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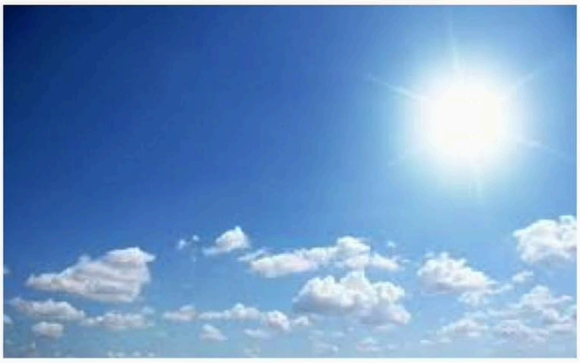
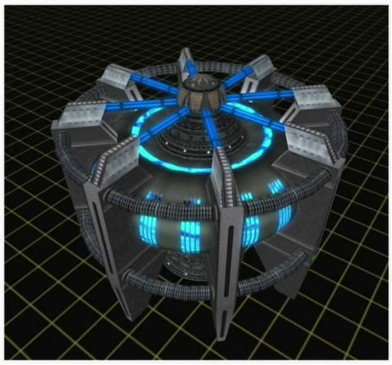


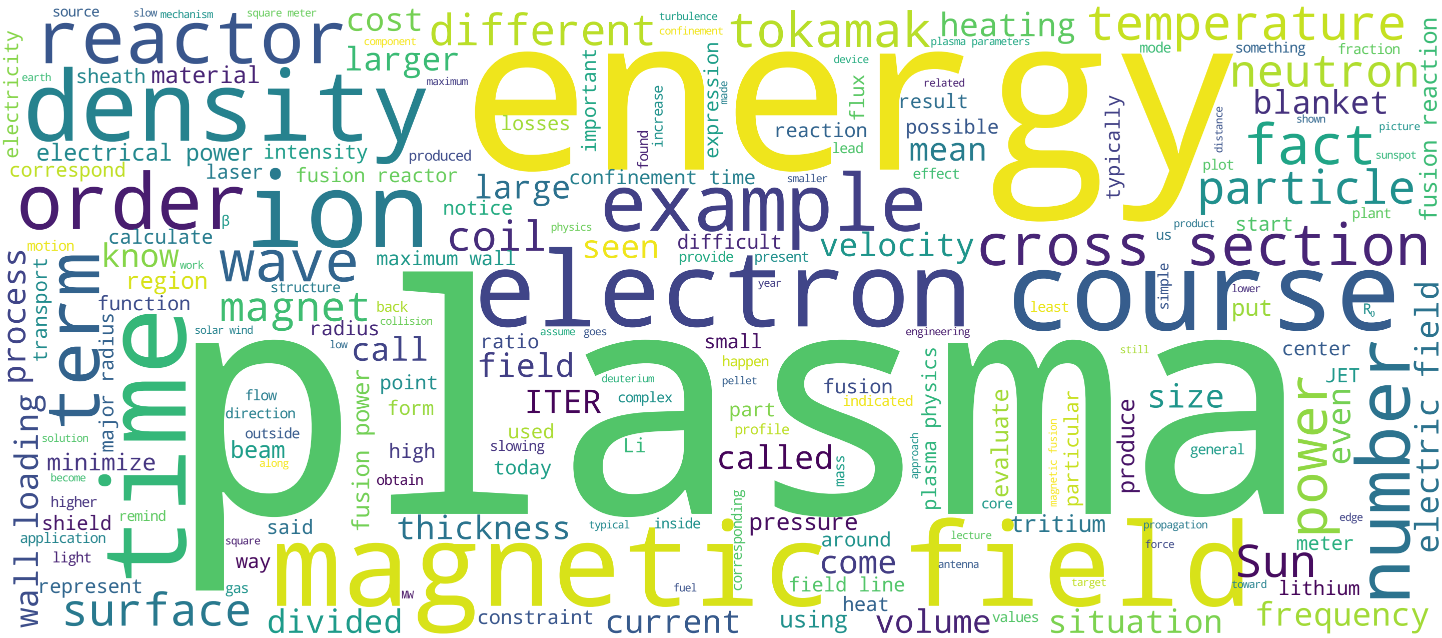


Simple design of a magnetic fusion reactor

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 6d

Ambrogio Fasoli



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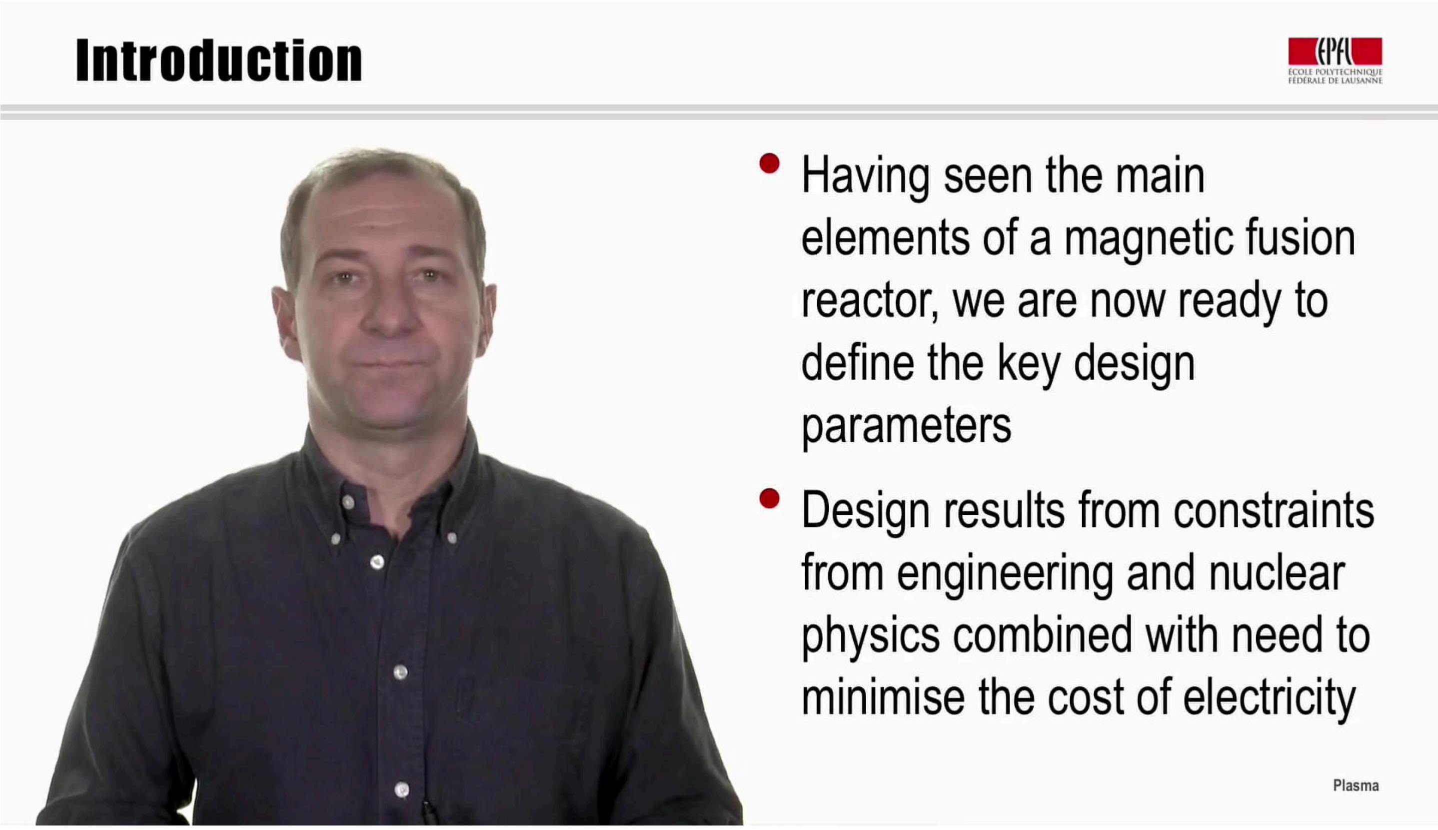


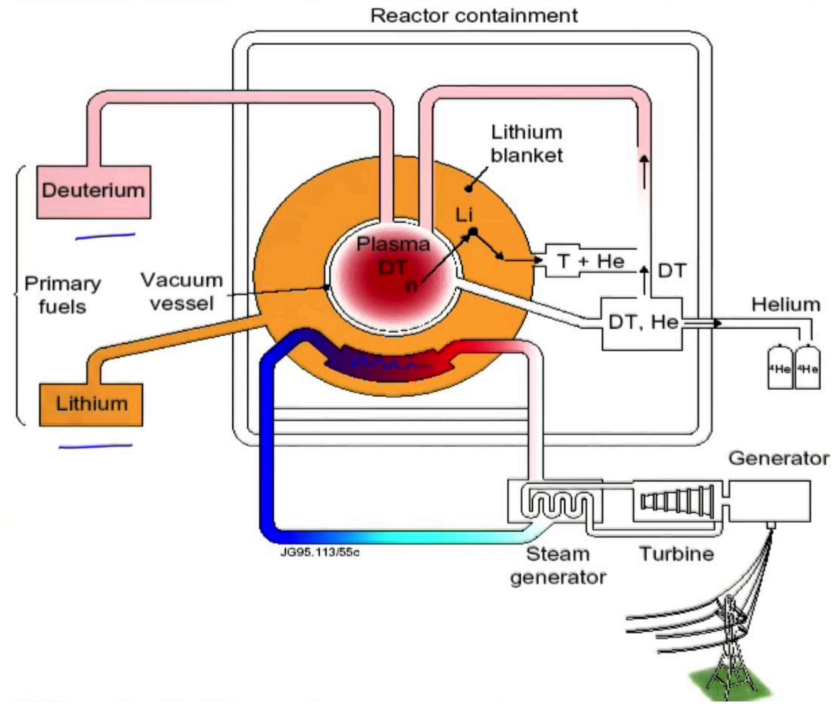
Welcome to the course on Plasma Physics and Applications. Today we will continue our discussion on thermonuclear fusion by taking a very simple approach to design the main elements of a magnetic fusion reactor. We have seen together what the main elements are for the reactor, but we're now ready to really define the key design parameters. The design parameters result from constraints that are coming from engineering, but also from nuclear physics. And this will be combined with the need to minimize the cost of electricity to lead to a generic, and very simple design of a reactor.

