

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

EPFL, Lausanne, Switzerland

7.7. Helping Humans



EPFL



Week 7 – part 2b : Firing patterns and phase plane analysis



✓ 7.1 What is a good neuron model?

- Models and data

7.2 AdEx model

- Firing patterns and analysis

7.3 Spike Response Model (SRM)

- Integral formulation

7.4 Generalized Linear Model

- Adding noise to the SRM

7.5 Parameter Estimation

- Quadratic and convex optimization

7.6. Modeling in vitro data

- how long lasts the effect of a spike?

7.7. Helping Humans

So this part b, the follow-up of lecture two of week seven of the class neuronal dynamics.

Notes

Summary



0m 04s

AdEx model

correct

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \mathcal{G}}{\Delta}\right) - R w + R I(t)$$

correct

$$\tau_w \frac{dw}{dt} = a (u - u_{rest}) - w + b \tau \sum_f \delta(t - t^f)$$

wrong

Throughout this lecture 7.2b, the τ in the differential equation for w should have on both sides an index w (here correct on the left, wrong on the right)

$$\tau \rightarrow \tau_w$$

Be work with an AdEx model. We just have two equations.

Notes

Summary



0m 12s

AdEx model

after each spike
u is reset to u_r

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - R w + R I(t)$$

$$\tau_w \frac{dw}{dt} = \underline{a} (u - u_{rest}) - w + \underline{b} \tau \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount b

parameter *a* – slope of *w*-nullcline

Can we understand the different firing patterns?

Spikes are generated at the moment when the voltage, which is controlled by the first equation, hits a threshold. At the moment of a spike, or more precisely, immediately after the spike, the second variable jumps by an amount *b*-- that's this parameter here-- while the voltage variable is reset to a reset value, *u_r*. Now there's a further important parameter, and that's this parameter *a*. It corresponds to the slope of the *w* nullcline. If you do a nullcline analysis, you can forget about spikes. You solved a nullcline equation by setting the left hand side to 0. And the result is *w* equals *a* times *u* minus *u_{rest}*. And that means the *w* nullcline is a linear function of the voltage. So let's look at the nullclines.

Notes

Summary



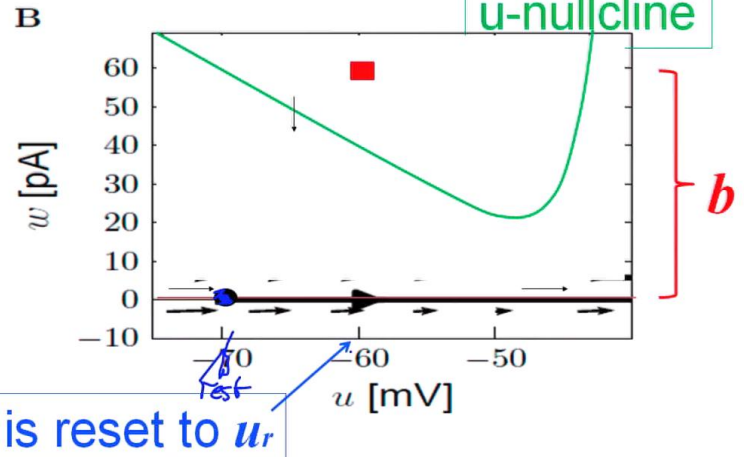
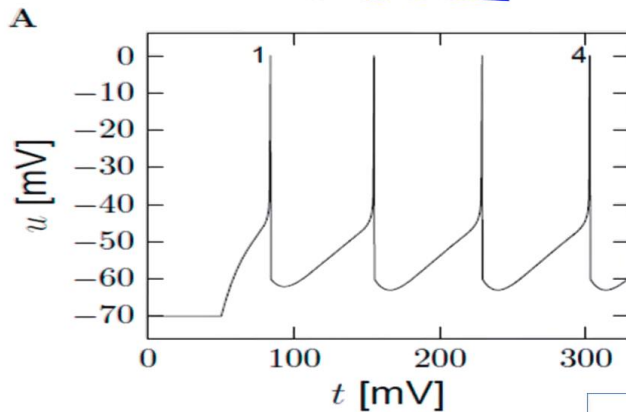
0m 17s

AdEx model – phase plane analysis: **large b**

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau \frac{dw}{dt} = a (u - u_{rest}) - w + b \tau \sum_f \delta(t - t^f)$$

