ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

PLAS@PAR Plasmas à Paris

Propagation and absorption 1

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 8b

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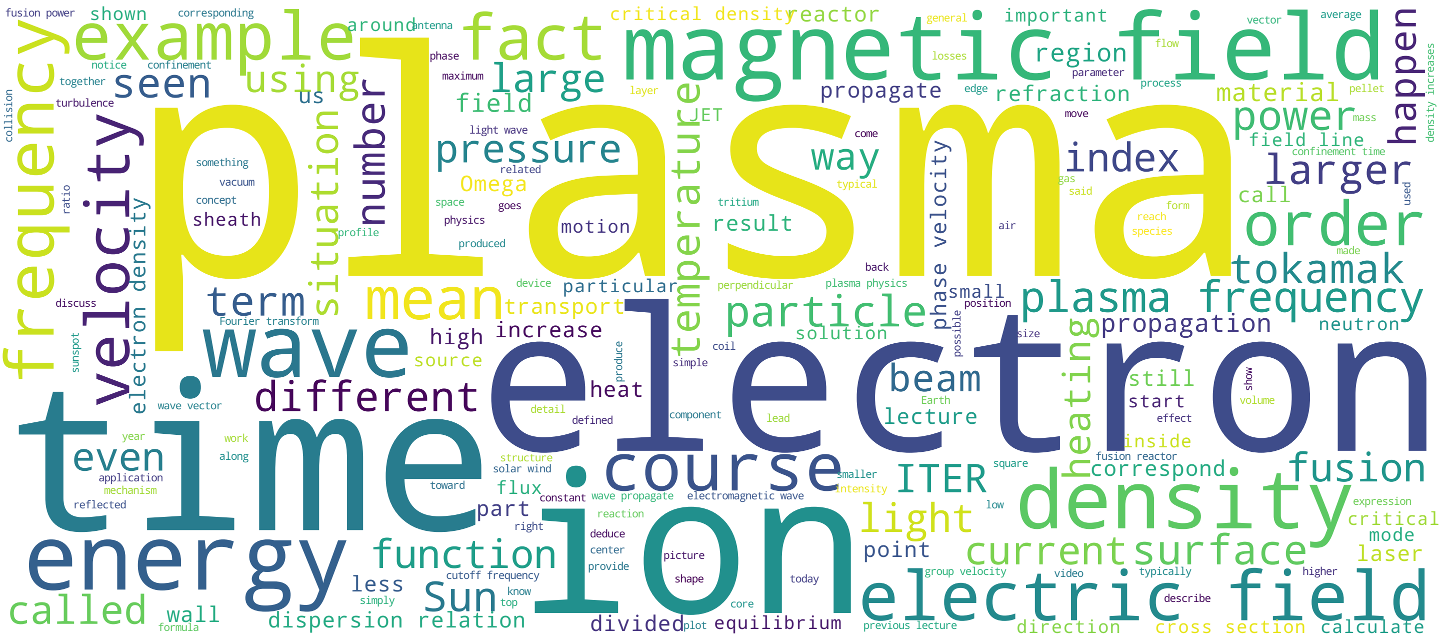

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Video

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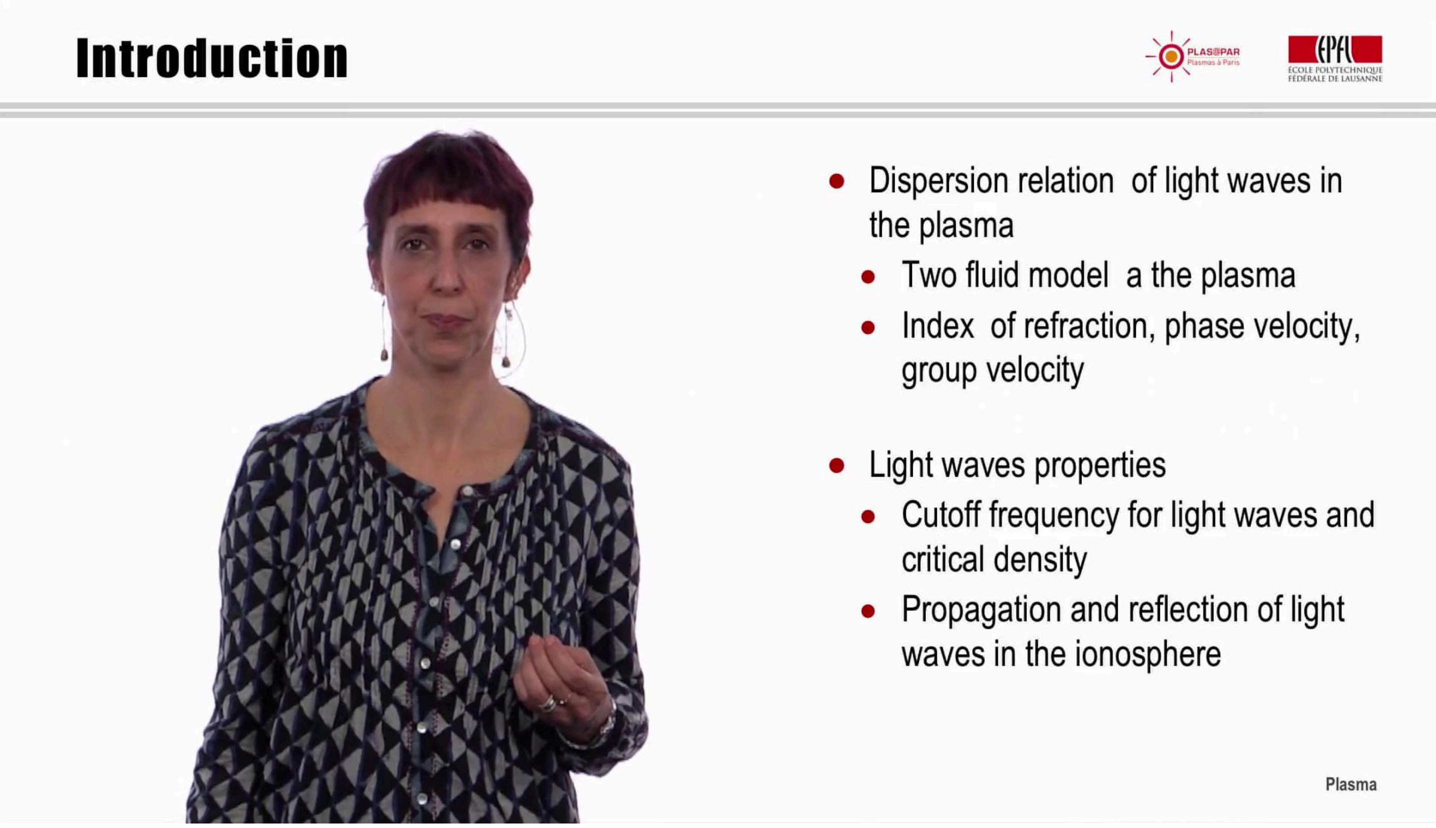


Welcome. In today's lecture, we will derive the dispersion relation of light waves in an unmagnetized plasma using the same technique presented in previous lectures by Professor Richie. With the two fluid model, we will will define an equilibrium and look for a small perturbation in the form of a wavelike solution. We will study in some detail how the plasma behaves, and we will calculate the index of refraction, the phase and group velocity of the wave. Finally, we will introduce the cutoff frequency for the light waves and the critical density and consider a simple model for the propagation of light in the ionosphere.

Notes

Summary

0m 04s

Electromagnetic waves in two fluid model

Equilibrium for an unmagnetized, cold, homogeneous plasma at rest

$s = e, i \qquad \vec{u}_{s0} = \vec{0} \qquad T_s = 0$

$n_{e0} = Z n_{i0} \longrightarrow \vec{E}_0 = \vec{0}$

$\vec{B}_0 = \vec{0} \qquad$ isotropic

Maxwell equations

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}_1}{\partial t} + \mu_0 \vec{J}$$

