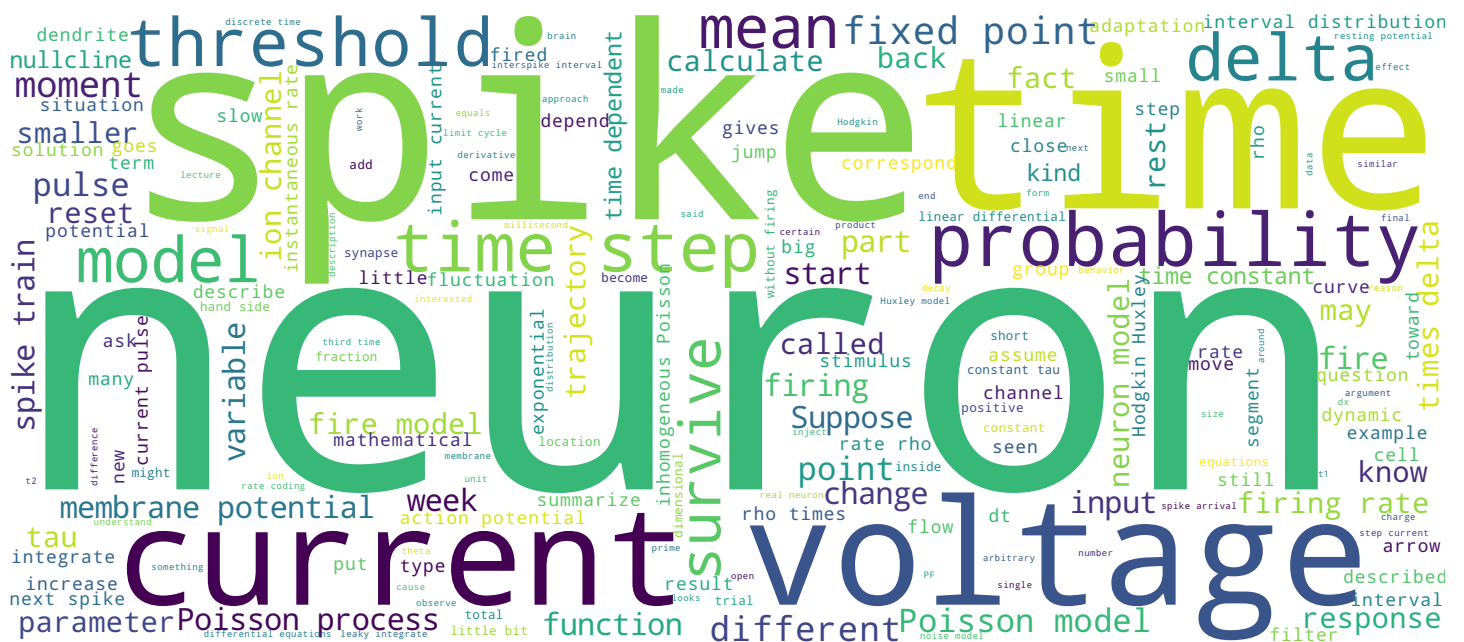


ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

EPFL, Lausanne, Switzerland

- subthreshold and superthreshold



EPFL



Week 5 – part 3b :Poisson Model



✓ 5.1 Variability of spike trains

- experiments

✓ 5.2 Sources of Variability?

- Is variability equal to noise?

5.3 Three definitions of Rate code

- Poisson Model

5.4 Stochastic spike arrival

- Membrane potential fluctuations

5.5. Stochastic spike firing

- subthreshold and superthreshold

Welcome back to the continuation of part three of week five. I've just finished the discussion of the different definitions of rate coding. Rate coding is linked to the idea of stochastic firing. Stochastic firing can be described by a Poisson model. And the Poisson model is the topic of this video.