~~$a=0$~~



And to do this, we start with a simple case. We set this parameter a , the coupling between the voltage and w , to 0. Recall that for adaptation, the standard input is a step current. So before the step, the input is 0. And after a step, it has some positive value. The w nullcline is a straight line with slope, a equals 0, which means it's a horizontal line. Now the u nullcline for I equals 0 is just this curve here with a linear and exponential part. That's the shape that we know from the exponential, and you're going to fire model. Before, the current is switched on. There's only one stable fixed point here. That's the resting value. But now we switch on the current. If you switch on the current, we now have a constant value I_1 . And the result of this is that the u nullcline is shifted vertically upwards. The fixed point, therefore, disappears. And the voltage will move. It'll follow the w nullcline until it hits the reset threshold about here. But then we know at the moment of reset, the w variable jumps by an amount b , while the voltages reset to u_r . So let's take u_r equals minus 60 millivolt. The jump, the reset, ends at this point.

Notes

Summary

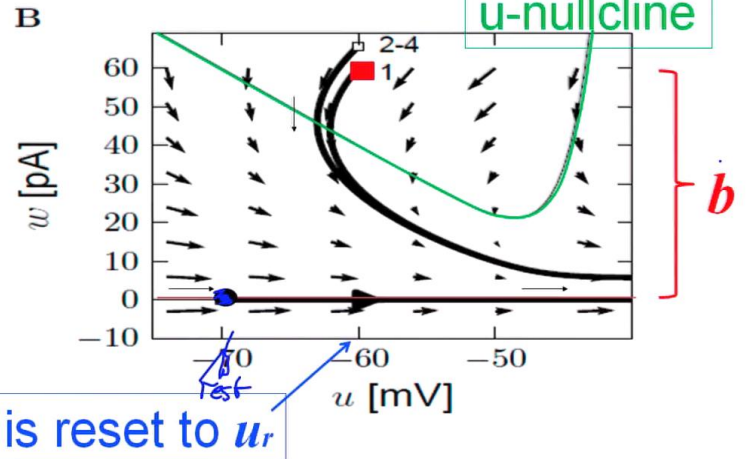
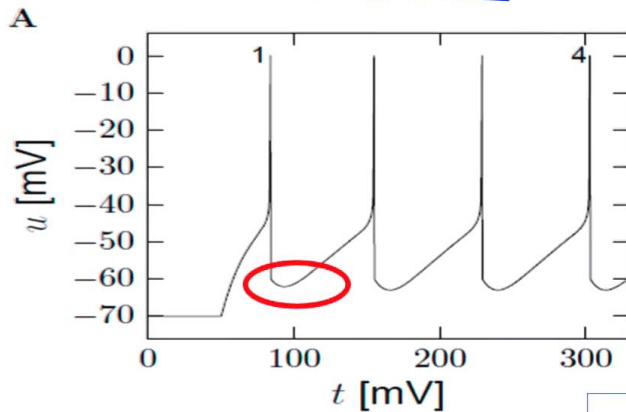


AdEx model – phase plane analysis: **large b**

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau \frac{dw}{dt} = a (u - u_{rest}) - w + b \tau \sum_f \delta(t - t^f)$$

~~$a=0$~~



And from there on, we have to follow the arrows. Arrows are vertically on the u nullcline and slightly diagonal downward to the left in this region up here. That means if you project down to the voltage equation, the voltage initially decreases before it increases again. And that's what you see here. Initially, the voltage decreases after the reset. And then it increases again. Thus, this choice of parameter gives fairly regular tonic firing with characteristically a dip in the voltage trajectory after the reset. So that's one choice of parameters, a fairly large jump size b .

