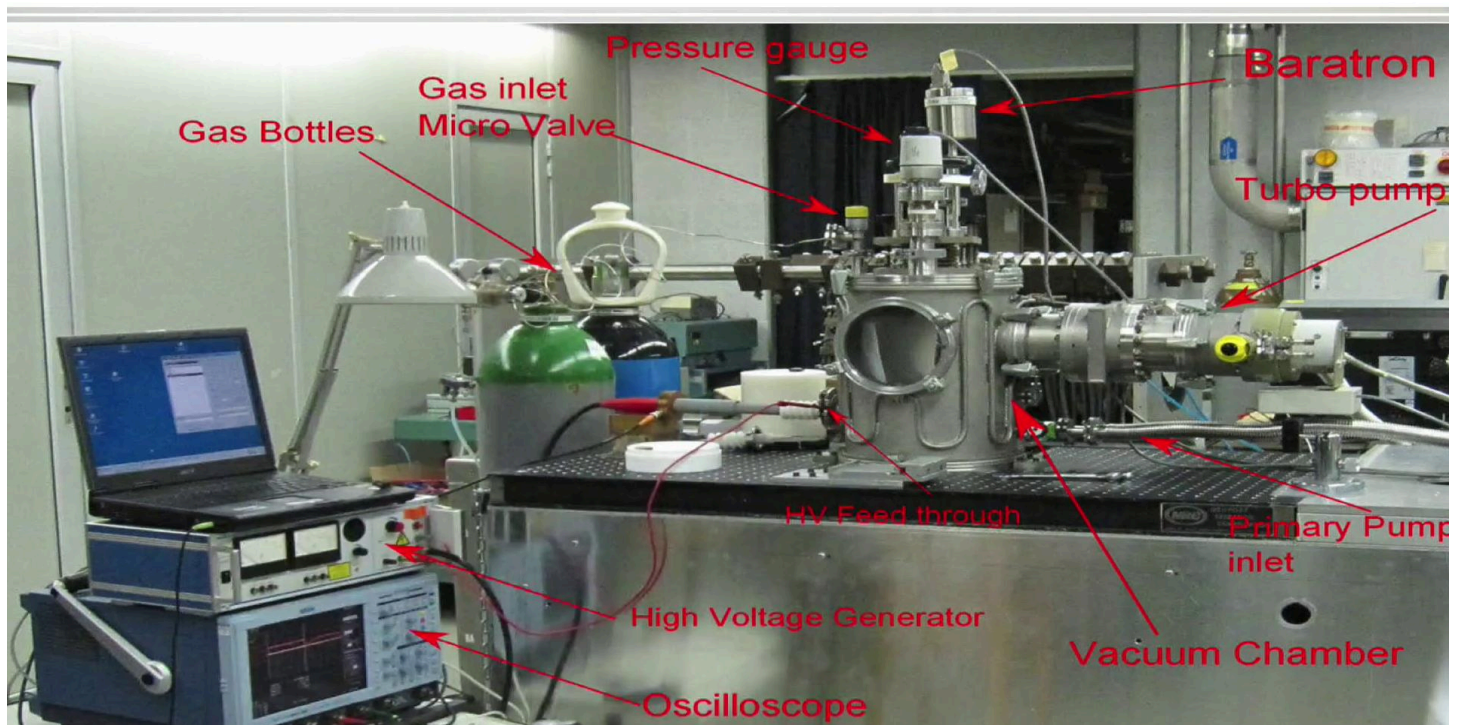




# Introduction



Welcome to the course on plasma physics and applications. In the second module on electrical breakdown in low pressure gases, we will complete the theory and compare it with experimental measurements made in the laboratory.

Notes

Summary





- Townsend criterion & first coefficient
- Paschen's law
- Breakdown measurements
- Vacuum breakdown
- Numerical model

Plasma

We will quickly revise Townsend's criterion and Townsend's first coefficient. We will use these to derive Paschen's law. We will then consider breakdown measurements and to compare measurements with theory we will need vacuum breakdown and finally, a numerical model to compare experiments and theory in the real world.

Notes

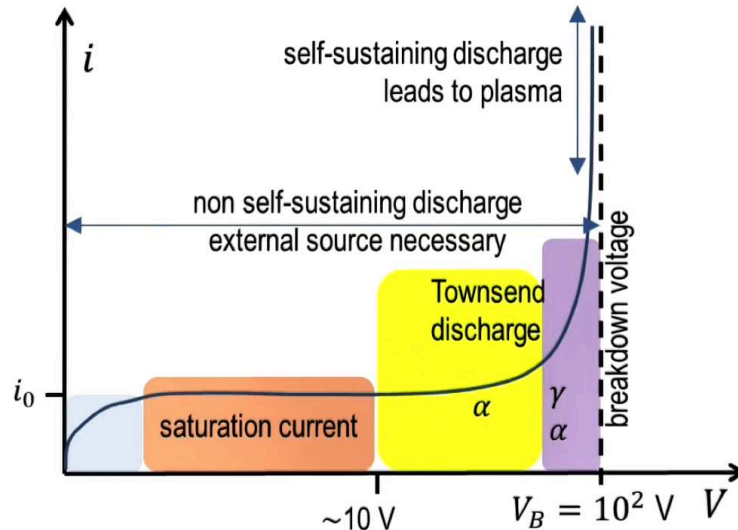
Summary



0m 20s

# Townsend's breakdown criterion

The Townsend criterion for breakdown:  $\gamma e^{\alpha d} = \gamma + 1$



Plasma

For revision, we saw that the Townsend's criterion for breakdown is  $\gamma e^{\alpha d} = \gamma + 1$  where the non self-sustaining discharge eventually leads to a plasma at the breakdown voltage.