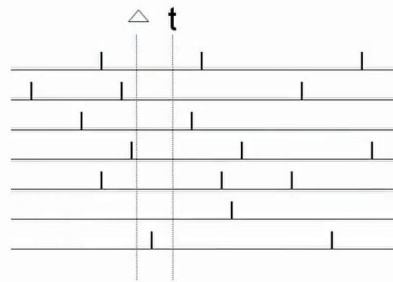
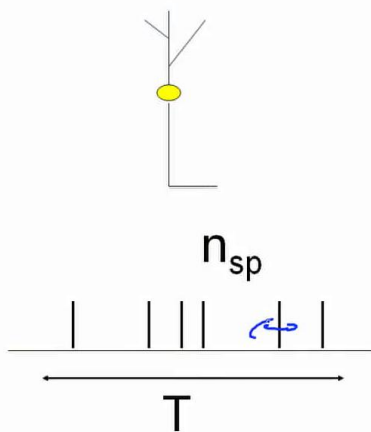
Notes

Summary

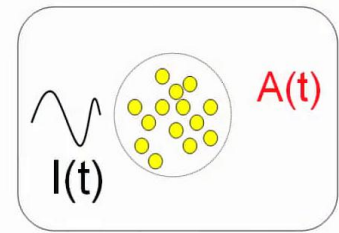


0m 04s

Neuronal Dynamics – 5.3b. Inhomogeneous Poisson Process



$$PSTH(t) = \frac{n(t; t + \Delta t)}{K \Delta t}$$



$$A(t) = \frac{n(t; t + \Delta t)}{N \Delta t}$$

population activity

Pure rate code = stochastic spiking → Poisson model

So we have seen that there are all these different definitions of rate coding. And I would like to convince you that if you think of a rate code where the exact timing of a spike doesn't play a role, where spikes can jitter a little bit, then a Poisson model is an appropriate description.

Notes

Summary

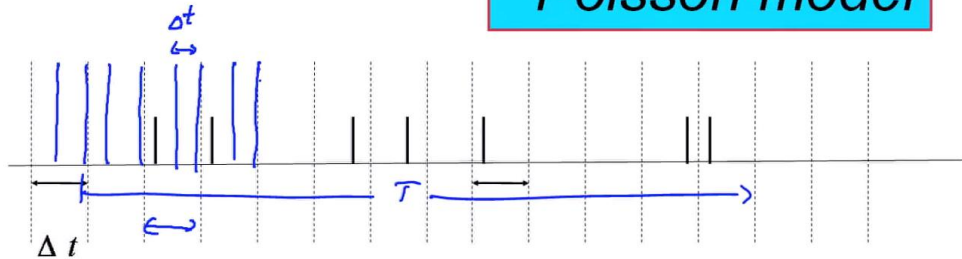


Neuronal Dynamics – 5.3b. Poisson Model

Homogeneous Poisson model: constant rate

90

*Math detour:
Poisson model*



Probability of finding a spike $P_F = \rho_0 \Delta t$ for very small Δt .

Pure rate code = stochastic spiking → Poisson model

So what is a Poisson model? Suppose I have observed a spike train. We assume that during the whole period, big T, the neuron fires at a constant rate. This is called a homogeneous Poisson process. Now, we discretize time. So we use short time bins, delta t. And then we say, well, if the neuron fires at a constant rate, which I call rho 0, then the probability of finding a spike in such a small interval is just rho 0 times delta t. In other words, if I decrease my temporal resolution, if I take smaller time steps, like this-- so now the new delta t is smaller-- then the probability of finding a spike in one of these segments is also smaller. This is expressed by this relation that the probability is proportional to delta t for very small delta t.

Notes

Summary

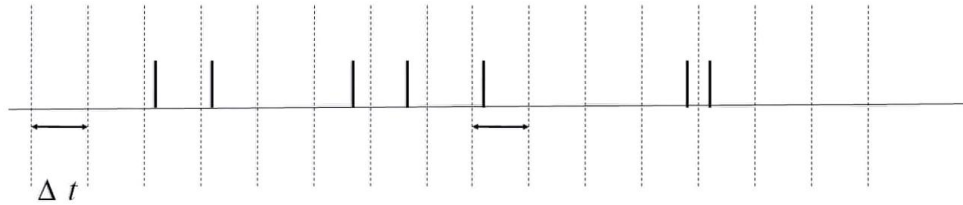


0m 53s

Neuronal Dynamics – 5.3b. Poisson Model

Probability of firing:

$$P_F = \rho_0 \Delta t$$



Take $\Delta t \rightarrow 0$

$$\lim_{\Delta t \rightarrow 0} \frac{P_F(\text{in } \Delta t)}{\Delta t} = \rho_0 \text{ rate}$$