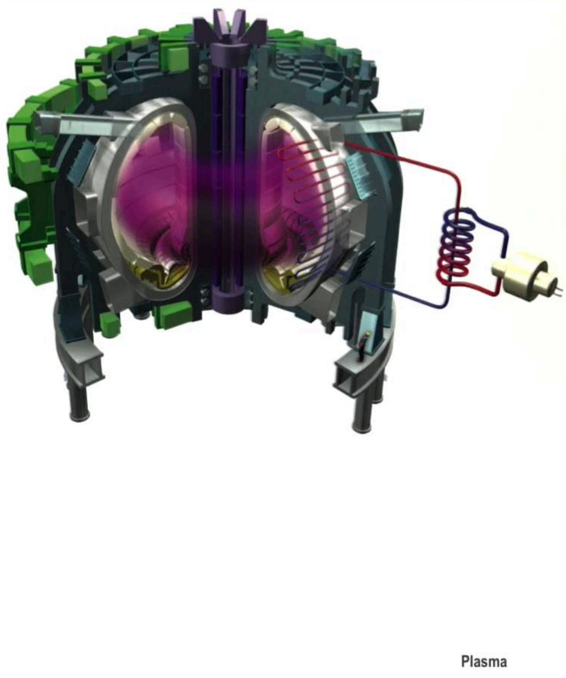
Notes

Summary

0m 05s

Reminder - schematic of a fusion reactor





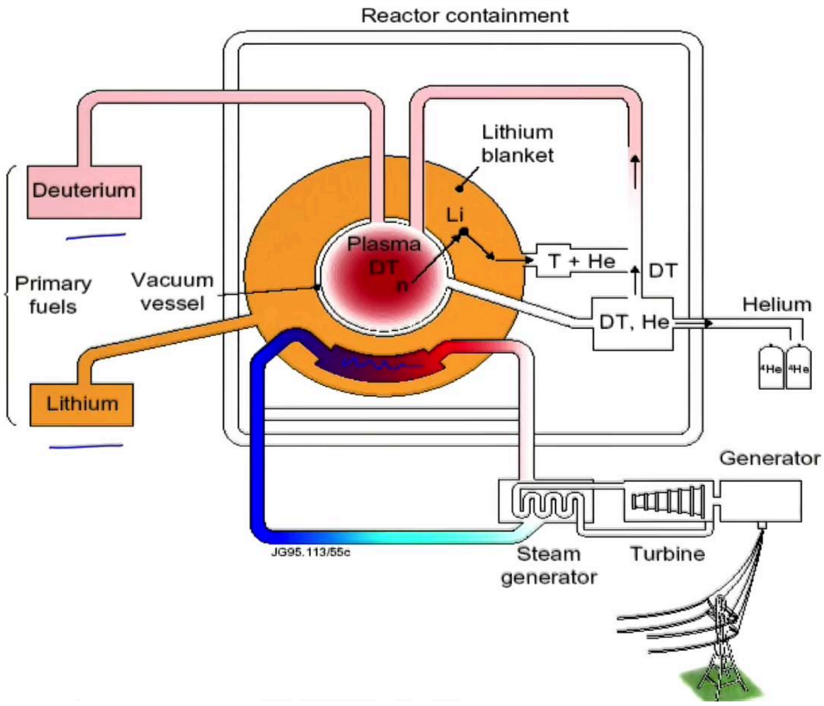
I'd like to start the discussion by reminding ourselves what the schematic is of a fusion reactor, at least as we anticipate it today. In general, we have fuels that come in from outside in the form of deuterium and lithium. They'll be injected to the system. The deuterium will be going into the plasma, which will form the core of the reactor. The lithium will be part of what we call the *blanket*. I remind you that the blanket has two main jobs. First it has to get the heat from the neutrons issued by the fusion reactions. The heat will be collected in a heat exchanger, and therefore will be used to generate electricity by a steam generator. And second, the blanket needs to be producing tritium. The tritium will be produced by the reactions between the neutrons issued by the DT fusion reactions themselves, and the lithium that will be present in the blanket. So the tritium, once more, will be only present in a closed loop within the reactor, it will not come in from outside, nor will it go out from the reactor.

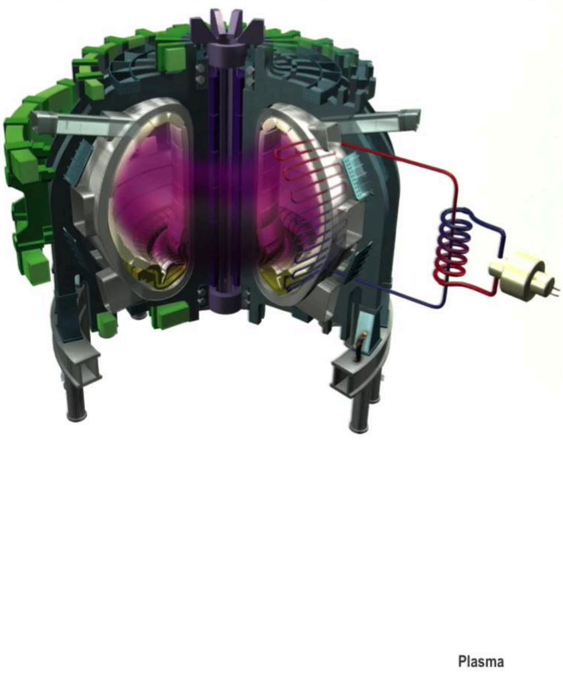
Notes

Summary

0m 42s

Reminder - schematic of a fusion reactor





So this is a generic scheme, with the plasma forming the hot core, of course, of the whole system, but as we said in previous lectures, we will concentrate in this course mostly on the magnetic fusion approach, and that is the approach in which we confine a plasma using magnetic fields, so we need to consider that around the plasma not only will there be a blanket, but it will also be a magnet, as illustrated in this 3D view of a fusion reactor, a magnet that then will produce the field with which we'll confine the plasma.

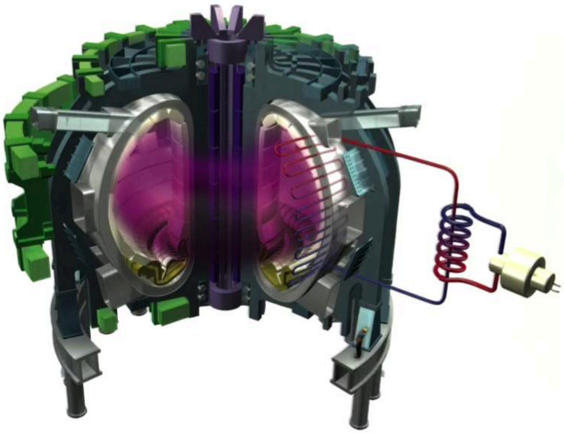
Notes

Summary

1m 53s

Simple approach to the design of a fusion reactor

- The first reactors will operate between breakeven (Q=1) and ignition (Q=∞)
  $n\tau_E \sim 1 - 6 \times 10^{20}\ m^{-3}s$ and $T \geq 10keV$
- Design goals
  - Minimise cost of reactor, to minimise cost of electricity
  - Minimise requirements on plasma performance (τ_E and β)
- We need to optimise
  - The size and geometry of the reactor and the value of B-field
  - The combination of n, T and τ_E
- Considering physics and engineering constraints



Plasma

We have said in the previous lecture that the first reactors will operate between what we call *breakeven* and what we call *ignition*. I remind you that breakeven is a situation in which the fusion power produced by the plasma is exactly equal to the input power, so the capital Q, which is the fusion gain, is equal to 1. Ignition is a situation in which there is no input power from outside at all, and so all the heating from the plasma comes from the alpha particle produced by the fusion reactions themselves, so that corresponds to a fusion gain of infinity. In terms of the confinement quality, we can quantify that as the product of the density of the plasma, and the confinement time for the energy, n times τE, and for the two situations we have about $1x10^{20}\ m^{-3}s$ for breakeven, and six times that, that is $6x10^{20}\ m^{-3}s$ for ignition, so we'll be in-between these two values. Of course, together with the value n τE we need to have a temperature that's large enough for the fusion reactions to take place, typically 10-15 keV. What are the design goals we are considering for this exercise? What we like to do is to minimize the cost of the reactor, because that will correspond to minimizing the cost of electricity.