$$\vec{J} = -e n_e \vec{u}_e + Z e n_i \vec{u}_i$$

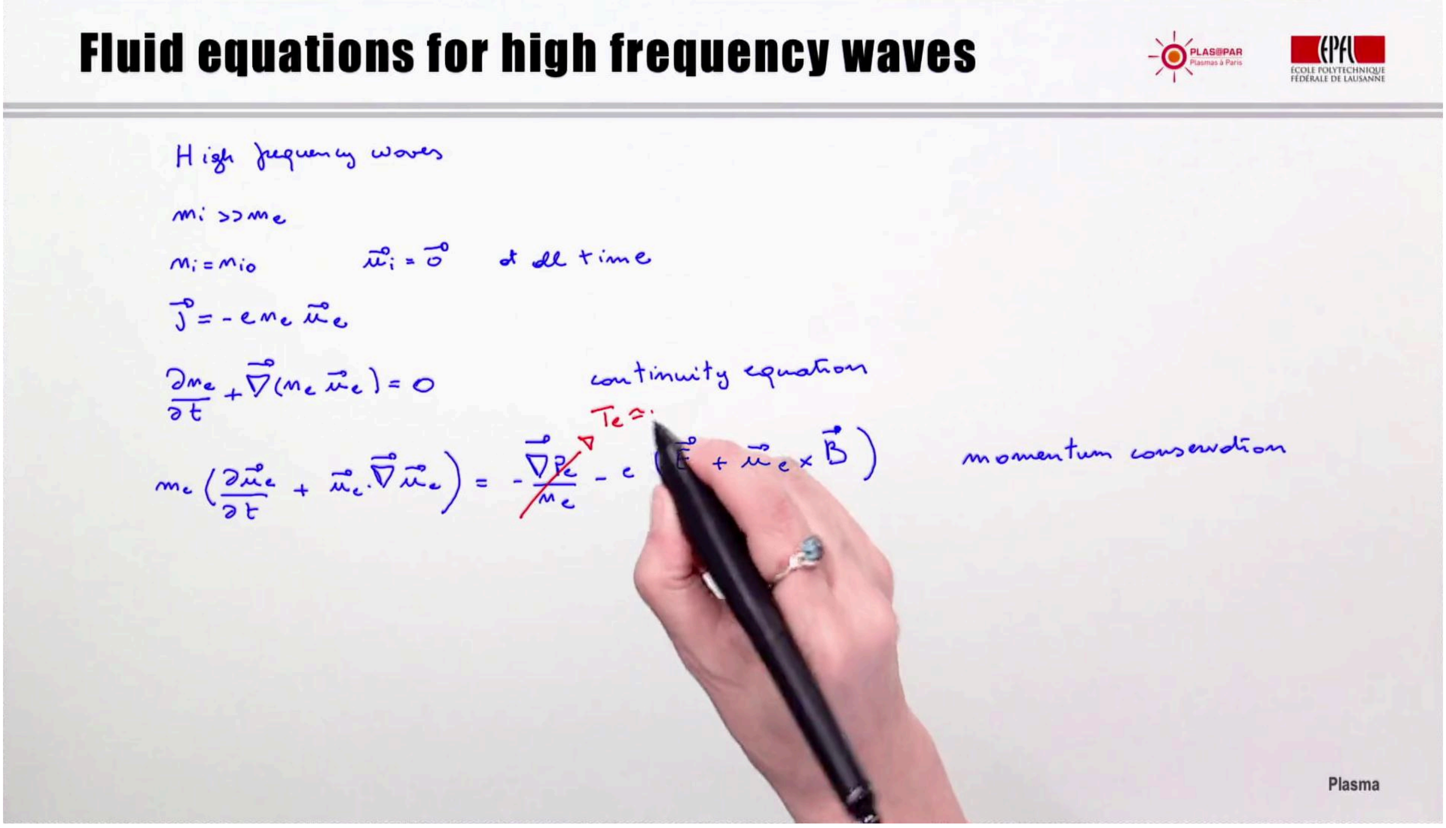
Plasma

First of all, we write the equilibrium for the plasma. The plasma is overall neutral, and to simplify the problem, we consider only two species, the electrons and one ion species. The plasma is at rest and is cold. So for both species, the equilibrium velocity and the equilibrium temperature will be zero. Then we write the neutrality condition for a homogeneous plasma. This means that for both electrons and ions the density is constant and the total charge density is zero. In such a plasma, there is no electric field at equilibrium. Finally, the plasma is unmagnetized, so there is no equilibrium magnetic field. This plasma behaves as an isotropic system. This means that we only have one solution for the electromagnetic wave. That is the analogy of the ordinary mode in a magnetized plasma. We can now write Maxwell equations. For an electromagnetic wave, we need both Faraday's law and Maxwell Ampere equation J, the current here is the global current for the twofold system, and it can be expressed in terms of the electron and ion density and velocities.

Notes

Summary

0m 42s

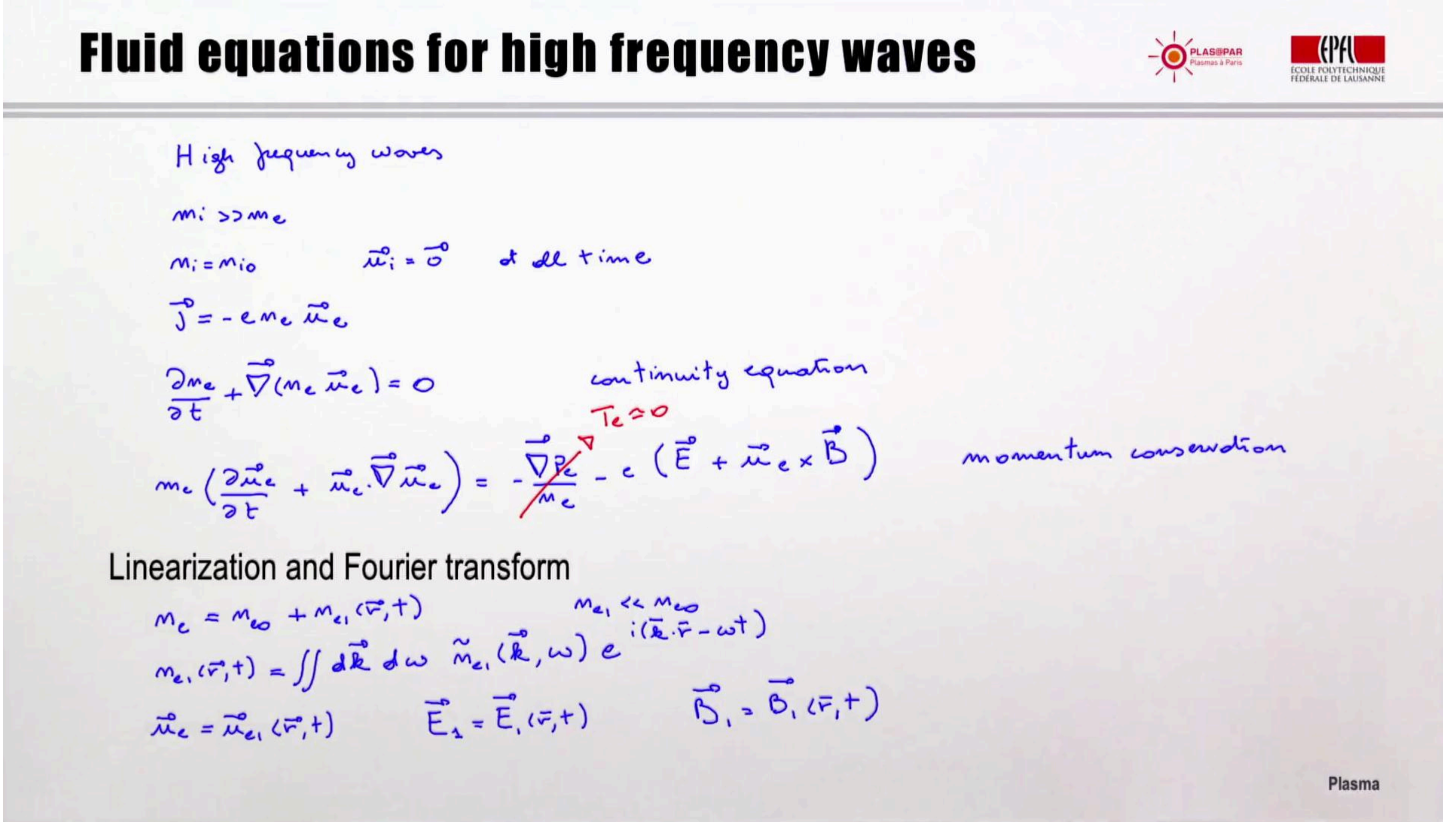


As we will see a posteriori, these are high frequency waves that is, their typical oscillation frequency is of the same order or larger of the electron plasma frequency. In this situation, since the mass of the ions is much larger than electron mass this wave propagate on such fast time scales that the ions do not have time to move and they can be considered as an immobile neutralizing background. For this reason, the density of the ion and the velocity stays constant all the time and equal to the initial value. In this situation, we can write again, the current that now will just be a function of the electron density and the electron velocity. In order to describe the propagation of the waves, we now need to write the equations for the evolution of the electron density and electron velocity. This is given by the fluid equation. First of all, continuity equation that is also conservation of particles and then the momentum conservation equation in this equation, since the electrons behave as a charge fluid responding to electromagnetic fields and pressure fields, we have a pressure term. However, in our case we consider a cold plasma and we can neglect this term.

