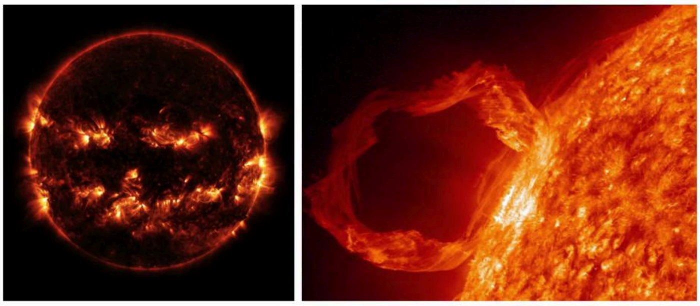


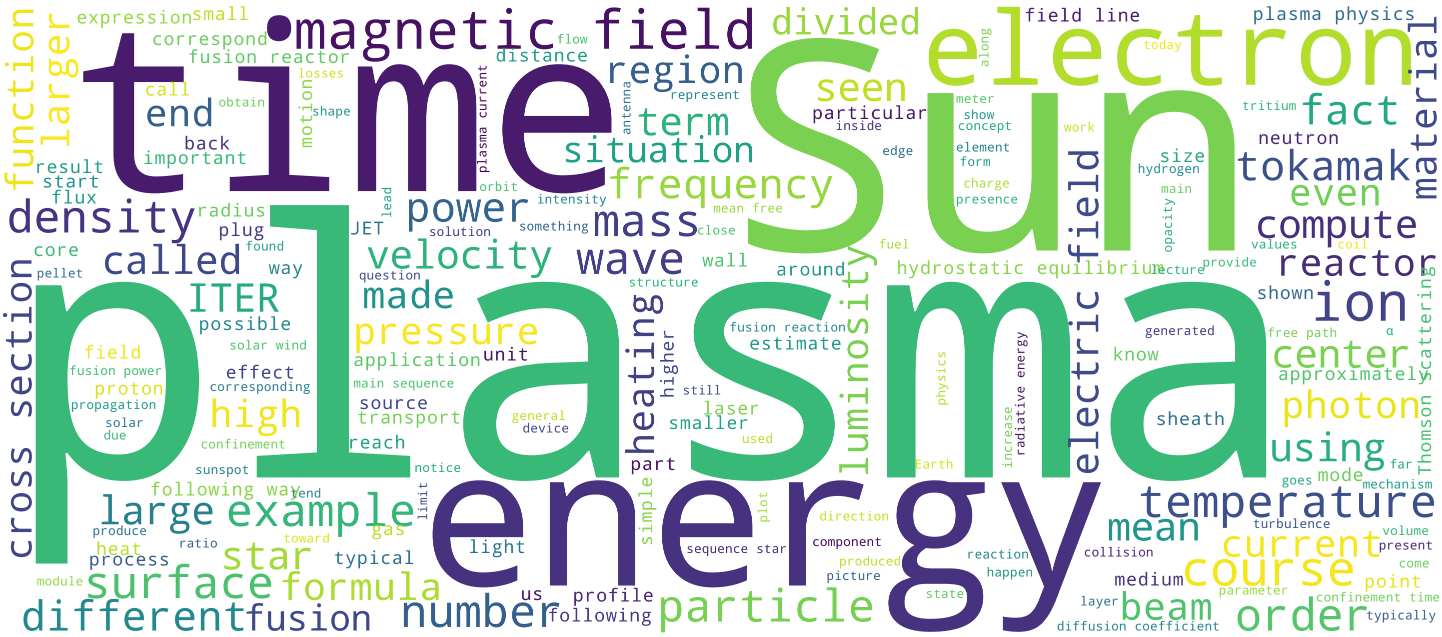
Courtesy of NASA

The Sun: a gravitationally confined fusion reactor

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 4a

Ivo Furno



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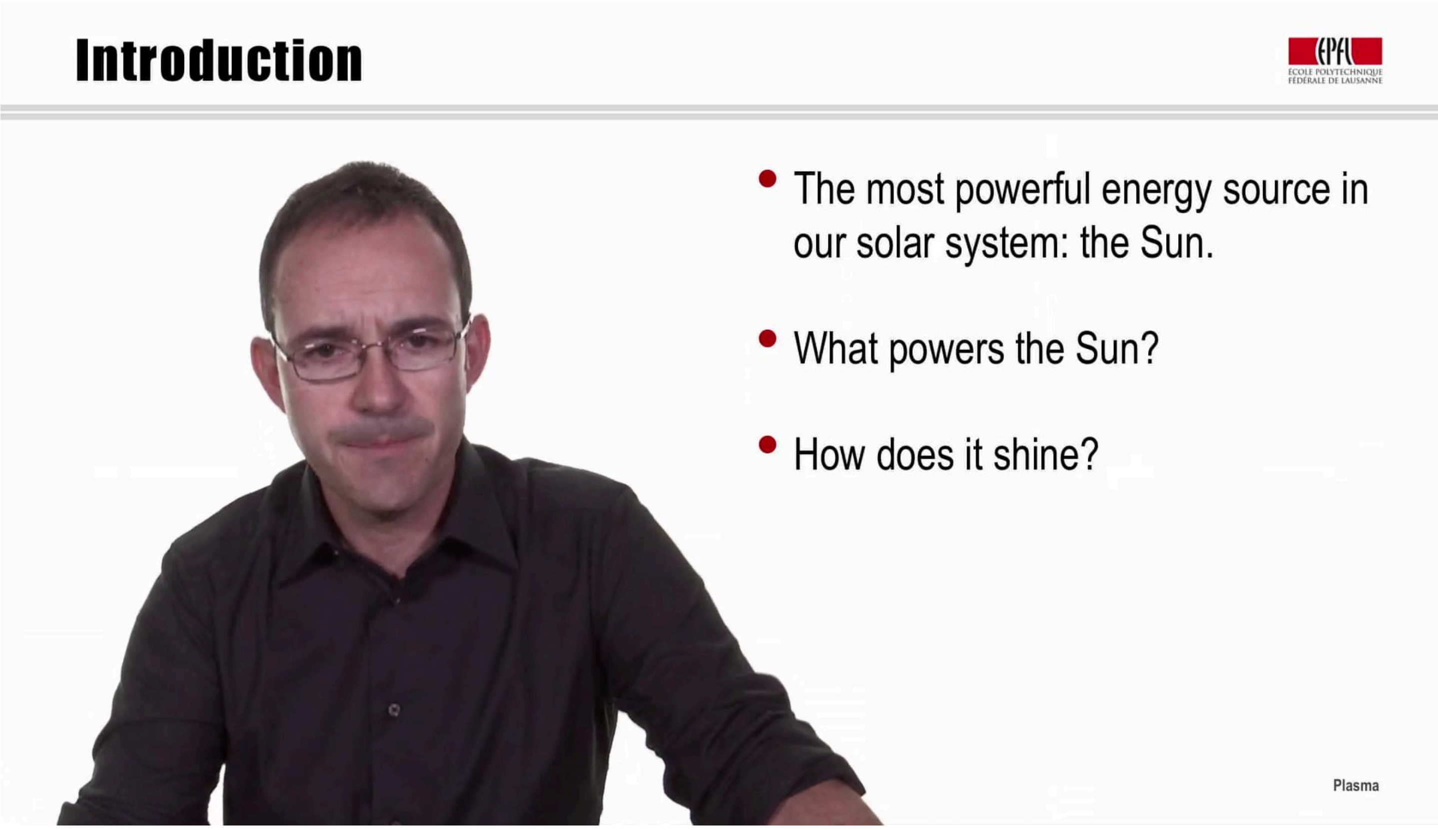


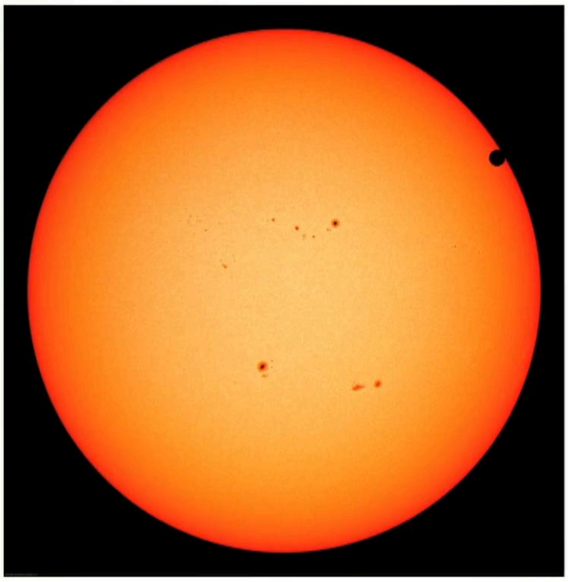
Welcome. Welcome back to the course on Plasma Physics and Applications to Fusion Energy, Astrophysics, and Industry. In this first module on plasma astrophysics, we will deal with the most powerful source of energy in our solar system: the Sun. Today we will answer some fundamental questions about our Sun. What powers the Sun? How does it shine? We will address these questions in a simplified way by using the fundamental tools of plasma physics that you have learned so far.

