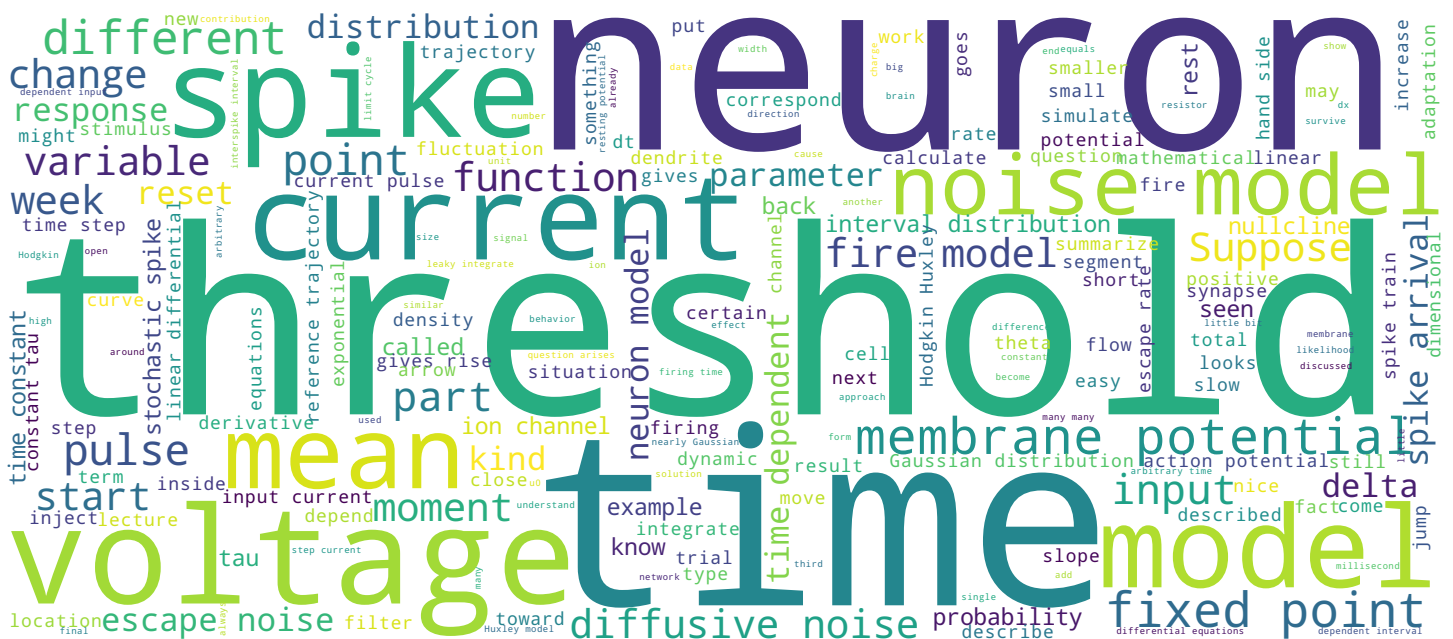


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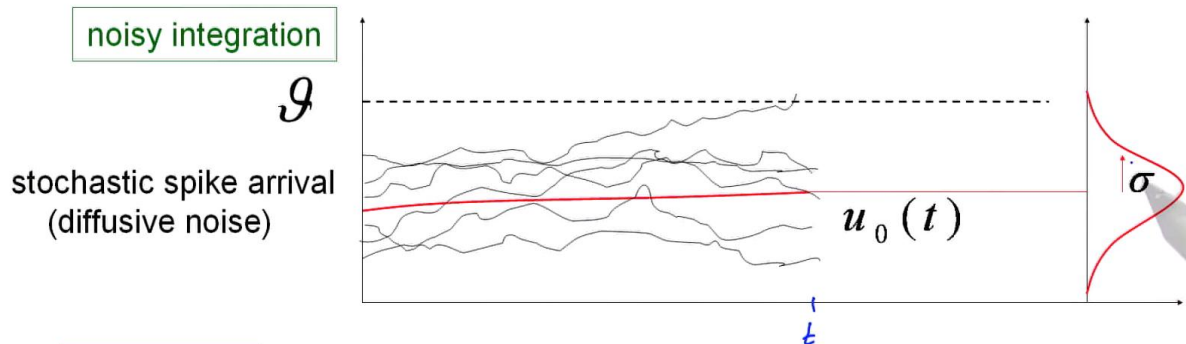
- stochastic resonance



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Noise models: from diffusive noise to escape rates



escape rate

$$\rho(t) = f(u_0(t) - \mathcal{G})$$

Well, we have seen last week that the diffusive noise is nice, because it can be described stochastic spike arrival. It can describe the noise generated in a network of neurons, therefore it looks like an attractive noise model. On the other hand, as just discussed, it's difficult to calculate quantities such as the time-dependent interval distribution, whereas this is easy for the escape noise model considered this week. So the question arises whether they are systematic methods to go from diffusive noise to an escape noise model. Now this is going to be an approximation. It's not the same noise model. But the question arises, is this a good approximation? So let's see how this could work. As we have seen in this lecture 6.4, for stochastic spike arrival models, we can calculate the deterministic trajectory. And the real trajectory stays nearby, and builds up some kind of a Gaussian distribution. So if you project at this moment t , if you project all the membrane potentials out there, we could find a Gaussian distribution-- a nearly Gaussian distribution with some standard deviation σ . We know that in reality, it cannot be a Gaussian distribution, because the density of the distribution at threshold must be 0.