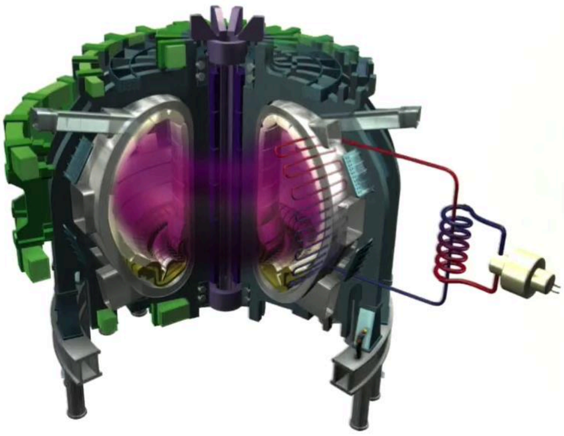
Notes

Summary

2m 29s

Simple approach to the design of a fusion reactor

- The first reactors will operate between breakeven (Q=1) and ignition (Q=∞)
  $n\tau_E \sim 1 - 6 \times 10^{20}\ \mathrm{m^{-3}s}$ and $T \geq 10\,\mathrm{keV}$
- Design goals
  - Minimise cost of reactor, to minimise cost of electricity
  - Minimise requirements on plasma performance (τ_E and β)
- We need to optimise
  - The size and geometry of the reactor and the value of B-field
  - The combination of n, T and τ_E
- Considering physics and engineering constraints



Plasma

We also like to minimize the requirements that the design imposes on the plasma performance. We know it's difficult to confine the energy and the particles in the plasma for a long time, so we don't want to make it more difficult by imposing too large values for τE, the confinement time for the energy. It's also difficult to hold the plasma stable if the plasma has very large pressure. That is the β of the plasma, which is the ratio between the plasma pressure and the magnetic field pressure, has to be as small as we can from the engineering point of view, in order not to make it too difficult for plasma physics. So these are the goals. What we need to optimize are the size and the geometry of the reactor, and the value of the intensity of the magnetic field that will provide the confinement. In a somewhat more sophisticated part of the exercise, one can also optimize the combination of a density temperature and confinement time for the plasma core. And what we need to consider in this optimization exercise, are the constraints that come in from physics and engineering.

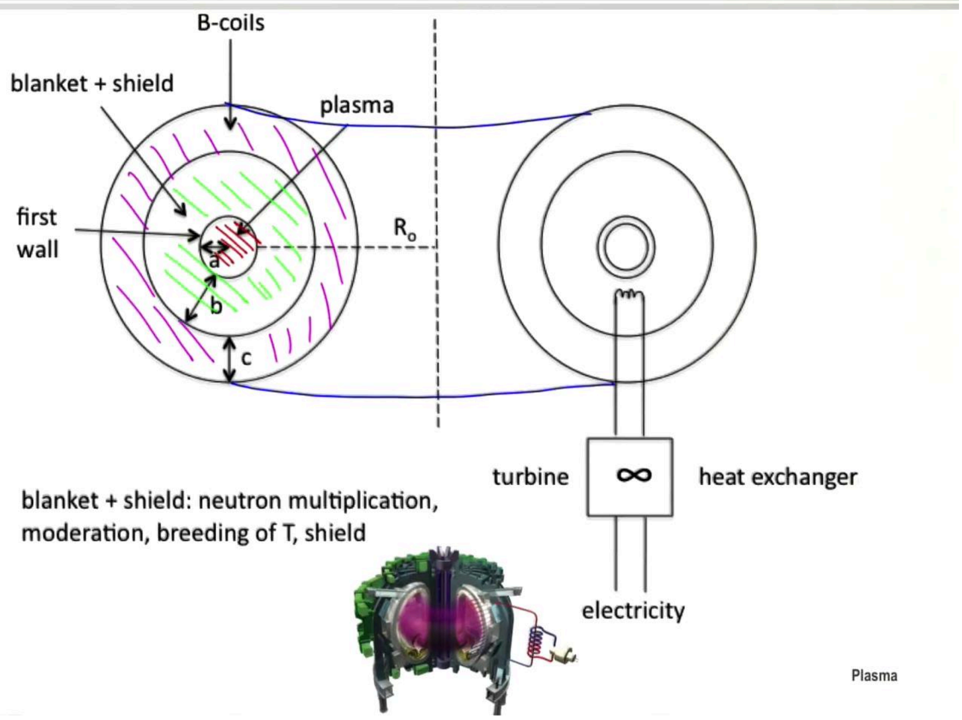
Notes

Summary

4m 01s

Simplified geometry for magnetic fusion reactor

- Toroidal shape
- Circular cross-section
- Parameters
  - Fusion power
  - Geometry: a, R_0, b, c
  - Magnetic field B
  - Plasma: τ_E, n, T, β



So we'll start by looking at the simplified geometry for our magnetic fusion reactor. We'll consider a toroidal shape, for example that of a tokamak, a toroidal shape characterized by a major radius, that's the radius of the torus. We have the size of the plasma, here, indicated by *a*. So let's draw the plasma in red. We have the size of the blanket and the shield that will surround the plasma, indicated by *b*, and here in green. And we have the thickness of the magnetic field coils, or magnet, that I represent here in purple, *c*. And notice that I used a circular cross-section for the plasma and the magnets, just to keep it simple. The other parameters that I need to consider are, of course, the fusion power or the electrical power that we require for the reactor, the intensity of the magnetic field, and the plasma parameters that would be typically: the confinement time, density, temperature and the value of β.

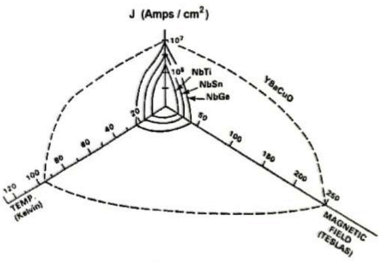
Notes

Summary

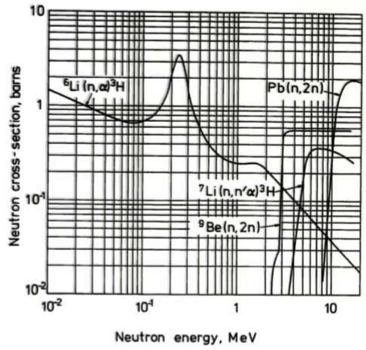
5m 18s

Constraints for the design of a fusion reactor

- Engineering
  - Electrical power $P_E \sim 1GW$
  - Wall loading $L_W \leq L_W^{max} \sim 4\text{-}5MW/m^2$ (plasma losses and neutrons)
  - Magnets – superconducting
    - Must remain below a curve in J, B and T space (in practice B_{max} at coil ~ 13T)



- Nuclear physics
  - Value of $<\sigma v>_{DT}$ (e.g. at T~15keV)
  - Blanket + shield
    - Neutron multiplication
    - Neutron slowing down
      - Cross-section for tritium breeding from Li^6 is much larger at low temperatures (and larger than for Li^7)
    - T-breeding with slowed down neutrons
    - Shielding of coils from neutrons



Plasma

What are the constraints? Let's look at them one by one. First one from the engineering point of view. We can call that a constraint, it's only the requirement for the power plant to produce a certain amount of energy, and power. We may require something like a GW of electrical power, for example. The second constraint is the *wall loading*, as we call it, Lw. That's the amount of power we can cope with, with the materials that face the plasma. Typically that cannot exceed something like 4 or 5MW per m², and that power comes from the plasma losses and the neutrons. We want the magnets to be superconducting, in order for the recirculating power in the plant not to be too large, and in order, of course, to have an optimal production of energy from our plant. In order for the magnets to be superconducting, and to remain superconducting, they must be below a certain curve in the J, B and T space, *J* is the current density, *B* is the magnetic field intensity, and *T* is the temperature, in this case. So in practice, for the materials that we'd like to use, at least in the first generations of reactors, that means that at the coil position, the maximum field cannot exceed something like about 13T.

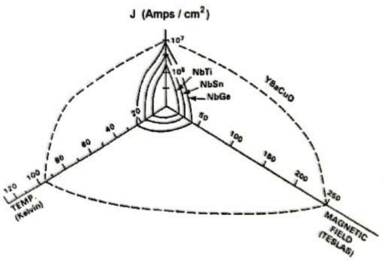
Notes

Summary

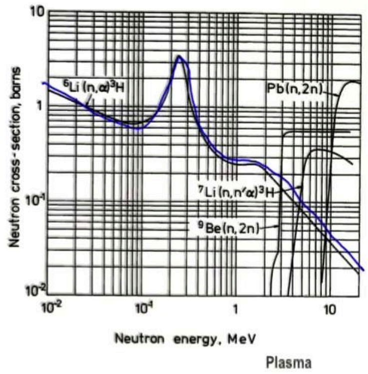
6m 33s

Constraints for the design of a fusion reactor

- Engineering
  - Electrical power $P_E \sim$ 1GW
  - Wall loading $L_W \leq L_W^{max} \sim$ 4-5MW/m² (plasma losses and neutrons)
  - Magnets – superconducting
    - Must remain below a curve in J, B and T space (in practice B_{max} at coil ~ 13T)



- Nuclear physics
  - Value of $<\sigma v>_{DT}$ (e.g. at T~15keV)
  - Blanket + shield
    - Neutron multiplication
    - Neutron slowing down
      - Cross-section for tritium breeding from Li⁶ is much larger at low temperatures (and larger than for Li⁷)
    - T-breeding with slowed down neutrons
    - Shielding of coils from neutrons



We also have constraints from a physics side, or nuclear physics in particular. We have the value of the fusion rate, determined by the fusion cross-section, for example the value of temperature that we're interested in, such as 15keV, that's a value given by nature, and also we have the various processes that have to happen in the blanket and in the shield, which are represented by cross-sections, as well. The processes are the neutron multiplication; we need to multiply neutrons before we breed the tritium, by reacting with lithium. We need to multiply them because of course we can't just rely on a one-to-one correspondence between the number of fusion reactions and the number of tritons (atoms of T) we'll produce, because there may be losses of neutrons, there *will* be losses of neutrons. We need to slow them down, and the reason for that is that we have to breed tritium by having the neutrons reacting with lithium, and the cross-section for such a reaction is much, much larger at low energies. The plot on the right illustrates that: if I take the cross-section here, that I'm highlighting with my blue pen in blue. This is the cross-section from breeding tritium by reacting with Li-6.