Notes

Summary

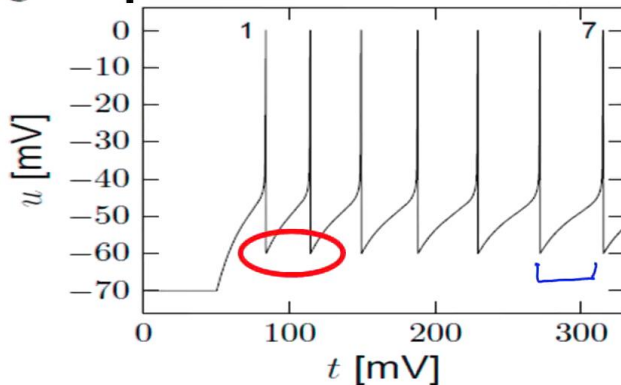


AdEx model – phase plane analysis: **small b**

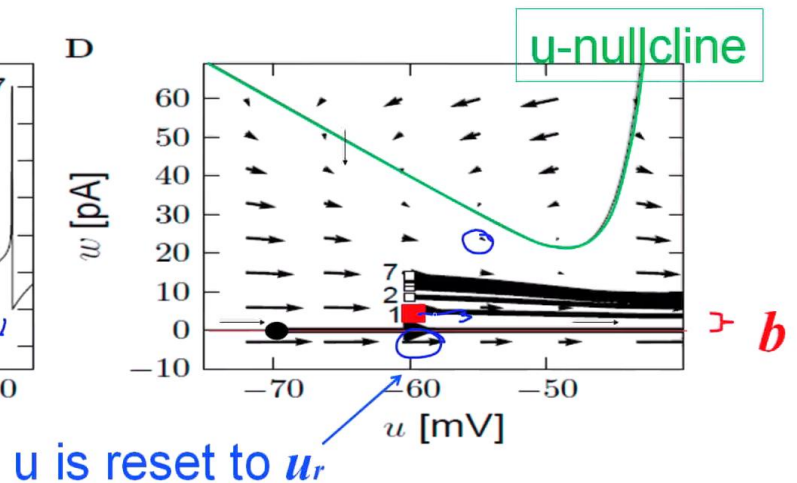
$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau \frac{dw}{dt} = a (u - \cancel{u_{rest}}) - w + b \tau \sum_f \delta(t - t^f)$$

C adaptation



D



Let's now look at a different choice of b . Again, a 's set to 0, so that the w nullcline is a horizontal line. Once the current is switched on, the voltage moves towards the upper threshold. And now w jumps by an amount b , while u is reset to u_r . So the new starting point is here. The same thing happens again. b jumps again, second trajectory. Now b jumps again by amount b , and again, and again. So the result is that you have repetitive firing. However, if the trajectory starts up here, the size of the arrows is actually smaller than down here. That means initially, the movement is a bit slower. And that's why in the spike intervals, after reset number six and reset number seven, are longer than at the beginning. Also note that there's no dip in the voltage trajectory. Once we have reset, the voltage moves immediately again upwards. So, this choice of parameter will lead to adaptation.

Notes

Summary

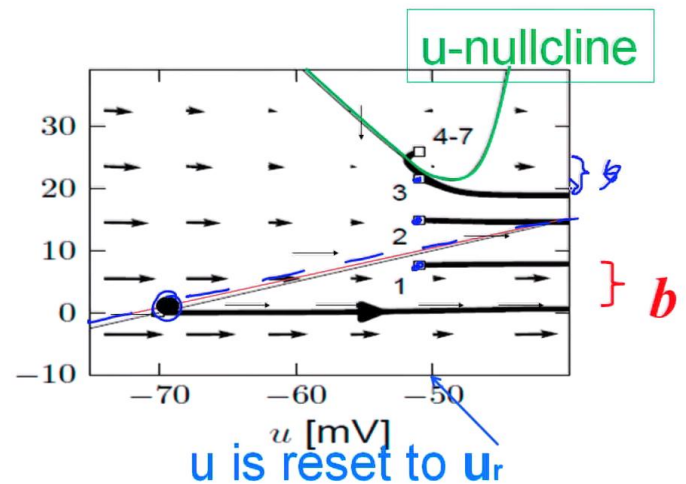


4m 44s

AdEx model – phase plane analysis: $a > 0$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau \frac{dw}{dt} = \underbrace{a}_{\text{blue wavy}} (u - u_{rest}) - w + b \tau \sum_f \delta(t - t^f)$$



Let's have another example. Let's now consider the more general case that this coupling parameter is positive. So the u nullcline now has a positive slope, characterized by this parameter a . Before switching on the current, the neuron was sitting at its resting potential, a stable fixed point. But after the switch, this fixed point has become unstable. So the voltage will move upwards. A spike is initiated. And then you have reset from this value to a new value. The voltage is now reset to, say, minus 50 millivolt, and b is in the medium range. So after the first spike, the trajectory starts here. And it's reset again. It will start here. It's reset again. It starts here. And it's reset again, and again by an amount b , starting from this point here.