Notes

Summary



# Townsend's first coefficient $\alpha$

Consider: *Townsend's first ionization coefficient*,  
 $\alpha$  = number of ionizing collisions per electron, per unit length along  $E$   
 $\alpha = Ap \exp(-Bp/E)$

Let's reconsider Townsend's first ionization coefficient. Where  $\alpha$  is the number of ionizing collisions per electron per unit length along electric field  $E$ . In this phenomenological expression for alpha, which is found from experiment.  $A$  and  $B$  are constants which depends on the gas type.  $p$  is the pressure of the gas and  $E$  is the electric field. Now let's see if we can get a physical understanding for this alpha.

Notes

Summary



1m 02s



# Townsend's first coefficient $\alpha$

Consider: *Townsend's first ionization coefficient*,  
 $\alpha$  = number of ionizing collisions per electron, per unit length along  $E$   
 $\alpha = Ap \exp(-Bp/E)$

The number of ionization collisions per electron in distance  $dx$  is  $\alpha dx$ .

By definition of mean free path  $\lambda$ : The number of collisions in a distance  $dx$  is  $dx/\lambda$

The probability that a collision causes ionization is  $\sim \exp(-\varepsilon_i/(eE\lambda))$ .

The number of ionizations in distance  $dx$  is  $\sim \frac{dx}{\lambda} \exp(-\varepsilon_i/(eE\lambda))$ .

Equating:  $\alpha = \frac{1}{\lambda} \exp(-\varepsilon_i/(eE\lambda))$ , and since  $p \propto \frac{1}{\lambda}$ ,

$$\alpha = Ap \exp(-Bp/E)$$

For parallel plates,  $E = V/d$ , therefore  $\alpha = Ap \exp(-Bpd/V)$

Plasma

I repeat, the number of ionization collisions per electron in distance  $dx$  is  $\alpha dx$ . By definition of mean free path  $\lambda$ . The number of collisions in distance  $dx$  is  $dx/\lambda$ . The probability that each of these collisions causes ionization is a type of Boltzmann factor where  $\varepsilon_i$  is the ionization energy of that atom and  $eE\lambda$  is the energy gained by an electron in crossing a mean free path. So this is a probability that each collision causes ionization. Therefore, the number of ionizations in a distance  $dx$  is given by number of collisions times the probability of ionization. Now we simply equate this expression with the definition of alpha and we find this expression and since pressure is simply related to the inverse of the mean free path, then we rederive this phenomenological expression for alpha. Since this whole discussion has been limited to parallel plates, the electric field is simply the voltage over distance between the plates and therefore we substitute for  $E$  to find this expression for first Townsend's coefficient.

Notes

Summary



1m 38s

# Paschen's law - DC breakdown; parallel plates

breakdown criterion:  $\gamma e^{\alpha d} = \gamma + 1$ ;  
 $\alpha d = \ln\left(1 + \frac{1}{\gamma}\right)$ ; substitute  $\alpha = Ap \exp(-Bpd/V)$   
 $Apd \exp\left(-\frac{Bpd}{V_B}\right) = \ln\left(1 + \frac{1}{\gamma}\right)$ ;  
 solve for breakdown voltage  $V_B$

$$V_B = \frac{B \cdot pd}{\ln(A \cdot pd) - \ln[\ln(1 + 1/\gamma)]}$$

Plasma

Now let's reconsider our breakdown criterion.  $\gamma e^{\alpha d} = \gamma + 1$  We rearrange and substitute our expression for the first Townsend coefficient. Now if this is the breakdown criterion, then this voltage is the breakdown voltage substituting  $V_B$  for the breakdown voltage. We can solve this equation and find the breakdown voltage, gives this expression where  $A$  and  $B$  are the constants which depend on the gas. Gamma is Townsend's second coefficient and its dependence on pressure and distance is the product of pressure and distance everywhere in this equation.

Notes

Summary



3m 04s

# Paschen's law - DC breakdown; parallel plates

Paschen's Law: The DC breakdown voltage between parallel plates depends only on the product  $pd$  for a given gas,

$$V_B = \frac{B \cdot pd}{\ln(A \cdot pd) - \ln[\ln(1 + 1/\gamma)]}$$

or equivalently,  $V_B = \frac{B \cdot pd}{\ln(C \cdot pd)}$ ; where  $C = \frac{A}{\ln(1+1/\gamma)} = \text{cst.}$

Plasma

This is Paschen's Law. It means that the breakdown voltage between parallel plates depends only on the produce  $pd$  for a given gas. This can be rearranged as the breakdown voltage is  $B$  times  $pd$  over the natural log of  $Cpd$ , where  $C$  is simply another constant given in terms of  $A$  and  $\gamma$ .