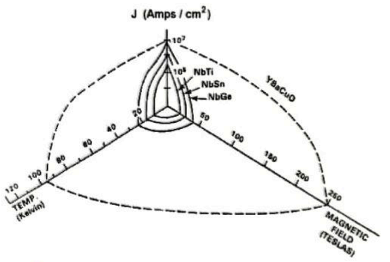
Notes

Summary

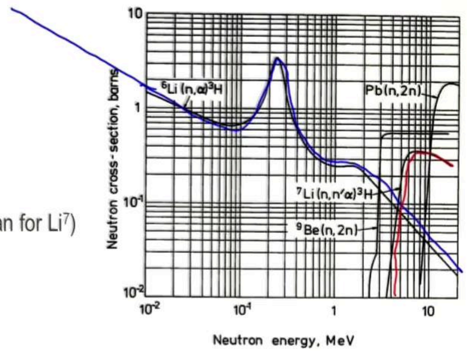
7m 52s

Constraints for the design of a fusion reactor

- Engineering
  - Electrical power $P_E \sim 1GW$
  - Wall loading $L_W \leq L_W^{max} \sim$ 4-5MW/m^2 (plasma losses and neutrons)
  - Magnets – superconducting
    - Must remain below a curve in J, B and T space (in practice B_{max} at coil ~ 13T)



- Nuclear physics
  - Value of $<\sigma v>_{DT}$ (e.g. at T~15keV)
  - Blanket + shield
    - Neutron multiplication
    - Neutron slowing down
      - Cross-section for tritium breeding from Li^6 is much larger at low temperatures (and larger than for Li^7)
    - T-breeding with slowed down neutrons
    - Shielding of coils from neutrons



You can notice that as I go down in energy, this cross-section increases very significantly. In fact, I can keep going down in energy and the cross-section will become much, much larger, So I need to slow down the neutrons in order to arrive at an optimal breeding of the tritium. The other thing I notice in this plot, I'd like to remind you of that, is that if I consider the breeding reaction with Li-7, which is this cross-section, the value of the cross-section is much, much lower than for Li-6, so we'll only consider Li-6, and we in fact probably don't even need to enrich lithium in the actual production of the blanket, because of such a huge difference between the two cross-sections. So once we have slowed down the neutrons, --of course we need to do it in breeding, but we don't have to forget that we need to shield the superconducting coils, and in fact the rest of the plant, and the rest of the environment, from the neutrons issued by the nuclear reactions. So these are the four jobs that the blanket and shield elements need to perform.

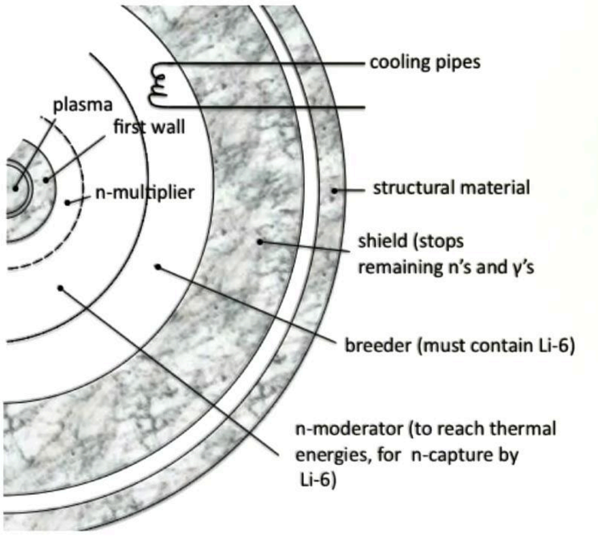
Notes

Summary

9m 18s

Neutron physics → thickness of blanket /shield

- Thickness needed for each process =
  = 1/(number density × cross section)
  - Neutron multiplication
    Ex. Be: $\lambda_{mult} = 1/(n_{Be}\,\sigma_{mult}) \sim 1/(1.2\times10^{29} \times 0.6\times10^{-28}) \sim 13\text{cm}$



in practice different layers are combined

Plasma

Let's look at them one-by-one, and also in a quantitative way, to evaluate the thickness that we need for each process. Here again I put the schematic view of the blanket and shield, the different layers that do the different tasks are indicated, but I'd like to underline that in practice, of course, and in engineering design for actual reactors, the different layers would be combined. So the thickness that's needed for each process, as in fact in all collisional processes, can be evaluated by 1 over the number density of the target that provides the process, times the cross section that represents the probability of occurrence of that process. The first process we need to evaluate is the neutron multiplication. We can take the example of beryllium (Be), beryllium is a neutron multiplier that's used in a variety of applications. So the thickness that we need for this process is given by 1 over the density of beryllium, times the cross-section that represents this process. So if we take the values for the density and the cross-section, we come out with a relatively small thickness of about 13 cm.

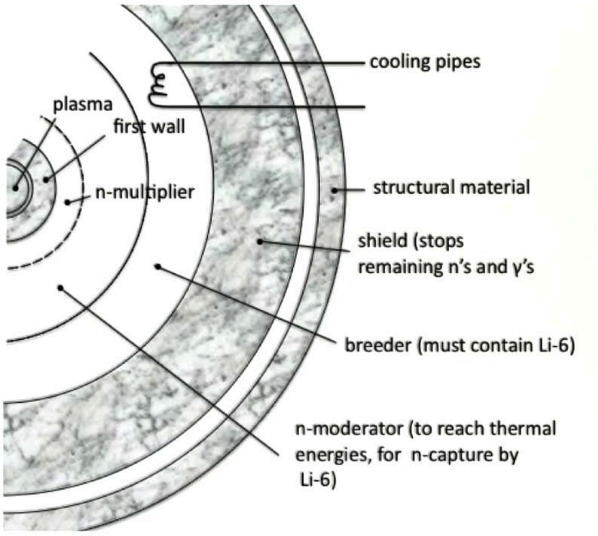
Notes

Summary

10m 31s

Neutron physics → thickness of blanket /shield

- Thickness needed for each process =
  = 1/(number density × cross section)
  - Neutron multiplication
    Ex. Be: $\lambda_{mult} = 1/(n_{Be}\ \sigma_{mult}) \sim 1/(1.2\times10^{29} \times 0.6\times10^{-28}) \sim 13$cm
  - Neutron slowing down
    Ex. Li: $\lambda_{sd} = 1/(n_{Li}\ \sigma_{sd}) \sim 1/(4.5\times10^{28} \times 10^{-28}) \sim 20$cm
  - Tritium breeding with slowed down neutrons
    Ex. at 0.025eV, Li in its natural composition (7.5% Li^6):
    $\lambda_{br} = 1/(n_{Li6}\ \sigma_{br}) \sim 1/(0.075\times5\times10^{28} \times 950\times10^{-28}) \sim 0.2$cm



in practice different layers are combined

Plasma

As we said before, we need to slow down the neutrons in order for the breeding to be more efficient, so we can use lithium itself, not only for breeding tritium, but also for slowing down neutrons, so we can evaluate the length with which this slowing-down process will take place as one over the density of the lithium, times the slowing-down cross-section, which is about one barn, or 10^{-28} square meters. So that is corresponding to about 20 cm, a value that's not insignificant, but again not unrealistically mountable in a blanket, in a practical reactor. Third process is the breeding of tritium, finally, with slowed-down neutrons, so suppose we are able to go down to room temperature, which is about one-fortieth of an eV, and here I consider lithium in its natural composition, so I don't enrich the lithium blanket with Li-6. So Li-6 is only 7.5%, so the thickness is calculated in the usual way, one over the density of Li-6 times the breeding cross-section, and for the density of Li-6 I then, I take the density of lithium times 6 itself, as if it was pure, times the isotopic fraction, so that's 7.5%.

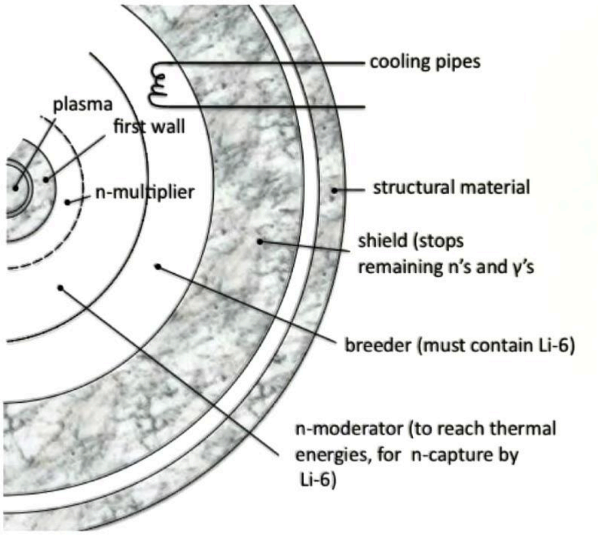
Notes

Summary

11m 44s

Neutron physics → thickness of blanket /shield

- Thickness needed for each process = = 1/(number density × cross section)
  - Neutron multiplication
    Ex. Be: $\lambda_{mult} = 1/(n_{Be}\ \sigma_{mult}) \sim 1/(1.2\times10^{29} \times 0.6\times10^{-28}) \sim 13$cm
  - Neutron slowing down
    Ex. Li: $\lambda_{sd} = 1/(n_{Li}\ \sigma_{sd}) \sim 1/(4.5\times10^{28} \times 10^{-28}) \sim 20$cm
  - Tritium breeding with slowed down neutrons
    Ex. at 0.025eV, Li in its natural composition (7.5% Li^6):
    $\lambda_{br} = 1/(n_{Li6}\ \sigma_{br}) \sim 1/(0.075\times5\times10^{28} \times 950\times10^{-28}) \sim 0.2$cm
  - Shielding of coils from neutrons (and structural material)
    Ex. reduce flux by factor of 100
    $I \sim I_0\exp(-x/\lambda_{tot}) \sim I_0\exp(-x/\lambda_{sd})$
    $I/I_0 \sim 0.01 \rightarrow x_{shield} \sim \ln(100) \times\lambda_{sd} \sim 1$m



in practice different layers are combined

Plasma

But as said a couple of times already, but now underlined by the quantitative factors here, the cross-section for the breeding is so large at these room temperatures that even 7.5% of Li-6 doesn't add a significant thickness to my blanket for the tritium breeding to take place, so that's only about.2 cm. So the key here is that the breeding is actually relatively easy, as long as we have slowed down the neutrons significantly. Finally, we need to shield the coils and the rest of the plant from the neutrons, and that in fact is the task that imposes the largest of the thicknesses. For example, let's say we want to reduce the flux by a factor of 100, so we only allow 1% of the neutrons issued by fusion reactions to arrive to the coil surfaces, so the intensity of the neutron flux is represented by I here, is going down exponentially with the distance, and the characteristic length in the exponent is a combination of different processes, but we can assume is dominated by the slowing-down process, as we have seen before.

