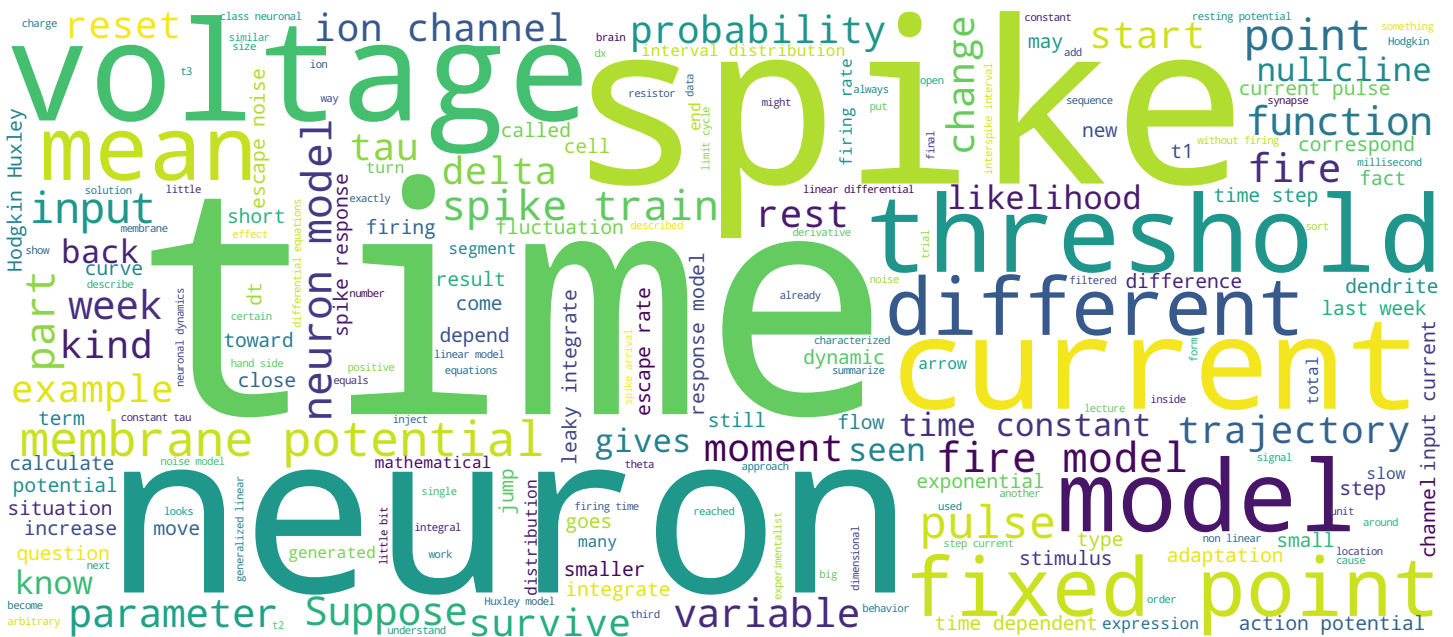


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-



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Week 7 – part 4 :Generalized Linear Model (GLM)



✓ 7.1 What is a good neuron model?

- Models and data

✓ 7.2 AdEx model

- Firing patterns and analysis

✓ 7.3 Spike Response Model (SRM)

- Integral formulation

7.4 Generalized Linear Model

- Adding noise to the SRM

7.5 Parameter Estimation

- Quadratic and convex optimization

7.6. Modeling in vitro data

- how long lasts the effect of a spike?

7.7. Helping Humans

This is part four of week seven of the class neuronal dynamics. In the previous lecture, I've introduced a deterministic version of the spike response model. Now let's add noise.

Notes

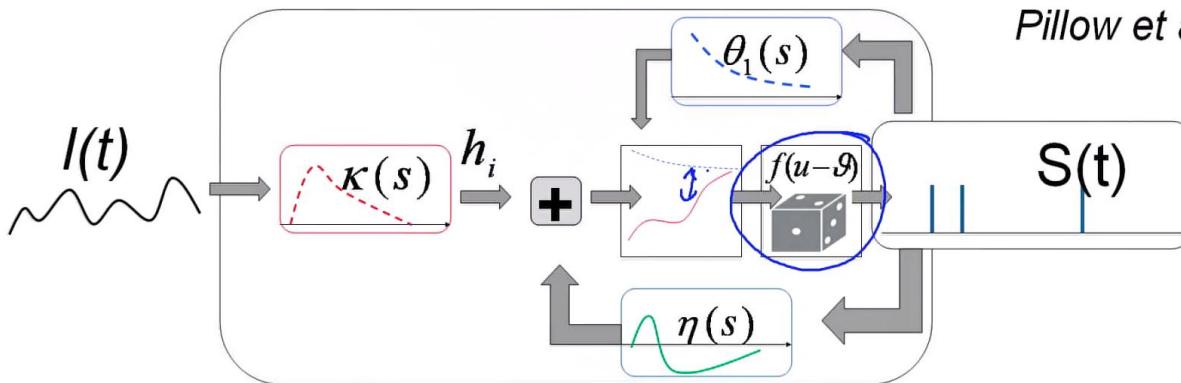
Summary



0m 04s

Spike Response Model (SRM) Generalized Linear Model GLM

Gerstner et al.,
1992, 2000
Truccolo et al., 2005
Pillow et al. 2008



potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $g(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - g(t))$