Notes

Summary

1m 57s

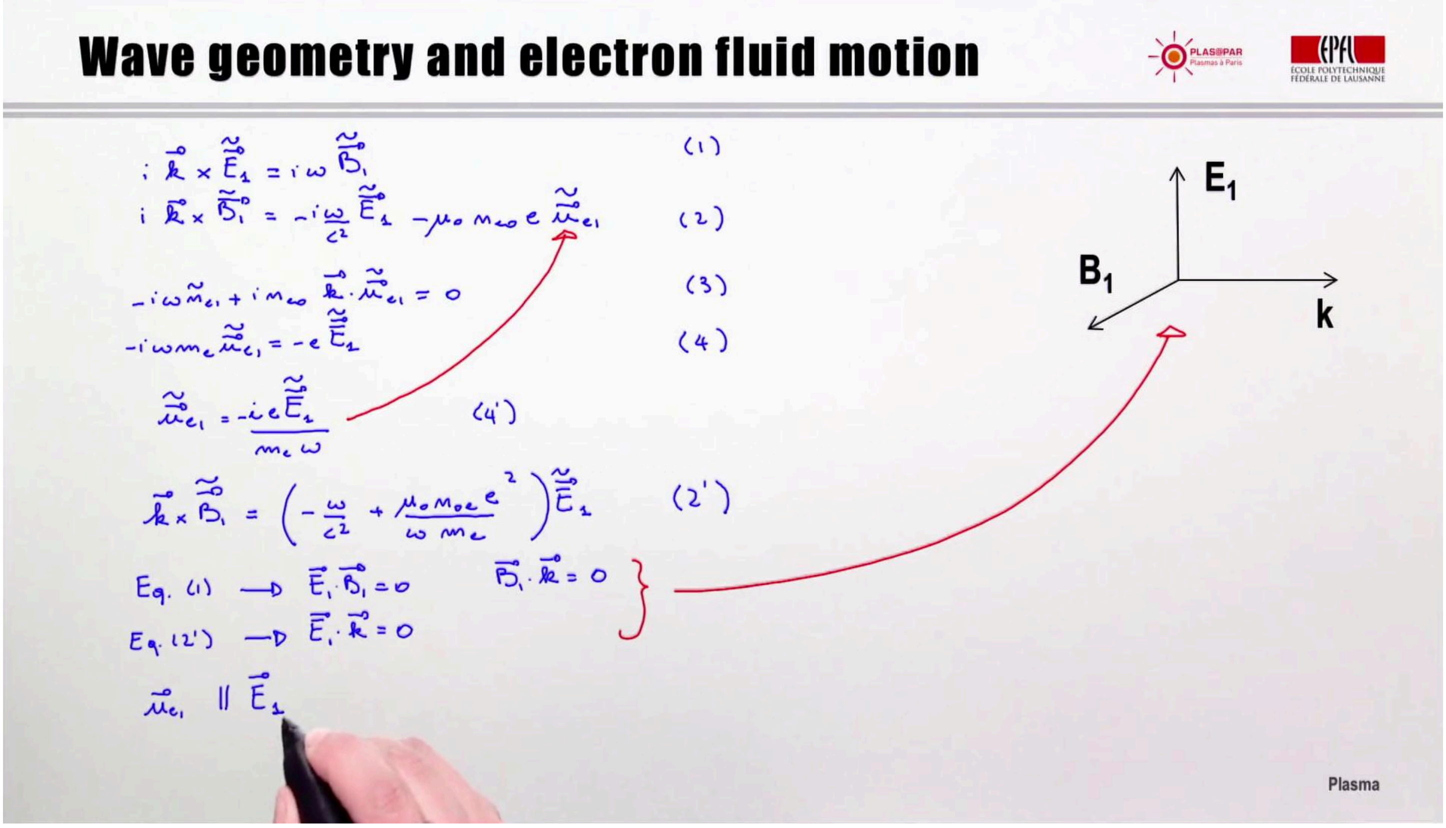


We can now consider a small perturbation and linearize our equations, the fluid equations and the Maxwell equations. At the same time, we will take the Fourier transform, as we have seen in previous lecture by Professor Richie. First of all, we define the small perturbation and the Fourier transform for the electron density. So the perturbation is much less than the equilibrium quantity, and the Fourier transform is defined in this way. We can do the same thing for the velocity, the electric field, and the magnetic field. However, all these quantities are zero at equilibrium, so we'll simply have... And so when you will see it in the sign like here on top of U, E or B, you will know that we are now considering the Fourier transform of these quantities.

Notes

Summary

3m 24s



We can now write Maxwell equations for the Fourier transform quantities. Then we can write and linearize the fluid equation for the Fourier transform of the electron density and velocity. So this is continuity equation linearized, and this is the momentum conservation equation linearized. We have now as many equations as unknowns, one scalar and three vectors. We can then solve this system. To do so, we will express the velocity of the electrons as a function of the electric field. Then we can replace this in equation two. Equation two now becomes... Now we can deduce the number of properties. First of all, from equation one, we can deduce the electric field and the magnetic field are perpendicular and also the magnetic field and the K vector are perpendicular. From equation two prime that we just wrote, we can deduce that the electric field is also perpendicular to the K vector. This means that inside the plasma, an electromagnetic wave still consists of a triad of orthogonal vectors as in vacuum. So it's a purely transverse wave propagating inside the plasma. On top of this, from equation four prime, we can see that the velocity of the electron is parallel to the electric field.

Notes

Summary

4m 22s

Wave geometry and electron fluid motion

$$i\,\vec{k}\times\tilde{\vec{E}}_1 = i\omega\,\tilde{\vec{B}}_1 \qquad (1)$$

$$i\,\vec{k}\times\tilde{\vec{B}}_1 = -i\frac{\omega}{c^2}\tilde{\vec{E}}_1 - \mu_0 n_{e0}\, e\,\tilde{\vec{u}}_{e1} \qquad (2)$$

$$-i\omega\tilde{n}_{e1} + i n_{e0}\,\vec{k}\cdot\tilde{\vec{u}}_{e1} = 0 \qquad (3)$$

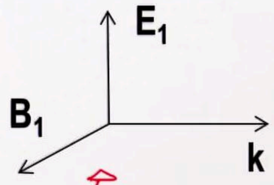
$$-i\omega m_e\tilde{\vec{u}}_{e1} = -e\,\tilde{\vec{E}}_1 \qquad (4)$$

$$\tilde{\vec{u}}_{e1} = \frac{-ie\tilde{\vec{E}}_1}{m_e\omega} \qquad (4')$$

$$\vec{k}\times\tilde{\vec{B}}_1 = \left(-\frac{\omega}{c^2} + \frac{\mu_0 n_{0e} e^2}{\omega m_e}\right)\tilde{\vec{E}}_1 \qquad (2')$$

$$\left.\begin{array}{ll} \text{Eq. (1)} \longrightarrow \vec{E}_1\cdot\vec{B}_1 = 0 & \vec{B}_1\cdot\vec{k} = 0 \\ \text{Eq. (2')} \longrightarrow \vec{E}_1\cdot\vec{k} = 0 & \end{array}\right\}$$

$$\vec{u}_{e1} \parallel \vec{E}_1 \qquad \tilde{\vec{u}}_{e1}\cdot\vec{k} = 0 \longrightarrow \tilde{n}_{e1} = 0 \qquad \text{Eq. (3)}$$



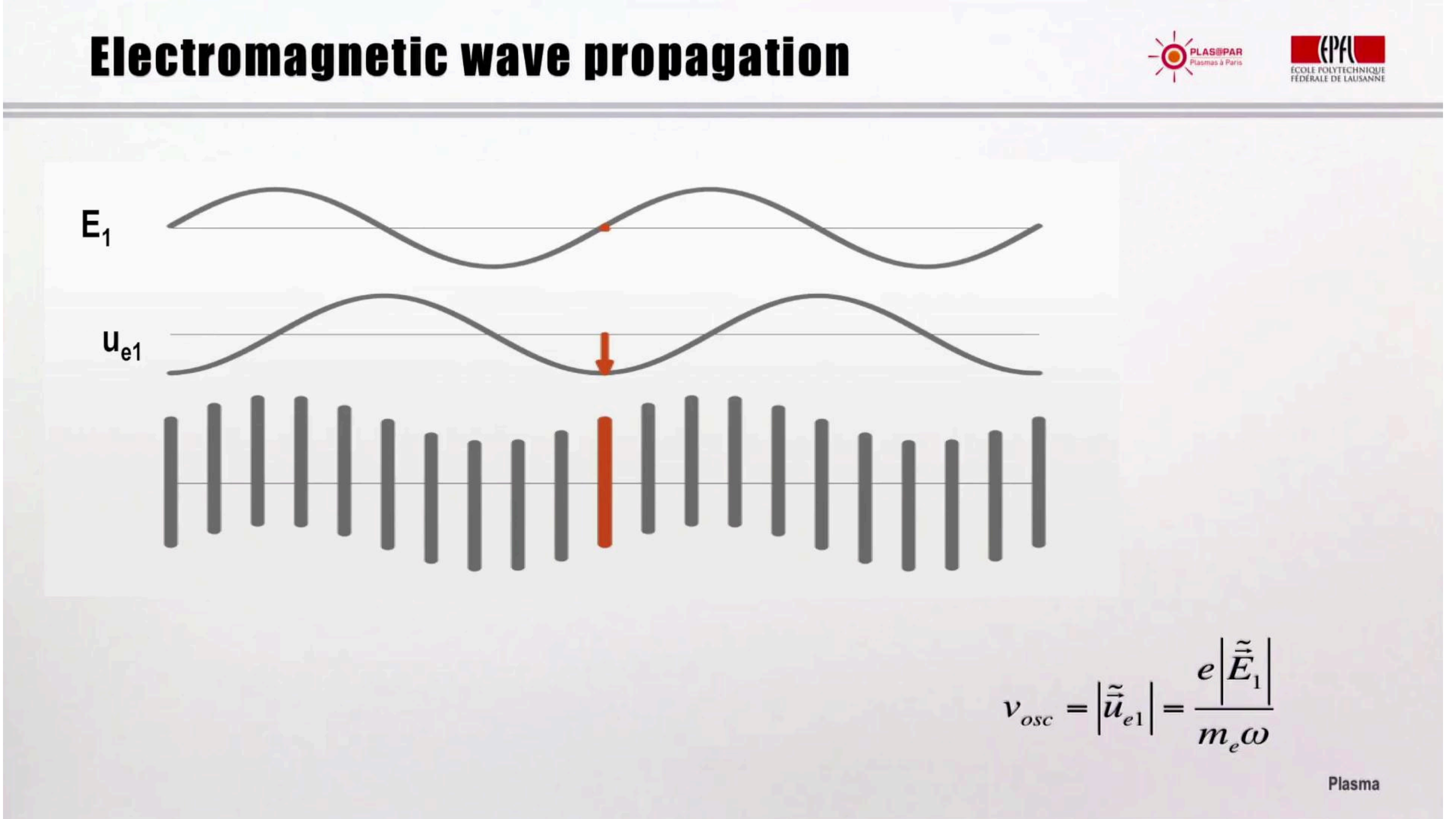
Plasma

This means that the velocity of the electron is also orthogonal to the K vector and U.K is zero. From this, from equation three, we can deduce that the density perturbation is also zero. This means that while the wave propagates in the plasma, there is no motion of the electron density and the electrons just move as rigid plane up and down along the electric field.

