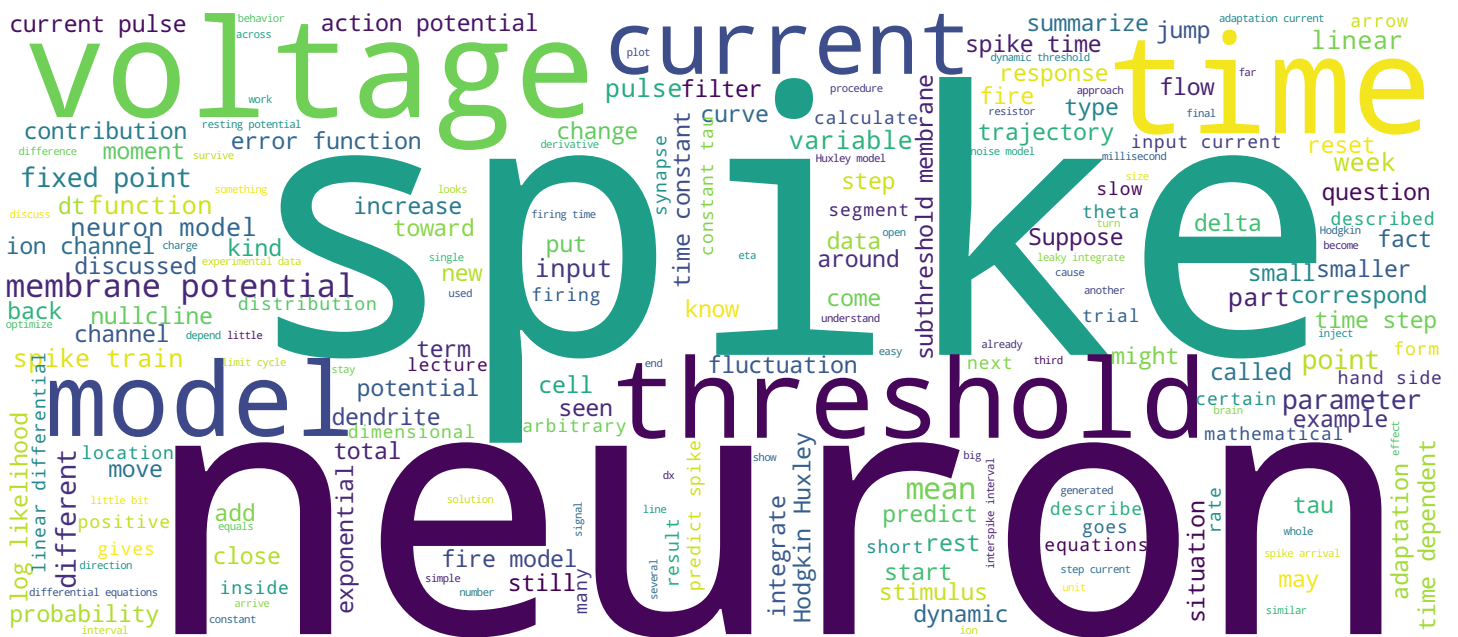


ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

EPFL, Lausanne, Switzerland

7.7. Helping Humans



EPFL



Week 7 – part 5b : Parameter estimation for spike times



✓ 7.1 What is a good neuron model?

- Models and data

✓ 7.2 AdEx model

- Firing patterns and analysis

✓ 7.3 Spike Response Model (SRM)

- Integral formulation

✓ 7.4 Generalized Linear Model (GLM)

- Adding noise to the SRM

7.5 Parameter Estimation

- Quadratic optimization: subthreshold
- convex optimization: spike times

7.6. Modeling in vitro data

- how long lasts the effect of a spike?

7.7. Helping Humans

So this is part 5b of week seven. So far, we have been limited to quadratic error functions that arise when we optimize the parameters for predicting subthreshold potential. However, from a good neuron model, we also expect that we should be able to predict spike times.