Notes

Summary



3m 49s

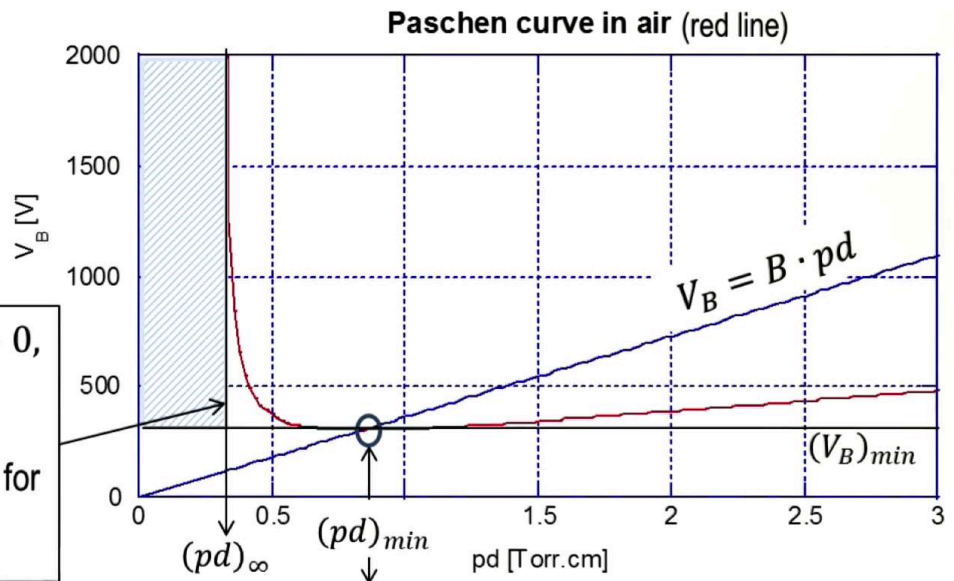


# Paschen's law - DC breakdown; parallel plates

$$V_B = \frac{B \cdot pd}{\ln(C \cdot pd)}$$

where  $C = \frac{A}{\ln(1+1/\gamma)} = \text{cst.}$

1) As  $C \cdot pd \rightarrow 1, \ln C \cdot pd \rightarrow 0$ , and  $V_B \rightarrow \infty$ .  
"No" breakdown is possible, i.e. "infinite" breakdown voltage for  $pd < (pd)_{\infty} = \frac{1}{A} \ln \left( 1 + \frac{1}{\gamma} \right)$ .



2)  $V_B$  is minimum for  $\ln(C \cdot pd) = 1$ ,  
where  $(V_B)_{min} = B \cdot (pd)_{min}$  and  $(pd)_{min} = e^1 \cdot (pd)_{\infty}$  Plasma

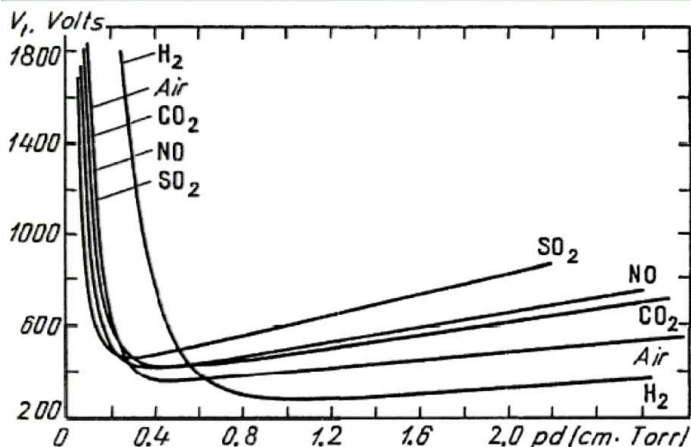
So let's reconsider our Paschen's Law, expressed in this form. It gives this red curve here in air, where the axes are the pressure times the distance and the horizontal axis in units of Torr centimeter here and the breakdown voltage on the Y axis. In air,  $A$  and  $B$  have these values for iron electrodes  $\gamma$  is 0.01 for an air plasma. Now we can see immediately, two strong results from this Paschen curve. The first one, is that as  $C$  times  $pd$  goes to one. That is as the log goes to zero, then the breakdown voltage becomes infinite. Therefore, a result of Paschen's curve is that no breakdown is possible. That is there's an infinite breakdown voltage for all pressure times distance below a critical value given by this expression here. That means at very low pressure times distance, at very low values breakdown should not occur. We also see that there's the minimum in the Paschen's curve, which occurs for  $\log Cpd = 1$ . This means there is a minimum breakdown voltage given by this expression  $B$  times  $pd$  the minimum breakdown voltage where  $pd$  at the minimum breakdown voltage is the natural base of natural logs,  $pd$  infinite.

Notes

Summary



# Paschen's law - DC breakdown; parallel plates



Why is the breakdown voltage the same for  $p \cdot d = \text{constant}$ ?