Notes

Summary

6m 24s

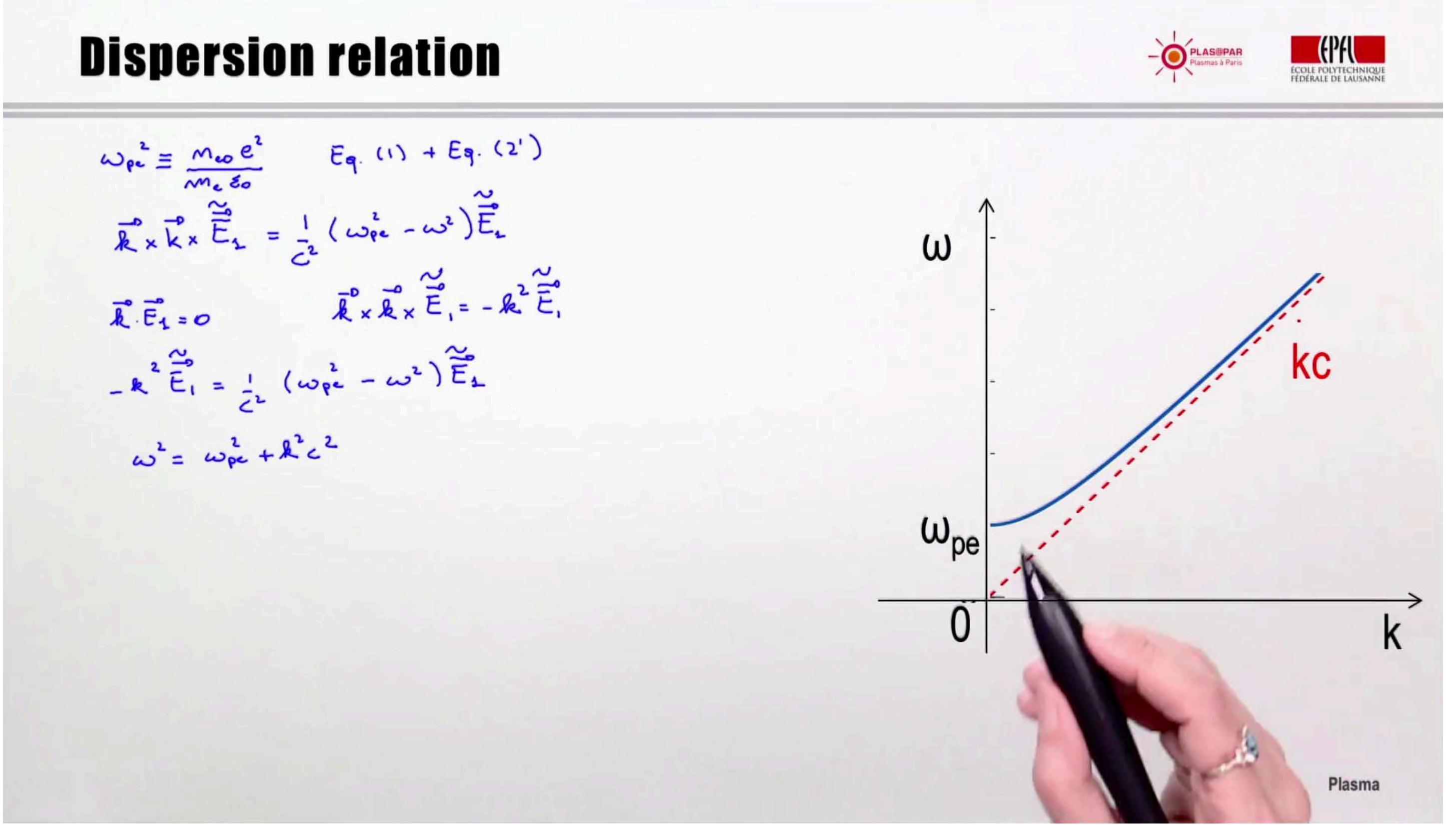


In this slide we can summarize what we just learned about an electromagnetic wave propagating inside the plasma. What is shown here is the electric field together with the electron velocity as a function of space and time. In the last line you can see planes of electron oscillating up and down according to the electric field. The movement of oscillation is characterized by a typical velocity that is called the oscillations velocity. That is defined here is the absolute value of the velocity of the electron, the maximum value that it can take is the oscillation velocity and is given by the charge times the electric field over the mass times the frequency.

Notes

Summary

7m 03s



We have found that electromagnetic wave is a purely transverse wave as in vacuum, and we have seen how the electrons behave when the wave propagates. Now we still need to find the dispersion relation of this wave in the plasma. That is how the frequency is related to the wave vector K. To do this, let us define the plasma frequency that you have already seen in previous lectures. By using this definition and by combining equation one and equation two prime, we can write an equation for the electric field only. Since as we have seen the electric field perpendicular to the wave vector, we can simplify this product and we just have where now K is the absolute value of the K vector. If we use this condition, the equation becomes simply... This equation can be satisfied only if the right hand side and the left hand side are identical, and this leads to the dispersion relation. As we can see, these waves are indeed high frequency waves, since the frequency is always larger than the plasma frequency. So the hypothesis of considering the ion as immobile is justified a posteriori. This dispersion relation is plotted here where we show Omega as a function of K.

Notes

Summary

7m 44s

Dispersion relation

$$\omega_{pe}^2 \equiv \frac{n_{e0} e^2}{m_e \varepsilon_0} \qquad \text{Eq. (1) + Eq. (2')}$$

$$\vec{k}\times\vec{k}\times\tilde{\vec{E}}_1 = \frac{1}{c^2}\left(\omega_{pe}^2-\omega^2\right)\tilde{\vec{E}}_1$$

$$\vec{k}\cdot\vec{E}_1 = 0 \qquad \vec{k}\times\vec{k}\times\tilde{\vec{E}}_1 = -k^2\tilde{\vec{E}}_1$$

$$-k^2\tilde{\vec{E}}_1 = \frac{1}{c^2}\left(\omega_{pe}^2-\omega^2\right)\tilde{\vec{E}}_1$$

