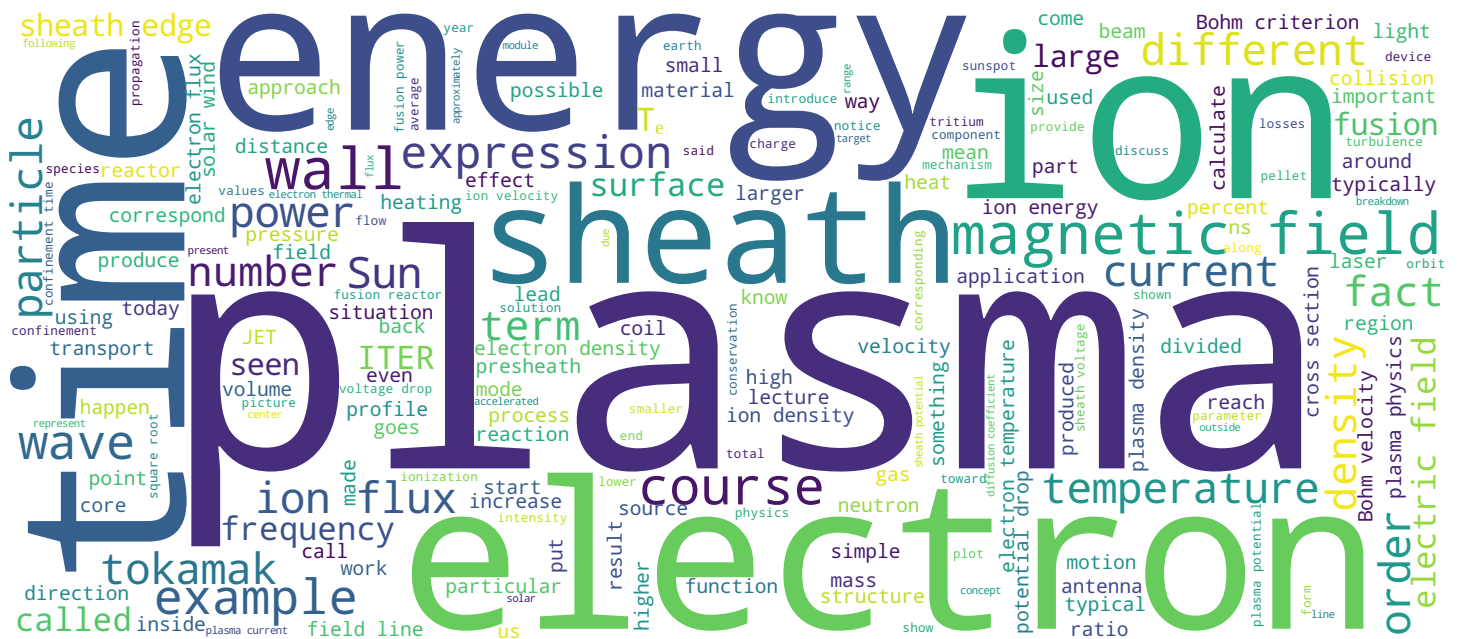


Sheaths and plasma etching: part II

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 5e

Alan Howling





- Sheath potential and density profiles
- Ion flux across a collisionless sheath
- Bohm criterion
- Ion energy to a floating wall

Plasma

Welcome to the course on plasma physics and applications. This is the fifth and last module on basic industrial plasmas. In the previous module we made a qualitative description of plasmas in contact with a wall and now we will introduce a mathematical description of sheath physics. We will show how to calculate the profiles of potential and density across a sheath. We will find the ion flux across a collisionless sheath. We will calculate the Bohm criterion. And also the ion energy across a sheath to a floating wall.