Notes

Summary

0m 05s

A few properties of the Sun



- Earth mean distance: $d_o = 1.5 \times 10^{11}$ m = 1 AU
- Mass: $M_o = 2 \times 10^{30}$ kg
- Radius: $R_o = 7 \times 10^{8}$ m
- Luminosity: $L_o = 3.84 \times 10^{26}$ W
- Elemental abundances are typical for cosmic sources. By mass:

  73.4% Hydrogen

  25.0% Helium

  1.6% everything else ("metals")

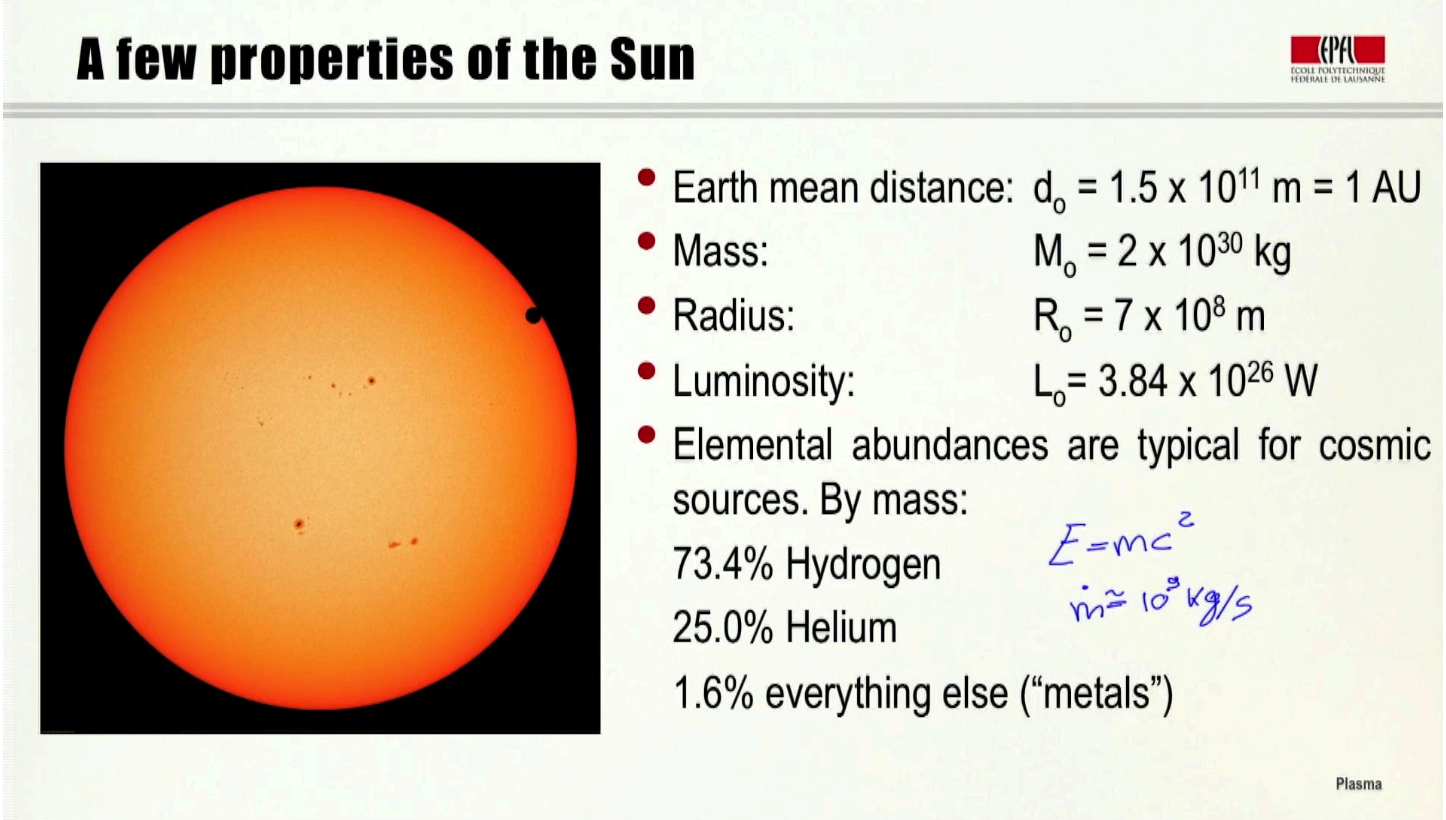
Plasma

Let me introduce first a few properties of the Sun. The solar distance from Earth is called the astronomical unit. It is approximately 1.5 on average 10^{11} meters. The astronomical unit is a basic unit in the solar system and even beyond. The mass of the sun, which is approximately 2 x 10^{30} kilograms, is a typical value for a normal star, and a normal star is an object that shines steadily for a fair long time by burning hydrogen. The Sun's radius, which is approximately 7 x 10^{8} meters is the radius of the visible disc which, we notice is almost perfectly round. The diameter of the Sun is a little bit more than half a degree as we see it from Earth. Note that the shape of the Sun is extremely sharply defined because almost all the light we receive on earth originates in a thin layer, which is a few hundred kilometers thick, the photosphere. In fact, if we move from the center of the sun, going outwards, the matter changes rapidly from being almost completely opaque to almost completely transparent in the photosphere. The sun emits electromagnetic radiation and its luminosity is approximately 3.84 x 10^{26} watts.