Notes

Summary

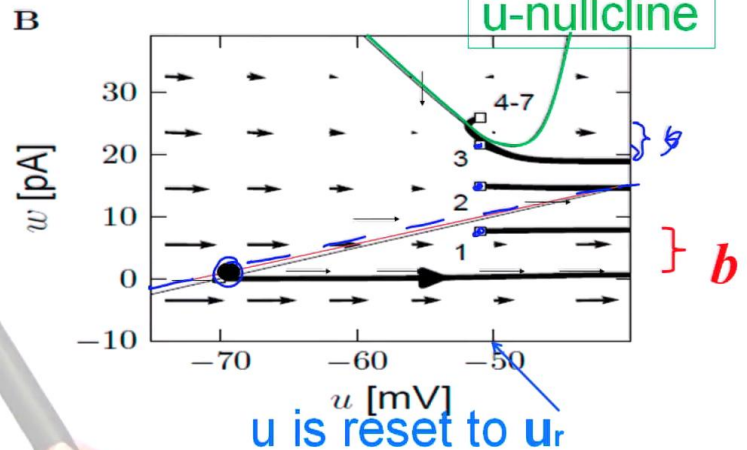
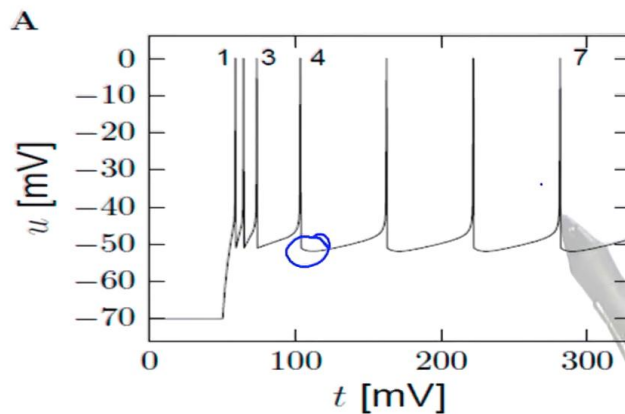


6m 08s

AdEx model – phase plane analysis: $a > 0$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau \frac{dw}{dt} = \underbrace{a}_{\text{blue wavy}} (u - u_{rest}) - w + b \tau \sum_f \delta(t - t^f)$$



So the result is that we have initially rapid sequence of spikes. It's only after spike number four that we see this dip in the voltage. And that's related to the fact that the trajectory starts above the green curve. The arrows are pointing slightly leftward. So let's look at this trajectory. Well, I would classify this as initial burst followed by tonic firing.

Notes

Summary



7m 21s

Neuronal Dynamics – 7.2 AdEx model and firing patterns

$$\left\{ \begin{array}{l} \tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - R w + R I(t) \\ \tau_w \frac{dw}{dt} = a (u - u_{rest}) - w + b \tau \sum_f \delta(t - t^f) \end{array} \right.$$

after each spike u is reset to u_r

after each spike

w jumps by an amount b .

parameter a – slope of w nullcline

Firing patterns arise from different parameters!

See Naud et al. (2008), see also Izhikevich (2003)

⌂ / ☰ ↻

So we have discussed three examples. All three examples used the same neuron model. A neuron model is containing two equations, a voltage regulation and a second equation for the adaptation variable with reset conditions. u is reset to u_r . w jumps by an amount b . And this same neuron model can account for different firing patterns, just by choosing the parameters a , b , and u_r in the appropriate range. And we can predict appropriate range by the phase plane analysis methods that we have discussed together.

Notes

Summary



7m 55s

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

$$(1) \quad \tau \frac{du}{dt} = f(u) + R I(t)$$

(2) If $u = \theta_{reset}$ then reset to $u = u_r$

Best choice of f : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right)$$

✎ ✎ ✎ ✎

Thus, let's take a non-linear integrating fire model. And the best choice is linear plus exponential.

Notes

Summary



8m 49s

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

$$(1) \quad \tau \frac{du}{dt} = f(u) + R I(t)$$

(2) If $u = \theta_{reset}$ then reset to $u = u_r$

Best choice of f : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \theta}{\Delta}\right)$$

BUT: Limitations – need to add

- ✓ -Adaptation on slower time scales
- ✓ -Possibility for a diversity of firing patterns
- Increased threshold θ after each spike
- Noise

✎ ✎ ✎ ✎

Let's combine this with adaptation variables on a slower time scale. Then we can account for the diversity of firing patterns. However, so far, this parameter θ was still fixed. But we can make this parameter dynamic.