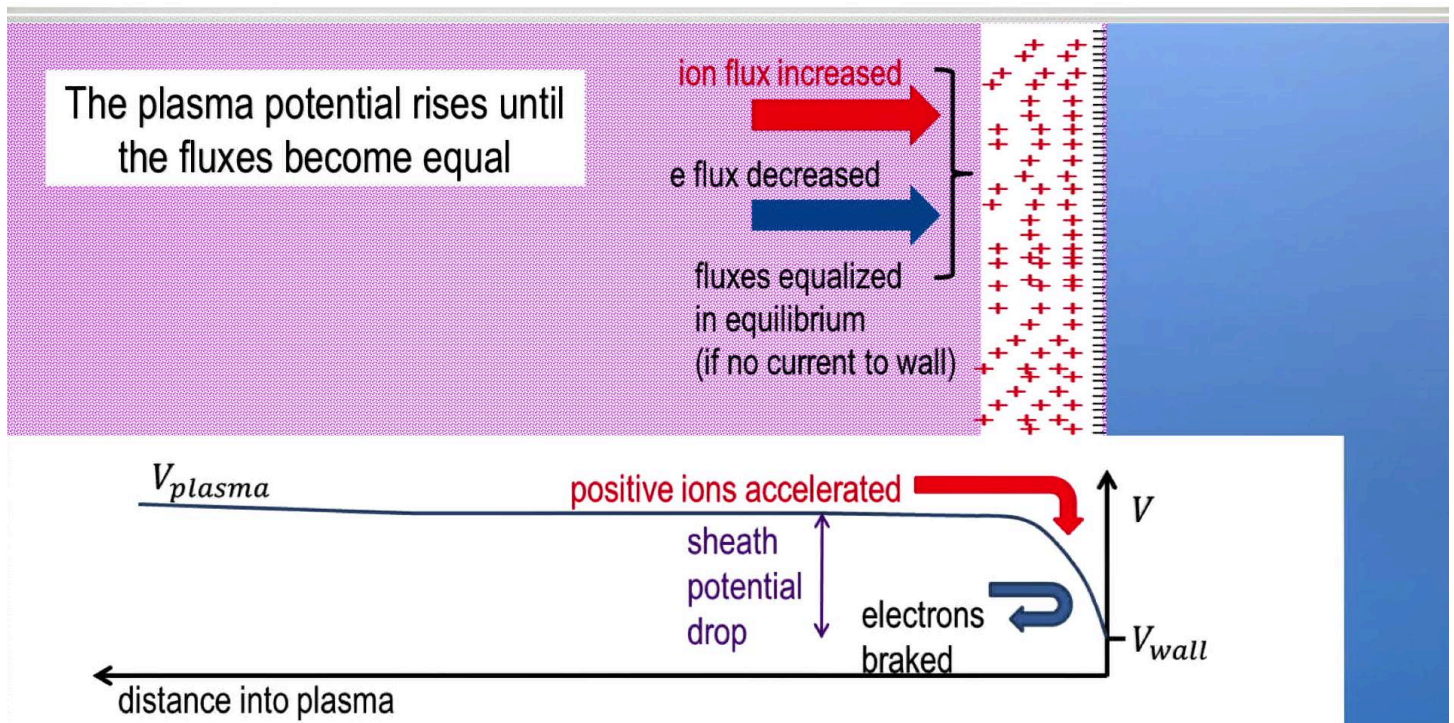
Notes

Summary



0m 05s

Formation of a sheath - revision



This is a revision of the sheath from the previous module. We saw that the plasma potential rises until the fluxes become equal. So we have equal ion and electron flux to the wall. The positive ions are accelerated in the drop of sheath potential from the plasma potential down to the wall potential. This same sheath potential drop has the effect of braking electrons to equalize the charge particle fluxes.