1) no. of collisions for an electron between the electrodes =  $d/\lambda \propto p \cdot d$

2) the collision energy =  $eE\lambda = eV\lambda/d \propto 1/(p \cdot d)$

Answer: Identical ionization physics for  $p \cdot d = \text{constant}$  (for a given gas)

Plasma

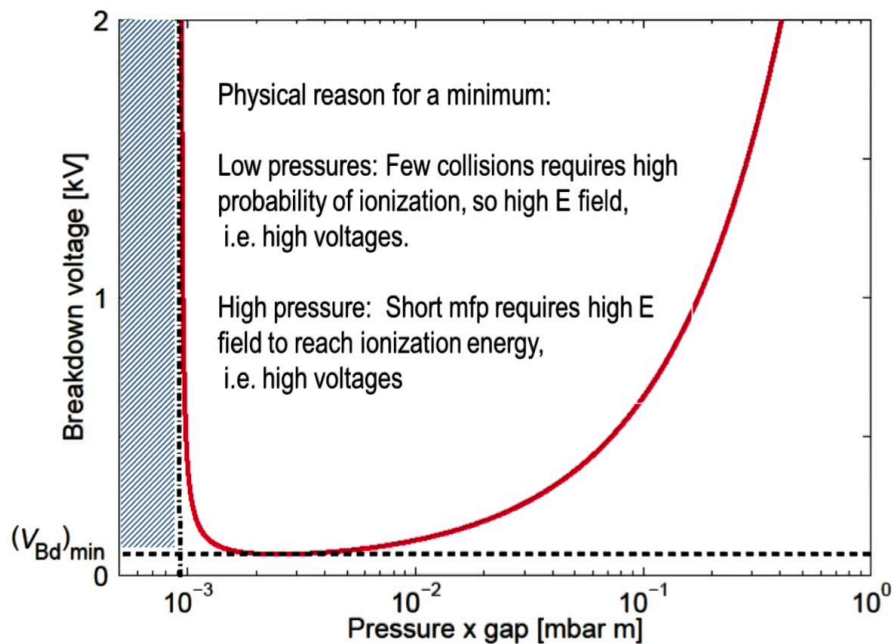
This graph gives an example of Paschen's curve for different gases. We know that the form of the curve depends on alpha, which is the Townsend's coefficient, depends on the gas type, and it also depends on gamma, which depends on the gas type and the type of material. Now we could ask ourselves why is the breakdown voltage the same for constant pressure times distance? Well we can look at this in a phenomenological way by noticing that the number of collisions for an electron, crossing the electrodes is the electrode gap divided by the mean free path, which is proportional to pressure times distance. We also note that the collision energy for each electron is the force times the distance for every mean free path travelled and therefore this collision energy is a function also only of pressure times distance. And therefore a reason as to why pressure times distance is a constant in Paschen's Law is that we have identical ionization physics for a given gas when pressure times distance is a constant.

Notes

Summary



# Paschen's law - DC breakdown; parallel plates



Let's consider a physical reason why there's a minimum in the Paschen's curve. It's because at low pressures there are very few collisions and therefore there has to be a high probability of ionization, which means you have to have a high electric field. That means high voltages are necessary to breakdown at low pressures. Whereas at high pressure, there's only a short mean free path and therefore electrons have to gain all their energy in crossing a very short distance. Therefore, again, you need a high electric field and therefore high voltages. This is why there is a minimum in the Paschen's curve.