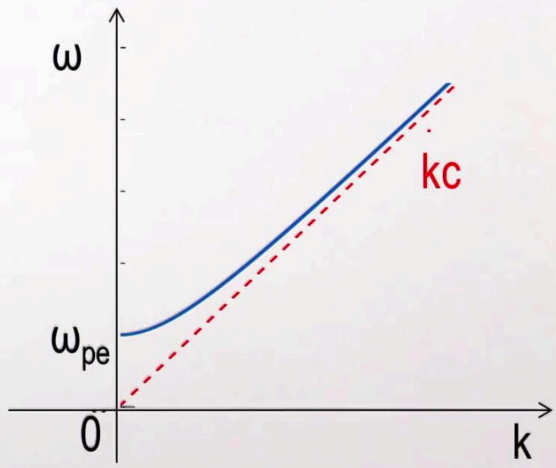
$$\omega^2 = \omega_{pe}^2 + k^2c^2$$

If $\omega > \omega_{pe}$

$$k = \frac{\omega}{c}\sqrt{1-\frac{\omega_{pe}^2}{\omega^2}} \qquad //$$

If $\omega < \omega_{pe}$ $\longrightarrow k = i k_i$

$$k_i = \frac{\omega}{c}\sqrt{\frac{\omega_{pe}^2}{\omega^2} - 1}$$



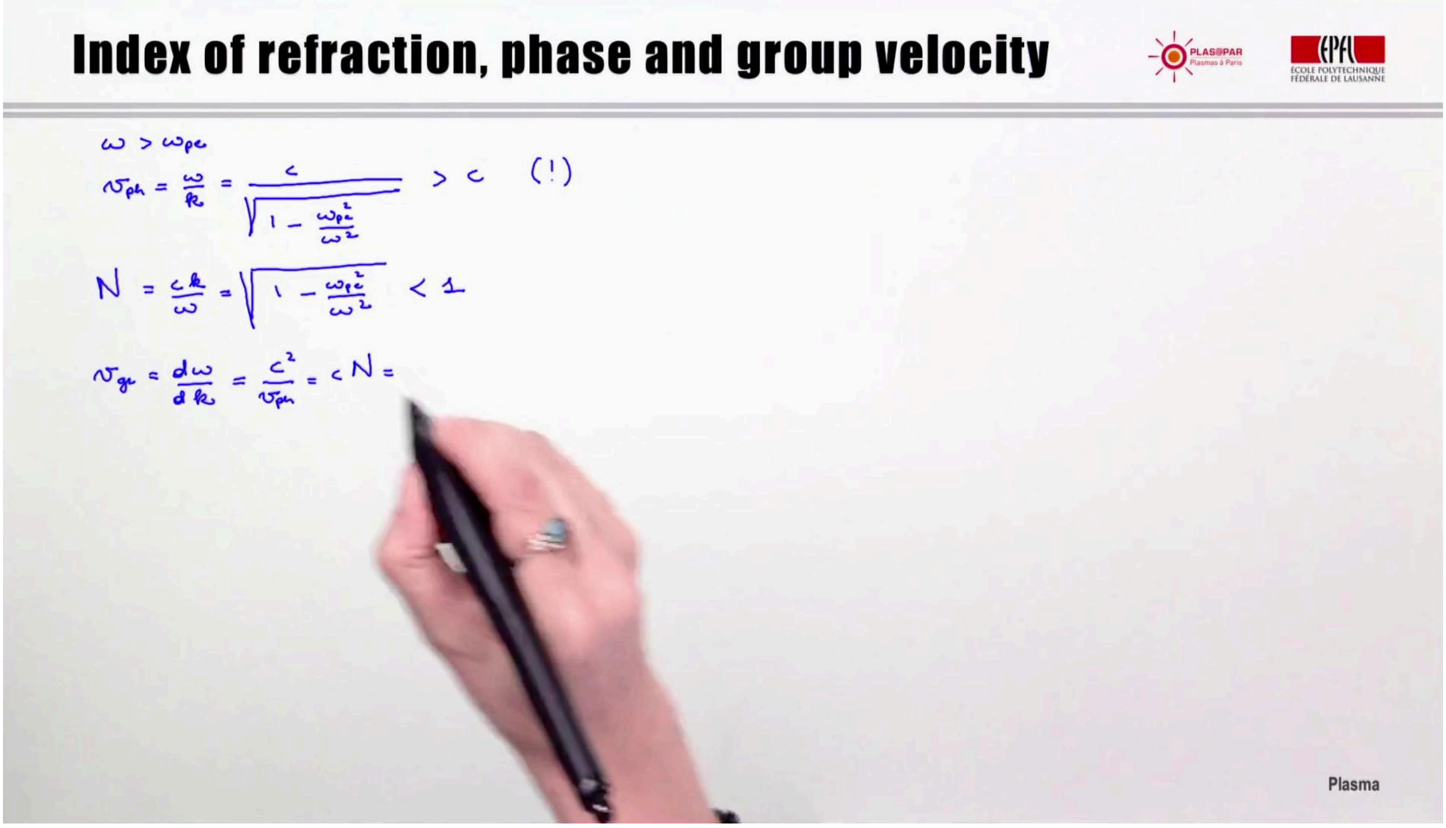
Plasma

So the frequency is always larger than the plasma frequency and when the K vector is large, so this term is dominant, or if the plasma frequency is very weak, we have that this dispersion relation asymptotically is equal to the dispersion relation of light in vacuum as one could expect. We can now solve for K as a function of Omega, where K is the real quantity only if Omega is larger than Omega P. However, we can still find a solution for K even in the opposite limit, but in this case, K will be purely imaginary and the imaginary contribution is equal to... We will discuss later the meaning of this solution, and for the moment, we just focus on the previous case, that is on the case Omega larger than Omega P and K real. That is the case of a propagating wave.

Notes

Summary

9m 08s

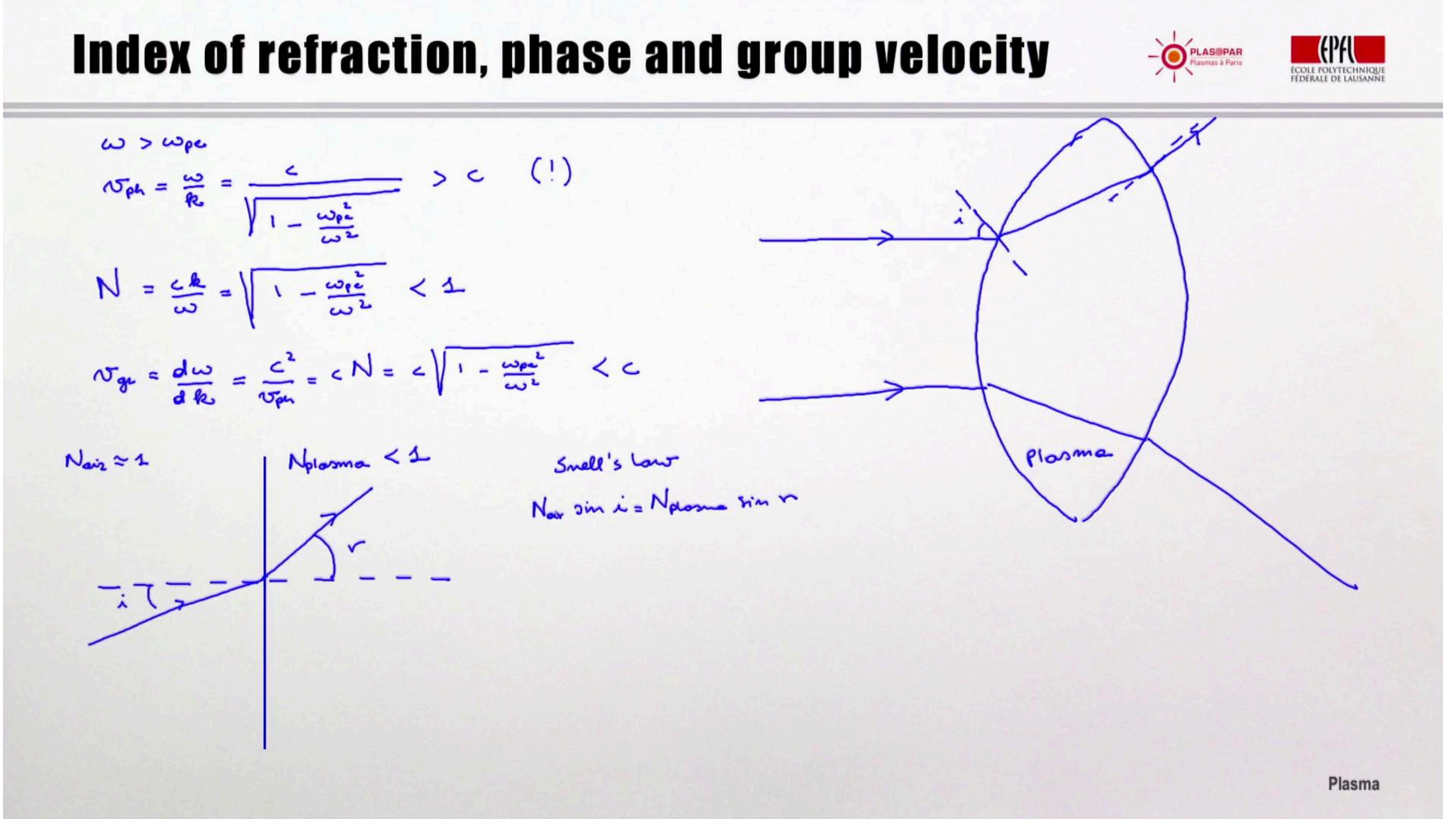


We are now in a position of calculating a number of quantities that are related to the electromagnetic wave in the plasma. Just for the case Omega larger than Omega P. That is a propagating wave. We can calculate first of all, the phase velocity. The definition of phase velocity is Omega over K, that is, by the dispersion relation. Since Omega is larger than Omega square, the denominator will be less than one, and the phase velocity is larger than the velocity of light. We can also calculate the index of refraction that is defined as the velocity of light over the phase velocity, and in this case, as opposed to the index of refraction in materials like water or glass, the index of velocity of waves in the plasma will be less than one. The condition the phase velocity is larger than C should not worry you because it's not really a violation of relativity. In fact, the transport of energy information they are not associated with phase velocity but with group velocity. So even if the phase velocity is larger than C, the system is still a physical system and well behaved. However, if we calculate the group velocity that is defined as the Omega over the K, and for light wave in a plasma is equal to C square over the phase or C times the index of refraction.