↩ ↪ ↻ ↺

So let's describe this more formally. So I say the firing probability in the time step Δt is P_F . Now, I'm interested-- but P_F will depend on Δt . I'm interested in a definition which is independent of Δt . Now, if Δt is small enough, then P_F is proportional to Δt . So formally, I can define a rate by the limit $\Delta t \rightarrow 0$ of the measurement P_F , the measured of probability of firing a spike in Δt . I divide by Δt . And this would provide me with an estimate of the firing rate, ρ_0 . So this is the rate of my Poisson model.

Notes

Summary



2m 32s

Neuronal Dynamics – 5.3b. Interval distribution

Probability of firing:

$$P_F = \rho_0 \Delta t$$

(i) Continuous time

prob to 'survive'

$$\frac{dS}{dt} = -\rho \cdot S(t)$$



(ii) Discrete time steps

$$\Delta t \rightarrow 0$$

Let's suppose now that we have a model, and this neuron model is a Poisson process. So it really fires in each time step Δt with this probability. And so it has fired here. This is the moment when it has fired. Another question is can I calculate something like the interval distribution? So given that it fired here, what's the probability that it survives for all this time up to here without firing, survive without finding again? Now, instead of considering the single neuron, let's for the moment switch to a group of neurons so I have here my group of 500 neurons, say, and for some reason-- because I stimulated them or whatever-- I manipulate it such that all 500 neurons have fired a spike in this moment in time, in this little time step. And I ask what's the probability that they survive for a certain duration t ? Now, this probability of surviving will decay. Well, S is the fraction of neurons that have survived after time t . Well, they decay. At each moment in time, there's a certain rate of firing which corresponds to a decay of the fraction of neurons that still survive. So the probability to survive will decrease, according to this differential equation.

Notes

Summary



3m 46s

Neuronal Dynamics – 5.3b. Interval distribution

Probability of firing:

$$P_F = \rho_0 \Delta t$$

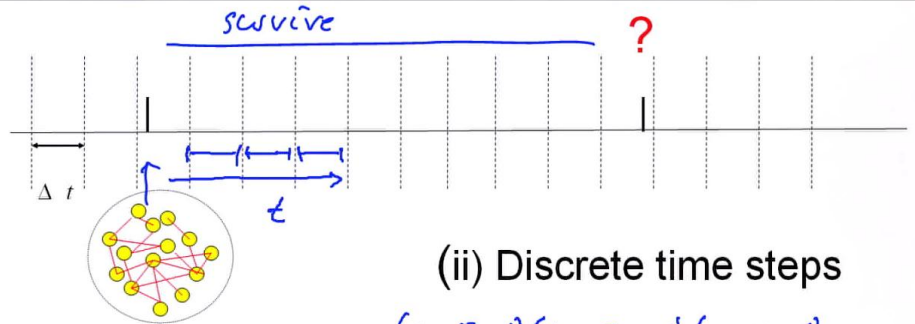
(i) Continuous time

prob to 'survive'

$$\frac{dS}{dt} = -\rho \cdot S(t)$$

$$S(t) = e^{-\rho \cdot t}$$

$$S(0) = 1$$



(ii) Discrete time steps

$$(1 - \rho_0 \cdot \Delta t)(1 - \rho_0 \cdot \Delta t)(1 - \rho_0 \cdot \Delta t)$$

$$\Delta t \rightarrow 0$$

And the solution is S of t -- the fraction of neurons that survives from that group that has fired here is e to the minus ρ times t . All neurons have fired here. The probability to survive at 0 in the same time step is 1. So this condition is that S of 0 is equal to 1. So this probability to survive is completely analogous to the case of radioactive decay. If a large group of particles have a certain decay rate-- this is my ρ . And because of this spontaneous decay, the group that still survives will decay exponentially. This is the continuous time description. But we can do the same thing, also, in discrete time. So the probability to fire in a small time step is ρ_0 times Δt . So the probability to survive in this first time step is 1 minus ρ_0 times Δt . The probability to survive in the second time step is the same thing. The neuron, the probability to survive in the third time step is, again, the same value. So it has to survive during the first time step. Then has to survive during the second time step. And it has to survive during the third time step. So the probability to survive without firing for three time steps is the product of these individual terms.