Notes

Summary

0m 37s

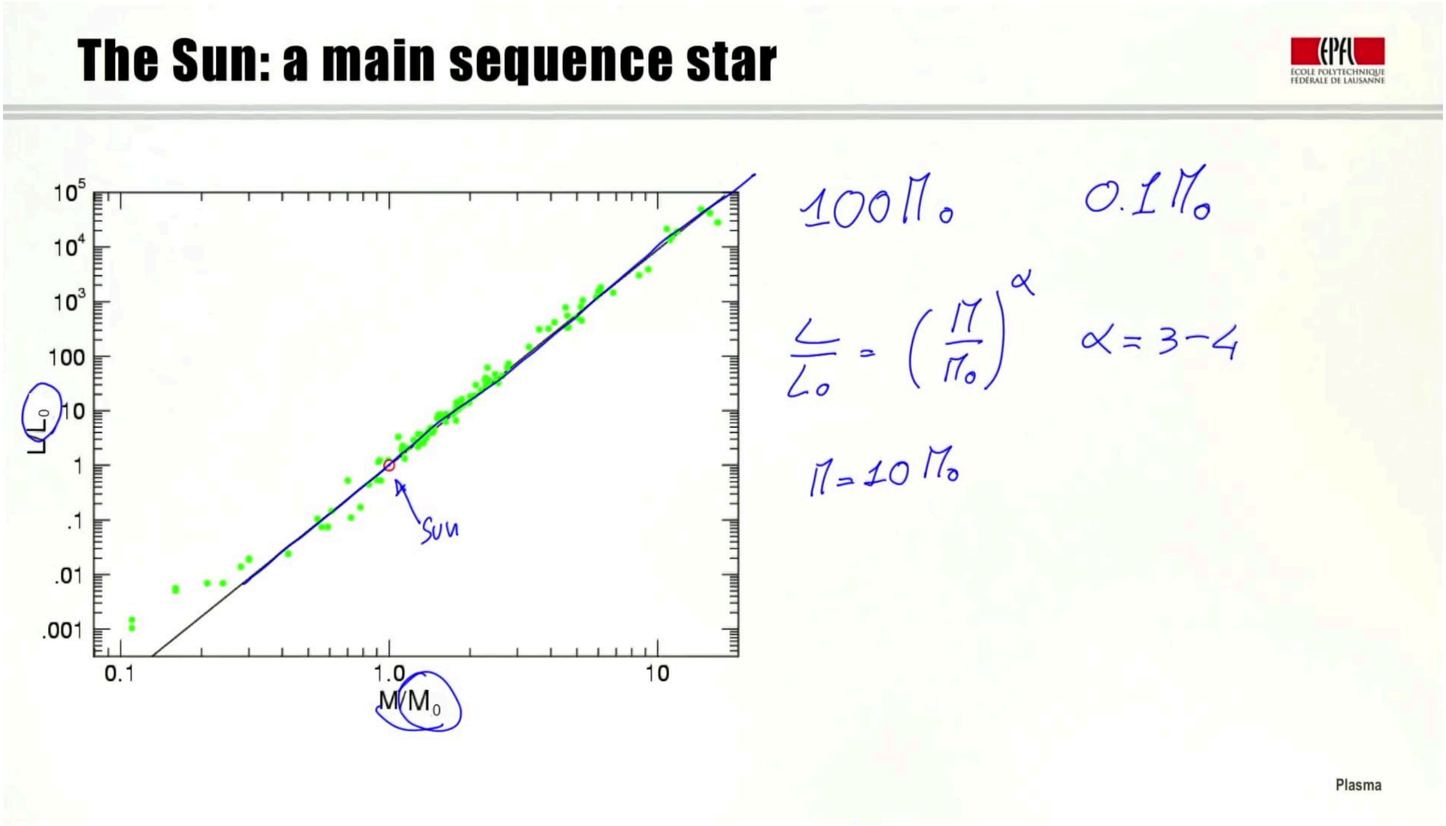


Using the famous E=mc² relation, this yields a mass loss of about 4 x 10⁹ kilograms per second, which results in a negligible perturbation to the total mass of the sun. And finally the elemental abundances of the Sun are typical for cosmic sources. The Sun is made approximately, in mass, by 73.4% of hydrogen, 25% of helium, and then 1.6% of everything else, mostly metals.

Notes

Summary

2m 09s

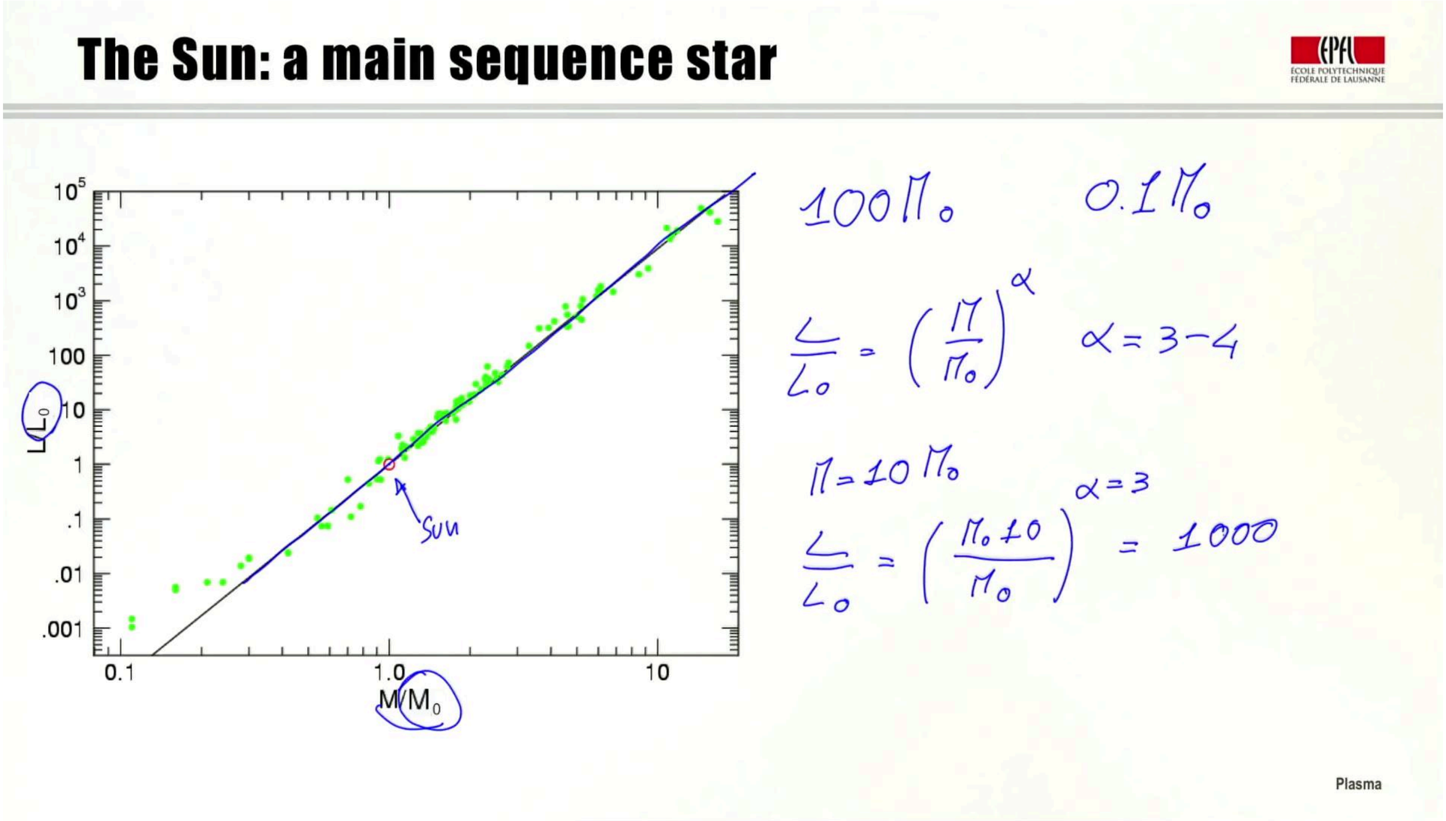


The Sun is a main sequence star. That is a star that has been shining steadily for a fair amount of time. The largest mass observed for a main sequence star is of the order of approximately 100 times the mass of our Sun while the lower limit is slightly below 0.1 times the mass of our Sun. An interesting property of main sequence stars is that they follow well-defined empirical laws. One such law is shown in this plot here where the star luminosity is shown as a function of the star mass. This is a logarithmic plot and we are using normalized units where the mass of the star is normalized to the mass of our Sun and the luminosity normalized to its luminosity. In fact, this red dot here in the middle of the plot indicates the position of our Sun. If we now try to interpolate this data with a straight line, we observe an interesting property of a main sequence's star. The luminosity of the star is related to the mass of the star through a scaling law, where the exponent α varies between three and four. This is telling us that the luminosity of the star increases much faster than the mass of the star. Let's see an important consequence of this scaling law. Let's consider a star of 10 solar masses.

