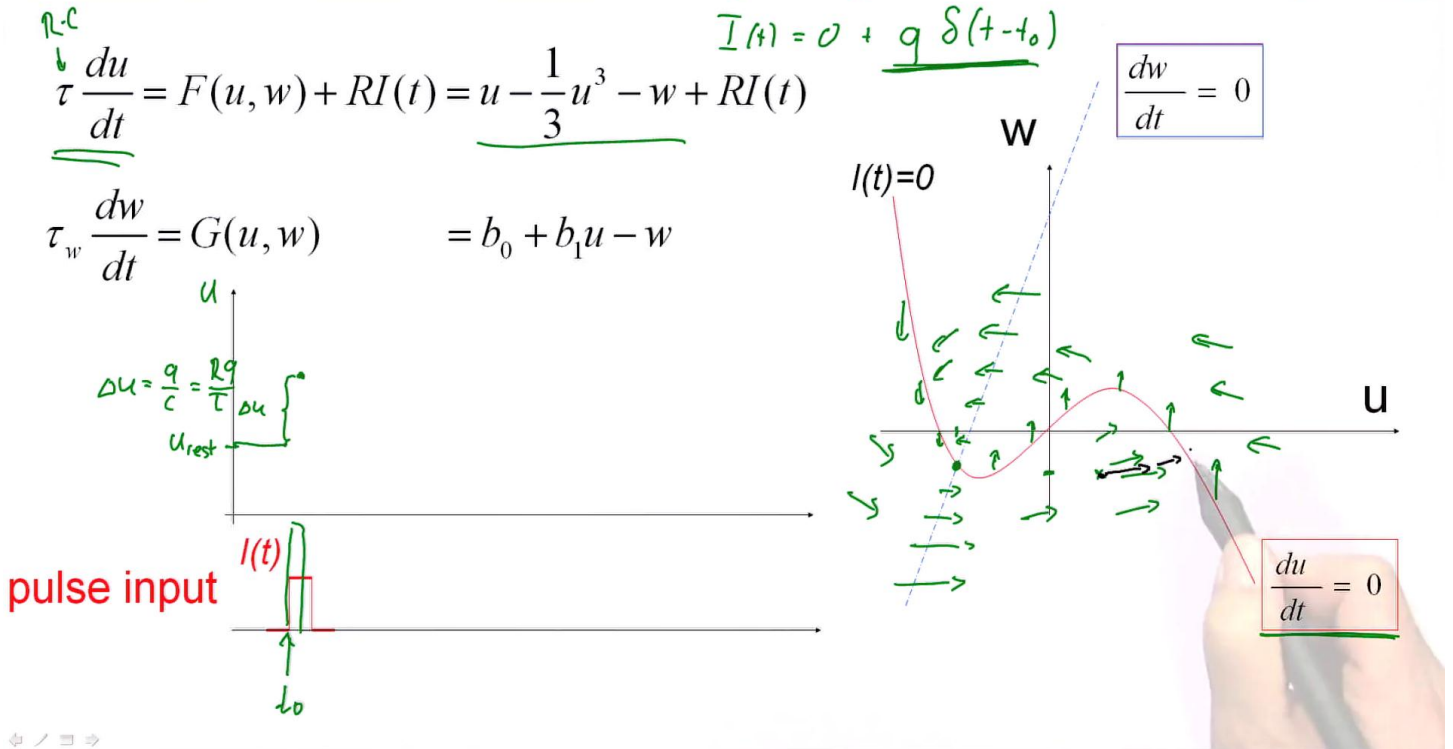
Notes

Summary

1m 06s



Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input



And this new value is delta u. It's just q over c or I can also write r times q over tau, where tau is r times c. In the face plane, which I have drawn here on the right hand side, a jump in the voltage means that the value of w-- this was w rest. The value of w does not change, but the voltage jumps to new value, say, to a new point here. So the pulse input acts like a jump in the voltage while w remains unchanged. Now what do we know? Now we are here. Well, we know from the previous part that on the u-nullcline, the flow arrows are vertical. And with a set of parameters that we have considered, the flow will be upward and would turn around, and here it's downward. The flow on the w-nullcline is horizontal, and arrows get bigger as we move away from the fixed point. Now in between, there's something sort of upward movement and leftward movement. So the arrows may look like this. In between here, arrows may look like this. Now with this preparation, let's draw the trajectory that arises from this new initial condition. Let's change colors. From this new initial condition, the direction flow will be like this. We continue like this. We know that on the u-nullcline, it's vertically upward.