So we have the same filter description as before. Input is filtered by some filter called kappa. Output is filtered and fed back in two ways. There is a refractive filter, eta of s, accounting for refraction as an adaptation by a modulation of the membrane potential. And then there is a threshold contribution, a threshold filter, theta 1, accounting for an increased threshold immediately after a spike. The only difference is here. We now say that firing is a stochastic process with a firing intensity that depends on the momentary distance between the membrane potential-- that would be the red trace here-- and the threshold. So even though the membrane potential hasn't reached a threshold yet, there is already a probability that the neuron may fire.

Notes

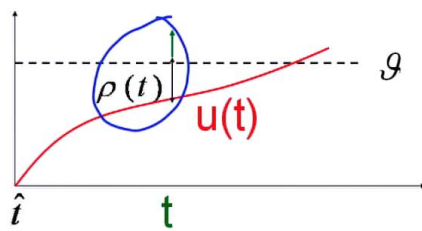
Summary



0m 18s

Neuronal Dynamics – review from week 6: Escape noise

escape process

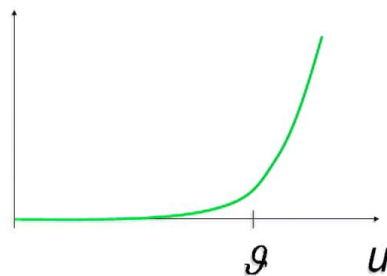


escape rate

$$\rho(t) = \frac{1}{\Delta} \exp\left(\frac{u(t) - g}{\Delta}\right)$$

escape rate

$$\rho(t) = f(u(t) - g)$$



Example: leaky integrate-and-fire model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\text{if spike at } t^f \Rightarrow u(t^f + \delta) = u_r$$

And this is exactly the notions that we have seen last week. A spike may be fired with a certain instantaneous rate, ρ of t , despite the fact that the membrane potential hasn't reached a threshold yet. And this model works, for example, for the leaky integrate and fire model or for leaky integrate and fire model with adaptation, or for non-linear integrate and fire model, and, in particular, it can be combined with the spike response model. A particularly useful choice is an exponential escape rate that was discussed last week.

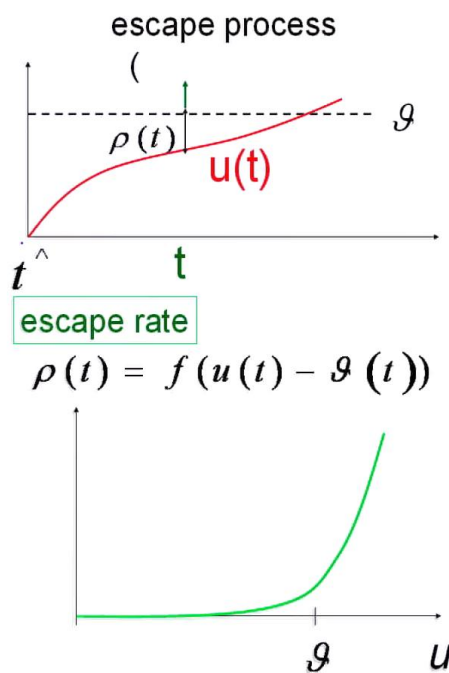
Notes

Summary



1m 21s

Neuronal Dynamics – review from week 6: Escape noise



Survivor function

$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t) S_I(t|\hat{t})$$

$$S_I(t|\hat{t}) = \exp\left(-\int_{\hat{t}}^t \rho(t') dt'\right)$$

Interval distribution

$$P_I(t|\hat{t}) = \underbrace{\rho(t)}_{\text{escape rate}} \cdot \underbrace{\exp\left(-\int_{\hat{t}}^t \rho(t') dt'\right)}_{\text{Survivor function}}$$

Good choice

$$\rho(t) = f(u(t) - g(t)) = \rho_0 \exp\left[\frac{u(t) - g(t)}{\Delta u}\right]$$

So last week we have seen that, with this noise model, we can construct quantities such as the survivor function. Given the last spike occurred at $t_{\hat{t}}$, what is the probability the neuron would survive up to time t without firing again? This probability of survival decays with the firing rate ρ , and, therefore, we can integrate this, and this is the result. Similarly, we may ask for the time dependent interval distribution, and it turns out, in order to fire the next spike at time t , the neuron has to survive from $t_{\hat{t}}$ to t , and then it has to fire. So we have an expression for the interval distribution. And this works for all sorts of escape rate functions. However, formulas turn out to be particularly nice if as, in the escape rate, we take the exponential one. Now that's cool, because the exponential one fits well experimental data measured in cortical slices.

