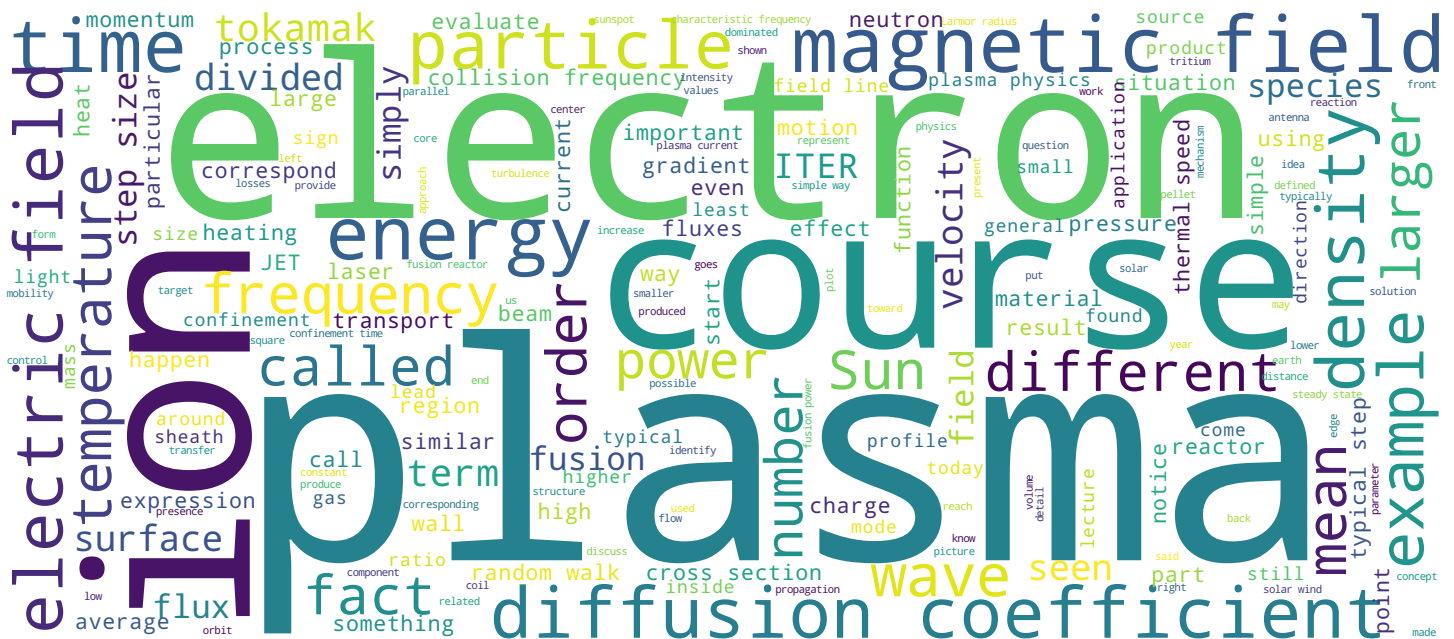


Ambrogio Fasoli





- Random walk and diffusion
- Un-magnetized plasmas
 - Prove of diffusive transport
 - Ambipolar diffusion
- Magnetized plasmas
 - Classical diffusion of particles and heat along and across \mathbf{B}_0

Plasma

Welcome to the course on Plasma Physics and Applications. Today we will discuss collisional transport in plasmas. Even if collisions play a relative minor role in plasmas that are hot, which is the case of course of plasmas for magnetic fusion, they're crucial for determining the plasma resistivity and the transport of particles and heat. The former characterize uniform plasmas, as you have seen in detail in the first parts of this course. But today we'll address the latter, which characterize non-uniform plasmas. In particular, we will look at how collisions determine transport in real space of particles and heat. We will start with a general discussion on the *random walk* and its relation to classical diffusion, both as a general reminder, and because this approach will enable us to treat the complex situation of the magnetically confined fusion plasmas, in a very simple way. We'll prove that the heuristic random-walk model does indeed correspond to a diffusion process in the case of an unmagnetized plasma. And we will see how the plasma sets up an intrinsic electric field to maintain quasi-neutrality called the *ambipolar field* and how this later diffusion that is equal for ions and electrons.

Notes

Summary



0m 05s



- Random walk and diffusion
- Un-magnetized plasmas
 - Prove of diffusive transport
 - Ambipolar diffusion
- Magnetized plasmas
 - Classical diffusion of particles and heat along and across \mathbf{B}_0

Plasma

This is the so-called *ambipolar diffusion*. We will treat the case of the magnetized plasmas on the basis of the random-walk concepts, so a very simple mathematical treatment, and we will distinguish particle and heat transport, and parallel and perpendicular directions with respect to the ambient magnetic field. Naturally, for a magnetized plasma cross-field transport is the crucial element for confinement.