Notes

Summary

13m 15s

Neutron physics → thickness of blanket /shield

- Thickness needed for each process = = 1/(number density × cross section)
  - Neutron multiplication
    Ex. Be: $\lambda_{mult} = 1/(n_{Be}\,\sigma_{mult}) \sim 1/(1.2\times10^{29} \times 0.6\times10^{-28}) \sim 13\text{cm}$
  - Neutron slowing down
    Ex. Li: $\lambda_{sd} = 1/(n_{Li}\,\sigma_{sd}) \sim 1/(4.5\times10^{28} \times 10^{-28}) \sim 20\text{cm}$
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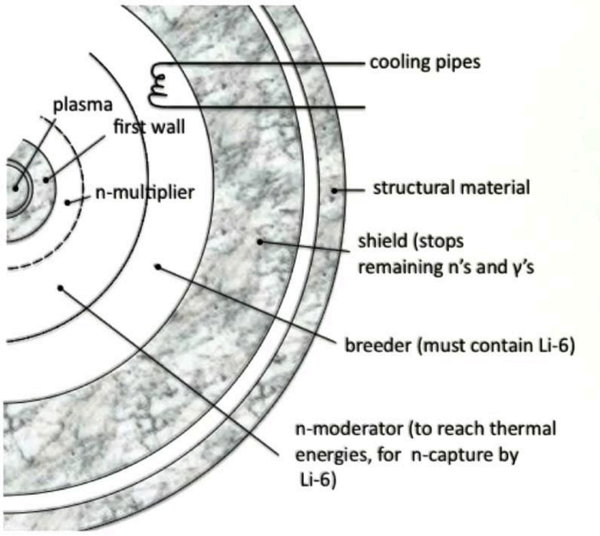
  - Shielding of coils from neutrons (and structural material)
    Ex. reduce flux by factor of 100
    $I \sim I_0\exp(-x/\lambda_{tot}) \sim I_0\exp(-x/\lambda_{sd})$
    $I/I_0 \sim 0.01 \rightarrow x_{shield} \sim \ln(100) \times\lambda_{sd} \sim 1\text{m}$



in practice different layers are combined

- Total thickness $b \sim \lambda_{mult}+\lambda_{sd}+\lambda_{br}+x_{shield} \sim 1.2\text{m}$

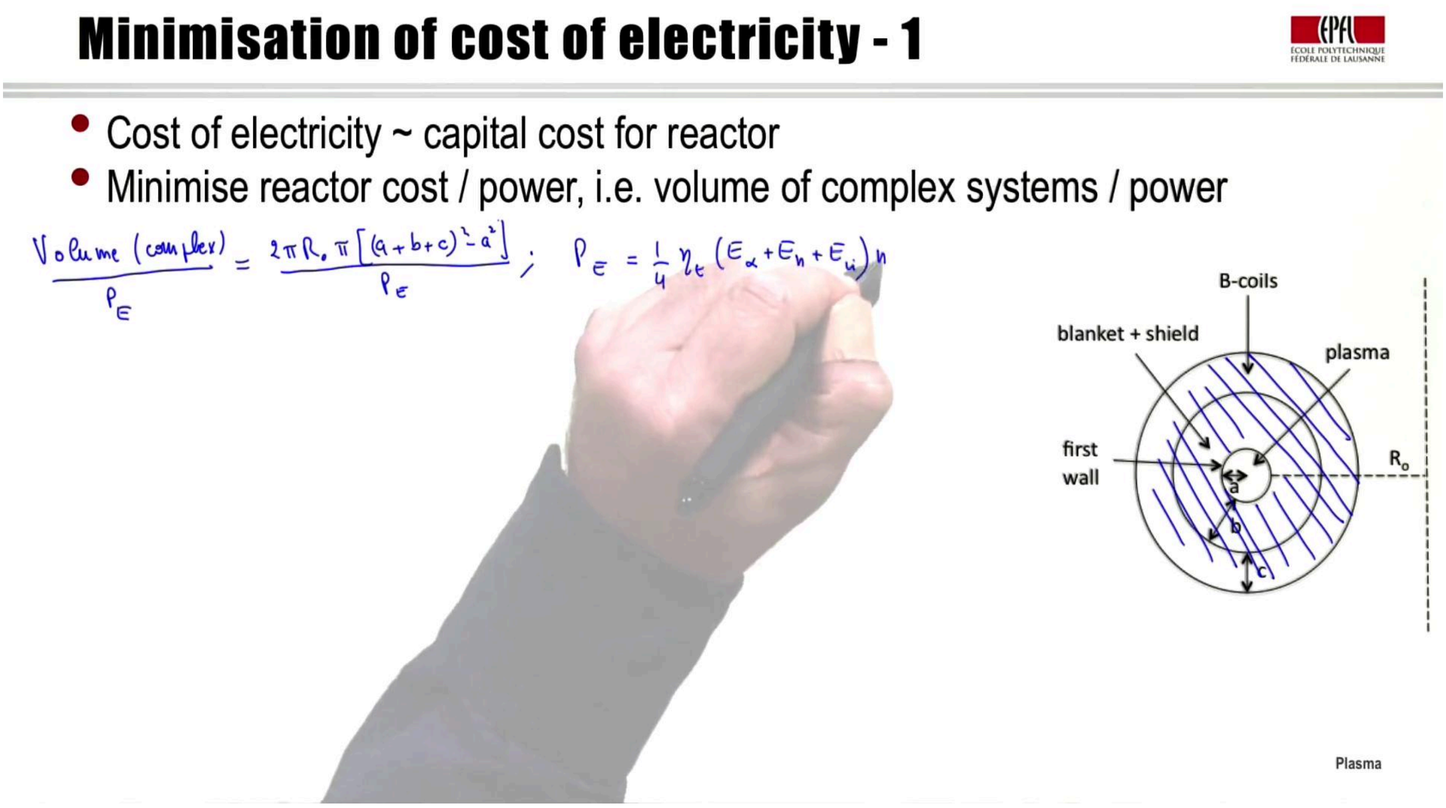
Plasma

So if you take that length, and we impose that the intensity goes down by a factor 100, we get the shielding thickness which is the log, or natural log of 100 times the slowing-down thickness we calculated before, that's about 1 meter, so really that's the dominant part of the thickness that we have found in this evaluation of the need for the blanket and shield to do all the four tasks that they are required to do in a reactor. The total thickness is therefore b, equal to the thickness required for multiplying neutrons, only a few centimeters, the thickness required to slow down neutrons that's significant, it's about 20 cm. The thickness needed to breed tritium with the slowed-down neutrons, and that's almost negligible, it's a fraction of a centimeter, and the thickness needed to shield the coils from the neutron flux, and that, in fact, is the dominant factor. So we get about 1.35 m of thickness for the blanket and shield. However, considering again that the different layers are combined, and that this exercise aims at providing only a rough estimate, we can assume a slightly lower value, say of about 1.2 meters.

Notes

Summary

14m 38s



We can now move to the minimization of the cost of electricity, and we notice that the cost of electricity in a fusion power plant will be proportional to the capital cost for the reactor. That's similar to the situation of fission plants, in fact, where the operational costs and the costs of the fuels are much lower than the capital investment in building the reactor. So when we want to minimize the cost of electricity we can minimize the reactor cost. Of course we have to minimize the reactor cost per unit power, but in a complex system that is actually corresponding to minimizing the volume of the complex systems divided by the unit power. So let's evaluate that. The volume of the complex system per power is given by the volume of everything in my reactor except the plasma, because the plasma is essentially only very few particles that we put in a vacuum, and they will be ionized, so it is not something we build with high technology. So I need to evaluate the volume of the system overall, minus the volume of the plasma. So that's $2\pi R_0$, which is the circumference of the torus, times π, times $[(a + b + c)^2 - a^2]$ and of course we need to divide that by the electrical power produced by the reactor. And the electrical power is equal to...

Notes

Summary

15m 59s

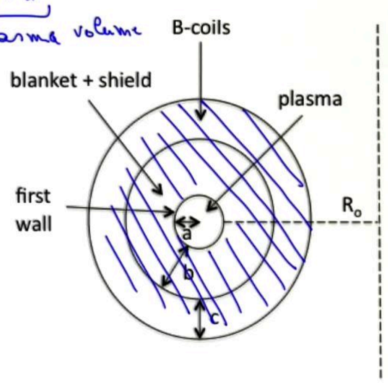
Minimisation of cost of electricity - 1

- Cost of electricity ~ capital cost for reactor
- Minimise reactor cost / power, i.e. volume of complex systems / power

$$\frac{\text{Volume (complex)}}{P_E} = \frac{2\pi R_0 \pi \left[(a+b+c)^2 - a^2\right]}{P_E} \; ; \quad P_E = \frac{1}{4}\eta_t \left(E_\alpha + E_n + E_{Li}\right) n^2 \langle\sigma v\rangle_{DT} \underbrace{2\pi R_0 \pi a^2}_{\text{plasma volume}}$$

Constraint on wall loading $\rightarrow R_0$ $\quad L_W^{max} \times \text{surface of plasma} = P_n$

$$P_n = \frac{P_E}{\eta_t} \frac{E_n}{E_\alpha + E_n + E_{Li}} \; ;$$



Plasma

to 1/4, times the efficiency (ηt) with which we transform fusion power into electricity, times the energy produced in each reaction, so that's the energy of the alphas (Eα), the energy of the neutrons (En), and the energy produced in the tritium breeding reaction in the blanket (Eli), with Li-6, times n^2, the density squared, the fusion reaction rate (<σv>DT), and, of course, the plasma volume. Now we use the constraint on the wall loading to get the value of the major radius R_0 And the way to use that is to take the maximum wall loading we can accept, times the area of the plasma, and that should be equal to the power in the plasma. Here I only take the neutron power, because I consider that as the only unavoidable part of the power that will go onto the wall, and of course it's an assumption to simplify the calculations. Now we know that the neutron power is equal to the fusion power, which is the electrical power produced by the plant, divided by the efficiency with which the fusion power is transforming into electricity, times the fraction of the energy in each reaction that's associated with the neutrons, so En divided by $E\alpha$ + En + Eli.