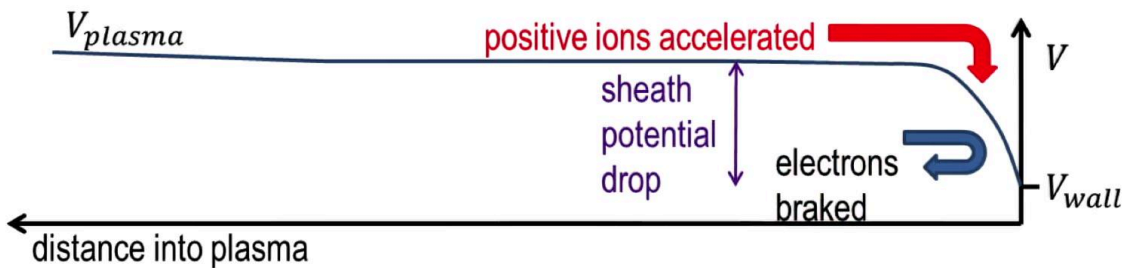
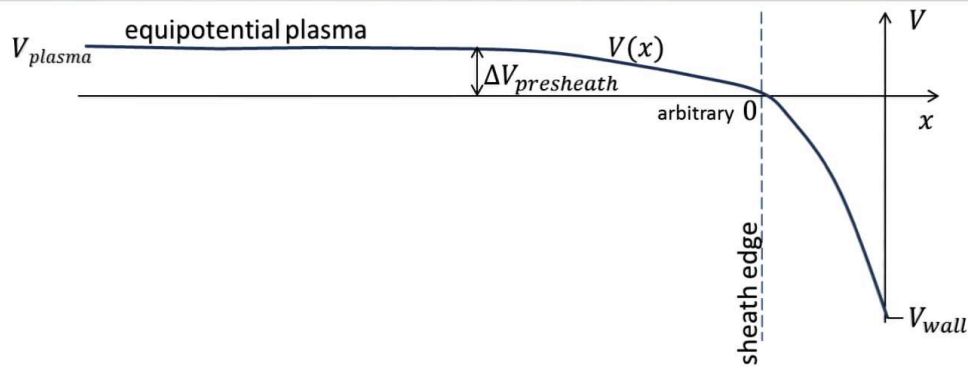
Notes

Summary



0m 47s

Sheath potential (qualitative)



Plasma

Now let's introduce some parameters onto our qualitative profile for plasma potential. We say that the potential on the wall is V_{wall} , the potential profile is $V(x)$, where x is the distance from the wall and V_{plasma} is the potential of the plasma, which is equipotential at a large distance from the wall. We will introduce an *arbitrary 0* of potential at the edge of the sheath, which will be defined later on and there is a drop from the plasma potential down to the sheath edge, which we will call $\Delta V_{presheath}$.

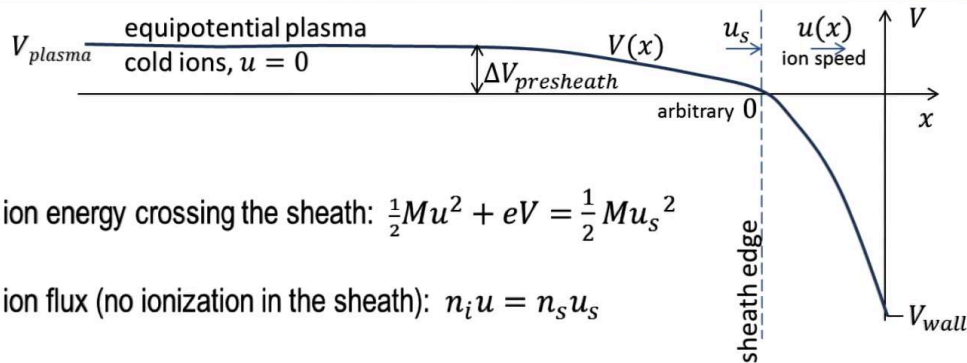
Notes

Summary



1m 20s

Ion density



Conservation of ion energy crossing the sheath: $\frac{1}{2}Mu^2 + eV = \frac{1}{2}Mu_s^2$

Conservation of ion flux (no ionization in the sheath): $n_i u = n_s u_s$

eliminate u :

$$\text{Ion density in sheath: } n_i = n_s \left(1 - \frac{2eV}{Mu_s^2}\right)^{-0.5}$$