Notes

Summary

10m 02s

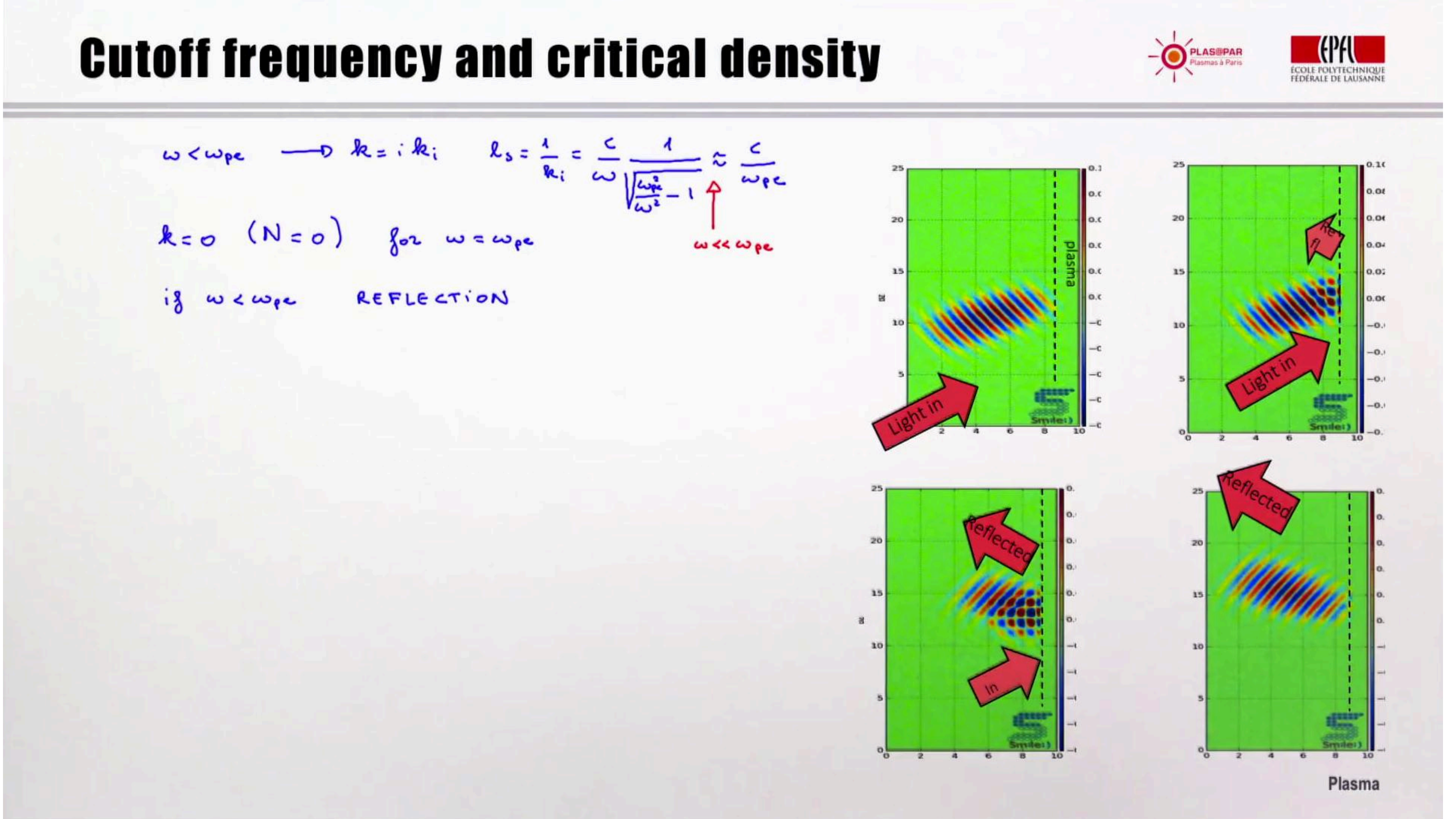


We see immediately that this quantity is well behaved and less than the velocity of light. We can now consider what happens when a wave propagates from air to a plasma. So this is air and this is the plasma. We can apply Snell's law and we will have that going from air to the plasma, the refraction angle is larger than the incoming angle is opposite to what happens when light goes from air to glass or air to water. Since the index of refraction of a plasma is less than one, this implies some peculiar behavior for optical objects. This is the normal on the surface and the wave would be refracted further away from the normal.

Notes

Summary

11m 39s

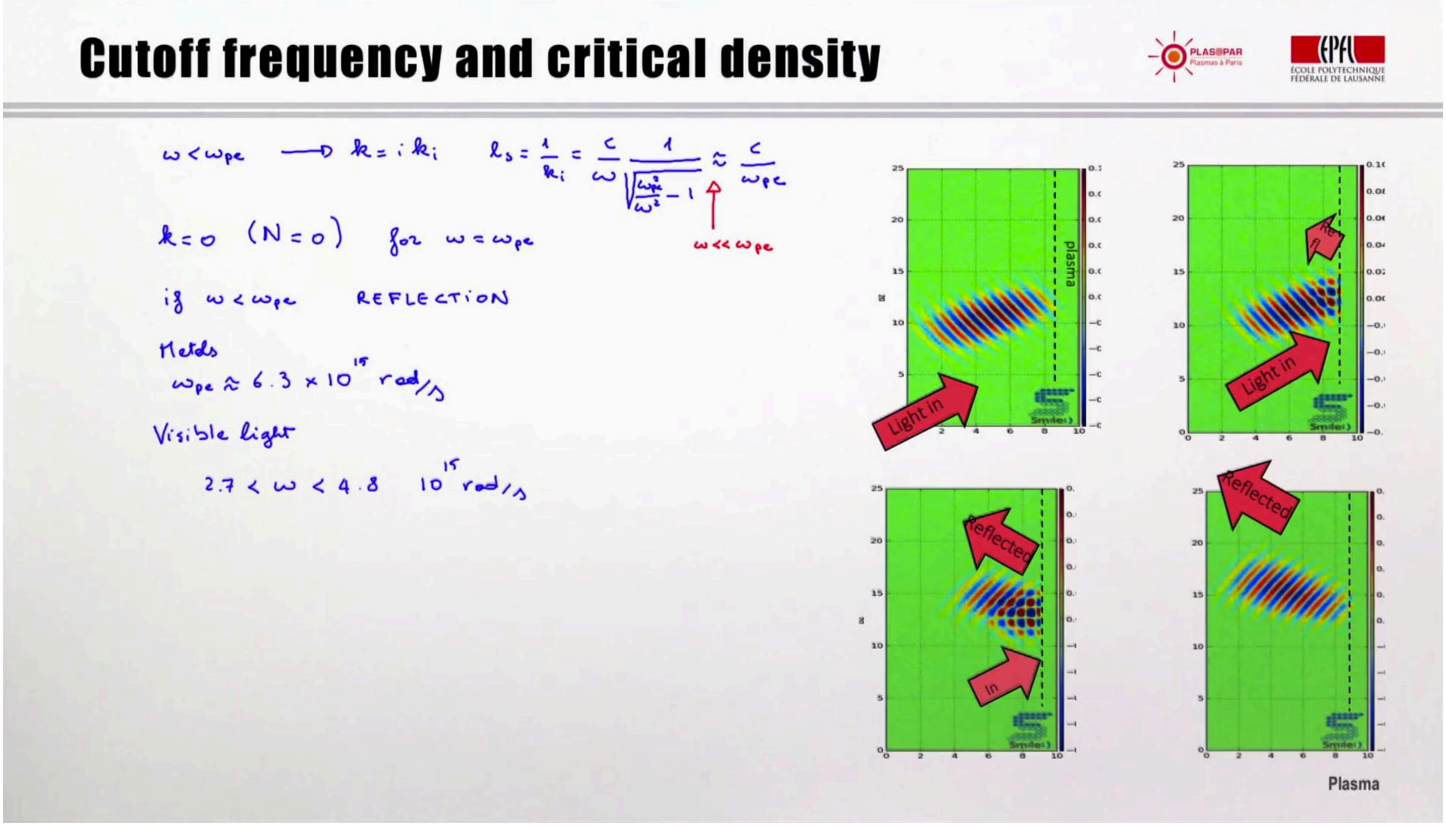


Let us now go back to the dispersion relation. If the frequency is below the plasma frequency, as we have seen, there can be no propagation and K becomes purely imaginary. An imaginary K means that the wave becomes evanescent and it will only exist in penetrating the plasma up to a small length that is called the skin depth length. This length is equal to the inverse of the imaginary K and if the frequency is much less than the plasma frequency is given by this simple formula that you might have already crossed. We are now in a position to define the cutoff frequency for light waves. This concept of cutoff frequency was already introduced by Professor Richie. So what does it mean? In fact, if we have a frequency such that for this frequency the wave vector K is equal to zero and the index of refraction is also equal to zero, this is called the cutoff frequency. For light wave, this happens for Omega equals Omega P, the plasma frequency and this frequency gives the cutoff between a propagating wave and the wave becomes evanescent at the plasma surface. What happens in fact, when the frequency is below the plasma frequency, the wave is reflected.