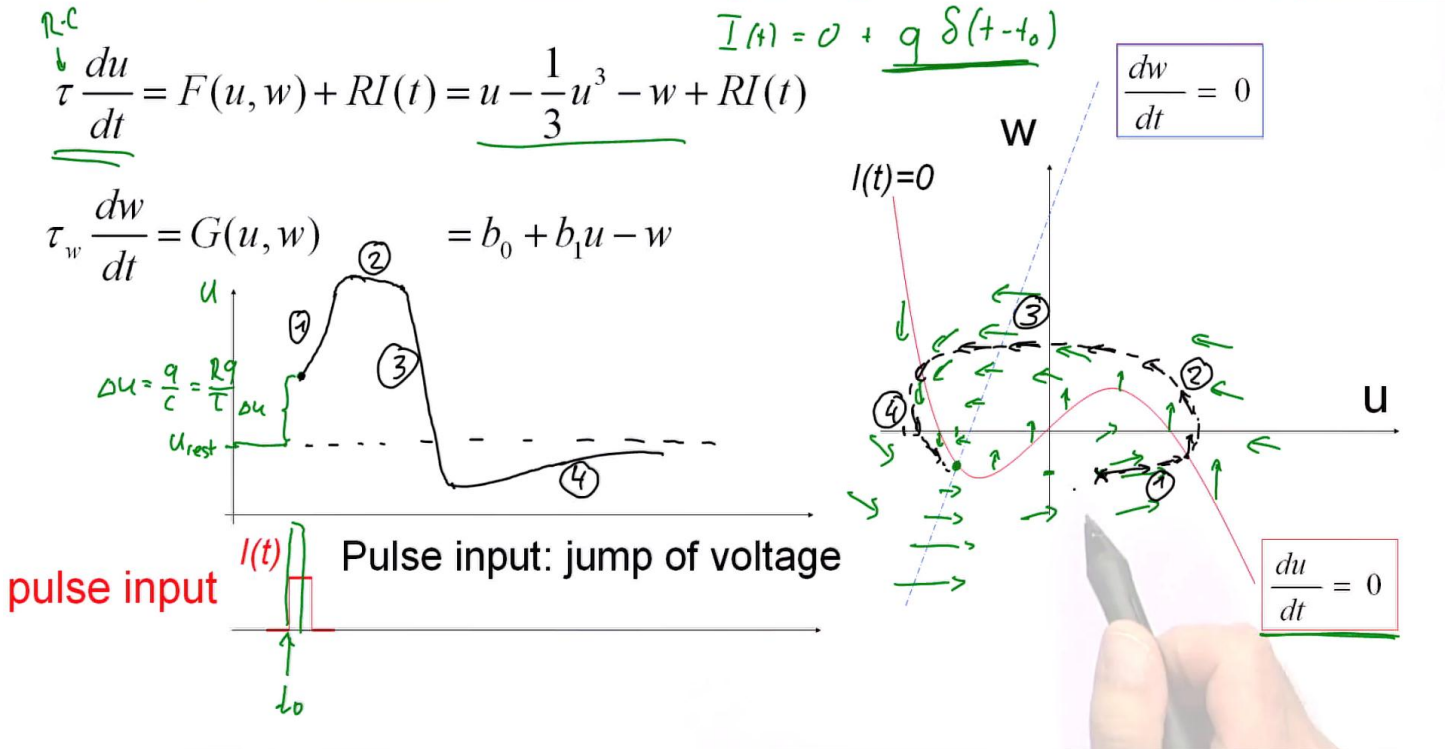
Notes

Summary



3m 15s

Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input



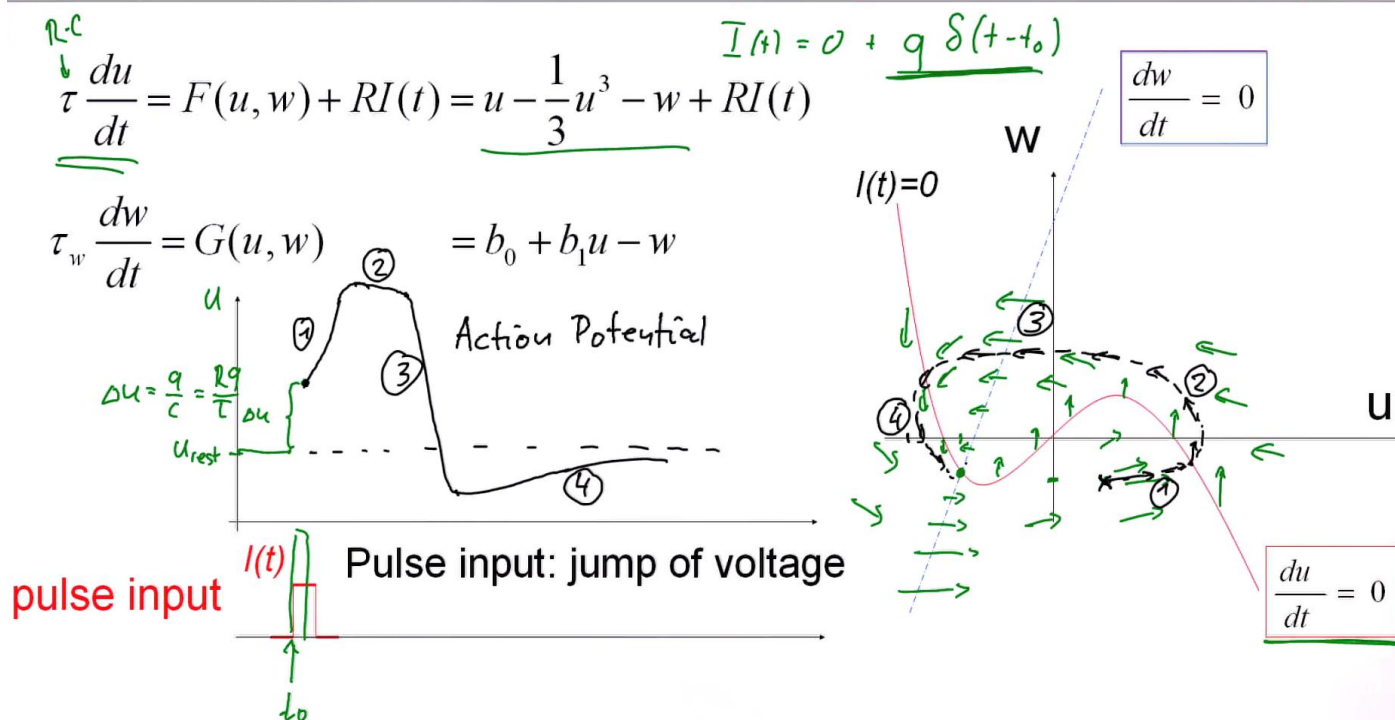
Then you cannot move too far away, because over here it will be pushed leftward. So the flow is probably something like this here. Then we turn around. We move here horizontally. We come down. We come down and we end up here. So this is what the trajectory to look like. So our voltage made a jump to the new value here. And from there on, it increases further. The voltage axis is horizontal. The voltage increases. So we know we increases further. That's the part one, the first part of the trajectory. But then it bends down. It bends over, now here the voltage doesn't move for a moment. And it comes down only slowly. This would be part two of the trajectory. Now here when it crosses the w-nullcline, it's actually fairly big arrows. So the downswing is quick. This will be phase three. And now you see, we have an undershoot. The voltage here is a lower than the voltage at rest. So in the end it will approach the resting value from below. And this would be phase four off the trajectory. So this little current pulse, called first a jump off the voltage to new initial condition. And then from there one we see a trajectory that corresponds to an action potential.

Notes

Summary



Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input



This is a tremendous an action potential in our two dimensional neuron model. There's an upswing. There's a time when it stays close to the maximum. And there's a rapid downswing. And then it comes back from below to the resting potential.