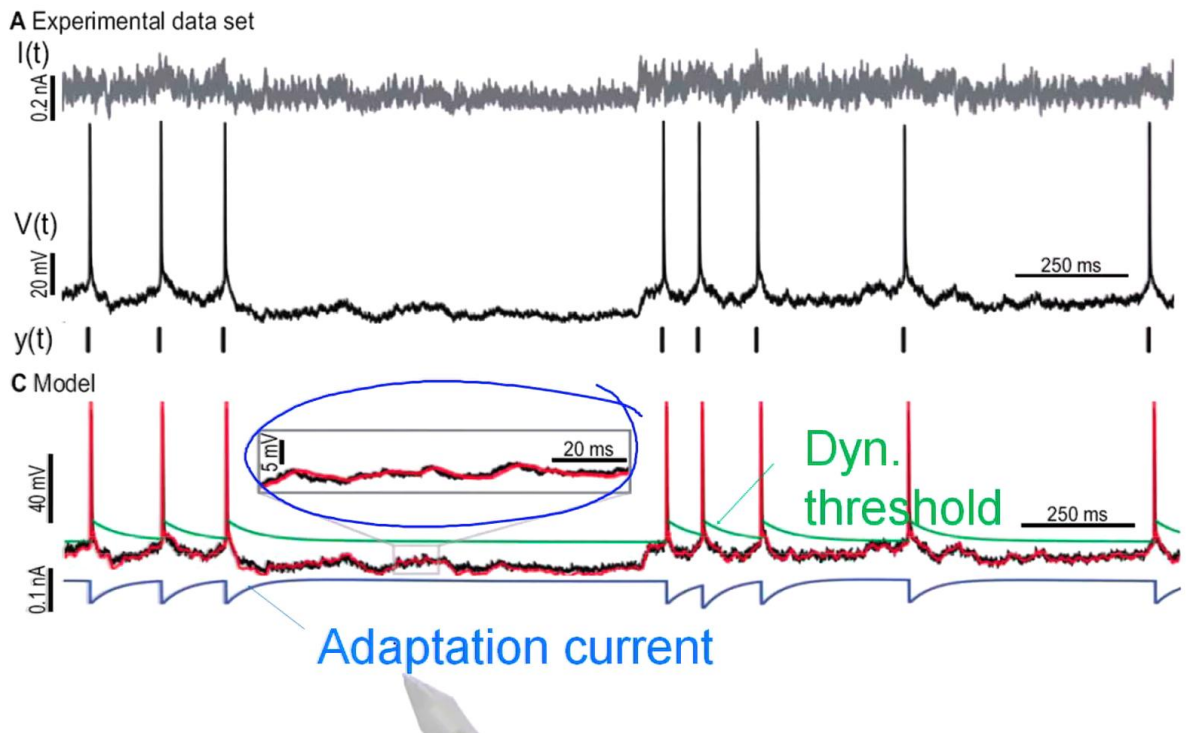
Notes

Summary



0m 04s

Fitting models to data: so far 'subthreshold'



So here's an example. We have an experimental data set with current injection. An experimental data set contains spikes, and it contains voltage traces. And so I've been mainly focused on the subthreshold membrane potential. And the model in red does very nicely in comparison to the data. Now, if you want to predict spike times, we'll need also adaptation. And we will also need a dynamic threshold. Now, the two things are a little different. As I explained in the previous part of this lecture, there's a subthreshold membrane potential caused by adaptation currents. And these adaptation currents can be optimized by looking at the membrane potential-- if we predict the membrane potential. However, if we also want to predict spike times and, at the same time, the membrane potential, we have to separate the notion of a dynamic threshold from the combination notion of adaptation currents, or equivalently, spike after potentials.

Notes

Summary



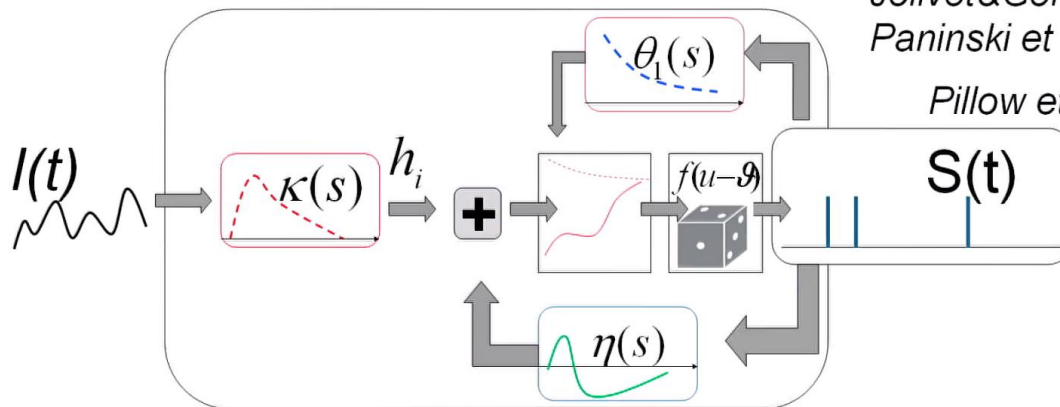
0m 27s

Neuronal Dynamics – 7.5 Threshold: Predicting spike times

Jolivet & Gerstner, 2005

Paninski et al., 2004

Pillow et al. 2008



potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $g(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - g(t))$

So here is our standard formulation with spike after potentials. The input is filtered with a filter kappa. Output spikes are generated stochastically. The feedback goes back as a contribution to the membrane potential that adds in here, as well as a contribution to the dynamics of the threshold that adds in here. And firing times are generated stochastically by the neuron model. And now, the aim is to not just predict the subthreshold membrane potential, which we can do by optimizing kappa and eta using what are the error functions of the membrane potential. Now, we also want to predict the spike times. But predicting spike times involves this stochastic firing intensity, which is a nonlinear function of the voltage.