Plasma

Let's consider the ion velocity in this plasma potential profile. At a far distance from the wall, these cold ions have zero drift velocity in the equipotential profile. As ions approach the sheath edge, they are accelerated across the presheath potential drop $\Delta V_{presheath}$ and they reach a velocity U_s at the sheath edge. They are further accelerated, their ion speed increases with a value $u(x)$ between the sheath and the wall. Let's consider the conservation of ion energy as they cross the sheath. The energy of the ions as they approach the sheath is a $\frac{1}{2} Mu_s^2$ and as they cross the sheath their energy is $\frac{1}{2} Mu^2 + eV$ as they cross the sheath. This voltage here is in fact negative. If we consider that the sheath is collisionless and therefore there are no collisions, no ionizations in the sheath, then conservation of ion flux simply gives the ion density times the velocity in the sheath is equal to the ion density at the sheath edge times the ion velocity at the sheath edge. Now let's simply eliminate the unknown ion speed in the sheath, u and we find an expression for the ion density in the sheath. Ion density in the sheath equals the ion density at the sheath edge times this expression.

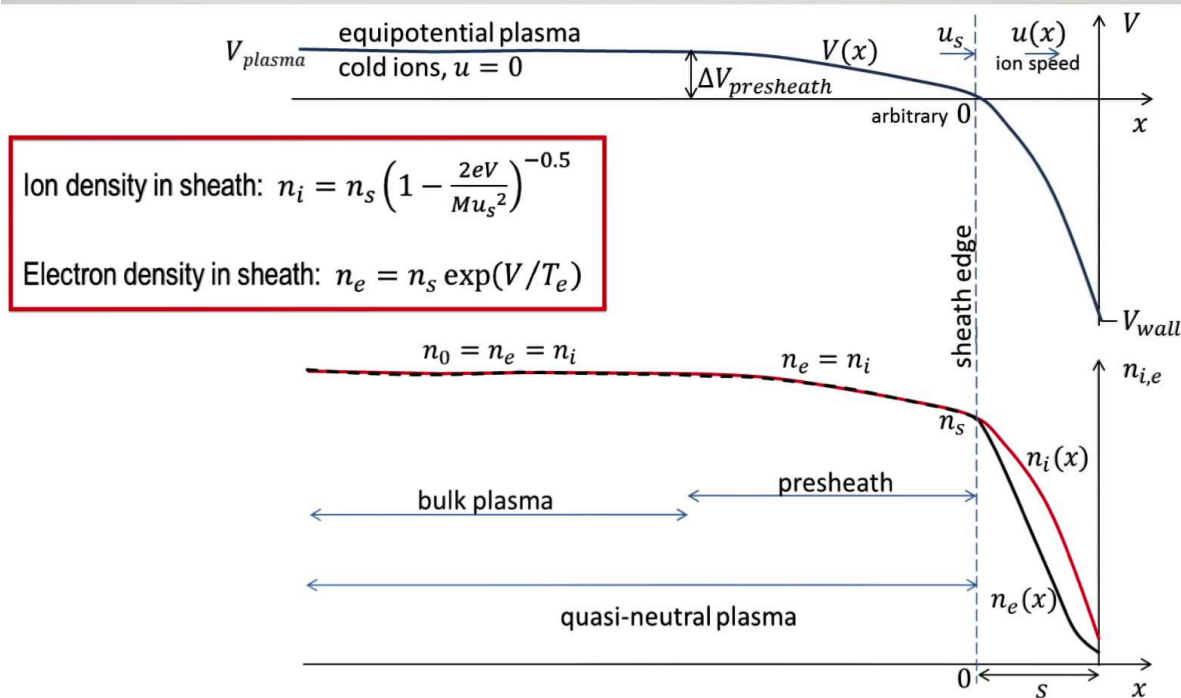
Notes

Summary



2m 03s

Ion & electron densities



Let's also introduce a profile for the charge particle densities in this situation. In the equipotential plasma, that is in the bulk plasma the densities of the electrons equal to the density of ions for a quasi-neutral plasma. $n_0 = n_e = n_i$ is the density in the plasma. In the presheath we consider that the plasma is still relatively equipotential and therefore we have electron density still equal, closely, to the ion density. At the sheath edge, therefore, the density of electrons and ions are both equal to n_s . This is the quasi-neutral plasma. However, in the sheath the electron density is strongly reduced as we saw due to the sheath potential drop. Therefore, we have the $n_e(x)$ electron density profile is lower than the $n_i(x)$ the ion density profile in the sheath, whose width is s . To summarize, we have the ion density in the sheath from our previous slides and using the Boltzmann relation, the electron density in the sheath is the electron density at the sheath edge n_s times the Boltzmann factor $\exp(V/T_e)$, V over electron temperature, in the sheath. Now these two expressions are different because the ion flux is determined by the drift speed of the ions whereas the electron thermal velocity is much greater than the drift speed and therefore still determined by the Boltzmann relation.