Notes

Summary



Neuronal Dynamics – 5.3b. Interval distribution

Probability of firing:

$$P_F = \rho_0 \Delta t$$

(i) Continuous time

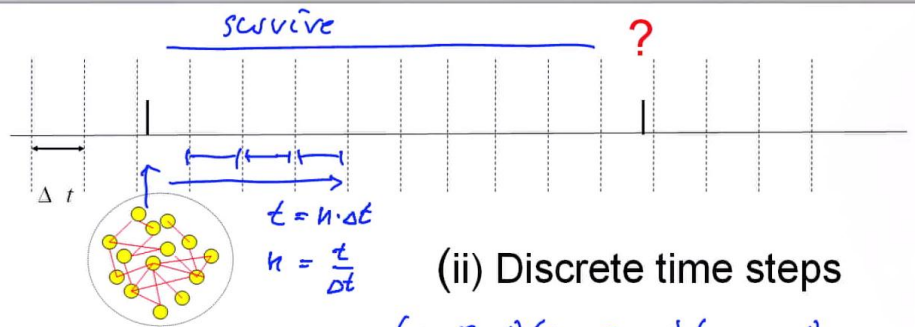
prob to 'survive'

$$\frac{dS}{dt} = -\rho \cdot S(t)$$

survivor

$$S(t) = e^{-\rho \cdot t}$$

$$S(0) = 1$$



(ii) Discrete time steps

$$(1 - \rho_0 \cdot \Delta t)(1 - \rho_0 \cdot \Delta t)(1 - \rho_0 \cdot \Delta t)$$

$$S = (1 - \rho_0 \cdot \Delta t)^n$$

$$S(t) = (1 - \rho_0 \cdot \Delta t)^{t/\Delta t}$$

$$\Delta t \rightarrow 0$$

It's a product because firing or not is an independent decision for a Poisson process at each moment in time. More generally, the probability to survive for n time steps for homogeneous Poisson process is simply $1 - \rho_0 \Delta t$, in brackets, to the power of n . Now, the survival time t is n times Δt . So I can write n as $t / \Delta t$. So S of t is $(1 - \rho_0 \Delta t)^{t/\Delta t}$. I put in the t that I forgot, so it's Δt . It's Δt . Now, I can take the limit $\Delta t \rightarrow 0$, and using the definition of exponential leads exactly back to this formula here. Thus, whether we start from discrete time or [INAUDIBLE] continuous time, we can calculate the probability to survive, also called the survivor function.

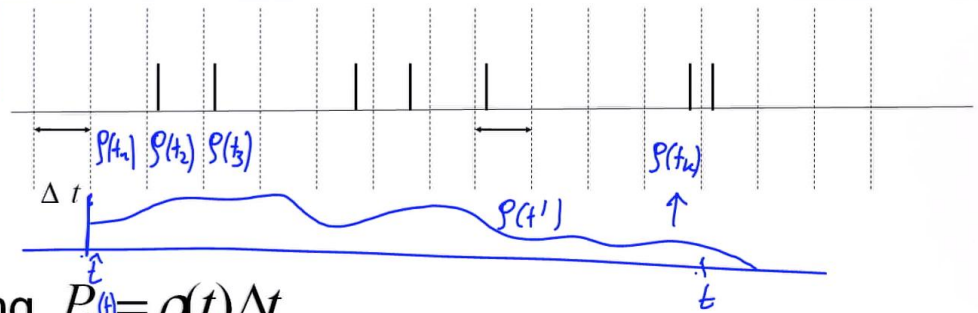
Notes

Summary



Neuronal Dynamics – 5.3b. Inhomogeneous Poisson Process

rate changes



Probability of firing $P_F(t) = \rho(t)\Delta t$

Survivor function $S(t | \hat{t}) = \exp\left(-\int_{\hat{t}}^t \rho(t') dt'\right)$