Notes

Summary

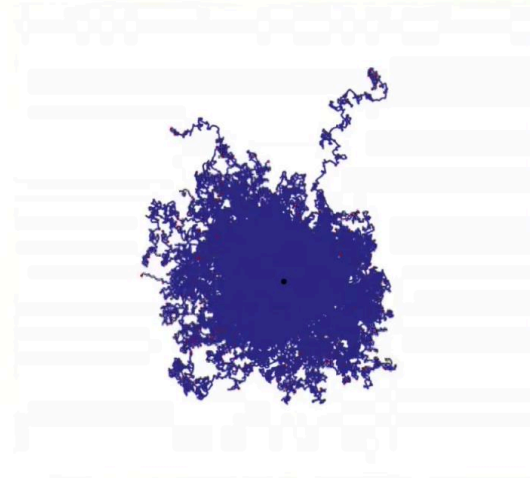
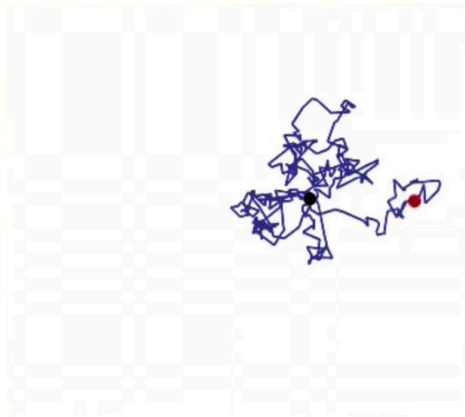


1m 19s

Random walk – reminder

N statistically independent steps ξ_i , each of size $|\xi_i| = \xi$

2D simulations



Courtesy of A. Bovet

Plasma

Just to start reminding ourselves what a random walk is, you can see here two simulations of a 2D situation in which we simulate on the left the evolution of a single particle which undertakes statistically independent steps. Each step has the same size, but of course in a different, randomly chosen direction. As opposed to the right-hand side where you see a movie that simulates the evolution of many particles each of which undergoes a random walk. So you can see the different steps on the left, and on the right, the spreading of a cloud in a sense that represents the spreading of the initially close particles under the influence of that random walk. Although the average displacement is zero, the mean square displacement as we will see immediately is actually non-zero, and that's the crucial element for assessing transport.

Notes

Summary



1m 47s

Random walk – reminder

N statistically independent steps ξ_i , each of size $|\xi_i| = \xi$

1D calculation

After N steps $x = \sum_{i=1}^N \xi_i$; $\bar{x} = 0$

$$\overline{x^2} \neq 0 : \quad \overline{x^2} = \sum_{i=1}^N \overline{\xi_i^2} + \underbrace{\sum_{i=1}^N \sum_{j \neq i} \overline{\xi_i \xi_j}}_{=0} = N \xi^2$$

$\overline{\xi_i \xi_j} = \bar{\xi}_i \cdot \bar{\xi}_j = 0$

τ : inter-collision time ; $N = \frac{t}{\tau}$

$\Rightarrow \overline{x^2} = N \xi^2 = \frac{t}{\tau} \xi^2 = \overline{v^2} \tau t$

Plasma

Having briefly seen the two-dimensional simulations now we are ready for a very simple one-dimensional calculation of the random walk. Again, we consider statistically independent steps ξ_i each of size ξ . So we can see what happens after N steps. After N steps, the displacement will be simply the sum of the steps undertaken. The average displacement will be zero, and average here means 'taken over many possible random walks'. It doesn't mean 'in time' for the same particle. But, as we said before, the average of the square of the displacement is not zero. And we'll evaluate it in a simple way. It will be equal to the sum of the average of the square of each step plus the crossed terms. But the crossed terms actually do not contribute because as we said, all the steps are statistically independent. Therefore the average of that product is equal to the product of the two averages which of course is zero. So we only end up with N times where N is the number of steps undertaken times ξ^2 where ξ is the size of each step. If we define the inter-collision time as τ that means that the number of steps N is equal to the total time divided by τ we can evaluate the mean square displacement as follows: $\langle x^2 \rangle$ is equal to $N \xi^2 = t/\tau \xi^2$ [Note: $\bar{x} = \langle x \rangle$] which is equal to the typical squared velocity averaged $\langle v^2 \rangle \tau t$.

Notes

Summary



Random walk – reminder

N statistically independent steps ξ_i , each of size $|\xi_i| = \xi$

1D calculation

After N steps $x = \sum_{i=1}^N \xi_i$; $\bar{x} = 0$

$$\overline{x^2} \neq 0 : \quad \overline{x^2} = \sum_{i=1}^N \overline{\xi_i^2} + \underbrace{\sum_{i=1}^N \sum_{j \neq i} \overline{\xi_i \xi_j}}_{=0} = N \overline{\xi^2} = N \xi^2$$

$\overline{\xi_i \xi_j} = \overline{\xi_i} \cdot \overline{\xi_j} = 0$

τ_c : inter-collision time ; $N = \frac{t}{\tau_c}$

$$\Rightarrow \overline{x^2} = N \xi^2 = \frac{t}{\tau_c} \xi^2 = \underbrace{\overline{v^2} \tau_c}_{D} t = D t \quad D : \text{diffusion coefficient}$$

$$D = \overline{v^2} \tau_c = \xi^2 / \tau_c = (\text{step size})^2 \times \text{collision frequency}$$

Plasma

And this product, $\langle v^2 \rangle \tau$ can be called D : *diffusion coefficient*. So $D = \langle v^2 \rangle \tau$ as we have just defined, or ξ^2 / τ and that is even simpler conceptually and also to apply to the different situations because that means we have simply the product of the typical step size in our random walk, squared, times a typical collision frequency which is of course the inverse of the intercollision time.