Notes

Summary



Bohm's criterion

Using 1D Poisson's equation in sheath: $\frac{d^2V}{dx^2} = -(n_i - n_e) e / \epsilon_0$,

substitute for the charge densities: $\frac{d^2V}{dx^2} = \frac{en_s}{\epsilon_0} \left[\exp\left(\frac{V}{T_e}\right) - \left(1 - \frac{2eV}{Mu_s^2}\right)^{-0.5} \right]$.

At the plasma-sheath interface, do a Taylor expansion in small x :

$$\frac{d^2V}{dx^2} \sim \frac{en_s}{\epsilon_0} \left[\left(1 + \frac{V}{T_e}\right) - \left(1 + \frac{eV}{Mu_s^2}\right) \right] = \frac{en_s}{\epsilon_0} \left[\frac{1}{T_e} - \frac{e}{Mu_s^2} \right] V.$$

For a physical solution (non-oscillatory), $\left[\frac{1}{T_e} - \frac{e}{Mu_s^2} \right] \geq 0$

Important Conclusion: The ion speed entering the sheath is

$$u_s \geq u_B = \sqrt{\frac{eT_e}{M}}$$

THIS IS BOHM'S CRITERION FOR THE ION VELOCITY LEAVING THE PLASMA

Plasma

Now we introduce Poisson's equation in the sheath. That is the second differential of voltage with respect to the distance is minus the charge density divided by the vacuum [permittivity]. The charge density is the ion density minus the electron density, multiplied by the magnitude of the electronic charge. We substitute for the charge densities from the previous slide. This is the electron contribution. This is the ion contribution. At the plasma-sheath interface, where x is small, we will do a Taylor expansion of these expressions. The exponential appears as $1 + V/T_e$ and the binomial expansion introduces this expression here and we find finally this expression for Poisson's equation. Now for a physical solution because we expect the voltage profile to be non-oscillatory with distance, we require the expression in the brackets must be greater than zero. From this inequality, we find an important conclusion that is that the ion speed entering the sheath, U_s has to be bigger than the square root of eT_e/M where T_e is the electron temperature in electron volts, therefore, eT_e is the electron thermal energy in Joules and M is the mass of the ions.

Notes

Summary



5m 38s

Bohm's criterion

Important Conclusion: The ion speed entering the sheath is

$$u_s \geq u_B = \sqrt{\frac{eT_e}{M}}$$

THIS IS BOHM'S CRITERION FOR THE ION VELOCITY LEAVING THE PLASMA

- The ion velocity depends on the electron temperature.
- The ions are accelerated to energy $\frac{1}{2}Mu_B^2 = eT_e/2$ in the presheath.
- The presheath potential drop $V_{presheath} = T_e/2$ (this is consistent with the Debye screening)
- The plasma density at the sheath edge is $n_s = n_0 \exp(-\Delta V_{presheath}/T_e) = n_0 e^{-0.5} = \frac{n_0}{\sqrt{e_{nat}}} = 0.61n_0$

Plasma

Therefore, we see the ion speed entering the sheath is determined by the ion mass and the electron temperature, not the ion temperature. This inequality, the lower limit is called the "Bohm velocity" for ions. This is the Bohm criterion for the ion velocity leaving the plasma and entering the sheath. The Bohm criterion is an important factor in sheath physics. Therefore, we will repeat it here and discuss the conclusions. That is that the ion velocity depends on the electron temperature. The ions are accelerated to an energy of $\frac{1}{2}Mu_B^2$ as they enter the sheath. If you work it out, this corresponds to an energy $eT_e/2$ which the ions gain in crossing the presheath. Now this corresponds to a presheath potential drop, $\Delta V_{presheath} = T_e/2$. Now this is consistent with Debye screening, which allows potentials of up to $T_e/2$ to penetrate an quasi-neutral plasma. Now the plasma density at the sheath edge, is n_s times the electron density in the equipotential plasma bulk times the Boltzmann factor crossing the presheath, electrons are negative, the presheath value is $T_e/2$ divided by the electron temperature. This gives n_0 the base of natural logarithms, one over the square root and this works out to be $0.61n_0$ that is the plasma density at the sheath edge is 61 percent of the plasma density and the plasma bulk far from the wall.