Notes

Summary

2m 49s

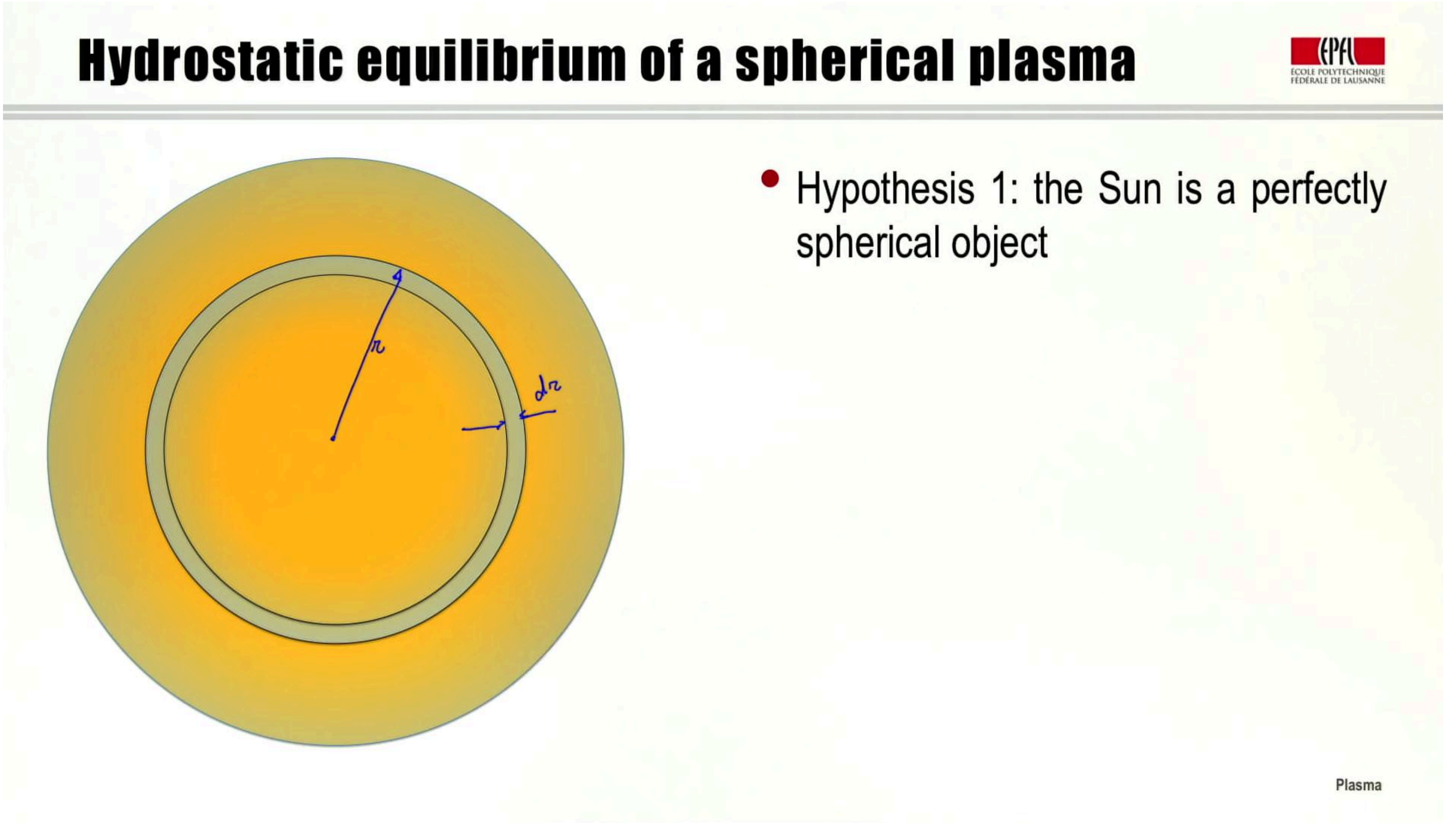


This means that this star has 10 times the fuel available since the fuel is proportional to the mass of the star. However, by using the scaling law, and assuming an exponent α equal to three, we obtain that the luminosity of this star will be one thousand times the luminosity of our Sun. This means that this star will burn its fuel one thousand times much faster than our Sun, and therefore, it will live one thousand times shorter.

Notes

Summary

4m 33s



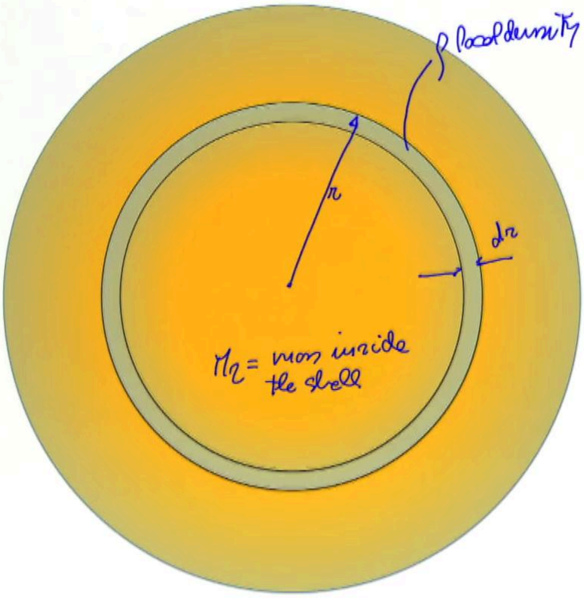
In the following, we will derive this fundamental property from basic principles. We know from everyday experience that a cloud of gas tends to fill all available space. On cosmic scale, this is not true because of the gravitation that tends to contract the gas under its own weight, heating it up eventually to a temperature which is high enough to ignite nuclear fusion. An equilibrium is reached when the gravitational force which acts to contract the star, and the gas pressure force which acts to expand the star are balanced. This is the so-called *hydrostatic equilibrium* and in a nutshell the Sun is a gravitationally self-confined nuclear fusion reactor. What we're going to do next is derive the hydrostatic equilibrium equation and compute the center temperature of the Sun. Now, to start deriving the equation for the hydrostatic equilibrium let's make a first hypothesis. That is the Sun is a perfectly spherical object. The radius of this sphere is the Sun's radius, which is 7×10^8 meters. Let's then consider a spherical shell, which is located at a distance r from the center of the sun and this shell has a radial thickness dr.

Notes

Summary

5m 08s

Hydrostatic equilibrium of a spherical plasma



- Hypothesis 1: the Sun is a perfectly spherical object

$$\rho \frac{\partial^2 r}{\partial t^2} = -G \frac{M_r \rho}{r^2} - \frac{dP}{dr}$$

- Hypothesis 2: acceleration can be neglected → hydrostatic equilibrium

$$\frac{dP}{dr} = -G \frac{M_r \rho}{r^2}$$

Plasma

Let's then indicate with a ρ the local density of the matter inside the shell and with Mr the total mass inside the spherical shell. We can write down the force balance in the following way where the first term describes the acceleration forces, the second term describes the gravitational forces and G is the gravitational constant in SI units and the third term describes the pressure forces. P is the pressure inside the Sun. Let's now make a second hypothesis by assuming that the acceleration can be neglected since the Sun has been in equilibrium for a very long time and therefore we assume an hydrostatic equilibrium by dropping the first term in this equation. In doing this we're balancing the pressure forces that tend to expand the matter inside the Sun with the gravitational forces that tend to contract the mass of the sun under its own weight.

Notes

Summary

6m 44s

Hydrostatic equilibrium of a spherical plasma

$$\frac{dP}{dr} = -\frac{G M_r \rho}{r^2}$$

Multiply by $4\pi r^3$ and integrate between $r=0$ and $r=R_0$

$$\int_0^{R_0} \frac{dP}{dr} 4\pi r^3 dr = -\int_0^{R_0} G M_r \rho 4\pi r \, dr$$

$$\cancel{P 4\pi r^3 \Big|_0^{R_0}}^{\,0} - \int_0^{R_0} P 12\pi r^2 dr = -3\int_0^{R_0} P 4\pi r^2 dr = -3\langle P\rangle \bar{V}$$

$\bar{V}$ → Total volume of the Sun

$P(R_0) = 0$

$$\langle P\rangle = \frac{1}{\bar{V}} \int_0^{R_0} P\, dr\, \pi 4 r^2$$

$$3\langle P\rangle \bar{V} = \underbrace{\int_0^{R_0} G \rho M_r 4\pi r \, dr}$$ → Gravitational energy of the Sun

Plasma

Let us now multiply by 4 π R³ and integrate by parts between the center of the Sun and R_0, which is the radius of the Sun. Let's now consider only the left-hand side of this equation and let's integrate by parts. We can easily observe that the first term goes to zero, since the pressure at R_0 is assumed equal to zero. Therefore we're left with the following expression, which we can write in the following way where we have defined an average pressure where V is now the total volume of the Sun. Now if we go back to the hydrostatic equilibrium equation and we plug what we just found into this equation, we end up with the following, where the right-hand side member represents the gravitational energy of the Sun.