Notes

Summary



5m 17s

Random walk and classical diffusion

Idea: if we identify a characteristic time or collision frequency and a typical step size in the different situations, we can evaluate the diffusion coefficient

Ex. electrons colliding with ions in fully ionized plasmas

characteristic frequency is that of the momentum exchange $\langle v_{e/i} \rangle$

step size $\lambda_{mfp} = 1/(n\sigma) = v_{the} / \langle v_{e/i} \rangle$

$$\rightarrow D_e = v_{the}^2 / \langle v_{e/i} \rangle$$

How to verify that this approach works and that D has really the meaning of a diffusion coefficient?

Plasma

Here's the idea that we will apply to much more complicated situations, those that actually are needed for fusion. If we identify a characteristic time or equivalently a collision frequency and a typical step size in various situations we need to assess, we can evaluate the diffusion coefficient. For example, if we take electrons that are colliding with ions in a fully ionized plasma, we can say the characteristic frequency is that of the momentum exchange, $\langle v_{e/i} \rangle$ as you have seen in previous parts of the course. I remind you that in this case the average symbol indicates an average over the Maxwellian distribution in the plasma. And we have a step size that corresponds to the mean free path for such collisions. Mean free path is always one over the density of the targets times the cross section for that process. And that's corresponding to the thermal speed for the electrons, divided again by the characteristic frequency for the momentum exchange, average of the distribution. So we get a simple evaluation of the diffusion coefficient which is the square of the thermal speed for the electrons in this case, divided by the effective collision frequency for the exchange of momentum. So this will help us in a number of situations. The question is: can we verify, at least in a simple case that this approach works, and that D has indeed the meaning of a diffusion coefficient.

Notes

Summary



6m 09s

Un-magnetized plasma: diffusion equation

Meaning of D in un-magnetized steady-state plasma with uniform temperature

Eq. of motion $m_j \frac{dv_j}{dt} = q_j E - \frac{\nabla p_j}{n_j} - \underbrace{m_j v_j \nu_j}_{\text{friction term}} ; \quad \text{steady-state } \frac{d}{dt} \equiv 0$

Uniform $T_j : \nabla p_j = T_j \nabla n_j$

$$0 = q_j E_j - T_j \nabla n_j - m_j v_j \nu_j \Rightarrow v_j = \frac{q_j}{m_j \nu_j} E - \frac{T_j}{m_j \nu_j} \frac{\nabla n_j}{n_j}$$

Plasma

And I propose to do that in the case of an unmagnetized steady-state plasma with uniform temperature. So as simple as we can get, but yet a typical case in which we'll try to interpret D as a real diffusion coefficient. We start by the equation of motion for electrons or ions. So the mass times the acceleration is of course equal to the electrical force $q_j E$, plus the pressure force which is $-\nabla p$ divided by the density $[n_j]$ plus the friction term which is minus the mass times the velocity times an effective collision frequency. So this is my friction term. I impose steady-state, so no time derivatives, I consider uniform temperature for any species. That means that ∇p_j is simply the temperature times the gradient of the density $[\nabla n_j]$. So with these assumptions, I can evaluate my equation of motion, so no time derivative, so that's zero on the left equals to $q_j E_j$, minus the pressure term simplified for uniform temperatures, and the friction term. That means I can have a steady-state velocity field, which is v_j , that I can express with two terms: the first term proportional to the electric field, and the second term proportional to the gradient of the density.

Notes

Summary



7m 45s

Un-magnetized plasma: diffusion equation

Meaning of D in un-magnetized steady-state plasma with uniform temperature

Eq. of motion $m_j \frac{dv_j}{dt} = q_j E - \frac{\nabla p_j}{n_j} - \underbrace{m_j v_j \nu_j}_{\text{friction term}} ; \quad \text{steady-state } \frac{d}{dt} \equiv 0$

Uniform T_j : $\nabla p_j = T_j \nabla n_j$

$0 = q_j E_j - T_j \nabla n_j - m_j v_j \nu_j$

$\Rightarrow v_j = \underbrace{\frac{q_j}{m_j \nu_j}}_{\text{mobility } \mu_j} E - \underbrace{\frac{T_j}{m_j \nu_j} \frac{\nabla n_j}{n_j}}_{\text{diffusion coefficient } D_j}$

Note $\frac{\mu_j}{D_j} = \frac{|q_j|}{T_j}$ "Einstein relation"

Flux $\Gamma_j = n_j v_j = n_j \left[\frac{q_j}{|q_j|} \mu_j E - D_j \frac{\nabla n_j}{n_j} \right]$

Plasma

The first term has a coefficient which is related to the mobility. I say *related* because it's a pretty much the mobility except the mobility $[\mu_j]$ is defined with an absolute value of the charge, there is no sign, as the charge in absolute value divided by the mass times the collision frequency, $m_j \nu_j$. The other term has a coefficient in front, which is called the diffusion coefficient. For the moment it's a relatively arbitrary definition but we will converge towards an interpretation for both cases. So that's D_j which is the thermal speed for species v_j^2 divided by the collision frequency once more. Of course the thermal speed squared is the ratio of the temperature divided by the mass. We note, as actually Einstein did, that the ratio of the mobility coefficient divided by the diffusion coefficient is simply equal to the ratio of the charge by the temperature. the so-called *Einstein relation*. Now we're interested in flux which we call Γ_j with species j . That's the density times the velocity. So we can use the expression with the velocity we have just found. And we get n_j times the sign of the charge, times μ_j , times electric field, minus the diffusion coefficient, times the gradient of the density.