Notes

Summary



1m 43s

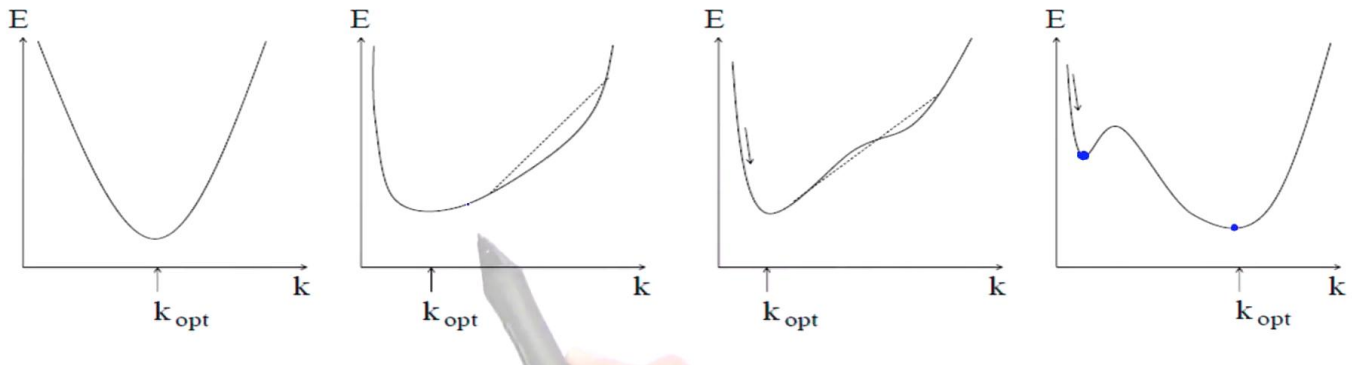
Neuronal Dynamics – 7.5 Generalized Linear Model (GLM)

$$\log L(t^1, \dots, t^N) = -\int_0^T \rho(t') dt' + \sum_f \log \rho(t^f) = -E$$

potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$



And here is a very nice result. Note that the membrane potential, as discussed, is linear in the parameters eta and kappa. We have this dynamic threshold. The dynamics threshold is, again, linear given the spike train. The spike train has been measured. I have discussed before that, given the spike train, predicting voltage is linear in the parameter. Given this spike train, optimizing the threshold is linear and the parameter is theta 1. It's just a convolution. But we have the nonlinearity in the firing intensity. This function f is nonlinear. Now, for the quadratic optimization problem, we always note that there is just a single minimum. If we have some arbitrary nonlinearities, in general, we could arrive at situations where we have multiple minima. And we would just locally look for minimum. You might get stuck in this local minimum, which is not the best one. Now, it turns out that if we take for f an exponential function-- as we should do, because it nicely fits theta-- if we put in an exponential function here, then the error function as a total, as a whole, is not of this form.

Notes

Summary



2m 47s

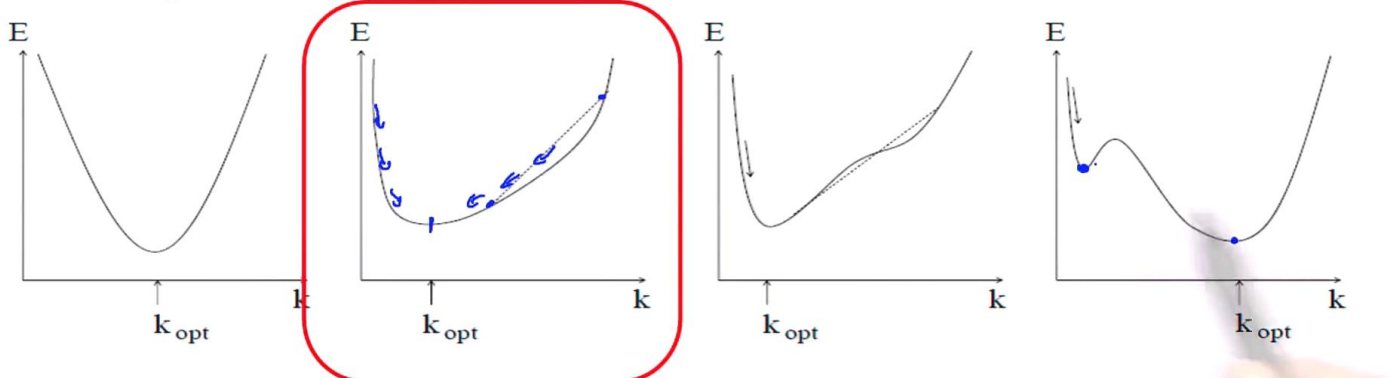
Neuronal Dynamics – 7.5 Generalized Linear Model (GLM)

$$\log L(t^1, \dots, t^N) = -\int_0^T \rho(t') dt' + \sum_f \log \rho(t^f) = -E$$

potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$



It's a convex function, which has only one single minimum, which means we can use standard optimization procedures to go to the minimum. Convex means that if I put a connecting line between two points, it stays above the error function. So this case here is not convex. And this is also not convex.