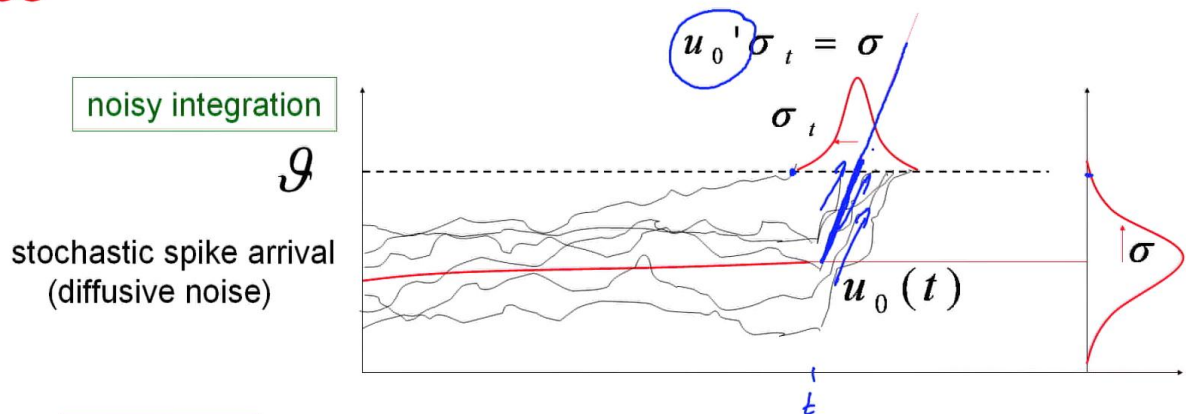
Notes

Summary



0m 00s

Noise models: from diffusive noise to escape rates



$$\rho(t) = f(u_0(t) - g) \propto \exp\left(-\frac{(u_0(t) - g)^2}{2\sigma^2}\right)$$

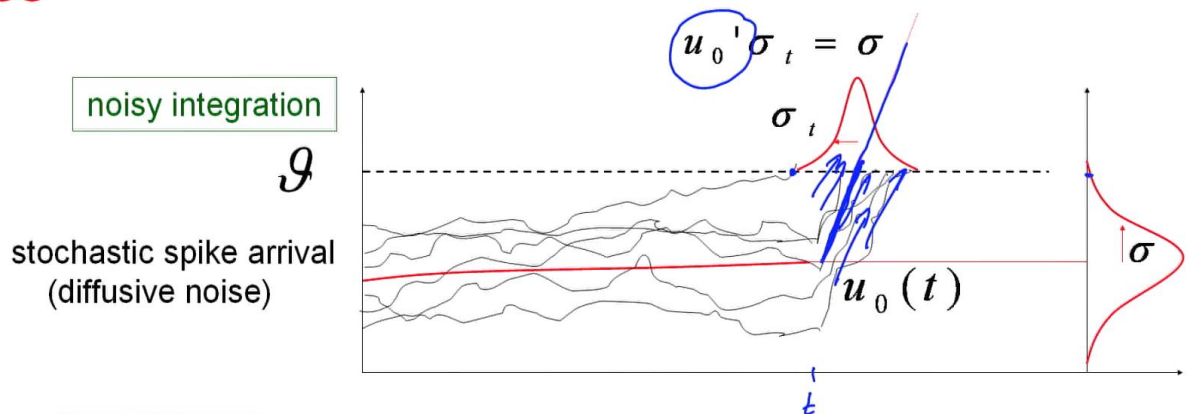
However, it's tempting to say, well, it's nearly Gaussian distribution, and the probability that a trajectory would be at the threshold should be something like the density of the distribution at threshold. So why not write down an escape rate [INAUDIBLE] say, the distribution is Gaussian, and the rate of crossing the threshold is proportional to the density at threshold? So we compare the value at threshold with u_0 . That's this difference normalized by the width of the distribution. That would give this density. Now this is only part of the story, because suppose I give a strong current drive. This drive will put the mean trajectory rapidly across the threshold. And in fact, all the other trajectories are driven towards the threshold. So the whole distribution of membrane potentials that we had at time t , this vertical distribution is rapidly pushed across the threshold, and gives rise to a distribution of firing times that's also nearly Gaussian. Now the width of this distribution depends on the slope. This is the slope of the reference trajectory. The more input I give, the steeper the slope, the more likely is that this membrane potential distribution is pushed across the threshold.