Notes

Summary



9m 47s

Un-magnetized plasma: diffusion equation

Meaning of D in un-magnetized steady-state plasma with uniform temperature

Eq. of motion $m_j \frac{dv_j}{dt} = q_j E - \frac{\nabla p_j}{n_j} - \underbrace{m_j v_j \nabla v_j}_{\text{friction term}} ; \quad \text{steady-state } \frac{d}{dt} \equiv 0$

Uniform T_j : $\nabla p_j = T_j \nabla n_j$

$$0 = q_j E_j - T_j \nabla n_j - m_j v_j \nabla v_j \Rightarrow v_j = \frac{q_j}{m_j v_j} E - \frac{T_j}{m_j v_j} \frac{\nabla n_j}{n_j}$$

mobility $\mu_j = \frac{|q_j|}{m_j v_j}$ diffusion coefficient $D_j = \frac{v_{thj}^2}{v_j}$

Note $\frac{\mu_j}{D_j} = \frac{|q_j|}{T_j}$ "Einstein relation"

Flux $\Gamma_j = n_j v_j = n_j \frac{q_j}{|q_j|} \mu_j E - D_j \nabla n_j ; \quad \text{if } E=0, \text{ we have } \Gamma_j = -D_j \nabla n_j \quad \text{Fick's law}$

Continuity eq. $\frac{\partial n_j}{\partial t} + \nabla \cdot \Gamma_j = 0 \Rightarrow \frac{\partial n_j}{\partial t} + \nabla \cdot (-D_j \nabla n_j) = 0$

$D_j = \text{constant}$ $\frac{\partial n_j}{\partial t} - D_j \nabla^2 n_j = 0$ Diffusion equation

Plasma

Now consider for a moment zero electric field, in this case we have Γ_j the flux is equal to the diffusion coefficient, with a minus sign in front, times the gradient of the density. This is the so-called *Fick's Law*. It tells us that particles tend to move down a density gradient, which is a very general property of course. Now we use the continuity equation without sources or sinks. So that's $\partial n_j / \partial t$ plus the divergence of the flux for species j equal to zero, and we use that to combine, to be combined with the definition of Γ_j or in fact with the result of Γ_j as a function of the gradient of the density. So that we come to a final equation, which will be the diffusion equation. Let's do that. Now I assume here that D_j is a constant so that I can take it out of the differential operator. This is just for the purpose of showing the final result, of course in real situations this may not be the case. And I get to the expression of $\partial n_j / \partial t - D_j \nabla^2 n_j$, diffusion coefficient times the second spatial derivative of the density, equals zero. What do I have here? I have a diffusion equation. So we have confirmed that at least in this simple case as an illustration, this coefficient D indeed is a diffusion coefficient because it's the coefficient that appears in a diffusion equation for the density of the plasma species j.

Notes

Summary



Un-magnetized plasma: ambipolar field and D_a

Quasi-neutrality implies $\Gamma_i \sim \Gamma_e$. As in general $D_e \neq D_i$, an *ambipolar* electric field is generated in the plasma. We derive it here for $\mathbf{B}_0=0$, but it is a general effect

Assume $n_e = n_i = n$ $\Gamma_i = \mu_i n E - D_i \nabla n = \Gamma_e = -\mu_e$

Plasma

Now a further consideration is that, plasmas tend to be quasi-neutral. In other words, the density of electrons and ions - presumed singularly charged - for simplicity, have to be equal. So that means that the fluxes need to be equal for electrons and ions, or quasi-neutrality will not be maintained. And in general, diffusion coefficients -as the mobilities- are different for ions and electrons, so what happens? What can the plasma do? The plasma in fact, will set up an electric field that will tend to slow down the faster of the two species and accelerate the slower of the two species so that actually, the two fluxes will be balanced. This is a general property in the plasma, we'll derive it here, specifically for the case of unmagnetized plasmas or $B_0 = 0$ because it is simpler. So we assume equal densities again for ions and electrons, we call it n . Let's write the fluxes. For the ions, Γ_i is mobility of the ions $[\mu_i]$ n times the electric field. Of course electric field is common between two species, is a property of the plasma as a whole, minus the diffusion term $[D_i \nabla n]$. That has to be equal to Γ_e for electrons, which is minus μ_e .

Notes

Summary



14m 14s

Un-magnetized plasma: ambipolar field and D_a

Quasi-neutrality implies $\Gamma_i \sim \Gamma_e$. As in general $D_e \neq D_i$, an *ambipolar* electric field is generated in the plasma. We derive it here for $\mathbf{B}_0=0$, but it is a general effect

Assume $n_e = n_i = n$ $\Gamma_i = \mu_i n E - D_i \nabla n = \Gamma_e = -\mu_e n E - D_e \nabla n$

$\Rightarrow E = \frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n}$; resulting flux for both species $\Gamma = \Gamma_i = \Gamma_e$

$\Gamma_e = -\mu_e n E - D_e \nabla n = -\mu_e n \left(\frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n} \right) - D_e \nabla n = \frac{-\mu_e D_i + \mu_e D_e - \mu_i D_e - \mu_e D_e}{\mu_i + \mu_e} \nabla n = -\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \nabla n$