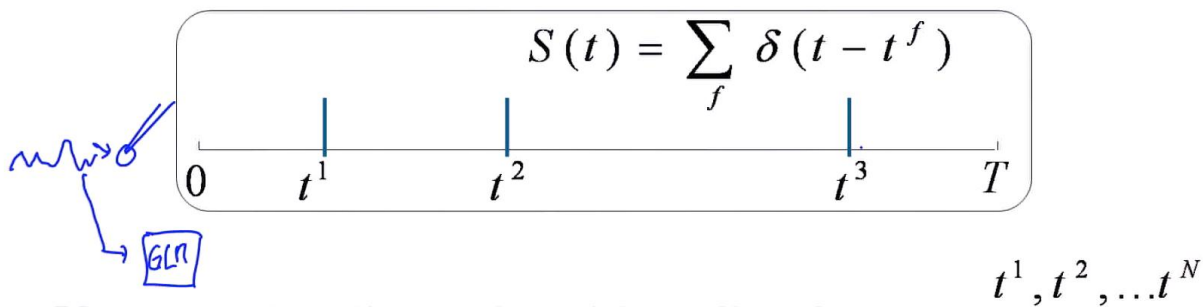
Notes

Summary



2m 04s

Neuronal Dynamics – 7.4 Likelihood of a spike train in GLMs



Measured spike train with spike times

Likelihood L that this spike train could have been generated by model?

$$L(t^1, \dots, t^N) = \exp\left(-\int_0^{t^1} \rho(t') dt'\right) \rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t') dt'\right) \dots$$

Now, after these preparations, we can now turn to the likelihood of a spike train in this kind of noise neuron models that are called generalized linear models. Suppose an experimentalist has injected some time-dependent current, a known current, into a neuron, and he has observed a sequence of spikes. Suppose we inject the same current in our neuron model. For example, a spike response model with escape noise, also called generalized linear model. Then what is the likelihood that this spike train with our specific spike times could have been generated by our model? And as we have seen last week, we have an expression for this. And this is what this expression looks like.

Notes

Summary



Neuronal Dynamics – 7.4 Likelihood of a spike train

$$S(t) = \sum_f \delta(t - t^f)$$

$$L(t^1, \dots, t^N) = \exp\left(-\int_0^{t^1} \rho(t') dt'\right) \underbrace{\rho(t^1)} \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t') dt'\right) \underbrace{\rho(t^2)} \dots \exp\left(-\int_{t^N}^T \rho(t') dt'\right)$$

$$L(t^1, \dots, t^N) = \exp\left(-\int_0^T \rho(t') dt'\right) \prod_f \underbrace{\rho(t^f)}$$

$$\log L(t^1, \dots, t^N) = -\int_0^T \rho(t') dt' + \sum_f \log \rho(t^f)$$

Let me just go over it again quickly. So here's a copy of the expression that we had on the previous slide. So the neuron has to survive from t^1 to t^2 without firing. That gives this term here. But before that, the neuron model had to fire at time t^1 . Has to fire again at time t^2 . Has to fire again at time t^3 . t^1 is supposed to be different spike, so the neuron model has to survive from zero to t^1 without firing and, again, it has to survive from t^3 to end of the measurement, capital T , which gives the final term over here. We can collect all these different firing times, and they appear up here as a product. And we can collect the integrals and exponent. It first runs from zero to t^1 and from t^1 to t^2 , and from t^2 to t^3 , and then from t^3 to T , capital T , while all this gives one single integral. And, as we have seen, sometimes we consider the logarithm instead of the likelihood itself. Now, this formulism works for arbitrary stochastic point processes that are characterized by this instantaneous intensity or instantaneous firing rate or hazard of firing.

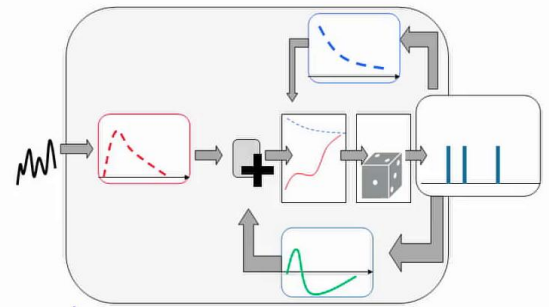
Notes

Summary



4m 35s

Neuronal Dynamics – 7.4 SRM with escape noise = GLM



-linear filters
-escape rate
→likelihood of observed
spike train

→parameter optimization
of neuron model

We have seen that if you take a spike response model with its three linear filters, and if we add escape noise, then we can write down the likelihood of an observed spike train, the likelihood that such a spike train could have been generated by the model. Thus, we have a generative model for spike trains. That's a technical term used in the statistical and machine learning literature. And this generative model has the properties that it depends on parameters, and we can use this dependence to find the optimal set of parameters to explain the data. That will be the topic of the next lecture. Just remember, a spike response model, together with escape noise, can be formulated in the framework of generalized linear models, short, GLM.

Notes

Summary



6m 27s