Notes

Summary

17m 34s

Minimisation of cost of electricity - 1

- Cost of electricity ~ capital cost for reactor
- Minimise reactor cost / power, i.e. volume of complex systems / power

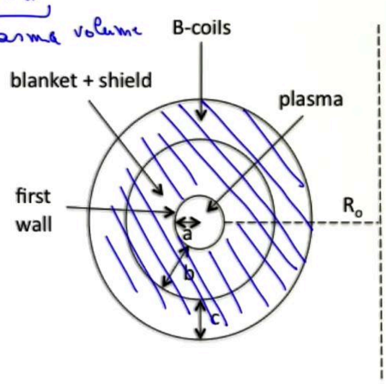
$$\frac{\text{Volume (complex)}}{P_E} = \frac{2\pi R_0 \pi\left[(a+b+c)^2 - a^2\right]}{P_E} \; ; \quad P_E = \frac{1}{4}\eta_t \left(\mathcal{E}_\alpha + \mathcal{E}_n + \mathcal{E}_{Li}\right) n^2 \langle\sigma v\rangle_{DT} \underbrace{2\pi R_0 \pi a^2}_{\text{plasma volume}}$$

Constraint on wall loading $\rightarrow R_0$ $\quad L_W^{max} \times$ surface of plasma $= P_n$

$$P_n = \frac{P_E}{\eta_t}\frac{\mathcal{E}_n}{\mathcal{E}_\alpha+\mathcal{E}_n+\mathcal{E}_{Li}} \; ; \quad L_W^{max} \times 2\pi a \, 2\pi R_0 = \frac{P_E}{\eta_t}\frac{\mathcal{E}_n}{\mathcal{E}_n+\mathcal{E}_\alpha+\mathcal{E}_{Li}}$$

$$\rightarrow R_0 = \frac{P_E}{\eta_t}\frac{\mathcal{E}_n}{\mathcal{E}_n+\mathcal{E}_\alpha+\mathcal{E}_{Li}}\frac{1}{4\pi^2 a L_W^{max}} \overset{\eta_t \sim 0.4}{\simeq} 0.04\frac{P_E}{a L_W^{max}}$$

$$\rightarrow \frac{\text{Volume (complex)}}{P_E} = \frac{2\pi^2}{\cancel{P_E}}\left[(a+b+c)^2-a^2\right]\times \underbrace{0.04\frac{\cancel{P_E}}{a L_W^{max}}}_{R_0} \simeq 0.8\frac{(a+b+c)^2-a^2}{a L_W^{max}}$$



Plasma

So let's go on with the evaluation of the product of the maximum wall loading times the surface. Again, the maximum wall loading times $2\pi a$ $2\pi R_0$ which is the surface of the plasma, is equal to the neutron power. So we can extract R_0 So R_0 is P over ηt times the fraction of the energy that's in the neutrons, divided by $4\pi^2$ times the radius of the plasma, times the maximum wall loading we can cope with. Now these are numbers that we know, and we can assume a value for the thermal efficiency ηt, let's say we take 40% for ηt, so we get to value for R_0 the major radius of the plasma, which is about.04, times the electrical power of the plant, divided by the radius of the plasma, and the maximum wall loading that we can tolerate. We can now insert the value of R_0 we have just found, in the ratio we need to minimize, that is the ratio between the volume and the power. So this ratio is now $2\pi^2$ divided by the power, times the geometrical factor, times R_0 expressed as.04 PE / (a Lw^max). If I just put the constants together, I get .8 ((a + b + c)² - a²) divided by (a Lw^max), and I notice that the electrical power, PE, has been simplified.

Notes

Summary

19m 07s

Minimisation of cost of electricity - 1

- Cost of electricity ~ capital cost for reactor
- Minimise reactor cost / power, i.e. volume of complex systems / power

$$\frac{\text{Volume (complex)}}{P_E} = \frac{2\pi R_0 \pi\left[(a+b+c)^2 - a^2\right]}{P_E} \; ; \quad P_E = \frac{1}{4}\eta_t\left(E_\alpha + E_n + E_{Li}\right) n^2 \langle\sigma v\rangle_{DT} \underbrace{2\pi R_0 \pi a^2}_{\text{plasma volume}}$$

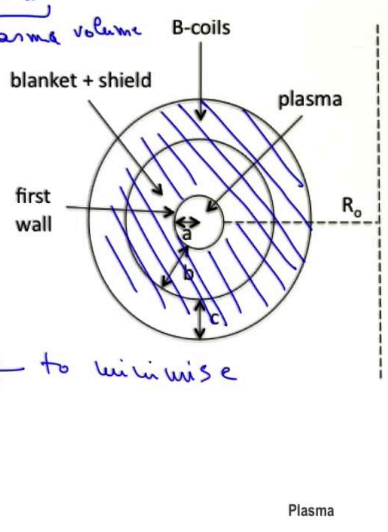
Constraint on wall loading $\rightarrow R_0$ $\quad L_W^{max} \times \text{surface of plasma} = P_n$

$$P_n = \frac{P_E}{\eta_t}\frac{E_n}{E_\alpha + E_n + E_{Li}} \; ; \quad L_W^{max} \cdot 2\pi a \, 2\pi R_0 = \frac{P_E}{\eta_t}\frac{E_n}{E_n + E_\alpha + E_{Li}}$$

$$\rightarrow R_0 = \frac{P_E}{\eta_t}\frac{E_n}{E_n + E_\alpha + E_{Li}}\frac{1}{4\pi^2 a L_W^{max}} \overset{\eta_t \sim 0.4}{\simeq} 0.04\frac{P_E}{a L_W^{max}}$$

$$\rightarrow \frac{\text{Volume (complex)}}{P_E} = \frac{2\pi^2}{\cancel{P_E}}\left[(a+b+c)^2 - a^2\right] \times \underbrace{0.04\frac{\cancel{P_E}}{a L_W^{max}}}_{R_0} \simeq \boxed{0.8\,\frac{(a+b+c)^2 - a^2}{a L_W^{max}}} \leftarrow \text{to minimise}$$

b is determined already $\qquad$ a ? c



This is the quantity to minimize, and I notice that a key parameter in the whole exercise is and will be the maximum wall loading, so it's a very simple minimization, of course, in terms of the maximum wall loading - The more wall loading we tolerate, the better off we will be with the reactor design, in the sense that we'll be automatically minimizing the volume of the complex system, that is the cost per unit power. Now we are ready to continue with the determination of the parameters of the reactor, because b we know, b is determined already from the blanket physics. We need to find now a and c, that is the size of the plasma, and the size of the magnet.

Notes

Summary

21m 14s

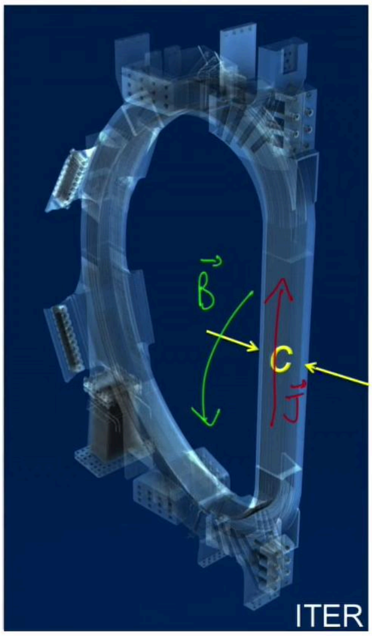
Thickness of magnet

- Magnet thickness should be minimum, compatibly with the maximum allowed stress on its structure from J×B forces
- A calculation using tensile stress gives

$$\boxed{c = \frac{2\alpha}{1-\alpha}(a+b)} \quad \text{with} \quad \alpha = \frac{B_{coil}^2}{4\mu_0\sigma_{max}}$$

- Knowing that it is difficult for the plasma to reach high β, we take $B_{coil} = B_{max}$~13T; as σ_{max}~300MPa

→ α ~ 0.1 and c ~ 0.22(a+b)



Plasma

In terms of the thickness of the magnet, here I remind you what we're talking about, in a more realistic situation than our schematic drawing. This in fact is the magnet designed for ITER, so the thickness is really the term c here. We would like that to be minimum, but we would like it to be compatible with the maximum allowed stress on the structure of the magnet itself, stress that comes from the JxB force. In the magnet, of course there will be current, and that will flow, and this current will produce a magnetic field with which it will interact to produce a JxB force. This is the force that is possible for the stress. If we do a simple calculation, which we don't report here, using only the tensile part of the stress, we obtain a value of c, indicated here, related to a parameter α, and related to a and b, so related to the other two geometrical parameters we discussed already. The parameter α contains, essentially, the intensity of the field at the coil, squared, divided by the maximum stress that we can tolerate (σ_max) for the integrity of the coil itself.