Plasma

Remember that the mobility was defined with no sign, so I have to put a minus sign here, times electric field minus the *Fick's Law*, if you like, term, that is the diffusion term. So the two have to be equal, that means I can extract the value of the electric field for that to happen, which is simply the difference between the diffusion coefficients $[D_i - D_e]$ divided by the sum of the mobilities, $[\mu_i + \mu_e]$ times $\nabla n / n$. So we have a flux that will be common for both species, that I can evaluate by replacing the electric field that we have just found in the expression for one of the two fluxes that I have for ions and electrons. It's equivalent because of course they have to be the same. Let's take Γ_e for example, so I have $-\mu_e n E - D_e \nabla n$ and I replace the value of E that we have just found. This is my electric field, a diffusion term and I simply do a simple calculation here. Everything will be multiplied times the ∇n so I put that on the right, and I have the different terms, $-\mu_i D_i + \mu_e D_e - \mu_i D_e - \mu_e D_e$. There are two terms that drop and I'm left with something like minus sign in front, a combination of mobilities and diffusion coefficients, $\mu_i D_e + \mu_e D_i$ divided by the sum of the mobilities, times the gradient of n .

Notes

Summary



15m 49s

Un-magnetized plasma: ambipolar field and D_a

Quasi-neutrality implies $\Gamma_i \sim \Gamma_e$. As in general $D_e \neq D_i$, an *ambipolar* electric field is generated in the plasma. We derive it here for $\mathbf{B}_0=0$, but it is a general effect

Assume $n_e = n_i = n$ $\Gamma_i = \mu_i n \mathbf{E} - D_i \nabla n = \Gamma_e = -\mu_e n \mathbf{E} - D_e \nabla n$

$\Rightarrow \mathbf{E} = \frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n}$; resulting flux for both species $\Gamma = \Gamma_i = \Gamma_e$

$\Gamma_e = -\mu_e n \mathbf{E} - D_e \nabla n = -\mu_e n \left(\frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n} \right) - D_e \nabla n = \frac{-\mu_e D_i + \mu_e D_e - \mu_i D_e - \mu_e D_e}{\mu_i + \mu_e} \nabla n = -\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \nabla n$

D_a 'ambipolar'

As $\mu_e \gg \mu_i$ $D_a \sim \frac{\mu_e D_i + \mu_i D_e}{\mu_e} = D_i + \frac{\mu_i}{\mu_e} D_e \sim D_i + \frac{T_e}{T_i} D_i \sim 2D_i$ if $T_e \sim T_i$

Plasma

Now I notice that this is a flux which is proportional with the minus sign to the gradient of n , so that means that the term that's in front of the gradient of n is also a diffusion coefficient, is a new diffusion coefficient we call *ambipolar* D_a and this diffusion coefficient is a combination of the different terms from ions and electrons, it is a consequence of the electric field which we also call *ambipolar* that was set up by the plasma in order to maintain equal fluxes for the different species and therefore, quasi-neutrality. Generally, we can notice that the mobility of the electrons is much larger than that of the ions so we can simplify the expression for the ambipolar diffusion coefficient, and get a feeling for its dependences. So it's $\mu_i D_i + \mu_i D_e$ and then denominator. I neglect the ion mobility and that's equal to simply the ion diffusion coefficient plus the ratio of the two mobilities: μ_i / μ_e times the electron diffusion coefficient And if I remember the Einstein relations that relate mobility and diffusion coefficients, I can express this as D_i plus the temperature ratio, T_e/T_i D_i And if the temperatures are similar, that is simply two times the ion diffusion coefficients.

Notes

Summary



Un-magnetized plasma: ambipolar field and D_a

Quasi-neutrality implies $\Gamma_i \sim \Gamma_e$. As in general $D_e \neq D_i$, an *ambipolar* electric field is generated in the plasma. We derive it here for $\mathbf{B}_0=0$, but it is a general effect

Assume $n_e = n_i = n$ $\Gamma_i = \mu_i n E - D_i \nabla n = \Gamma_e = -\mu_e n E - D_e \nabla n$

$\Rightarrow E = \frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n}$; resulting flux for both species $\Gamma = \Gamma_i = \Gamma_e$

$\Gamma_e = -\mu_e n E - D_e \nabla n = -\mu_e n \left(\frac{D_i - D_e}{\mu_i + \mu_e} \frac{\nabla n}{n} \right) - D_e \nabla n = \frac{-\mu_e D_i + \mu_e D_e - \mu_i D_e + \mu_e D_e}{\mu_i + \mu_e} \nabla n = -\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \nabla n$

As $\mu_e \gg \mu_i$ $D_a \sim \frac{\mu_e D_i + \mu_i D_e}{\mu_e} = D_i + \frac{\mu_i}{\mu_e} D_e \sim D_i + \frac{T_e}{T_i} D_i \sim 2D_i$ if $T_e \sim T_i$

$D_i < D_a << D_e$

Plasma

That is to say that, in general, we can say that D_i is the smallest of all diffusion coefficients. The ambipolar diffusion coefficient will be larger than that but it will still be significantly lower than the electron diffusion coefficient. So that means that -as we anticipated intuitively-, the plasma is setting up electric fields so that the diffusion of both species will occur at a rate which will be larger than the ions, larger than that that the ions would have naturally, should I say, and lower than that of the electrons, or again that the electrons would have naturally in the absence of any plasma effect.