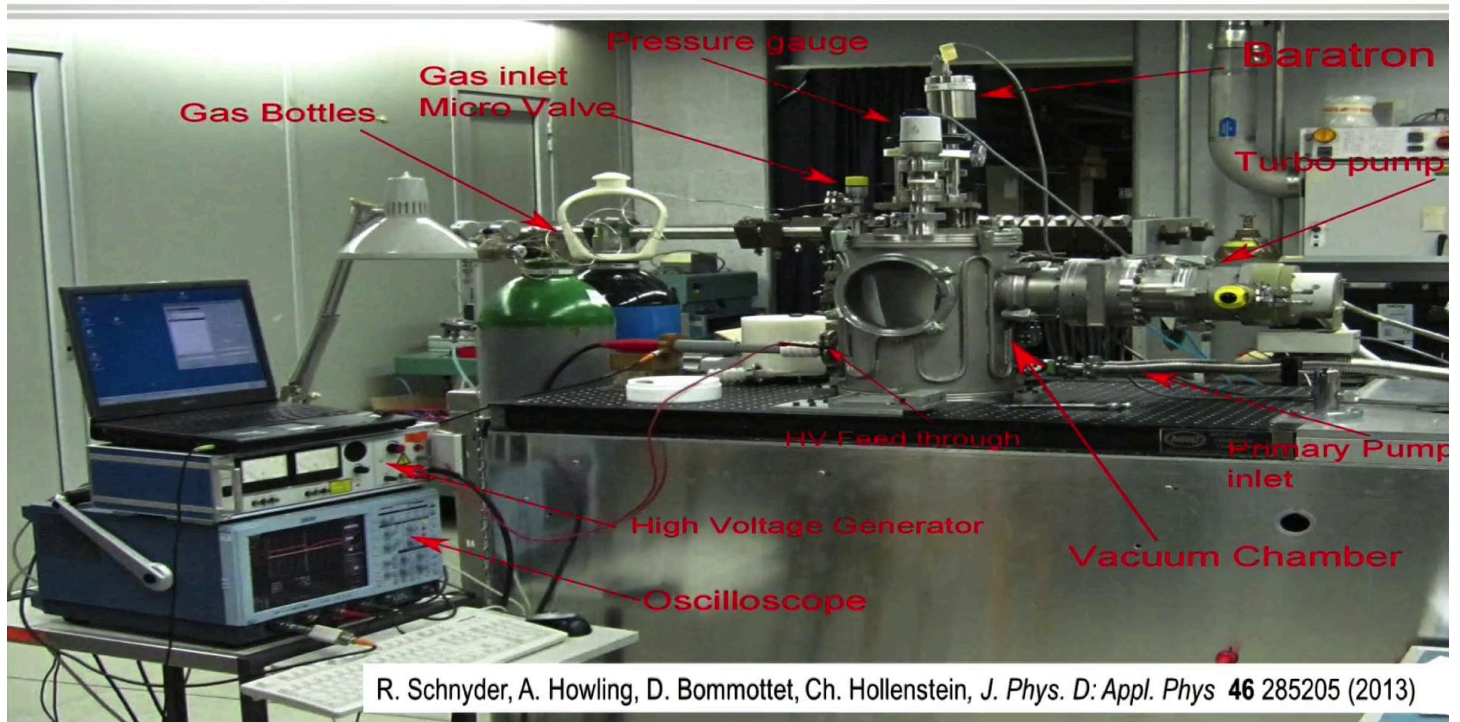
Notes

Summary



7m 12s

# Breakdown measurements for real applications



Now let's consider some breakdown measurements made in the lab for real applications on a slip ring assembly.

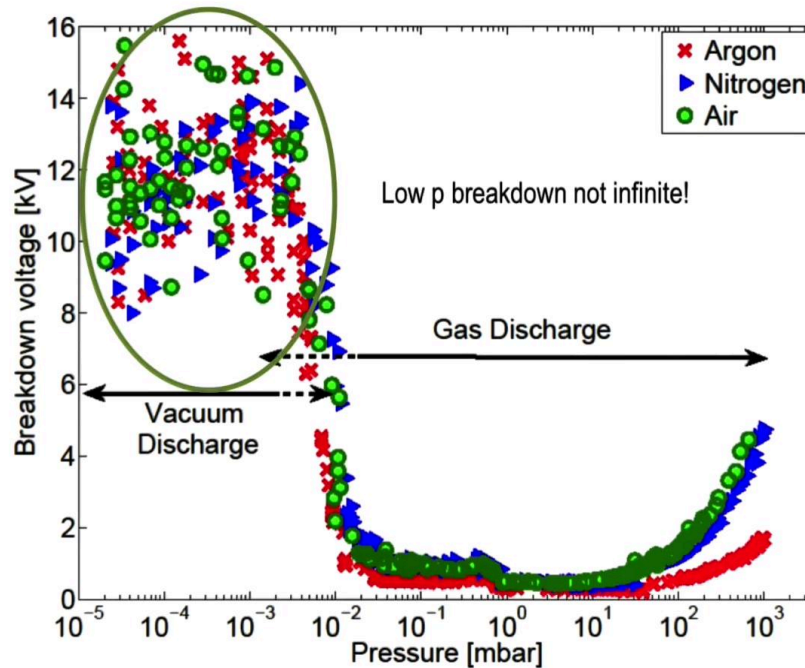
Notes

Summary



7m 53s

# Breakdown measurements



Plasma

When we look at the experimental measurements, we find that the Paschen's curve is more or less reproduced except that we notice we have some breakdown in the so-called vacuum region where Paschen's Law would say there is no breakdown. So the low pressure breakdown in fact is not an infinite voltage.

Notes

Summary



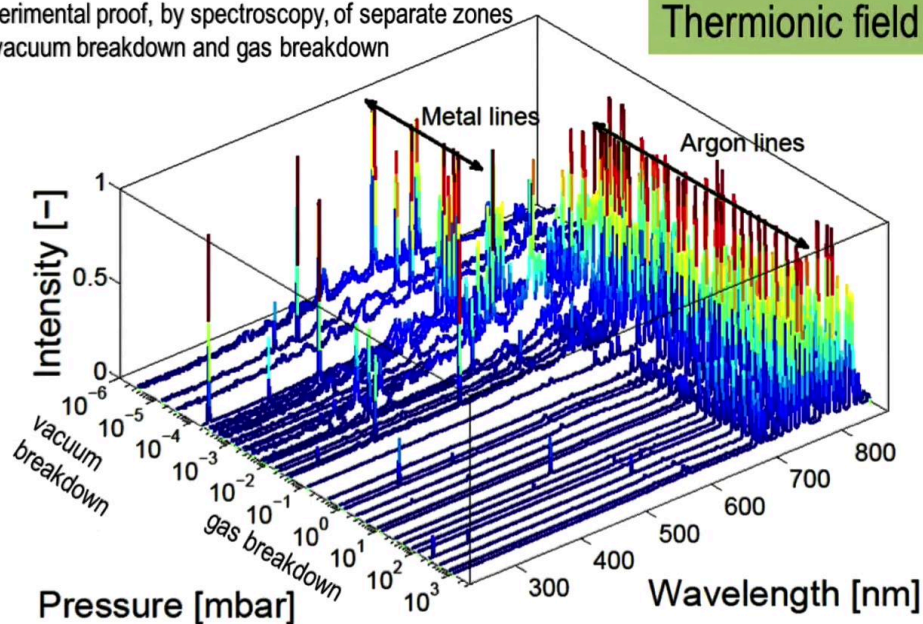
8m 00s



# Vacuum breakdown

Experimental proof, by spectroscopy, of separate zones for vacuum breakdown and gas breakdown

Thermionic field emission



Plasma

This is because in vacuum the electron material itself can breakdown and plasma can occur in the vapor of the metal electrodes. Now this is an experimental proof of that. Where along this axis we have the pressure between the electrodes and here we have the optical spectrum measured of the plasma created for every discharge. We see that in the gas breakdown region, which is from moderate values of pressure all these light emitted from the plasma is the light emitted from the Argon gas in the plasma. Whereas in the vacuum breakdown region, which is below the theoretical Paschen minimum, the lines correspond to vapor from metal elements. Therefore, this is experimental proof by spectroscopy that the vacuum breakdown and gas breakdown are separate phenomena. In fact, vacuum breakdown depends on something called Thermionic field emission, which we will not consider further in this module.