Notes

Summary

12m 40s

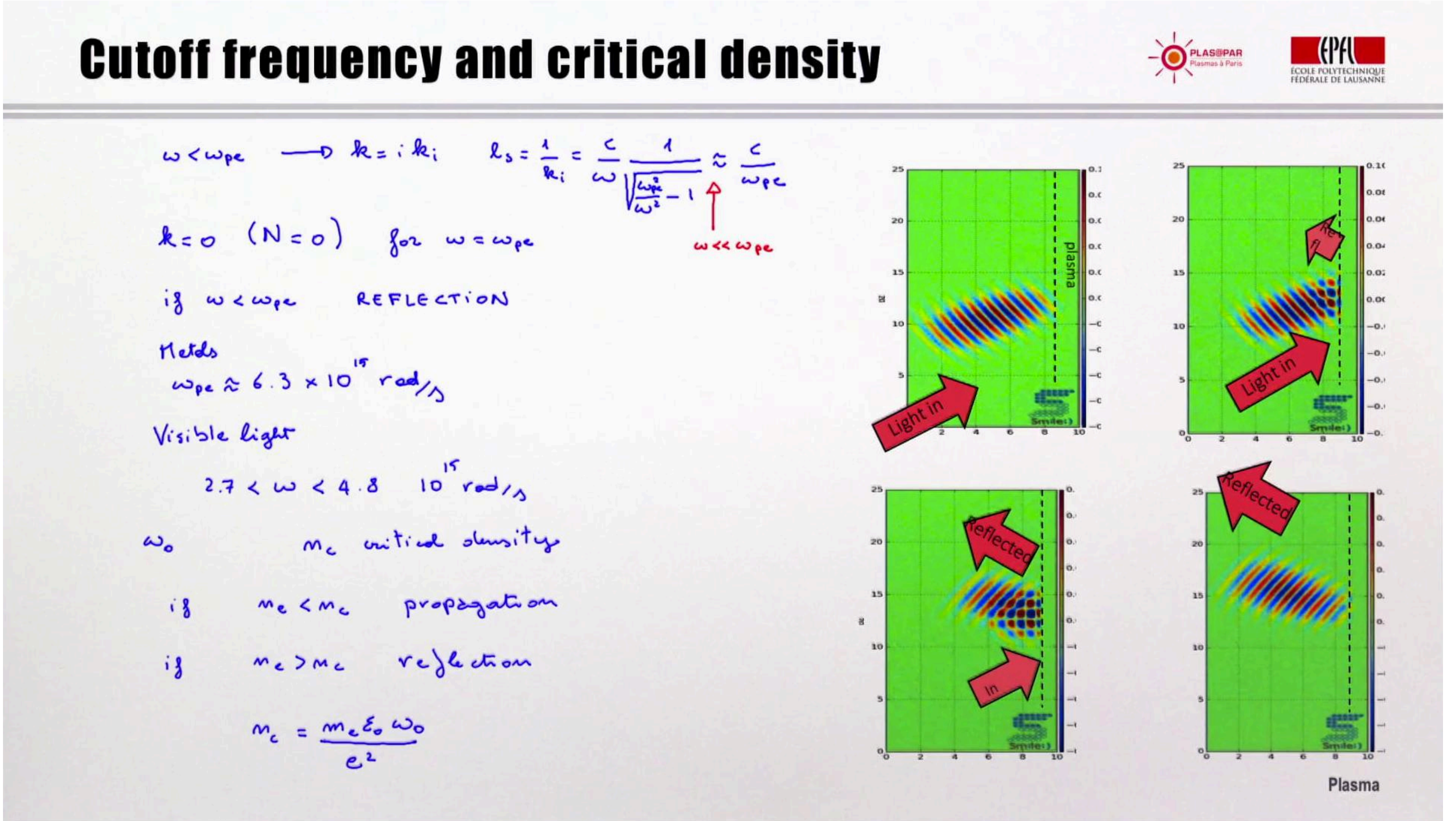


So here on the right we have an example of a simulation where a wave packet is impinging over on a plasma and the plasma is chosen such that its plasma frequency is above the cutoff frequency for the pulse. As you can see, the light gets in and then it starts to be reflected. It's fully reflected and leaves the plasma and the plasma stays unchanged. While the light interacts with the plasma, it will penetrate a little bit in the evanescent length, that is, however very small and is very visible in this simulation. A nice example that you are all familiar with that is related to the fact that a cutoff frequency exists for light wave in a plasma is the fact that the metals shine. The free electrons in the conduction band of the metal, in fact, can be treated as a cold plasma. And then we can calculate a plasma frequency associated to that. If we now consider the pulsation of visible light, it is between 2.7 and 4.8 10 to the 15 rod per second. So we can see that all visible light is below the plasma frequency of the conducting electron, and for this reason all visible light is reflected making metals shine. We can now look at the problem from another point of view.

Notes

Summary

14m 07s

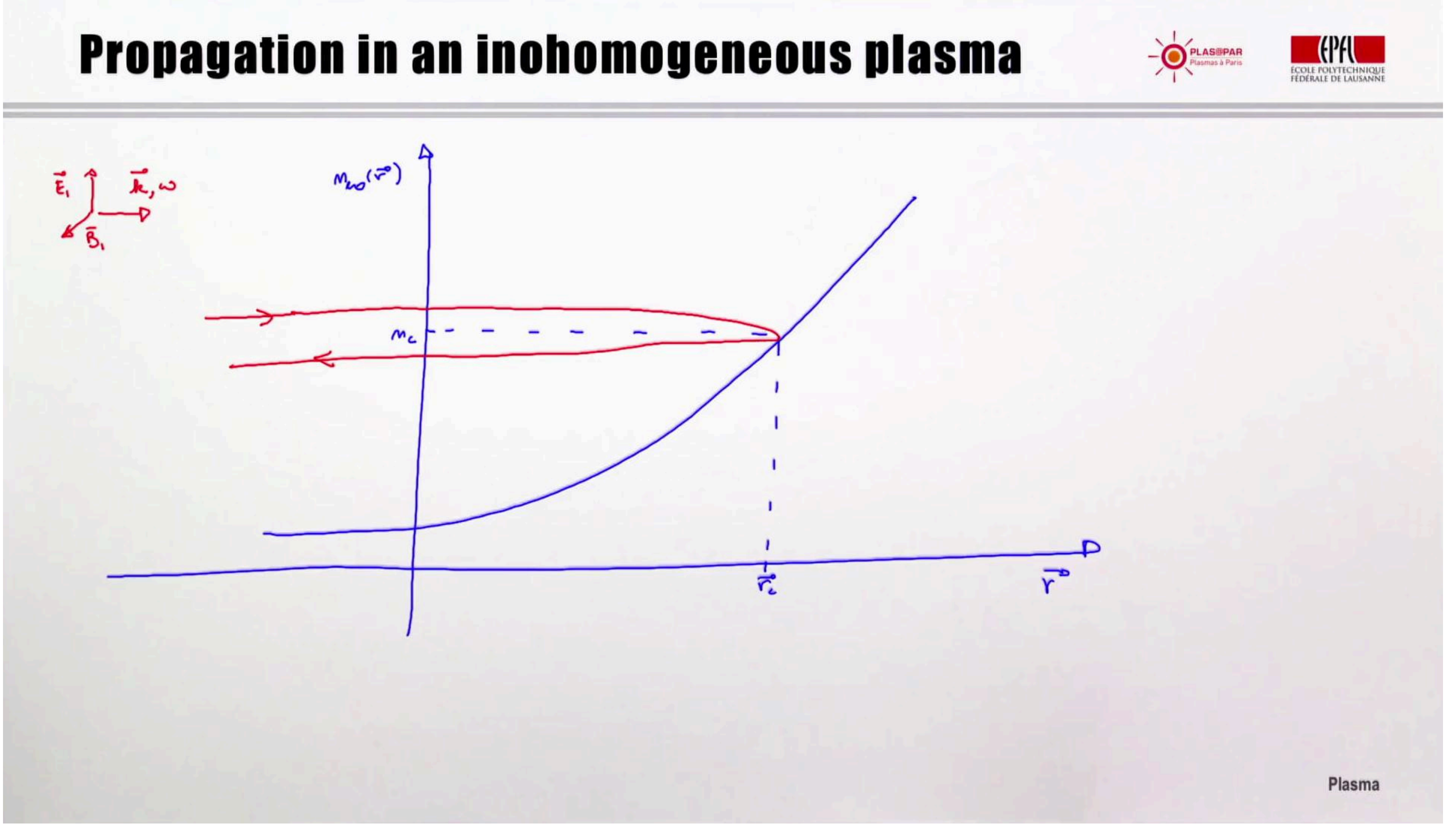


In fact, we can consider a given frequency Omega zero, a way with this frequency will be able to propagate in a plasma only if the density is below a critical value that is such that the plasma frequency is equal to this frequency. We can thus define a critical density that is such that if the density is below this critical value, there is propagation. If the density is above this critical value, there is reflection. The critical value is value of the density for which the plasma frequency is equal to the frequency of the considered wave, and is then given by this formula.

Notes

Summary

15m 33s

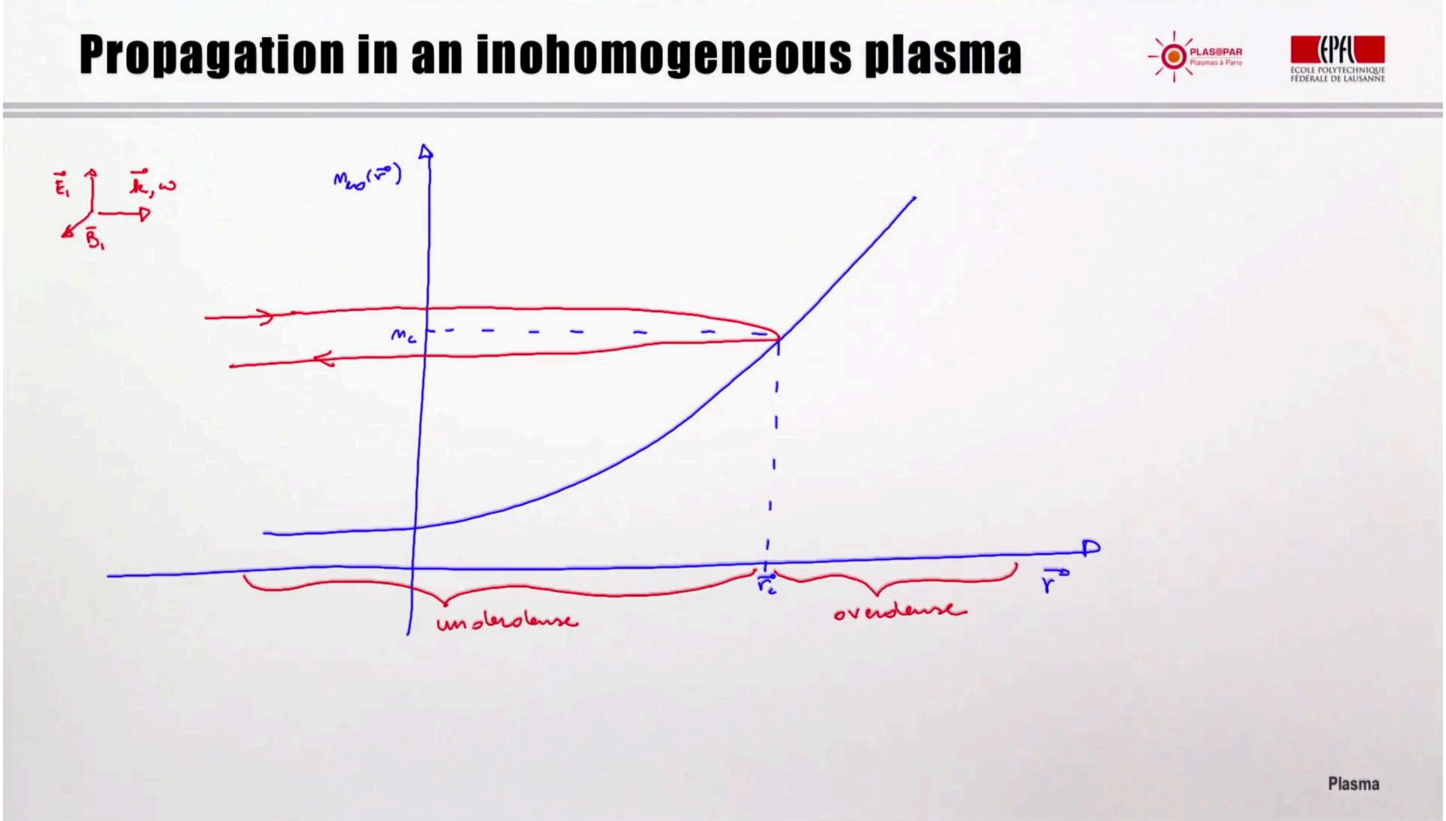


The notion of critical density is very useful if we consider that the plasma density is not homogeneous. So what I'm showing here is a density profiles that evolves with space. The equilibrium density is a function of space, and for example, it has a shape like that. Now I can consider an incoming wave that is propagating in the same direction of the density profile, for example, can be something like that. The light will propagate in the plasma and during the propagation, it will keep its frequency constant but instead the plasma frequency will vary as the density increases and then the plasma frequency increases. This means that the light can only propagate up to the point when its frequency is below the plasma frequency, and this is the critical density. For example, let's say that this point here corresponds to the critical density. With this configuration, light will propagate up to the critical layer and then it will be reflected. We will discuss in the next lecture what happens when light instead of propagating parallel to the density gradient propagates at an angle, oblique incidence. As we will see, the situation is a little bit different.