Notes

Summary



Magnetized plasma: particle diffusion

- The simple approach $D \sim (\text{step size})^2 \times \text{frequency}$ makes sense, with the caveat that there may be an ambipolar E-field to set equal fluxes for ions and electrons

With $B_0 \neq 0$ we must separate directions

- Along B_0 : very similar to $B_0=0$, except that ambipolar field and diffusion may be generated in a more complicated way, depending on specific 3D configuration
- Across B_0 : collisions are necessary for transport of particles (over distances larger than particle orbits)
 - Typical step size?

Plasma

Let's go now to the case that interests us the most, that is the case of a magnetized plasma, which is the case of fusion plasmas. What we have verified together is that the simple approach of taking a diffusion coefficient which is simply the product of a typical step size square times a typical frequency makes sense. With only the caveat we have just found that there may be an ambipolar electric field to set equal fluxes for the different species. But in general we can actually attempt evaluating the transport by the diffusion coefficient, evaluated in this very simple way. We notice that in the presence of a magnetic field we of course have different directions. We don't have an isotropic situation anymore. We have the direction along the field, for which the dynamics will be very similar to the case of unmagnetized plasmas, except perhaps that the ambipolar field, and therefore the diffusion, may be generated in a more complicated way which will depend on the specific 3D configuration. We will not go into that kind of detail but we just noticed that there will be ambipolarity, so this setting up of an electric field, but it will be more complicated than the simple, say, 1D case that we have illustrated together.

Notes

Summary



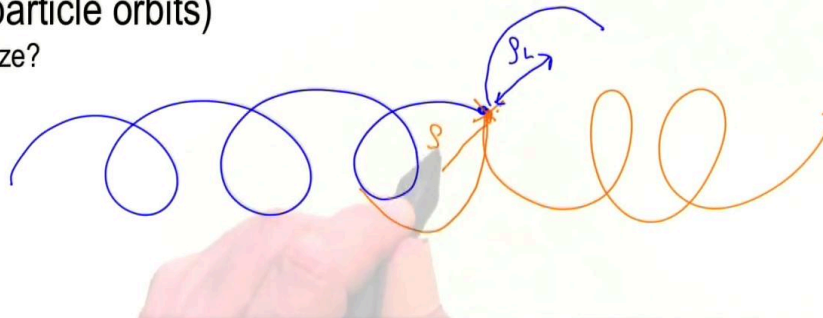
20m 24s

Magnetized plasma: particle diffusion

- The simple approach $D \sim (\text{step size})^2 \times \text{frequency}$ makes sense, with the caveat that there may be an ambipolar E-field to set equal fluxes for ions and electrons

With $B_0 \neq 0$ we must separate directions

- Along B_0 : very similar to $B_0=0$, except that ambipolar field and diffusion may be generated in a more complicated way, depending on specific 3D configuration
- Across B_0 : collisions are necessary for transport of particles (over distances larger than particle orbits)
 - Typical step size?



Plasma

Across the magnetic field is the more interesting direction for confinement. We are setting up a magnetic cage to trap the plasma to make fusion happen and of course a plasma via transport phenomena will tend to go, given the rate across the magnetic field and that's something we really need to keep under control. To go across the field, collisions are necessary. At least to go over distances that are larger than particle orbits. So let's play the game in which we try to identify the step size and the frequency, and therefore evaluating the diffusion coefficient in a simple way. What is the typical step size? So we have particles that undergo their gyro motion, and this particle will probably encounter another particle, with a different trajectory that will come maybe from another direction. And what happens when we have a collision between the two particles? There will be for the orange particle sort of a jump in its orbit, in this direction, and for the blue particle, it will be jumping the orbit in the other direction. So the displacement, that will be taken typically upon such a collision, will be of the order of the Larmor orbit size, ρ_L .

Notes

Summary

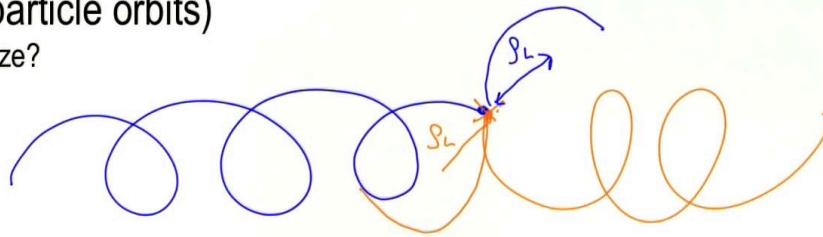


Magnetized plasma: particle diffusion

- The simple approach $D \sim (\text{step size})^2 \times \text{frequency}$ makes sense, with the caveat that there may be an ambipolar E-field to set equal fluxes for ions and electrons

With $B_0 \neq 0$ we must separate directions

- Along B_0 : very similar to $B_0=0$, except that ambipolar field and diffusion may be generated in a more complicated way, depending on specific 3D configuration
- Across B_0 : collisions are necessary for transport of particles (over distances larger than particle orbits)
- Typical step size?



Plasma

We are evaluating orders of magnitude of course, we don't pretend to be very precise in this evaluation but it's very important to give a typical step size for this concept of diffusion coefficient. So a typical step size is therefore the *Larmor orbit* for different particles.