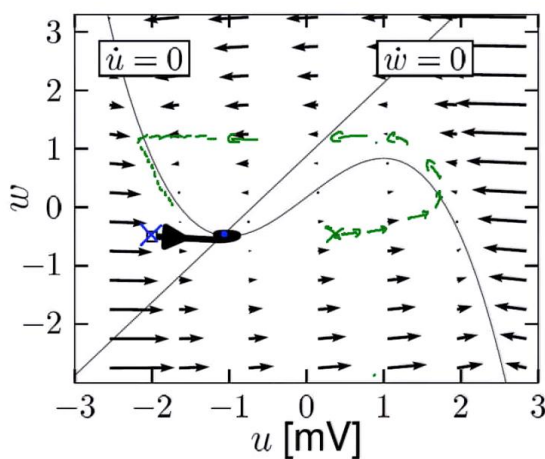
Notes

Summary



8m 03s

Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input



FN model with $b_0 = 0.9; b_1 = 1.0$

Pulse input: jump of voltage/initial condition

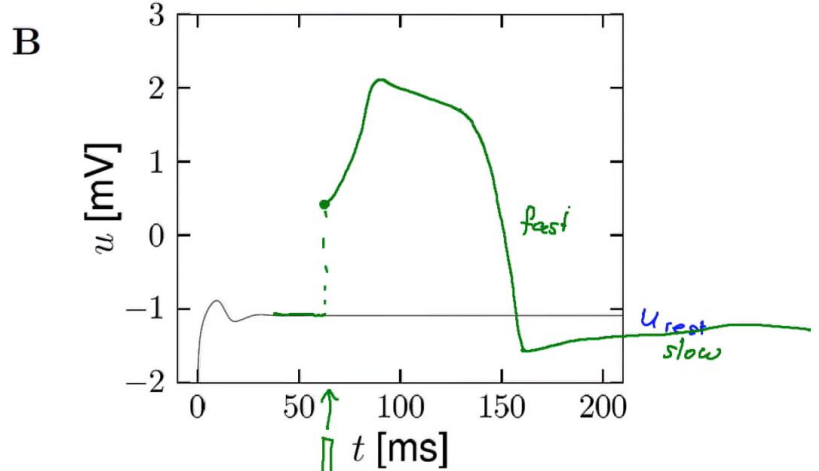


Image: Neuronal Dynamics,
Gerstner et al.,
Cambridge Univ. Press (2014)



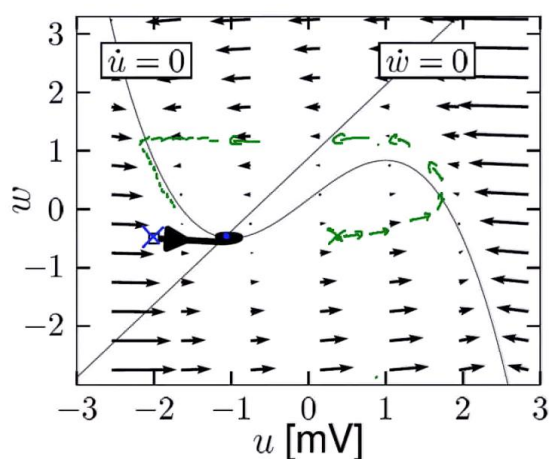
This was a hand drawn figure. Here is computer generated graph. The resting potential is here. Resting potential is about at minus 1.1. Now if you give a negative current pulse, the voltage will jump to a new initial condition. And from this new initial condition, it will approach the resting potential with a little over 3, with a little oscillation. And then we are here at the resting potential. Now suppose we give a positive current pulse, from this resting value, the potential would it be kicked upwards. It's a horizontal shift to a new starting condition, to a new initial condition. From here on, there will be a slow increase, the size of the arrows is still small. Then it will turn around. And then it will go down again. So suppose I give my positive current pulse here, there's a jump to, say, 0.5. From there on, we move on to our maximum of about 2. And at some point, arrows will increase, which indicates a rapid downswing, until here it will follow. It cannot go far away from this nullcline because it always pushed back by the nearly horizontal arrows. And so very slowly, it'll go back to the resting potential. So this part be slow. This part will be fast. That's a pulse input.

- Notes

Summary



Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model : Pulse input



FN model with $b_0 = 0.9; b_1 = 1.0$

Pulse input: jump of voltage/initial condition

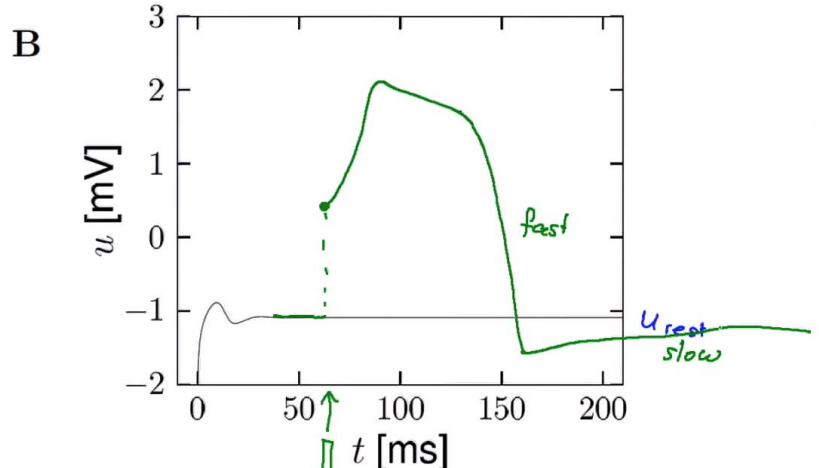


Image: Neuronal Dynamics,
Gerstner et al.,
Cambridge Univ. Press (2014)