Notes

Summary



4m 25s

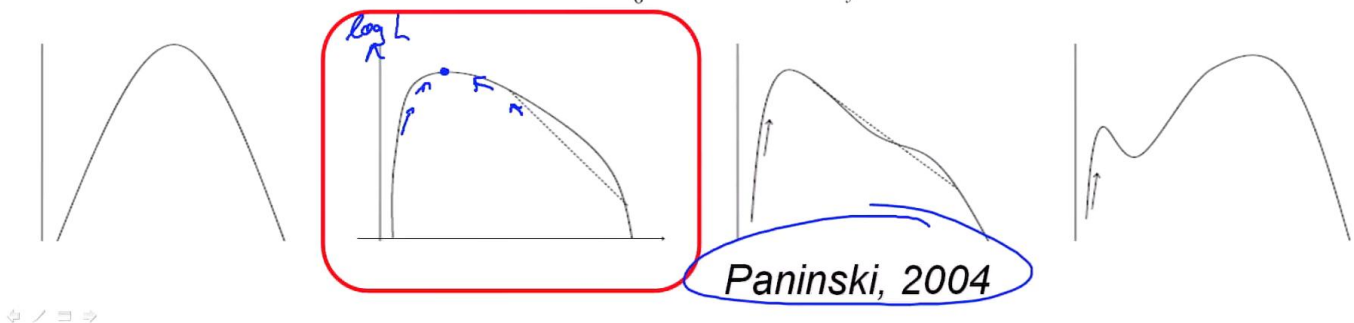
Neuronal Dynamics – 7.5 GLM: concave error function

potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\mathcal{G}(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \mathcal{G}(t))$

$$\log L(t^1, \dots, t^N) = - \int_0^T \rho(t') dt' + \sum_f \log \rho(t^f) - E$$



So how does this come about? Well, I have the membrane potential, linear parameters of the threshold, linear parameters. I have one single nonlinearity. Now, if I look at the log likelihood, then it's just the error function with the minus sign. So in fact, I just took the graphs on the previous slide and flipped them around. And so it's now finding a maximum rather than a minimum if you plot here the log likelihood. And the result, which I mentioned on the previous slide, which is due to Liam Paninski states that such a neuron model has a log likelihood function which is concave, which only has one single maximum. And that means parameters can be optimized in a systematic fashion-- by maximizing the log likelihood. In particular, the parameters with a threshold, which we still have to optimize, can be done in this systematic fashion. So in the procedure I discuss here, we optimize eta and kappa using the membrane potential. Then these are fixed, and we optimize theta so as to optimize the log likelihood of the spike times. With extracellular recordings so we don't have access to the subthreshold membrane potential, we can directly optimize log likelihood and find all three sets of parameters-- theta 1, eta, and kappa.

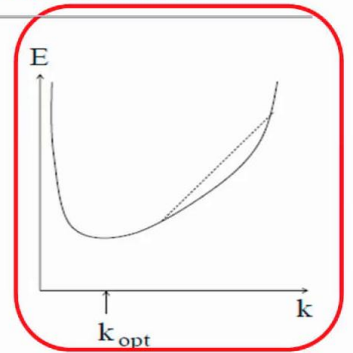
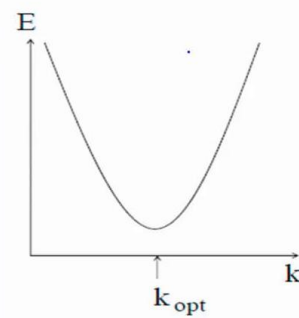
Notes

Summary



4m 59s

Neuronal Dynamics – 7.5 quadratic and convex/concave optimization



Voltage/subthreshold

- linear in parameters
→ quadratic error function

Spike times

- nonlinear, but GLM
→ convex error function

Let me summarize. In the procedure I have discussed here, if we have access to the subthreshold membrane potential, then we can optimize parameters of the filters-- kappa, the membrane filter, and of eta, the spike after potential-- using a quadratic error function. The problem is linear in the parameters and therefore easy. If we want to predict spike times, then we can optimize the likelihood that the eta spike train could have been generated by the model. And in this case, we have the problem that's concave, that has a single maximum as a function of the threshold parameters, the parameters of this filter, theta 1. And again, this means it's easy to solve.

Notes

Summary



6m 49s