Notes

Summary



23m 21s

Magnetized plasma: particle diffusion

- The simple approach $D \sim (\text{step size})^2 \times \text{frequency}$ makes sense, with the caveat that there may be an ambipolar E-field to set equal fluxes for ions and electrons

With $B_0 \neq 0$ we must separate directions

- Along B_0 : very similar to $B_0=0$, except that ambipolar field and diffusion may be generated in a more complicated way, depending on specific 3D configuration
- Across B_0 : collisions are necessary for transport of particles (over distances larger than particle orbits)
 - Typical step size: Larmor radius ρ_L
 - Characteristic frequency?

For particle transport, is that for the transfer of momentum, but only for *un-like* particles. The collisions among *like* particles only lead to a swap in the position of their guiding centers, i.e. no net transport.

$D_{\perp,e} \approx \rho_{Le}^2 \langle v_{ei} \rangle$; $D_{\perp,i} \approx \rho_{Li}^2 \langle v_{ie} \rangle$. Note: if $T_e \sim T_i$, then $D_{\perp,e} \sim D_{\perp,i}$ and there is no need for an ambipolar field

Plasma

So we identify that, and the next question of course is *what is the characteristic frequency?* Now we are considering the transport of particles. So the characteristic frequency here is that for the transfer of momentum. But we notice one thing, we need to consider only unlike particle collisions. Because the collisions among like particles will only lead to a swap in the position of their respective guiding centers, therefore no net transport. So those actually don't contribute to any transport at all. The end result of this very simple reasoning is that we have a diffusion coefficient that can be evaluated. For example for the electrons, and this perpendicular sign indicates, of course, that we're talking about the direction across the magnetic field. So that's the typical step size, the Larmor radius for electrons squared, times the frequency for the exchange of momentum between different species or between unlike particles. Similarly for the ions we get the ion Larmor radius and the frequency for the exchange in momentum between ions and electrons, the other way around.

Notes

Summary



23m 40s

Magnetized plasma: heat diffusion

- A diffusion equation similar to that for particles is valid for heat

$$\frac{3}{2} \frac{\partial T}{\partial t} - \chi \nabla^2 T = 0$$

- The heat diffusivity χ (here assumed constant) plays the same role as D , and is also obtained from a typical step size and a characteristic frequency

- Typical step size: Larmor radius ρ_L , as for D
- Characteristic frequency?

For heat transport, is that for the transfer of energy, v_E , but collisions between *all* particles (not only *un-like* particles) contribute to heat diffusion

- Along \mathbf{B}_0 : $\chi_{\parallel,e} \approx v_{the}^2 / [\langle v_E^{e/i} \rangle + \langle v_E^{e/e} \rangle] \sim 0.7 v_{the}^2 / \langle v_{e/i} \rangle$ and $\chi_{\parallel,i} \approx v_{thi}^2 / [\langle v_E^{i/i} \rangle + \langle v_E^{i/e} \rangle] \sim v_{thi}^2 / \langle v_{i/i} \rangle$
Parallel heat transport is dominated by electrons
- Across \mathbf{B}_0 : $\chi_{\perp,e} \approx \rho_{Le}^2 [\langle v_E^{e/i} \rangle + \langle v_E^{e/e} \rangle] \sim 2 \rho_{Le}^2 \langle v_{e/i} \rangle$ and $\chi_{\perp,i} \approx \rho_{Li}^2 [\langle v_E^{i/e} \rangle + \langle v_E^{i/i} \rangle] \sim \rho_{Li}^2 \langle v_{i/i} \rangle$
Perpendicular heat transport is dominated by ions

Plasma

If we say that the plasma is at the thermodynamic equilibrium for example with two temperatures for ions and electrons that are similar, then the perpendicular diffusion coefficients are also similar because actually the effective collision frequency difference compensates the difference between the two Larmor radii. So in that case, in fact we don't need, or the plasma does not need, to set up an ambipolar field to get equal fluxes. So we have a situation in which there's a so-called ambipolarity, in a natural way. I also notice, although we will not demonstrate that explicitly, that a diffusion equation that's similar to that for particles is valid for heat. So here's a diffusion equation for heat. The variable here is the temperature, and we have a coefficient χ which plays exactly the same role as D for particles. It's called the *heat diffusivity*. And in this case, as we have done before, I assumed that to be a constant so I could take it out of the differential operator. χ has the same characteristics as D and can also be obtained from identifying a typical step size and a characteristic frequency, or a typical collision time.

Notes

Summary



24m 55s

Magnetized plasma: heat diffusion

- A diffusion equation similar to that for particles is valid for heat

$$\frac{3}{2} \frac{\partial T}{\partial t} - \chi \nabla^2 T = 0$$

- The heat diffusivity χ (here assumed constant) plays the same role as D , and is also obtained from a typical step size and a characteristic frequency