Notes

Summary



8m 58s

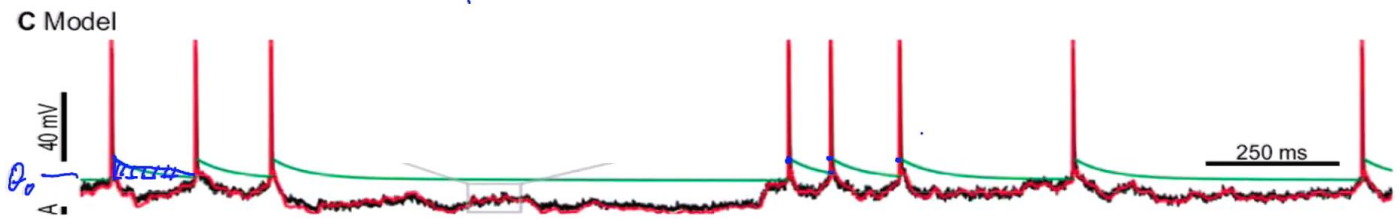
Neuronal Dynamics – 7.2 AdEx with dynamic threshold

Add dynamic threshold:

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - g}{\Delta}\right) - R \sum_k w_k + RI(t)$$

Threshold increases after each spike

$$g = \theta_0 + \sum_f \theta_1(t - t^f)$$



This is my theta. And it has a standard value, theta 0. And then after each spike, it's increased by some amount. And you see the shape of theta 1 in this little curve here. It goes up, and it goes down again. This is the value of theta 0. The difference between those two is theta 1. Now if there are several spikes with short intervals, then you can actually increase to higher and higher values. And this causes a different form of adaptation.

Notes

Summary



9m 20s

Neuronal Dynamics – 7.2 Generalized Integrate-and-fire



$$\tau \frac{du}{dt} = f(u) + R I(t)$$

If $u = \theta_{reset}$ then reset to $u = u_r$

add

- ✓ -Adaptation variables
- ✓ -Possibility for firing patterns
- ✓ -Dynamic threshold ϑ
- Noise

Let's summarize. You can start with the exponential integrate-and-fire model and add one or several adaptation variables. And suddenly, we are able to account for a whole variety of different firing patterns, firing patterns in response to step current input. So our new model is now very flexible. But it's still a deterministic model. You should also account for noisiness for variability in spike trains. And that's what's going to come in the next few lectures.

Notes

Summary



10m 03s

Neuronal Dynamics – Quiz 7.3. Nullclines for constant input

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + w + RI(t)$$

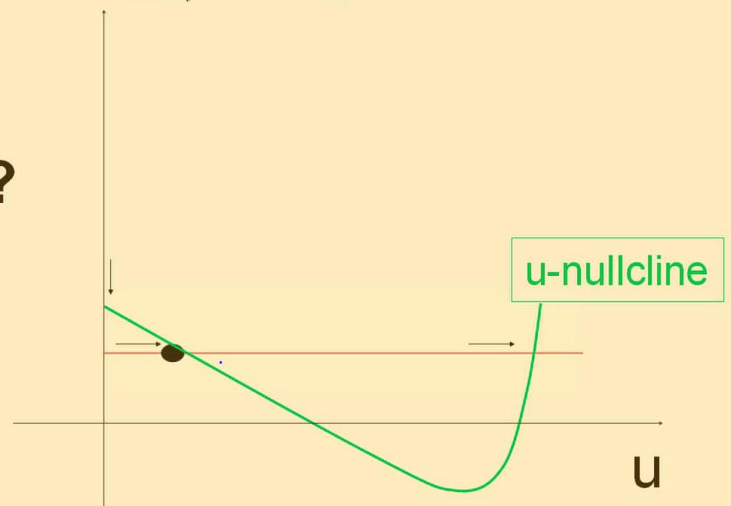
$$\tau \frac{dw}{dt} = a(u - u_{rest}) - w + b \tau \sum_f \delta(t - t^f)$$

a=0

Only during reset

What happens if input switches from $I=0$ to $I>0$?

- ☐ u-nullcline moves horizontally
- ☐ u-nullcline moves vertically
- ☐ w-nullcline moves horizontally
- ☐ w-nullcline moves vertically



10m 46s

Notes

Summary