Notes

Summary

22m 07s

Thickness of magnet

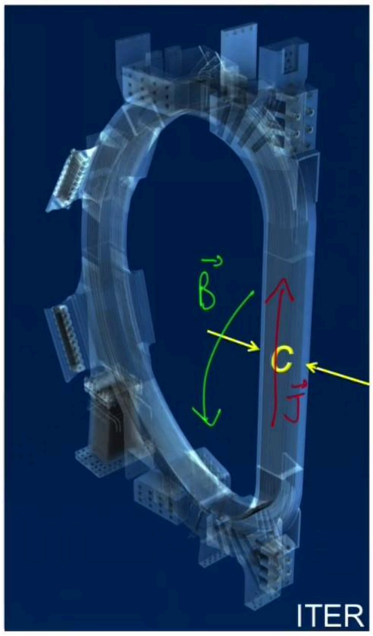
- Magnet thickness should be minimum, compatibly with the maximum allowed stress on its structure from J×B forces

- A calculation using tensile stress gives

$$\boxed{c = \frac{2\alpha}{1-\alpha}(a+b)} \quad \text{with} \quad \alpha = \frac{B_{coil}^2}{4\mu_0\sigma_{max}}$$

- Knowing that it is difficult for the plasma to reach high β, we take $B_{coil} = B_{max}$~13T; as σ_{max}~300MPa

$\rightarrow \underline{\alpha \sim 0.1}$ and $\boxed{c \sim 0.22(a+b)}$



Plasma

Now we can actually choose the intensity of the field at the coil, just observing that it is difficult for the plasma to reach high β, so if we don't want to make it too difficult for the plasma side, we need to take the maximum field we can have at the coil, so the pressure for the plasma, it will be easier to have that high enough without violating stability limits, for example, because that will correspond to large values of β. So we take the maximum field at the coil that we can have in order for the coil to remain superconducting, as we said before, that's about 13 T, at least in the magnets that we can consider today. So we take a maximum value for Bcoil. We know that typically the maximum stress we can tolerate is of the order of 200 MPa, so we insert that into the expression for α, and we get a value of α that's about.1 So with that we calculate c, and that gives c =.22 (a + b) So we have determined b from blanket physics, we have expressed c in terms of a and b from considerations on the magnet, now we go back to our minimization of the cost of electricity to evaluate a.

Notes

Summary

23m 36s

Minimisation of cost of electricity - 2

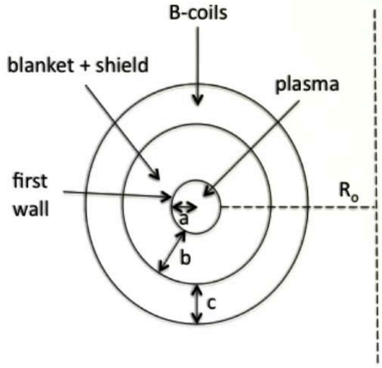
$$\frac{\text{Volume (complex)}}{P_E} \simeq \frac{0.8}{a L_w^{max}} \left\{ \left[a + b + \underbrace{0.22(a+b)}_{c} \right]^2 - a^2 \right\} \simeq \frac{0.4}{a L_w^{max}} \left(a^2 + 6ab + 3b^2 \right) = \frac{0.4}{L_w^{max}} \underbrace{\left(a + 6b + \frac{3b^2}{a} \right)}_{f(a)}$$

Def $f(a) = a + 6b + \frac{3b^2}{a}$;

Minimum of $f(a)$: $\frac{df}{da} = 0 = 1 - \frac{3b^2}{a^2} \Rightarrow \boxed{a = \sqrt{3}\, b}$

$$c = 0.22\,(a+b) = 0.22\,(\sqrt{3}+1)\, b \simeq 0.6\, b$$

$\boxed{b \simeq 1.2 \text{ m}}$ $\boxed{a \simeq 2.1 \text{ m}}$ $\boxed{c = 0.7 \text{ m}}$



Plasma

We write again the quantity we want to minimize, that's the volume of the complex system divided by the power, so we have written exactly what we have found before, except that instead of a c, we have put what we had just derived for the magnet thickness, that is.22(a + b). And by putting the numbers, and developing the square, we get approximately.4 divided by a times the maximum wall loading, times ($a^2 + 6ab + 3b^2$) Just divide by a, and we obtain the very simple expression, in terms of a and b. We define the function of a as $f(a) = a + 6b + 3b^2/a$ that is the parenthesis we have here, because that is the function we need to minimize. So in a minimum of f(a) is what we want. That's a trivial calculation, so it's $1- 3b^2/a^2$, and that has to be equal to 0, which gives that a has to be equal to the square root of 3, times b. So this minimization tells us what the value of a should be, in terms of the value of b. c we had already, it's.22(a + b), so that's.22 times the square root of 3 plus 1, times b, which is about.6b So we have everything now, and we can put the numbers. b is equal to about 1.2 meters, blanket thickness. a is about 2 meters, a little more, 2.1 meters. That's the radius of the plasma. c, the thickness of the magnet, is about.7 meters.

Notes

Summary

25m 08s

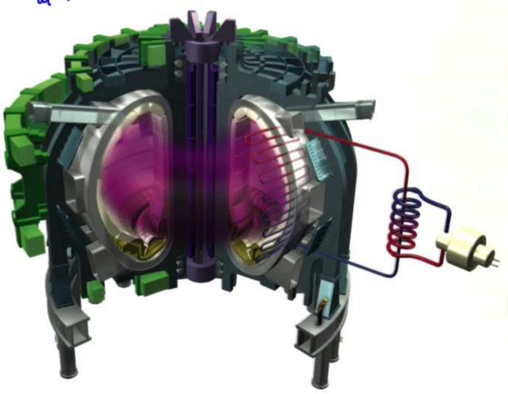
Resulting reactor geometry and plasma parameters

Major radius $R_0 = 0.04 \frac{P_E}{a L_W^{max}} \simeq \left(\begin{array}{l} P_E = 1\,GW \\ L_W^{max} = 4.5 \frac{MW}{m^2} \end{array} \right) = 0.04 \frac{10^9\,W}{2.1\,m \times 4.5 \times 10^6 \frac{W}{m^2}} = 4.2\,m$

Plasma surface $= 4\pi^2 R_0 a \simeq 350\,m^2$; Plasma volume $=$

$= 2\pi R_0 \pi a^2 \simeq 365\,m^3$

Power density $\frac{P_\alpha + P_n}{Volume} \simeq 6 \frac{MW}{m^3}$



Plasma

We are now ready to put down numbers for the resulting reactor geometry, and even the plasma parameters, the main plasma parameters. Major radius R_0 =.04 times the electrical power that we want, divided by the plasma radius, that we now know, and also divided by the maximum wall loading. Let's take two numbers, for these two parameters. Let's take 1GW for the electrical power, as we said at the beginning, and let's take 4.5 MW per square meter for the maximum wall loading. So that will allow us to calculate the value of the major radius of our plant, or tokamak. So we have 1GW on the top, 4.5 MW per square meter at the bottom, 2.1 meters, which is the value of a we found before, and a resullt is slightly more than four meters. We can therefore evaluate the plasma surface, that's $4\pi^2 R_0 a$ and that's about 350 m^2. It's interesting also to evaluate the plasma volume, it's $2\pi R_0$ times πa^2, that's about 365 m^3. So what's the power density in such plasma? That's the power in the alphas, plus the power in the neutrons, divided by the volume. If I put in the numbers I obtain something like 6 MW per m^3. The next parameter we'd like to calculate, it's a key parameter for plasma physics, that is β.

Notes

Summary

27m 15s

Resulting reactor geometry and plasma parameters

Major radius $R_0 = 0.04 \frac{P_E}{a L_W^{max}} \simeq \left(P_E = 1 GW ;\ L_W^{max} = 4.5 \frac{MW}{m^2} \right) = 0.04 \frac{10^9 W}{2.1 m \times 4.5 \times 10^6 \frac{W}{m^2}} = 4.2 m$

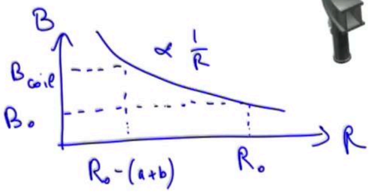
Plasma surface $= 4\pi^2 R_0 a \simeq 350 m^2$; Plasma volume =

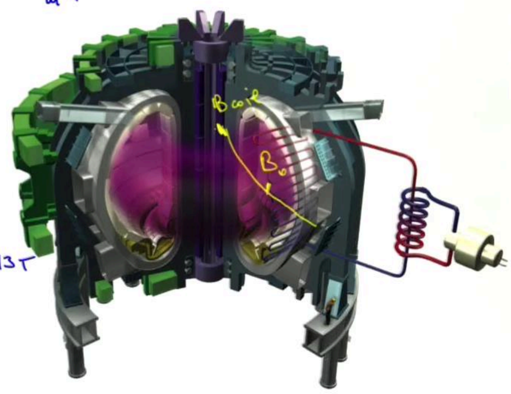
$= 2\pi R_0 \pi a^2 \simeq 365 m^3$

Power density $\frac{P_\alpha + P_n}{Volume} \simeq 6 \frac{MW}{m^3}$

For β, we need $B_0 = B(R = R_0)$; $B \propto \frac{1}{R}$; $B_{coil} = B_{max} = 13T$