Notes

Summary

8m 24s

Hydrostatic equilibrium leads to nuclear fusion

$$\int_0^{R_\odot} \rho G M_r 4\pi r \, dr = \int_0^{R_\odot} \rho^2 G 4\pi 4\pi \frac{r^4}{3} dr =$$

$$= \rho^2 \frac{(4\pi)^2}{3} G \int_0^{R_\odot} r^4 dr =$$

$$= \rho^2 \frac{(4\pi)^2}{3} G \frac{R_\odot^5}{5} = G \frac{m_p^2 N^2}{V^2} \frac{(4\pi)^2}{15} R_\odot^5 =$$

$$= \frac{m_p^2 N^2}{R_\odot} \frac{3}{5} G$$

$$\frac{m_p^2 N^2}{R_\odot} \frac{3}{5} G = 3 \langle P \rangle V = 3 \times 2 N k_B T$$

$$T = \frac{m_p^2 N G}{10 R_\odot k_B}$$

$$N = \frac{M_\odot}{m_p} \simeq 1.2 \, 10^{57} \text{ particles}$$

- Hypothesis 1: the Sun is a perfectly spherical object.
- Hypothesis 2: acceleration can be neglected → hydrostatic equilibrium.
- Hypothesis 3: the volume is filled with a plasma made of N protons and N electrons at temperature T.

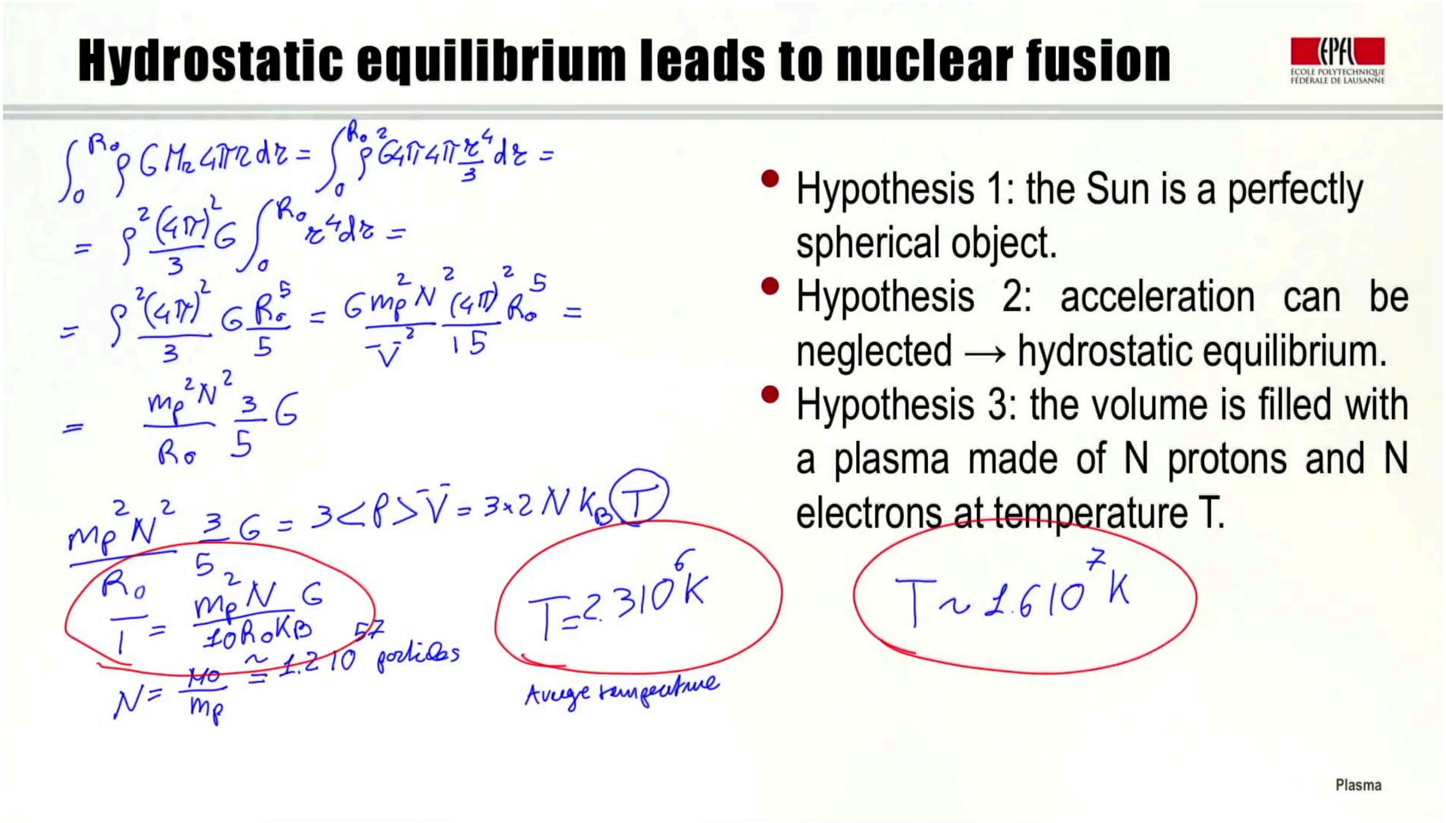
Plasma

Now in order to compute the average pressure, <P>, let's make a further hypothesis. We assume that the volume in the Sun is filled with a plasma, which is made out of N protons and N electrons at the temperature T, and we will then use the equation of state of an ideal gas. We can thus write the average pressure in the following way where KB is the Boltzmann constant. Let's make the following approximation that the mass density is a constant, and we can therefore write in the following form where N is the total number of protons inside the Sun and V is the total volume of the Sun and mp is the mass of a proton, which has the following value in SI units. Then let's compute the total mass inside the shell, the spherical shell, at the distance r in the following way. Now let's go back to the original hydrostatic equilibrium equation and let's plug what we just found in. Using this equation, we can now find an expression for the average temperature T of the Sun. We can also compute the total average number of particles by dividing the mass of the Sun by the mass of the proton and we end up with this amount of particles.

Notes

Summary

9m 41s



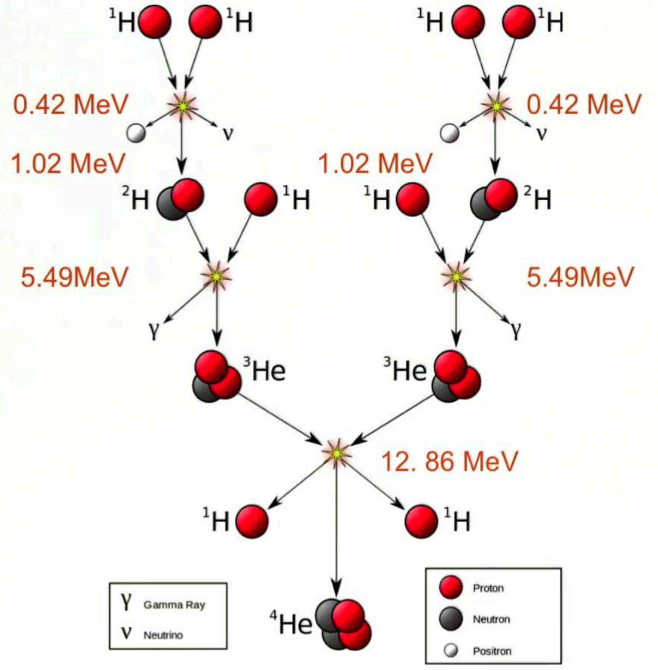
If we now plug the number -in SI units- we just found into the equation for the temperature, we end up with an average temperature of the Sun of approximately 2.3 million degrees Kelvin. This is, of course, an average temperature. In reality, more complex calculations which allow the temperature to vary according to a profile in the radial direction, end up with a much larger temperature, which is of the order of 1.6 x 10^7 degrees Kelvin.