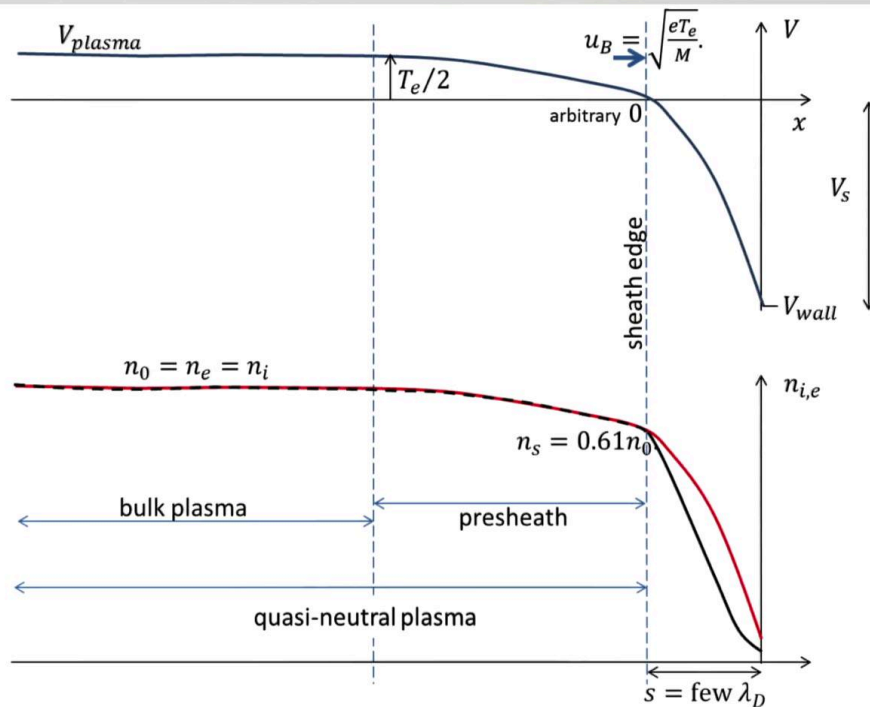
Notes

Summary



7/m 18s

Re-visit potential & density profiles



Plasma

Using this knowledge, we can now go back to our plasma profile and our density profile and start to put some expressions in for the unknown values. The presheath potential drop we found is $T_e/2$ from the plasma to the sheath edge. The ion velocity approaching the sheath edge is the Bohm criterion velocity. The electron ion and plasma density approaching the sheath edge is 61 percent of the charge particle density in the bulk plasma. Now Debye screening means that the sheath width can only be a few Debye lengths long.

Notes

Summary



9m 15s

Bohm consequence #1: Ion flux, etch rate

Bohm's criterion $u_B = \sqrt{\frac{eT_e}{M}}$ applies in "all" cases probably because the maximum potential drop to penetrate the presheath is $T_e/2$ generally

The ion flux into a sheath = ion flux through a sheath = ion flux to the wall
 $= n_s u_B = n_s \sqrt{\frac{eT_e}{M}} = \frac{n_0}{\sqrt{e_{nat}}} \sqrt{\frac{eT_e}{M}} = 0.61 n_0 \sqrt{\frac{eT_e}{M}} = \text{electron flux} = \text{plasma particle loss flux}$
 provided there is no current to the wall (i.e. an electrically floating wall)
 The ion flux controls the etch rate!