- Typical step size: Larmor radius ρ_L , as for D
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- Along \mathbf{B}_0 : $\chi_{\parallel,e} \approx v_{the}^2 / [\langle v_E^{e/i} \rangle + \langle v_E^{e/e} \rangle] \sim 0.7 v_{the}^2 / \langle v_{e/i} \rangle$ and $\chi_{\parallel,i} \approx v_{thi}^2 / [\langle v_E^{i/i} \rangle + \langle v_E^{i/e} \rangle] \sim v_{thi}^2 / \langle v_{i/i} \rangle$
Parallel heat transport is dominated by electrons
- Across \mathbf{B}_0 : $\chi_{\perp,e} \approx \rho_{Le}^2 [\langle v_E^{e/i} \rangle + \langle v_E^{e/e} \rangle] \sim 2 \rho_{Le}^2 \langle v_{e/i} \rangle$ and $\chi_{\perp,i} \approx \rho_{Li}^2 [\langle v_E^{i/e} \rangle + \langle v_E^{i/i} \rangle] \sim \rho_{Li}^2 \langle v_{i/i} \rangle$
Perpendicular heat transport is dominated by ions

Plasma

We have said together that the typical step size is the Larmor radius, just like for D . What is the characteristic frequency? Well, it will be the frequency of Coulomb collisions like in the previous case, but for heat transport, it would be that for the transfer of energy as opposed to for the transfer of momentum. That's the difference number one. the difference number two is that for heat transport, actually collisions between *all* particles, and not only unlike particles, contribute to diffusion. We don't care which particles collide as long as they transfer energy. So there is no effect leading to a zero net transport if particles are identical. So that means that the evaluation of the diffusion coefficient, although based on the same principle, is somewhat different in quantitative terms. Let's consider both directions along the magnetic field and across the magnetic field. Along the magnetic field, $\chi_{\parallel,e}$, for electrons will be the thermal speed for the electrons squared, divided by the sum of the relevant collision frequencies, again for the exchange of energy which I call $\langle v_E \rangle$ as you have done in the first part of the course, electron and ions or electrons and electrons.

Notes

Summary



26m 15s

Magnetized plasma: heat diffusion

- A diffusion equation similar to that for particles is valid for heat

$$\frac{3}{2} \frac{\partial T}{\partial t} - \chi \nabla^2 T = 0$$

- The heat diffusivity χ (here assumed constant) plays the same role as D , and is also obtained from a typical step size and a characteristic frequency

- Typical step size: Larmor radius ρ_L , as for D
- Characteristic frequency?

For heat transport, is that for the transfer of energy, v_E , but collisions between *all* particles (not only *un-like* particles) contribute to heat diffusion

- Along \mathbf{B}_0 : $\chi_{\parallel,e} \approx v_{the}^2 / [\langle v_E^{e/i} \rangle + \langle v_E^{e/e} \rangle] \sim 0.7 v_{the}^2 / \langle v_{e/i} \rangle$ and $\chi_{\parallel,i} \approx v_{thi}^2 / [\langle v_E^{i/i} \rangle + \langle v_E^{i/e} \rangle] \sim v_{thi}^2 / \langle v_{i/i} \rangle$
Parallel heat transport is dominated by electrons
- Across \mathbf{B}_0 : $\chi_{\perp,e} \approx \rho_{Le}^2 [\langle v_E^{e/i} \rangle + \langle v_E^{e/e} \rangle] \sim 2 \rho_{Le}^2 \langle v_{e/i} \rangle$ and $\chi_{\perp,i} \approx \rho_{Li}^2 [\langle v_E^{i/e} \rangle + \langle v_E^{i/i} \rangle] \sim \rho_{Li}^2 \langle v_{i/i} \rangle$
Perpendicular heat transport is dominated by ions

Plasma

And that corresponds to a constant of order of one, times the square of the thermal speed divided by the collision frequency for momentum exchange, which we normally refer to. For the ions, it will be something very similar, except of course that there will be the collision frequency between ions here and the ion thermal speed squared. And we can notice by simply looking at these two expressions that the parallel heat transfer will be dominated by electrons, in other words, electrons we contribute to the transport along the magnetic field of heat, in a much, much larger proportion because of the much, much larger value of the thermal speed, for similar temperatures than that of the ions. Let's look at the direction across the magnetic field. We do the exact same reasoning, considering the appropriate collision frequencies and we get something for the electrons that's of the order - although there's a factor here of ρ_{Le}^2 times the collision frequency for the momentum exchange. So it's not very different from the diffusion of particles. And for the ions, again, it's something that refers to the collision frequency between ions and ions. But because the Larmor radius of the ions is so much larger than that of the electrons, the perpendicular heat transfer will be dominated by the ions, so the opposite situation to that of the parallel heat transport.

Notes

Summary



Summary



- Classical diffusion results from random walks of ions / electrons undergoing Coulomb collisions
- With $B_0 \neq 0$ cross-field transport is dictated by Larmor orbit size
- Transport along (across) B_0 is dominated by electrons (ions)
- Next lecture: compare classical predictions with data and discuss other transport mechanisms

Plasma

In summary, in this lecture we have gone back to some of the basics of plasma physics to study the classical transfer mechanisms in fusion plasma. We have seen the classical diffusion results from the random walk of ions or electrons, that undergo Coulomb collisions, that is collisions dictated by the electric field. In the presence of a background magnetic field, the transport across this field is dictated by the size of the Larmor orbits of the particles. We have seen that the transport along the field is dominated by the electrons while the transport across the field is dominated by the ions. In the next lecture, we'll compare the predictions of this classical theory with the experimental data from fusion devices, and we'll discuss how we can actually go beyond the classical paradigm and explain the experimental data, and what are the transport mechanisms that actually play in the fusion devices.

Notes

Summary



29m 15s