Notes

Summary

11m 42s

The proton-proton chain reaction



- At the core temperature of more than 10^7 degrees, the main fusion reaction is the pp I chain.
- The complete pp I chain releases a total energy of ΔE = 26.22 MeV.

$$E = \Delta E \times \frac{N_0}{10}$$

$$T_{Sun} = \frac{\Delta E \times N_0/10}{L_0} \sim 10^{10}\,y$$

Plasma

At the core temperature of more than 10 million degrees Kelvin, fusion reactions become possible. The main fusion reaction in the core of the sun is the pp 1 chain which is represented here in this schematic diagram. The nuclear reaction proceeds in several steps and the net result is the conversion of four protons into a helium nucleus, which is made of two neutrons and two protons. The reaction also produces neutrinos which escape carrying away part of the energy and two positrons. The net result of the pp 1 chain is the release of a total energy [ΔE] of approximately 26.22 mega electron volts [MeV]. How long can the Sun radiate its luminosity by using this source of energy? We know that approximately 10% of the solar mass is available for hydrogen fusion. This provides an energy reservoir of ΔE times the total number of protons divided by 10. We can now estimate the lifetime of the Sun by dividing this energy reservoir by the Sun's luminosity. We end up with approximately 10 billion years telling us that the present Sun is roughly half way through its hydrogen burning phase.

Notes

Summary

12m 25s

The luminosity of the Sun

- A hot body in local thermal equilibrium at a temperature T contains photons of energy density

$$w_{ph} = \frac{4\sigma_s}{c} T^4$$

$\sigma_S = 5.67\,10^{-8}$ in SI units

- The total radiative energy can therefore be expressed as $W_{ph} = \frac{4\sigma_s}{c} T^4 V$

- ...and the luminosity as

$$L = \frac{W_{ph}}{\text{mean time } t_{ph} \text{ to reach the surface}}$$

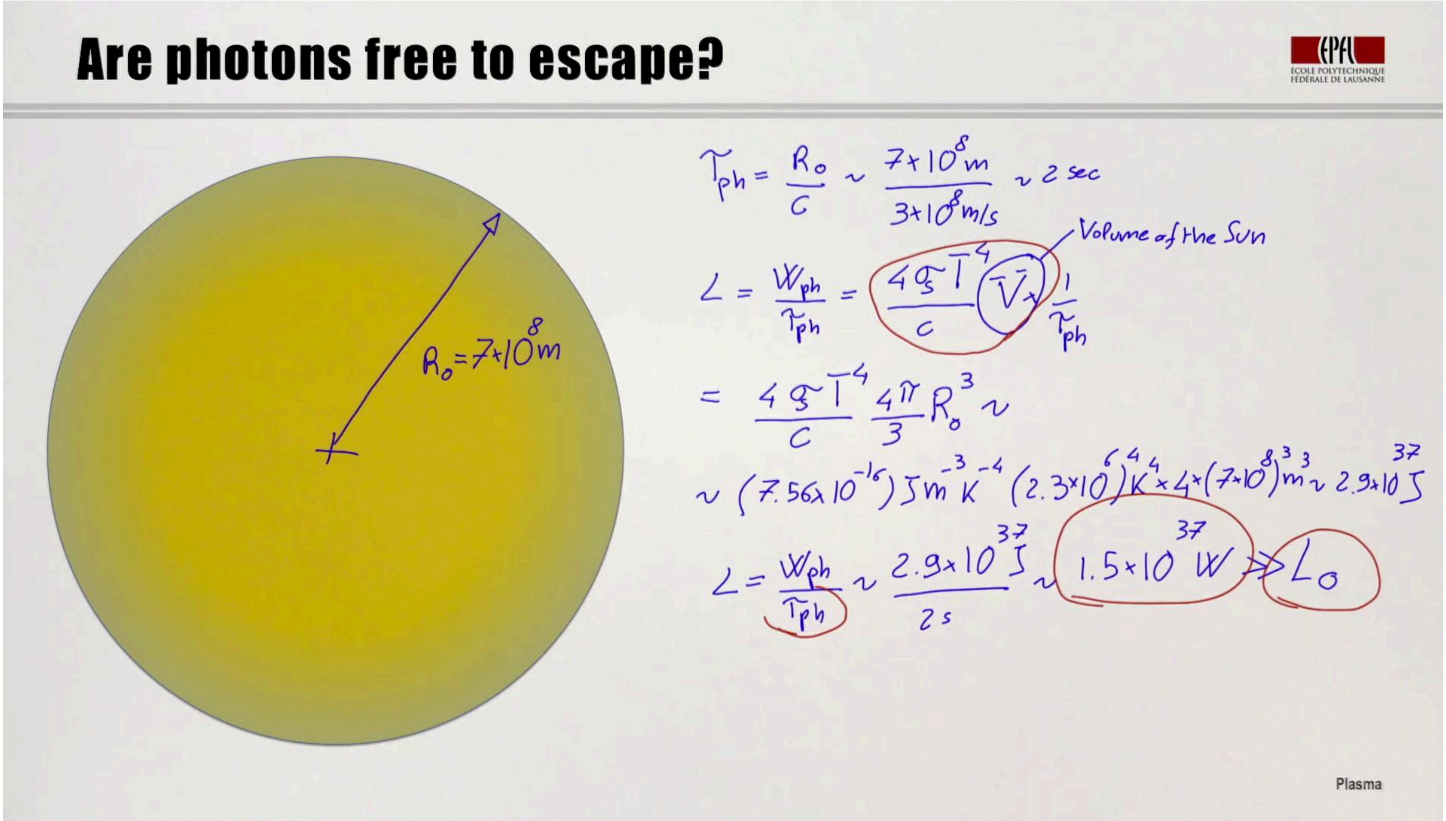
Plasma

Let me briefly summarize what we have done so far. We have approximated the sun as a spherical plasma, which is made of electrons and protons, which are in hydrostatic equilibrium. This simple assumption has allowed us to estimate the Sun's center temperature from its mass and its radius. This temperature is high enough to sustain thermonuclear fusion of hydrogen. Now in the following, let's now try to understand the origin of the solar luminosity. A hot body in local thermal equilibrium at a temperature T contains photons which possess an energy density expressed by this formula. The energy density of the photons depends on the fourth power of the local temperature. This constant σS is the so called Stefan–Boltzmann constant which has this value in SI units. Knowing the total volume of our body, we can compute the total radiative energy which can be therefore expressed in the following way: knowing the total radiative energy of the Sun, we can compute the luminosity simply by dividing the total radiative energy by the mean time t_photon [tph], that the photon will take to reach the surface of the Sun.