Plasma

Now that we have derived Bohm's criterion, we can use it as a tool to calculate other effects in the plasma. For our first example, we will use it to calculate the ion flux, which influences the etch rate in plasma etching. Now first we need to know that Bohm's criterion has some rather general applicability. It seems to apply in many cases, for example, radio frequency plasmas, probably because the maximum potential drop which can penetrate a presheath is generally $T_e/2$ and therefore Bohm's criterion tends to have some general validity in many cases. This is part of the great interest in Bohm's criterion. Now the ion flux into a sheath equals the ion flux through the sheath equals the ion flux to the wall because of conservation of ion flux crossing the sheath. That is, no ionization in the sheath. Therefore, the ion flux to the wall is the same as the ion flux into the sheath, which is n_s times the Bohm velocity, n_s times the Bohm velocity again n_s is the plasma density in the bulk divided by the square root of natural logs, which is 61 percent of the plasma density in the plasma.

Notes

Summary



Bohm consequence #1: Ion flux, etch rate

Bohm's criterion $u_B = \sqrt{\frac{eT_e}{M}}$ applies in "all" cases probably because the maximum potential drop to penetrate the presheath is $T_e/2$ generally

The ion flux into a sheath = ion flux through a sheath = ion flux to the wall
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 provided there is no current to the wall (i.e. an electrically floating wall)
 The ion flux controls the etch rate!

The ion current to a probe area $A = en_s u_B A = 0.61 e A n_0 \sqrt{\frac{eT_e}{M}} = \text{ion saturation current}$
 to a negatively-biased wall or probe
 (in this case the electron flux is ~ 0)

Plasma

Now we know that the ion flux to the wall is equal to the electron flux to the wall because the sheath potential appeared in order to guarantee equality of flux. And therefore, the electrons and ions, that is the plasma particle loss flux, is given by this expression. In making the statement that the electron flux and ion flux to the wall are both the same, this is the same as stating that there is no current to the wall, we have assumed no current to the wall. That is we have assumed the wall is electrically floating in this example. Now the ion flux controls the etch rate as we said at the beginning and we are now able to calculate also the ion current to a probe. A probe of area A will find a current $e n_s u_B A$ times the probe area times this expression. This is a factor very often seen in plasma physics is called the ion saturation current to a wall or probe. If the wall is negatively biased, then the Bohm velocity still determines the ion flux into the wall because the ions cannot arrive any faster. In this case, electron flux is zero.

Notes

Summary



11m 25s

Bohm consequence #2: Ion bombardment energy

For equal ion flux and electron flux to a wall,

$$\Gamma_i = n_s u_B = \Gamma_e = \frac{1}{4} n_s \bar{c}_e \exp\left(\frac{-V_s}{T_e}\right) = \frac{1}{4} n_s \sqrt{\frac{8eT_e}{\pi m_e}} \exp\left(\frac{-V_s}{T_e}\right)$$

Hence the sheath voltage drop is

$$V_s = \frac{T_e}{2} \ln\left(\frac{M}{2\pi m}\right) \sim 4.7T_e \text{ for an argon plasma}$$



Plasma

A second consequence of Bohm's criterion is that we can calculate the ion bombardment energy. For equal fluxes to the wall, the ion flux can be equated to the electron flux and the electron flux is given by the kinetic expression, which is $\frac{1}{4}$ times the electron density times the electron thermal speed. The electron density is given by n_s times the Boltzmann factor in crossing the sheath and the electron thermal velocity is $\sqrt{8eT_e/\pi m_e}$ where T_e is the electron volts. We use the kinetic expression for electrons and not for ions because the thermal velocity of electrons is much much greater than the drift velocity in these cases. We can now solve this equation for the sheath voltage drop. We find the sheath voltage drop depends on the electron temperature and the natural logarithm of the ratio of ion mass to electron mass, with a factor 2π , substituting the values for an argon plasma we find that this is about 4.7 times the electron temperature.