Interval distribution

Now, the homogeneous Poisson processes is a special case. We may also assume that the rate changes. So in the first time step, I have a rate ρ of t_1 . In the second time step, I have a rate ρ of t_2 . In the third time step, I have a rate ρ of t_3 . For the moment, we assume that this rate is constant during that time step. So evaluate at some moment in time, and I multiply by Δt . And then this gives me, again, the firing rate for this time step. So in an arbitrary time step, PK, PF of t is ρ of t_K times Δt , the time step at time t . Repeating the argument that I made on a previous slide, it's easy to see that we find, again, the exponential function. But now, it's this time dependent rate. The rate may be different. Suppose the first spike occurred here and I have a rate ρ of t which changes-- which may go up and down. If I want to know what's the probability to survive up to some time t , well, I just integrate over this function ρ of t' from $\hat{t}$ -- that's the moment when the last spike happened-- up to time t . In one of the earlier lectures this week, I pointed out the importance of the interval distribution. So what would be the interval distribution?

Notes

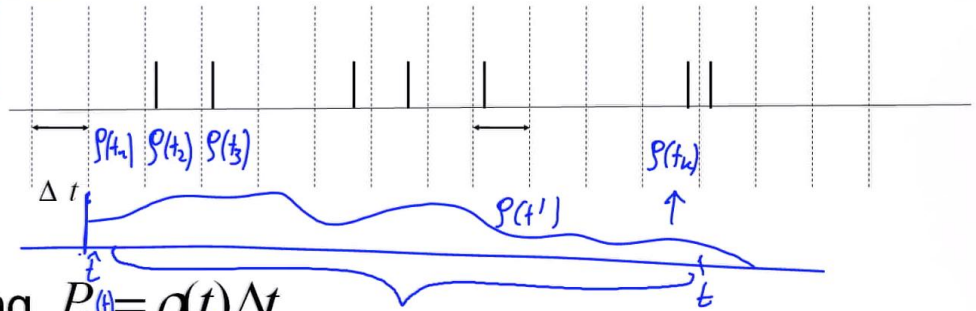
Summary



9m 21s

Neuronal Dynamics – 5.3b. Inhomogeneous Poisson Process

rate changes



Probability of firing $P_F^{(k)} = \rho(t_k) \Delta t$

Survivor function
$$S(t | \hat{t}) = \exp\left(-\int_{\hat{t}}^t \rho(t') dt'\right)$$

Interval distribution
$$\underline{P(t|\hat{t}) = \rho(t) \cdot S(t|\hat{t})}$$

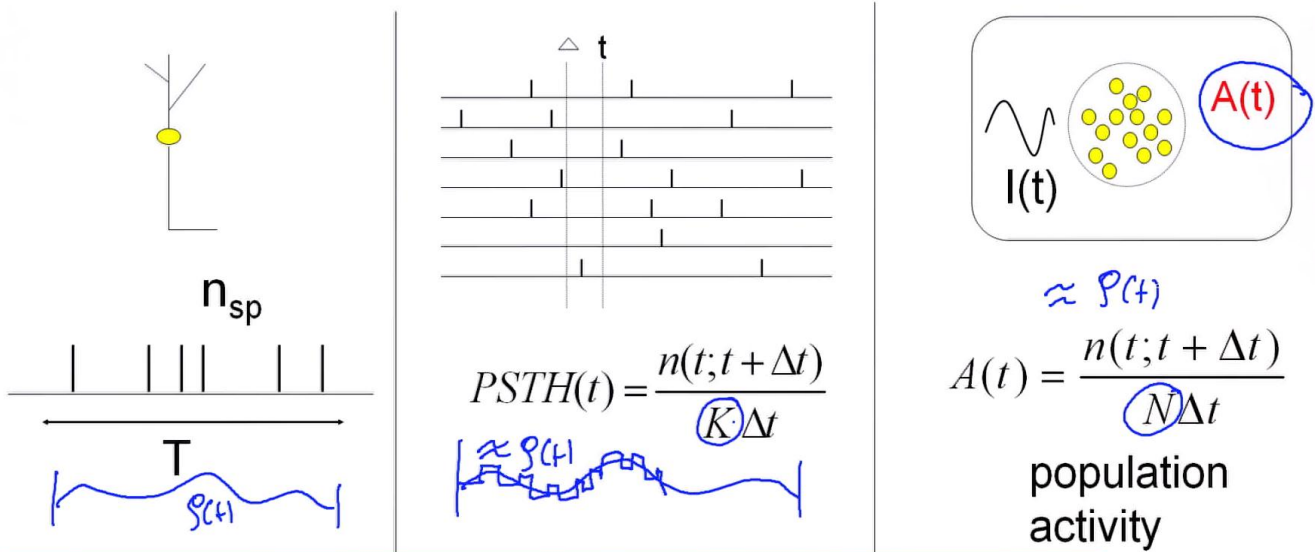
Suppose a neuron has fired here. We ask the question what's the probability that I would see the next spike over here? More precisely, it's the probability density. We find the next spike at t given the last spike has occurred at $\hat{t}$. Now, for the neuron to have the next spike at t we talk about interval. It has to be the next spike. That means the neuron has to survive from $\hat{t}$ to t without firing. So I can take the expression I just calculated. But then it actually has to fire at time t . But the instantaneous rate of firing at time t is ρ of t . So the interval distribution has an exponential term from the survivor function and then the instantaneous rate.