It uses a jump of the voltage. It leads to a new initial condition, either the blue one or the green one. From this new initial condition, we just follow the flow of the arrows. And this gives the voltage trajectory that we are expected to find.

Notes

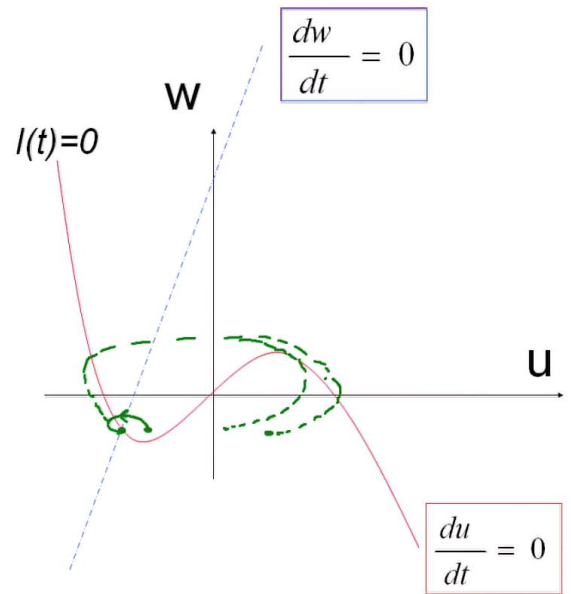
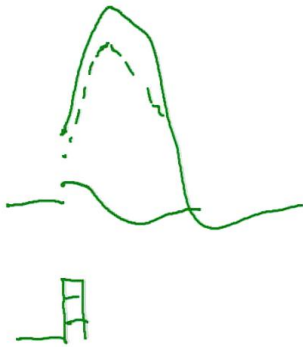
Summary



Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model

Pulse input:

- jump of voltage
- 'new initial condition'
- spike generation for large input pulses



Thus, we can understand the generation of action potentials. In a two-dimensional neuron model, action potentials will arise, if the current pulse is sufficiently large. So this is my initial condition. If I make a large current pulse, voltage trajectory will look like this. This corresponds to an action potential. This corresponds to a jump, followed by an action potential. If I make the current pulse smaller, I jump to a different value. And the trajectory will look different. If I jump to a much smaller value, I don't have an excursion at all, but it returns immediately to the resting value. Spike generation will occur for a large input pulses. So far pulse input--

Notes

Summary



11m 05s

Neuronal Dynamics – 4.3. FitzHugh-Nagumo Model

Pulse input:

DONE!

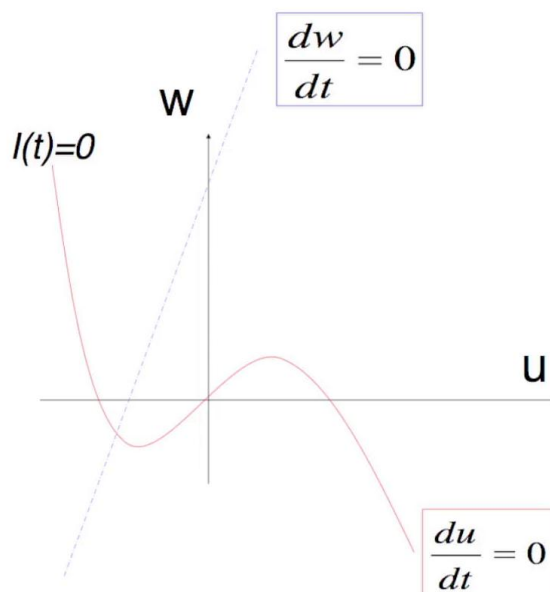
- jump of voltage
- 'new initial condition'
- spike generation for large input pulses

constant input:

- graphics?
- spikes?
- repetitive firing?

Lesson 4.3B!

...
Comes next



Notes

Summary



12m 21s