Notes

Summary



12m 50s

Bohm consequence #2: Ion bombardment energy

For equal ion flux and electron flux to a wall,

$$\Gamma_i = n_s u_B = \Gamma_e = \frac{1}{4} n_s \bar{c}_e \exp\left(\frac{-V_s}{T_e}\right) = \frac{1}{4} n_s \sqrt{\frac{8eT_e}{\pi m_e}} \exp\left(\frac{-V_s}{T_e}\right)$$

Hence the sheath voltage drop is

$$V_s = \frac{T_e}{2} \ln\left(\frac{M}{2\pi m}\right) \sim 4.7 T_e \text{ for an argon plasma}$$

For the total ion energy in Joules, add the ion energy gained across the voltages of the presheath and the sheath :

$$\varepsilon_i = e T_e / 2 + e V_s = \frac{e T_e}{2} \left\{ 1 + \ln\left(\frac{M}{2\pi m}\right) \right\} = \frac{e T_e}{2} \ln\left(\frac{\exp(1) \times M}{2\pi m}\right) = \frac{e T_e}{2} \ln\left(\frac{M}{2.3 m}\right) \sim 5.2 e T_e \text{ for an argon plasma,}$$

where $\exp(1) = 2.718 \dots$ is Euler's number.

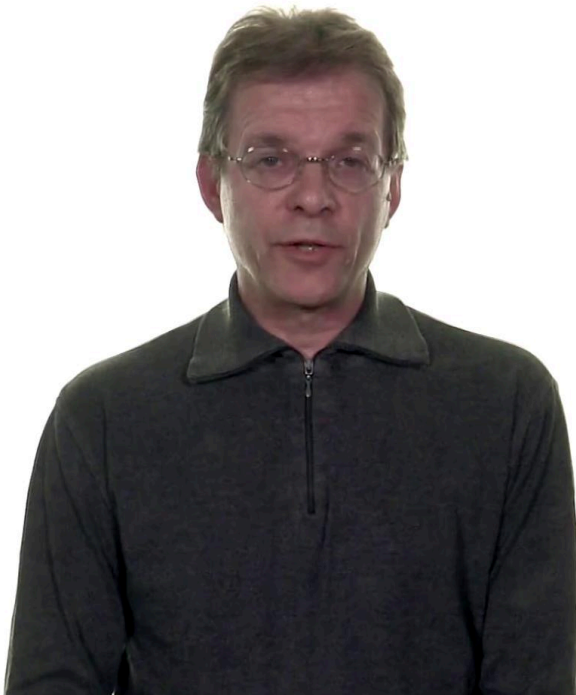
Plasma

Therefore, the sheath voltage drop is $4.7 T_e$ and this is equal to the ion energy drop across the sheath. To get the total ion energy, we add the presheath voltage drop to the sheath voltage drop because the ions are accelerated across the presheath plus acceleration across the sheath voltage V_s . Therefore, the total ion energy approaching the wall is equal to the energy they gained in the presheath plus the energy they gained crossing the sheath, which is $eT_e/2$ times one plus this expression here. We can put the one inside the natural log and combine it this way and we find an expression which is often given in the books $T_e/2 \log$ mass over 2.3 electron mass. If you put the numbers in, this is $5.2 T_e$ which of course is 0.5 plus 4.7. $T_e/2$ plus $4.7 T_e$ is $5.2 T_e$. In books and papers, if you see this expression of the ion energy then that's because we're talking about the ion energy crossing the sheath. If you see this expression for the ion energy, it's because we're talking about the ions from the plasma through the presheath, crossing the sheath, to the wall. That is the total ion energy. To summarize this module, we have calculated the Bohm velocity for ions into a sheath. We've seen that the ion flux to a wall is 61 percent of the ion density in the plasma times the Bohm velocity.

Notes

Summary





- Bohm velocity for ions into a sheath

$$u_B = \sqrt{\frac{eT_e}{M}}$$

- Ion flux to a wall (controls etch rate)

$$0.61n_0 \sqrt{\frac{eT_e}{M}}$$

- Ion energy to a wall

$$\varepsilon_i = \frac{eT_e}{2} \ln\left(\frac{M}{2.3m}\right) \sim 5.2eT_e$$

Plasma

And we have calculated the ion energy of an ion from the plasma to the wall, which is about 5.2 times the electron temperature in volts.

Notes

Summary



16m 01s