Notes

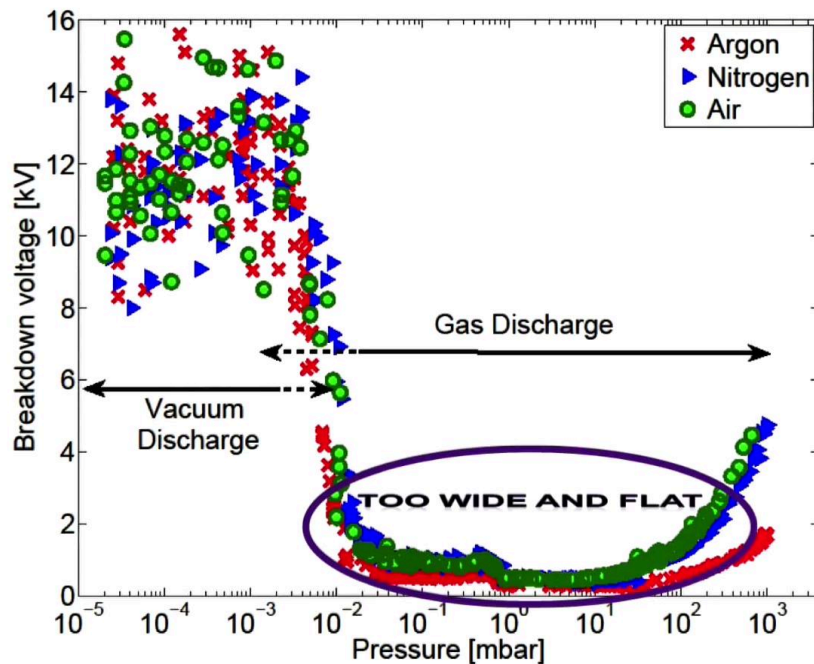
Summary



8m 21s



# Breakdown measurements



Plasma

Having explained the vacuum discharge region, we can now reconsider the gas breakdown where this looks like the Paschen's curve but it's rather too wide and flat compared to a typical Paschen's curve.

Notes

Summary



# Breakdown numerical simulation

## A two-fluid numerical model for breakdown in *arbitrary* geometry

Continuity equation for electrons and ions:  $\frac{dn_j}{dt} + \vec{\nabla} \cdot \vec{\Gamma}_j = S_j$ , the source term

The flux term  $\vec{\Gamma}_j = \vec{\Gamma}_j^{\text{Diffusive}} + \vec{\Gamma}_j^{\text{Convective}}$

$-D_j \vec{\nabla} n_j$

$n_j u_j$ ,  
 $u_j = \pm \mu_j E$   
is the drift velocity

$S_j = n_e \alpha u_e$ ,  
total ionization rate per unit  
volume due to electron  
collisions

$\vec{E} = -\vec{\nabla} V$      $\nabla^2 V = 0$   
Vacuum potential;  
Poisson's equation for negligible  
charge densities

... a simple closed set of  
equations for a practical  
application

Plasma

Remember that the Paschen's curve was derived only for the case of parallel electrodes but in the real world, such as a slip ring, we'll have to calculate the breakdown in arbitrary geometry. Here is a simple two-fluid numerical model to do that. We start with a continuity equation for electrons and ions. The flux term here is the sum of the diffusive term, which is Fick's law and the convective flux, which is density times the drift velocity. If we know the mobility of each species, electrons and ions, we can calculate the drift velocity. The electric field we can calculate from the vacuum electric field, Laplace's equation, because we know that the charge density is negligible, for which case the Poisson's equation becomes the Laplace equation. So once we know the electric field, we can calculate Townsend's first coefficient and then we have a total ionization rate per unit volume due to electron collisions. This is a simple closed set of equations for a practical application.

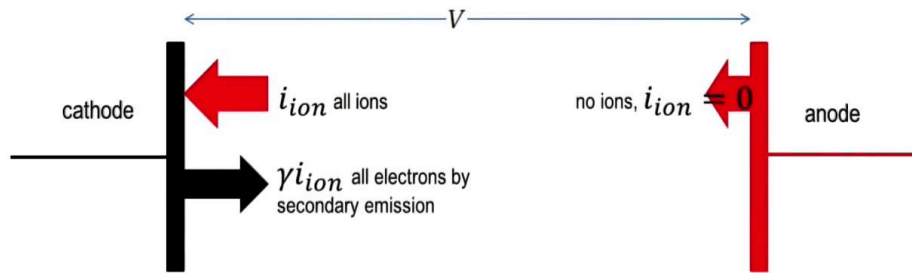
Notes

Summary



# Breakdown numerical simulation

A two-fluid numerical model for breakdown in *arbitrary* geometry



## Boundary conditions:

(we will use flux boundary conditions, instead of "density=0", to avoid numerical instabilities)