$B_0 = B(R=R_0) = B_{max} \frac{R_0 - a - b}{R_0} \simeq 2.7T$





Assume $\tau_E \sim 2s$; $n \sim 10^{20} m^{-3}$; $T \simeq 15 keV$

$\Rightarrow \beta = \frac{nT}{B_0^2/2\mu_0} \simeq 8\%$

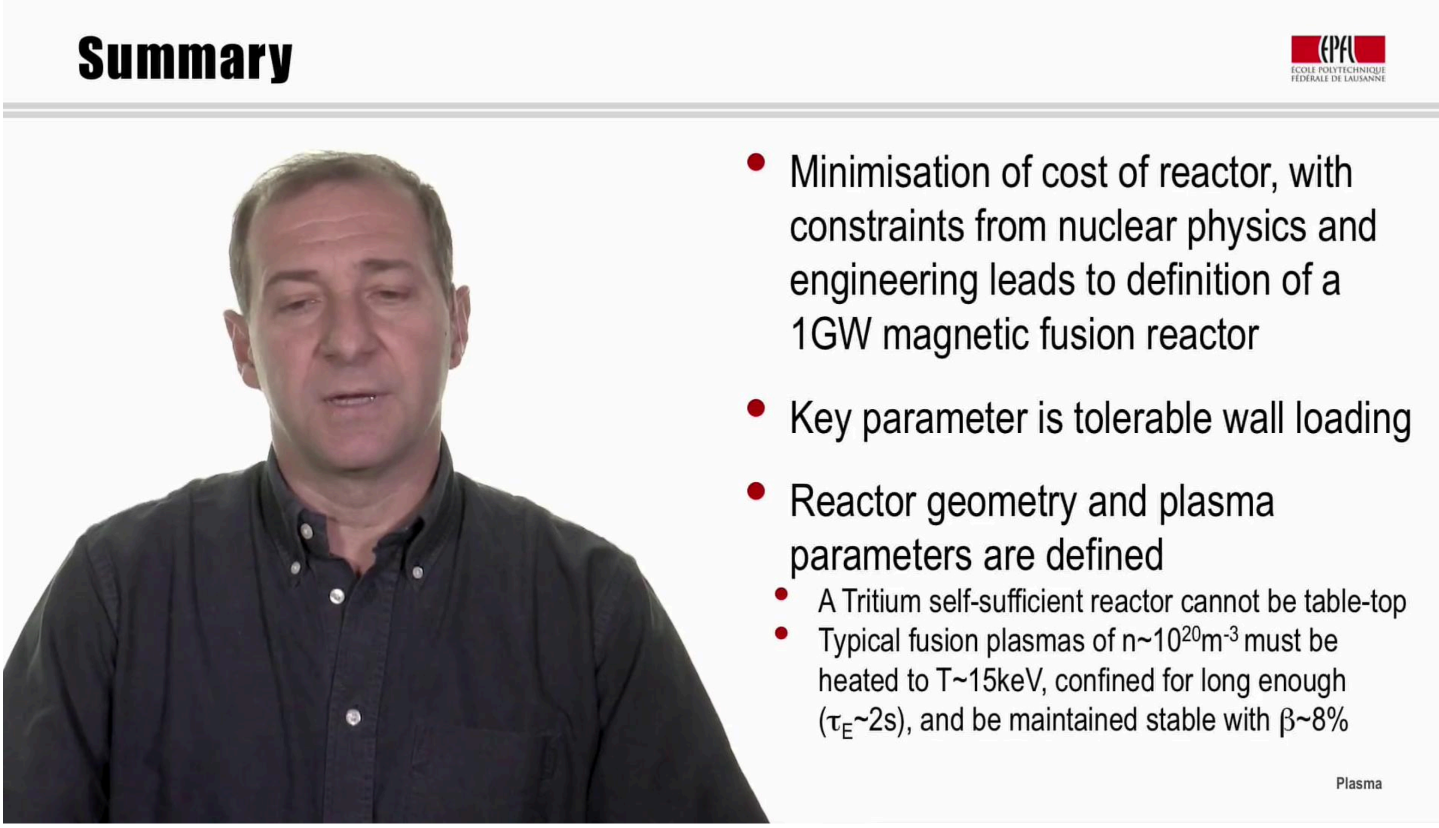
Plasma

But for that we need the value of the magnetic field in the plasma, which I call B_0 that's B at the center of the plasma, which is the value of the radial position equal to the major radius. How do I calculate that? I know that the field scales as 1/R, and I also know that the field at the coil has its maximum value that we took of about 13T. So we can just evaluate B_0 as the Bmax times the radius at which I have my coil, divided by the radius of the torus, that's about 2.7T. If I just represent what we've done graphically, we know the field scales as 1/R, we know the field at the coil, the radius of the coil, position of the coil is R_0- (a + b), and we calculate the value of the field, which we call B_0 at R_0. If I go on the layout of the tokamak, that implies knowing the field here at the coil, and going down as 1/R, and finding B_0 where the center of the plasma is. So with this value of 2.7T we can evaluate β by assuming a confinement time of about two seconds, density of about 10^{20} per m^3, and a temperature of about 15 keV. β is equal to nT divided by $B_0^2/2\mu_0$ and if I put the numbers, I get a value of about 8%.

Notes

Summary

29m 19s

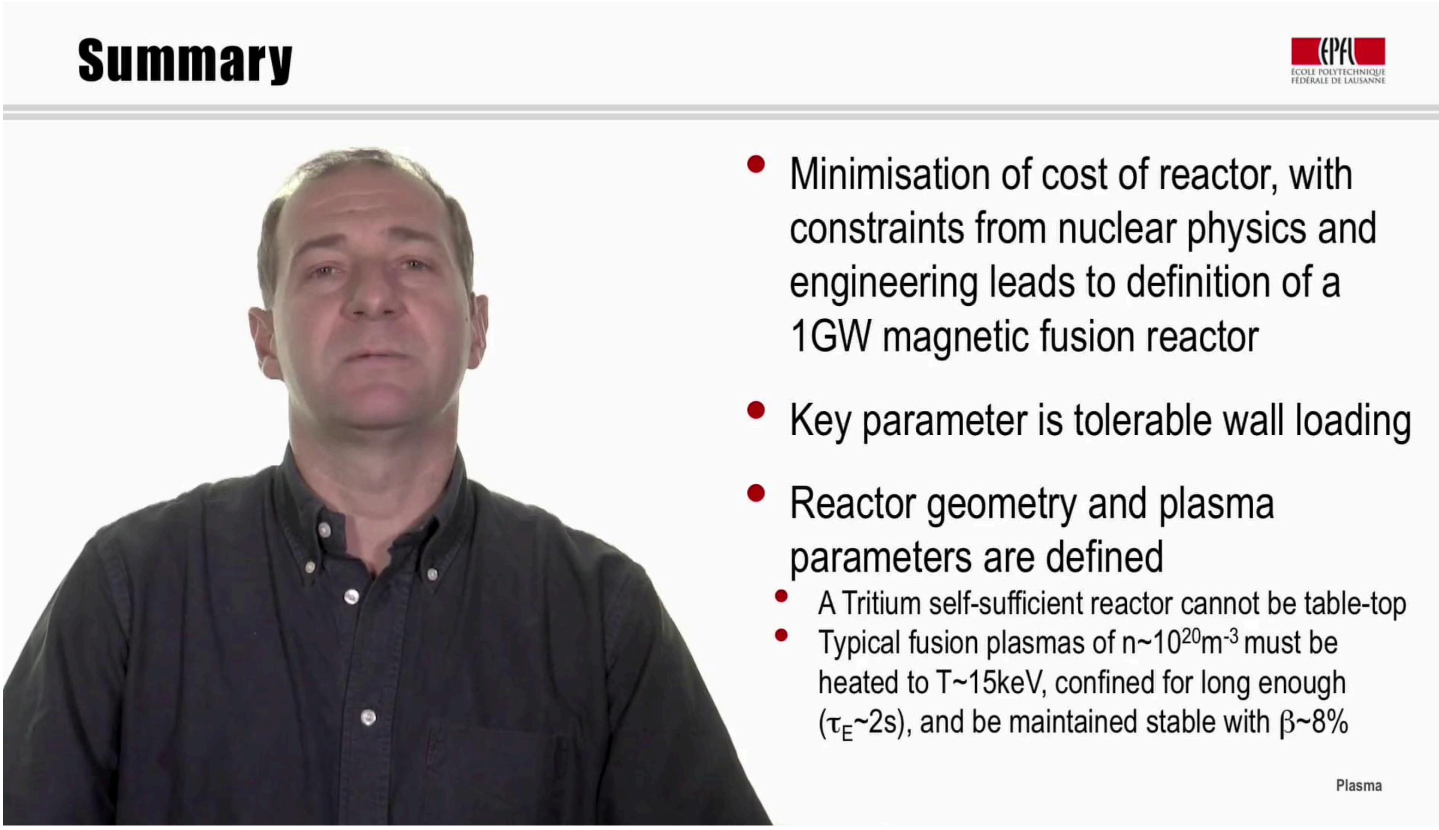


In summary, we have seen together, that a minimization of the cost of a reactor, together with constraints that come from nuclear physics and engineering, can lead, even in a very simple approach, to a general definition of magnetic fusion reactor design that is capable of producing about a 1GW of electrical power. We noticed together that one of the key parameters, perhaps the most important parameters, is the wall loading we can tolerate, that is the maximum amount of power per square meter that the surfaces that see the plasma first can tolerate without being destroyed. We have defined together the reactor geometry, and even the plasma parameters, or the main plasma parameters that the reactor needs to have. We noticed that the size of the reactor cannot be that small, so we can't really have a tabletop reactor if we want the reactor to be self-sufficient in terms of the production of tritium, and that size is dictated by the size of the blanket, we found something like 1.2 m. So for the challenges that this design poses to the plasma, we have summarized them in a few parameters. We need to have a fusion plasma of about 10^{20} per m^3 of density that's produced. It has to be heated up to about 15KeV.

Notes

Summary

31m 34s



It has to be confined for long enough, about a couple of seconds of confinement time for the energy, and it has to be maintained stable, with the plasma β, that is a ratio of the plasma pressure to the magnetic field pressure of about 8%. These are the challenges that plasma physics need to face in order for this reactor to comply with the engineering and physics constraints, and produce a significant amount of fusion power and electrical power.

Notes

Summary

32m 59s