Notes

Summary

14m 12s

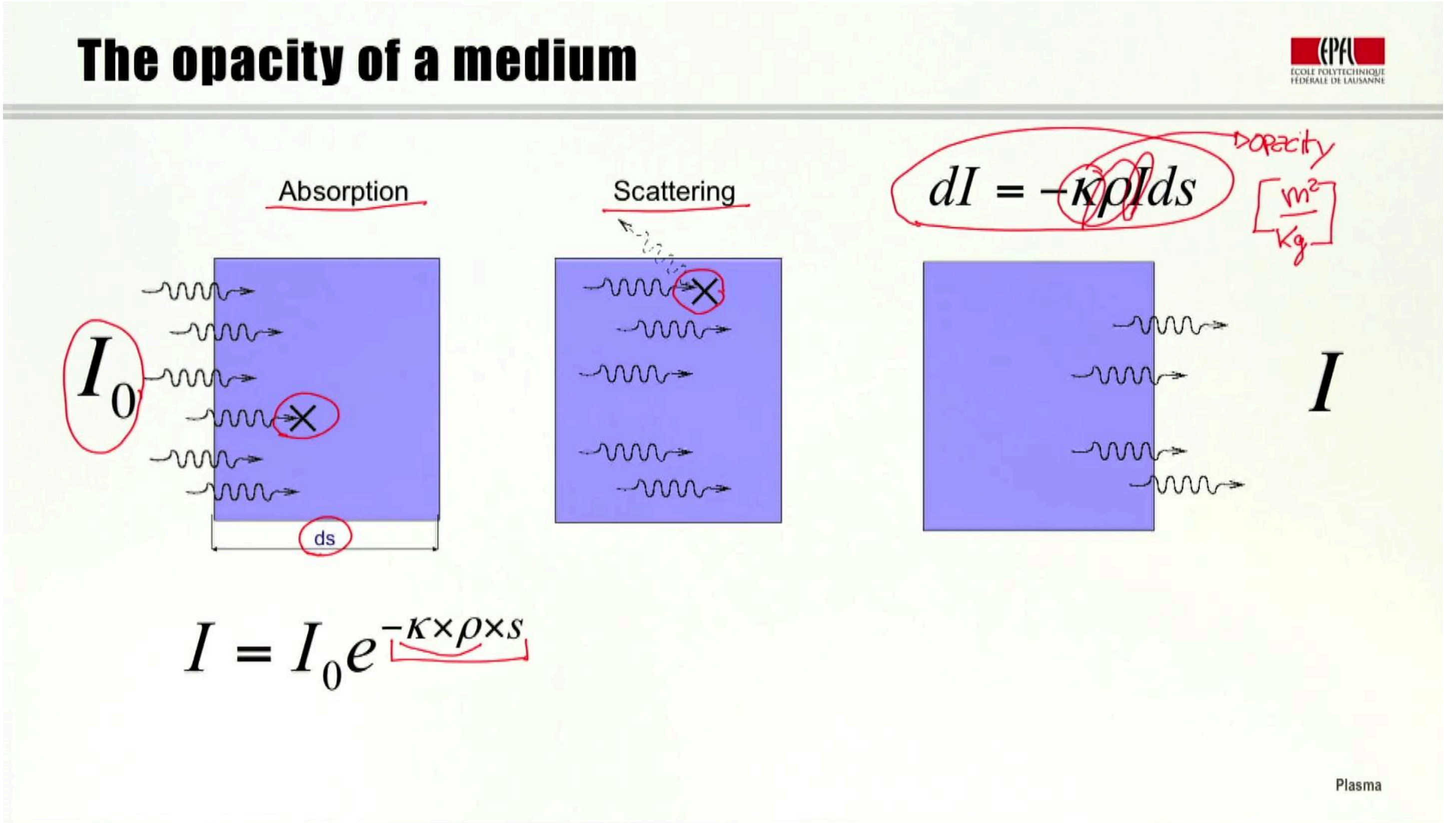


Let's now make a simple exercise and suppose that the photons are free to escape. In this case, a photon that is generated in the center of the Sun will reach the surface in a time that can be simply estimated in the following way. Let's now plug in the real number to estimate the time. By making this simple assumption, a photon that is generated in the center of the Sun will take only approximately 500 seconds to reach the surface. Let us now use the previous formula to compute the Sun's luminosity with these numbers. Let's first compute the total radiative energy of the Sun. Now by plugging the real numbers into this formula we end up with approximately 10^{56} Joules of radiative energy, which is contained in the Sun. Now if we go back to the formula to compute the luminosity and we divide just the number that we have found by the mean time a photon would take to escape from the center to the surface of the Sun, we end up with a value of the solar luminosity which is much larger than the real luminosity of the Sun. Where is the problem? We have assumed that a photon that is generated in the center of the Sun will free fly to the surface. However, the medium, the plasma which the Sun is made of is not transparent. It is opaque. Therefore the answer is in the opacity of the medium. When a beam of photon traverses a layer of thickness, ds, it will undergo a number of phenomena.

Notes

Summary

15m 48s



It can be, for example, absorbed by the medium or it could be scattered by the medium. The end result of this ensemble of processes is that the original intensity of the photon beam will be decreased by a certain fraction depending on the opacity of the medium. This fraction can be expressed as a function of the original intensity of the photon beam of the local density ρ of the medium and the opacity, K, of the medium. Which, in SI units, is measured in square meters per kilogram. This means that over a distance, ds, sufficiently small for K ρ to remain constant, the number of photons is decreased by this factor here. Which are the sources of opacity in the Sun? Strictly speaking, the opacity is a function of the frequency of the electromagnetic radiation, and also depends on a multitude of atomic processes. In the Sun, among the different interactions leading to opacity, the most important are the inverse Bremsstrahlung or free-free absorption, the bound-free absorption or photo-ionization, the bound-bound absorption or photo-excitation.

Notes

Summary

17m 59s

Sources of opacity in the Sun

- Strictly speaking κ is a function of frequency and depends on a multitude of atomic processes.
- Among the different interactions leading to Sun opacity the most important are:
  - **free-free** absorption (inverse Bremsstrahlung)
  - **bound-free** absorption (photo-ionization)
  - **bound-bound** absorption (photo-excitation)
  - **e^- scattering** (Thomson or Compton)
- For T > 10^7 K, the dominant process is Thomson scattering with a cross section:

$$\sigma_T = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-29}\, m^2$$

Plasma

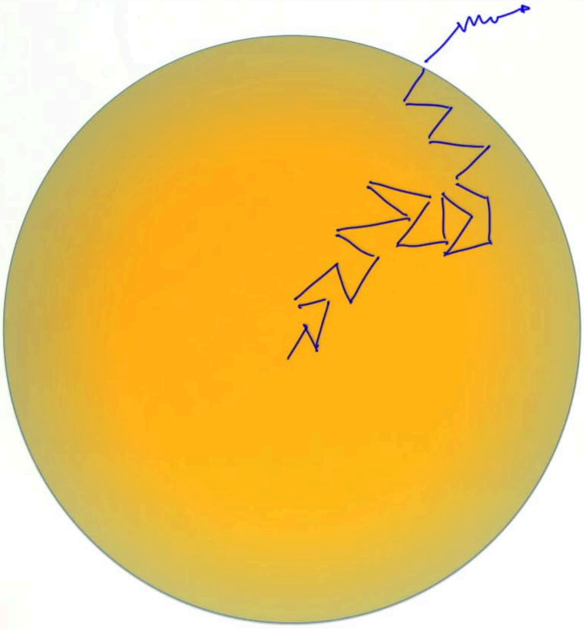
However, for temperatures larger than 10 million degrees the dominant process is Thomson scattering, the scattering that photons undergo over free electrons in the plasma. The Thomson scattering cross section is linked to the classical radius of the electron and it is extremely small, as can be seen in this formula. Using the Thomson scattering cross section, we can compute the mean free path of photons to undergo Thomson scattering on free electrons in the Sun's interior. The mean free path will depend inversely upon the local density of the plasma and the Thomson scattering cross section. Now if we plug the numbers into this formula, we realize that the mean free path for Thomson scattering is extremely small, of the order of a centimeter.

Notes

Summary

19m 40s

Diffusion of photons



- The mean-free path for Thomson scattering in the Sun's interior is:

$$\lambda = \frac{1}{n\sigma_T} = \frac{1}{N\sigma_T}\frac{4\pi R^3}{3} \sim 2cm \ll R_\odot$$

- In this limit, the photons undergo many scatterings before exiting the surface → diffusion process.

$$\langle R^2 \rangle = D\,\tau_{ph}$$

↳ Diffusion coefficient

$$D = \frac{\lambda^2}{\tau_{\text{between collisions}}} = \frac{\lambda^2}{\lambda/c} \Rightarrow \tau_{ph} = \frac{R^2}{\lambda c}$$