- For ions:  $\vec{n} \cdot \vec{\Gamma}_i^{\text{anode}} = 0$
- For electrons:  $\vec{n} \cdot \vec{\Gamma}_e^{\text{cathode}} = -\gamma_{se} \vec{n} \cdot \vec{\Gamma}_i^{\text{cathode}}$
- Convective flow to all other conducting or insulating surfaces
- Voltage across the electrodes,  $V$

Plasma

Of course we have to consider boundary conditions for our numerical model. For the ions, this is simply that the flux of ions leaving the anode is zero. The flux of electrons leaving the cathode is Townsend's second coefficient times the flux of ions arriving at the cathode. That is for all the ions arriving at the cathode, there are gamma times that number of electrons leaving the cathode. We assume convective flow to all of the surfaces and we apply voltage  $V$  across the electrodes.

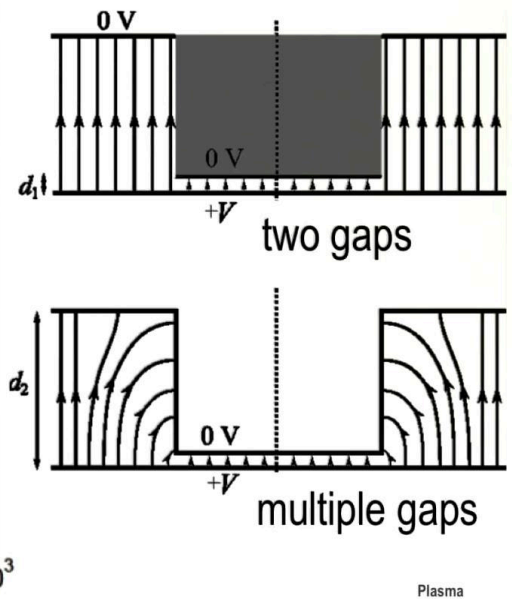
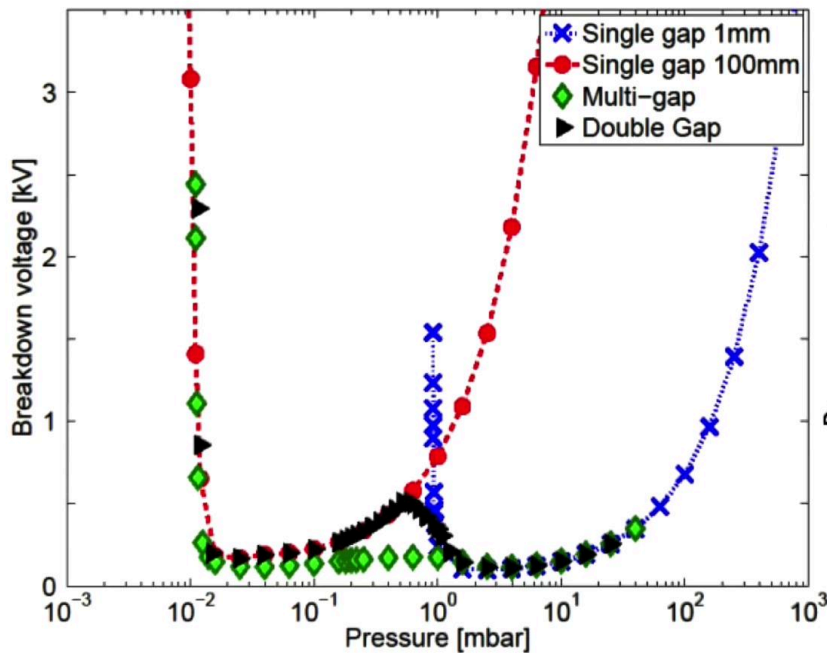
Notes