Notes

Summary



Neuronal Dynamics – 5.3b. Inhomogeneous Poisson Process



inhomogeneous Poisson model consistent with rate coding

← / □ →

With the notions of an inhomogeneous Poisson process, you can go back to definitions of the firing rate. Suppose you have an inhomogeneous Poisson process and it has a time dependent rate and you observe the total number of spikes accumulated over a time t . Well, the Poisson model allows you to calculate this. Suppose that I make several measurements across different repetitions of the same stimulus. The PSTH is, in fact, an estimate of my instantaneous firing rate of the Poisson process. So the PSTH will be a noisy discrete version which should, in principle, be very close to this instantaneous rate. In fact, if K is very big-- we have 10,000 repetitions-- and then you allow yourself to make Δt fairly small, then this becomes a very quick estimator of the firing rate ρ of t of the inhomogeneous Poisson model. Now, we said that this is an experimental measurement. Now, assume that I have a group of n neurons that all have the same time dependent firing rate. Then the instantaneous measurement of the population Activity A of t is, again, a good estimate of the underlying firing rate of the individual neurons.

Notes

Summary



12m 11s

Neuronal Dynamics – 5.3b. Inhomogeneous Poisson Process



Probability of firing

$$P_F = \rho(t) \Delta t$$

Survivor function

$$S(t | \hat{t}) = \exp\left(-\int_{\hat{t}}^t \rho(t') dt'\right)$$

Interval distribution

$$P(t | \hat{t}) = \rho(t) \exp\left(-\int_{\hat{t}}^t \rho(t') dt'\right)$$

To summarize, for a Poisson model-- for an inhomogeneous Poisson process-- I can calculate different quantities. We can define an instantaneous rate, which is closely related to the experimental procedures of estimating a rate. We can calculate the interval distribution. We can calculate the survival function. And therefore, the Poisson model is a first important description of spike trains, of fluctuations in neuron spike trains. In a Poisson model firing in step one, it's independent to firing in step two. A real neuron, if it fires a spike, it will have a dead time. It will be refractory. It's less likely to fire immediately afterwards. That's not captured by a Poisson model, neither by the homogeneous nor by the inhomogeneous Poisson process. Nevertheless, the Poisson model is an important reference model used widely in experimental and theoretical neuroscience.

Notes

Summary



14m 04s

Neuronal Dynamics – Quiz 5.3.

A Homogeneous Poisson Process:

A spike train is generated by a homogeneous Poisson process with rate 25Hz with time steps of 0.1ms.

- ☐ The most likely interspike interval is 25ms.
- ☐ The most likely interspike interval is 40 ms.
- ☐ The most likely interspike interval is 0.1ms
- ☐ We can't say.

B Inhomogeneous Poisson Process:

A spike train is generated by an inhomogeneous Poisson process with a rate that oscillates periodically (sine wave) between 0 and 50Hz (mean 25Hz). A first spike has been fired at a time when the rate was at its maximum. Time steps are 0.1ms.

- ☐ The most likely interspike interval is 25ms.
- ☐ The most likely interspike interval is 40 ms.
- ☐ The most likely interspike interval is 0.1ms.
- ☐ We can't say.

Before we continue, please have a look at the quiz.

Notes

Summary



15m 12s