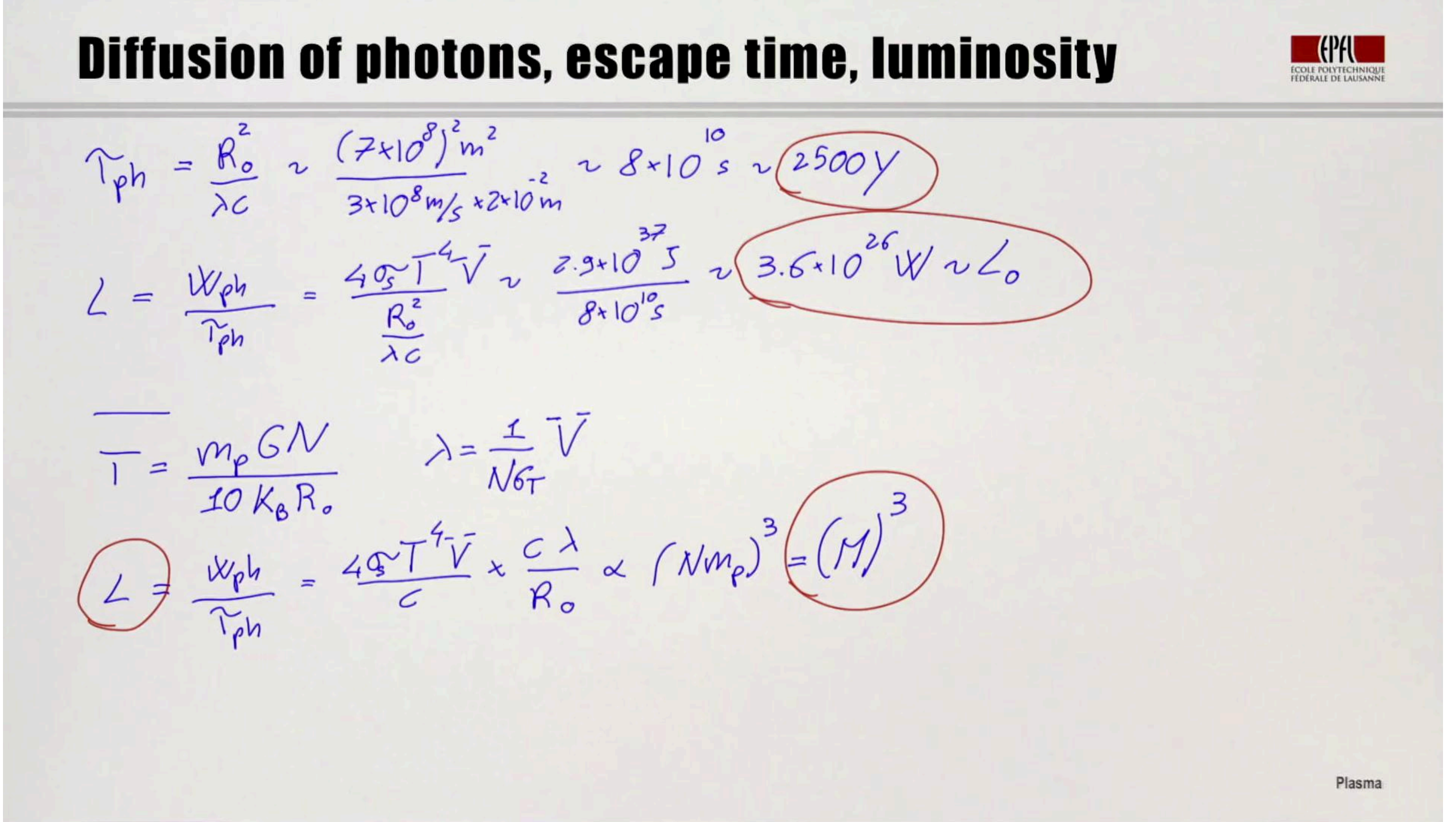
Plasma

This is much smaller than the Sun's radius and in this limit, the photons undergo many scatterings before exiting the surface. We can represent this process as a diffusion process. The spreading of an ensemble of photons that undergo a diffusion process can be represented in the following way, where D is a diffusion coefficient. The diffusion coefficient can be computed as the ratio of the mean free path between two collisions, divided by the effective time between these two collisions. [τ_(between collisions)] If we use this formula, we can compute an average time that it will take for an ensemble of photons to diffuse up to the surface of the Sun in the following way. The striking result is that it will take approximately 100 million years for a photon to escape from the center to the surface of the sun and this reduces dramatically the luminosity of the sun. Now going back to the luminosity, we can estimate it using the formulas and the numbers that we have just computed. If we do that, and we plug the numbers into this formula, we end up with a value which is very close to the actual luminosity of the sun.

Notes

Summary

20m 45s

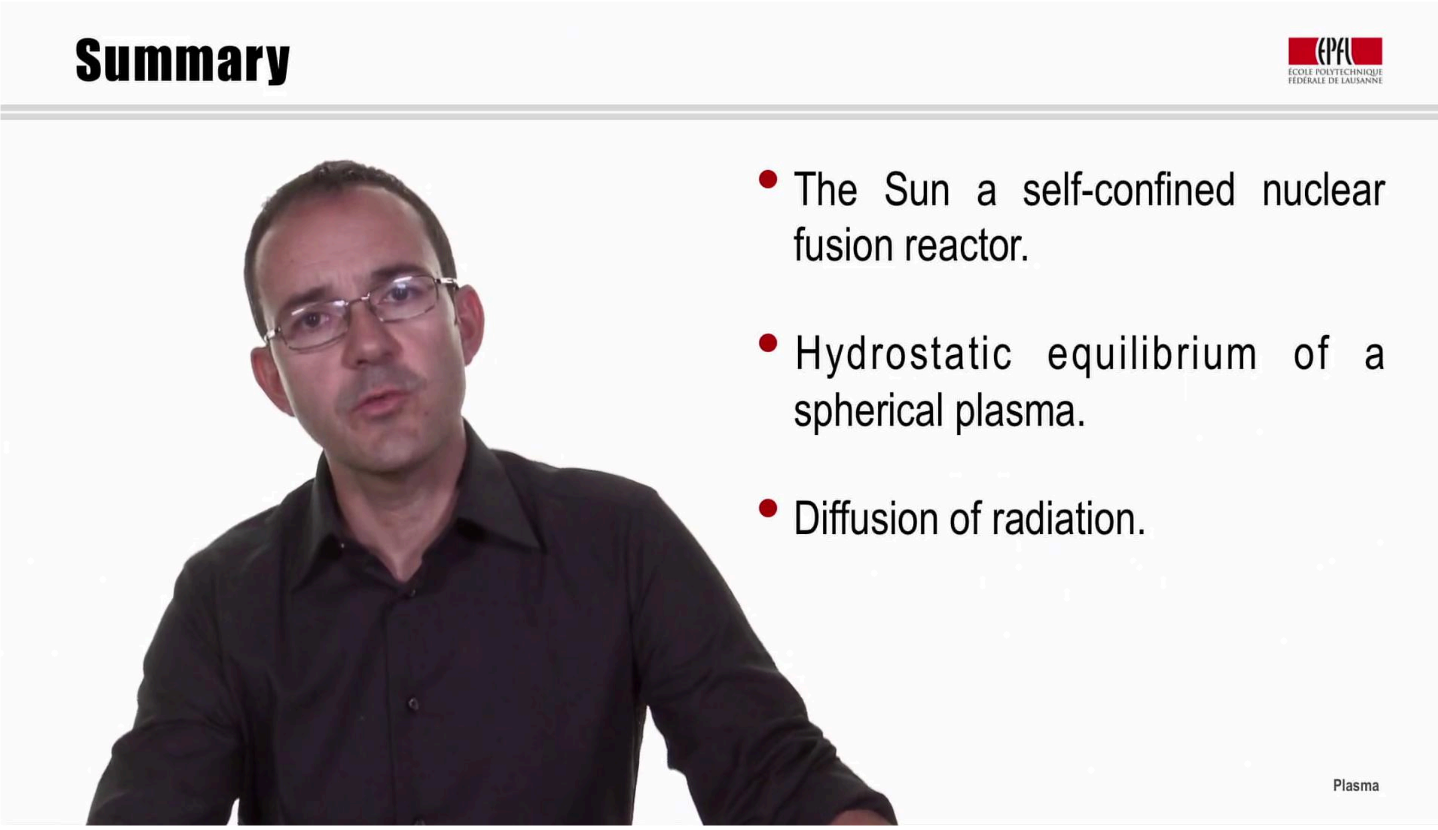


The second important result we can obtain is by recalling the effective average temperature of the sun that we have estimated from the hydrostatic equilibrium. Using the expression for the mean free path for Thomson scattering, we can thus find a relation between the luminosity and the mass of the star. In doing this, we realize that the luminosity scales like the third power of the mass. This is an extremely important result since from basic principles we have now recovered a scaling low, which is observed in nature from experimental data for main sequence's stars. In this module, we have derived estimates of solar properties from basic principles. We have made dramatic approximations to shed light on basic physics. We have made the assumption that the Sun is an homogeneous ball of protons and electrons, which are in hydrostatic equilibrium. We have also assumed an ideal equation of state, which is that of ideal gases. We have also made the assumption that the energy is transported solely by radiation. This has allowed us to compute some of the most fundamental properties of the Sun and in general of main sequence's stars.

Notes

Summary

22m 21s



However, the assumptions that we have made are not entirely satisfied. For example, the Sun contains other elements than hydrogen, and also it shows some internal structures and small deviation from the ideal gases law. Furthermore, the Sun has other important elements that contribute to its dynamics. One of these elements is the presence of magnetic fields, which we will deal with in the following modules.

Notes

Summary

23m 44s