Summary



11m 04s

# Breakdown numerical simulation



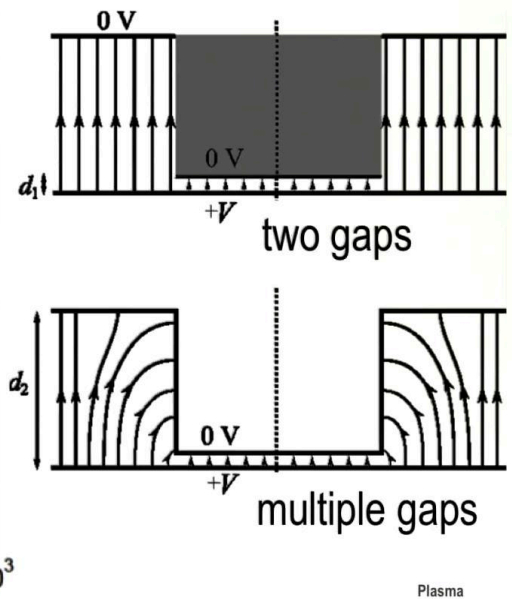
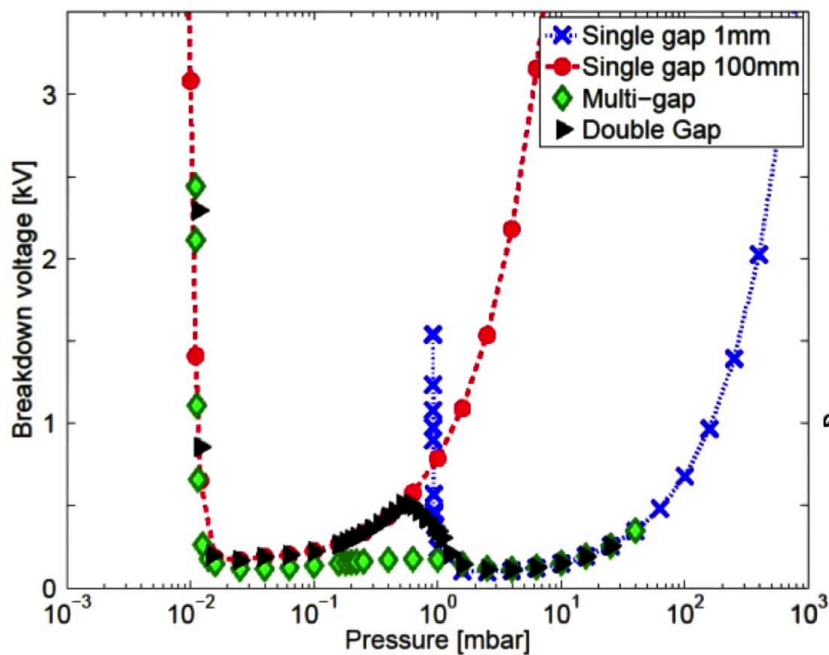
Here are some results from a numerical simulation. If we consider a single gap of parallel electrodes one millimeter apart, then we find the Paschen's curve which is this blue line here as expected. If we consider a single gap between parallel electrodes a hundred millimeters apart, then we find also Paschen's curve but which is displaced to lower pressures because on this graph the axis is pressure, not pressure times distance and therefore these two curves will superimpose on a Paschen plot of pressure times distance. Now let's consider a special case of electrodes where we have a large gap and simultaneously a small gap. So we have electrodes assembly with two gaps present and now the numerical simulation gives the physically reasonable answer, the breakdown at high pressure occurs in the small gap but as we get to lower pressures, the breakdown switches to breakdown in the large gap and therefore we get the black curve for numerical simulation, which is the breakdown in this double gap electrode geometry. Now if we consider electrodes which are conducting on every surface here, we find that the electric field is curved and gives a possibility of breakdown in a range of gaps, not only breakdown in the small gap but also breakdown in the large gap.

Notes

Summary



# Breakdown numerical simulation



And now the numerical simulation shows that the breakdown occurs in the small gap only at high pressure but then in intermediate pressures, the breakdown now will occur in the gap length, which gives the optimum lowest breakdown voltage instead of the double gap curve, which is black. Therefore, now the numerical simulation result goes from high pressure, remains at low values all the way down to the lowest pressure for Paschen breakdown.

Notes

Summary



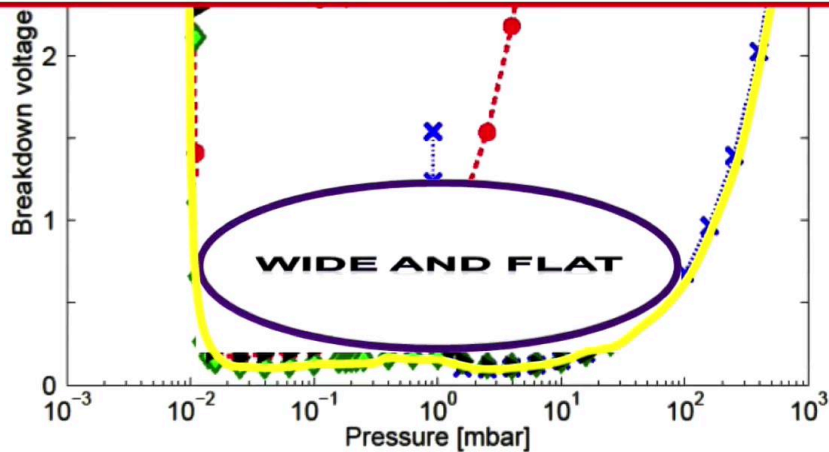
13m 25s

# Breakdown numerical simulation

resultant curve for multiple gaps:

**wide and flat**

due to availability of different gaps to obtain minimum breakdown voltage over a wide range of pressures.



Plasma

The resultant curve for multiple gaps shows that we have therefore breakdown at minimum breakdown voltage over a whole range of pressures. This gives a very wide and flat curve as we observe experimentally.

Notes

Summary





# Summary



- Townsend's first coefficient  
 $\alpha = Ap \exp(-Bp/E)$
- Paschen's law for parallel plates  
$$V_B = \frac{B \cdot pd}{\ln(C \cdot pd)}$$
- Breakdown measurements
- Vacuum, & multi-gap breakdown
- Numerical model for applications

Plasma

To summarize this module, we've derived an expression for Townsend's first coefficient. After this we find Paschen's Law for parallel plates where the breakdown is a function only of pressure times distance. By comparison with breakdown measurements we see that we also need to compare vacuum and breakdown in multiple gaps. We did this using a numerical model for the applications of a slip ring.

Notes

Summary



14m 16s