Notes

Summary



Noise models: from diffusive noise to escape rates



escape rate

$$\rho(t) = f(u_0(t), u_0'(t)) \propto \frac{\exp\left(-\frac{(u_0(t) - g)^2}{2\sigma^2}\right)}{\text{erf}\left((u_0(t) - g)/\sigma\right)} [1 + u_0'(t)]$$

So we expect that there should be a term that is proportional to the derivative. And this is how we can write this down. And then, we have to normalize it in some appropriate fashion. Now this is a heuristic argument. Suppose we simulate a diffusive noise integrated fire model, and we also simulate an integrated fire model with escape noise with this specific escape case rate, which depends on the reference trajectory, as well as its derivative.

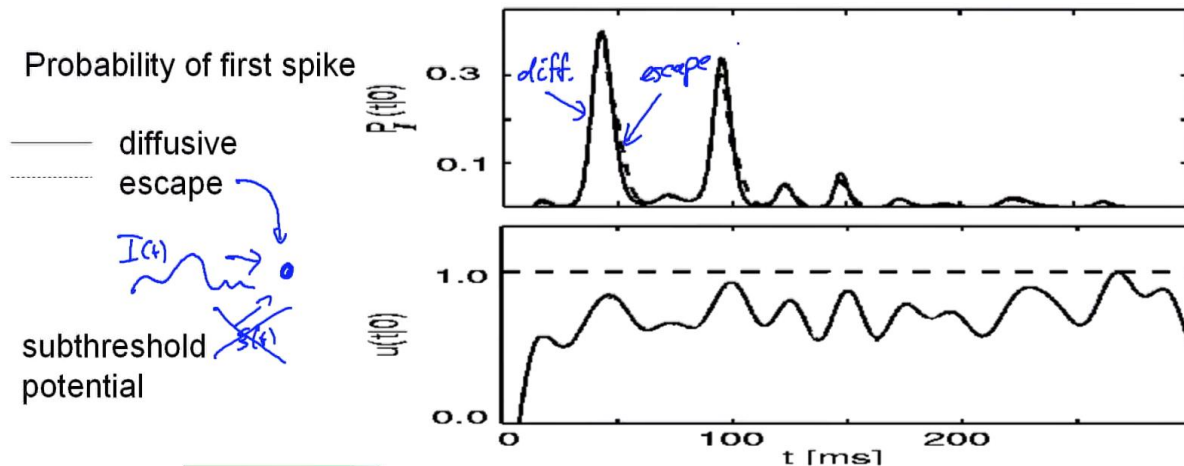
Notes

Summary



3m 17s

Comparison: diffusive noise vs. escape rates



escape rate

$$\rho(t) = f(u_0(t), u'_0(t)) \propto \exp\left(-\frac{(u_0(t) - \vartheta)^2}{2\sigma^2}\right) [1 + u'_0(t)]$$

And this is what we would find. We give some arbitrary time-dependent input current into the model. First, we simulate it with diffusive noise. The deterministic part of the input gives rise to this deterministic trajectory. Now for the diffusive noise model, we would have this extra noise term that makes a diffusion of the actual trajectories away from the reference trajectory. And so, you may expect that whenever the membrane potential is close to the threshold, the probability of firing is high. Now if you just repeat this same input stimulus for many, many, many times, then you would find this time dependent interval distribution. The solid line is for the diffusive noise model. But now we can inject the same deterministic current without the noise, and add escape noise instead. With. An escape rate given by the formula that I indicated before. And if you do this, we find, again, a time-dependent interval distribution, which can be calculated, and it's the dash line. And you see that the two interval distributions are nearly the same. Thus, the diffusive noise model-- which is nice, because it represents stochastic spike arrival-- is well approximated by an escape noise model with the appropriate rate. So let us summarize this lecture 6.4.

Notes

Summary



3m 54s

Neuronal Dynamics – 6.4. Comparison of Noise Models

Diffusive noise

- represents stochastic spike arrival
- easy to simulate
- hard to calculate

Escape noise

- represents internal noise
- easy to simulate
- easy to calculate
- approximates diffusive noise
- basis of modern model fitting methods



Let me summarize. So we have two different noise models. There's a diffusive noise model, which is nice, because it has an easy interpretation in terms of stochastic spike arrival. We can calculate distribution of membrane potentials, and it's easy to simulate on a computer, but it's difficult to calculate something like the inter-spike interval distribution for arbitrary time-dependent input. The escape noise model-- well, for an escape noise model, calculating this very same quantity is easy. It does not have the interpretation of stochastic spike arrival. At the start, it's more like an internal noise model. However, as we have seen with the appropriate choice of escape rate, the escape noise model gives a very good approximation to the diffusive noise model, and therefore indirectly accounts for stochastic spike arrival, as well. The advantage of being able to write down inter-spike interval distribution-- and hence, the likelihood-- is that we can use this as a basis for neuron model optimization. And that's what we'll do next week. But before we go there, let's have a look at some problems of neural coding.

Notes

Summary



5m 50s