Notes

Summary

16m 33s

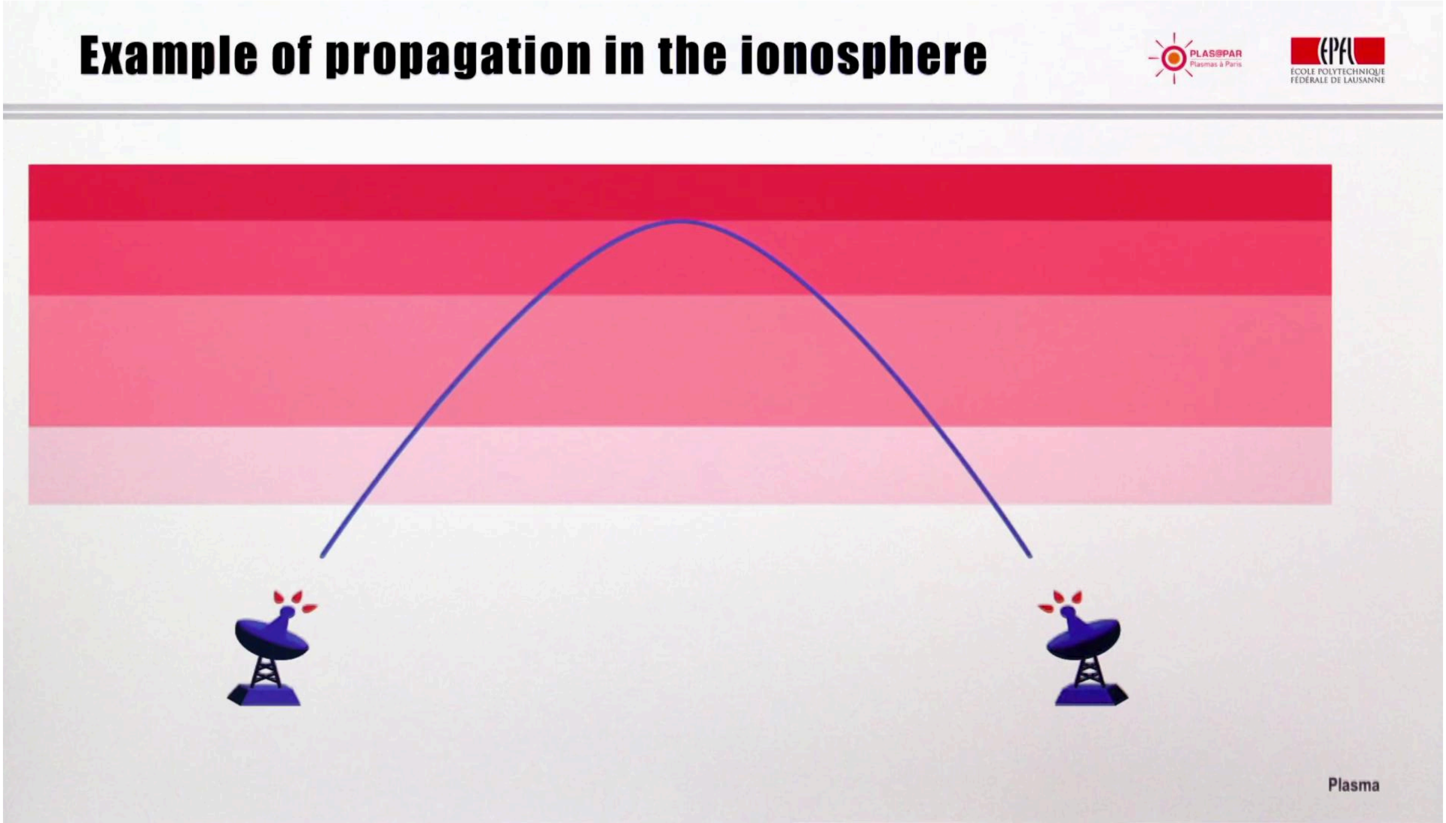


Finally, we can introduce a definition that you've already seen in the previous lecture. The part of the plasma where light can propagate is called underdense, and the part of the plasma where light cannot propagate is called overdense.

Notes

Summary

18m 06s

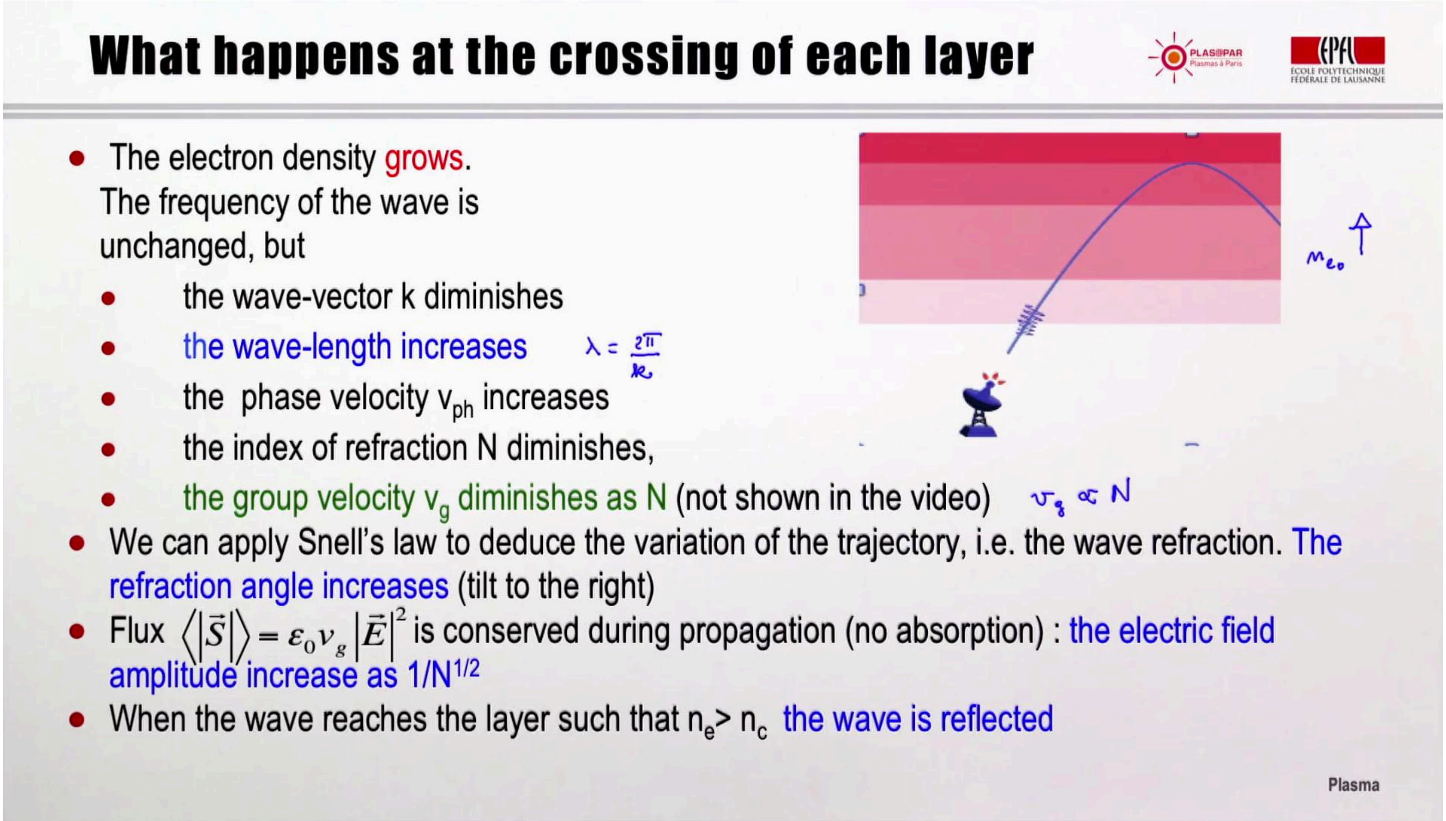


Most of the results that you learned in this lecture are summarized by this video that we can watch together and then we can discuss. What we show here is a simple model for the ionosphere. The ionosphere is the higher part of the atmosphere that is ionised. In reality, the electron density increases continuously as we go towards the highest layers of the Earth atmosphere. Since ionisation increases in the higher layers. In this animation instead, we consider many layers each at constant density and such that the density increases and the as color gets darker. At the crossing of each layer, you go from here, here, here we can apply what we just learned today in homogeneous plasma, and this will give a good example of the propagation of the wave. So let's have a look at the video.

Notes

Summary

18m 34s

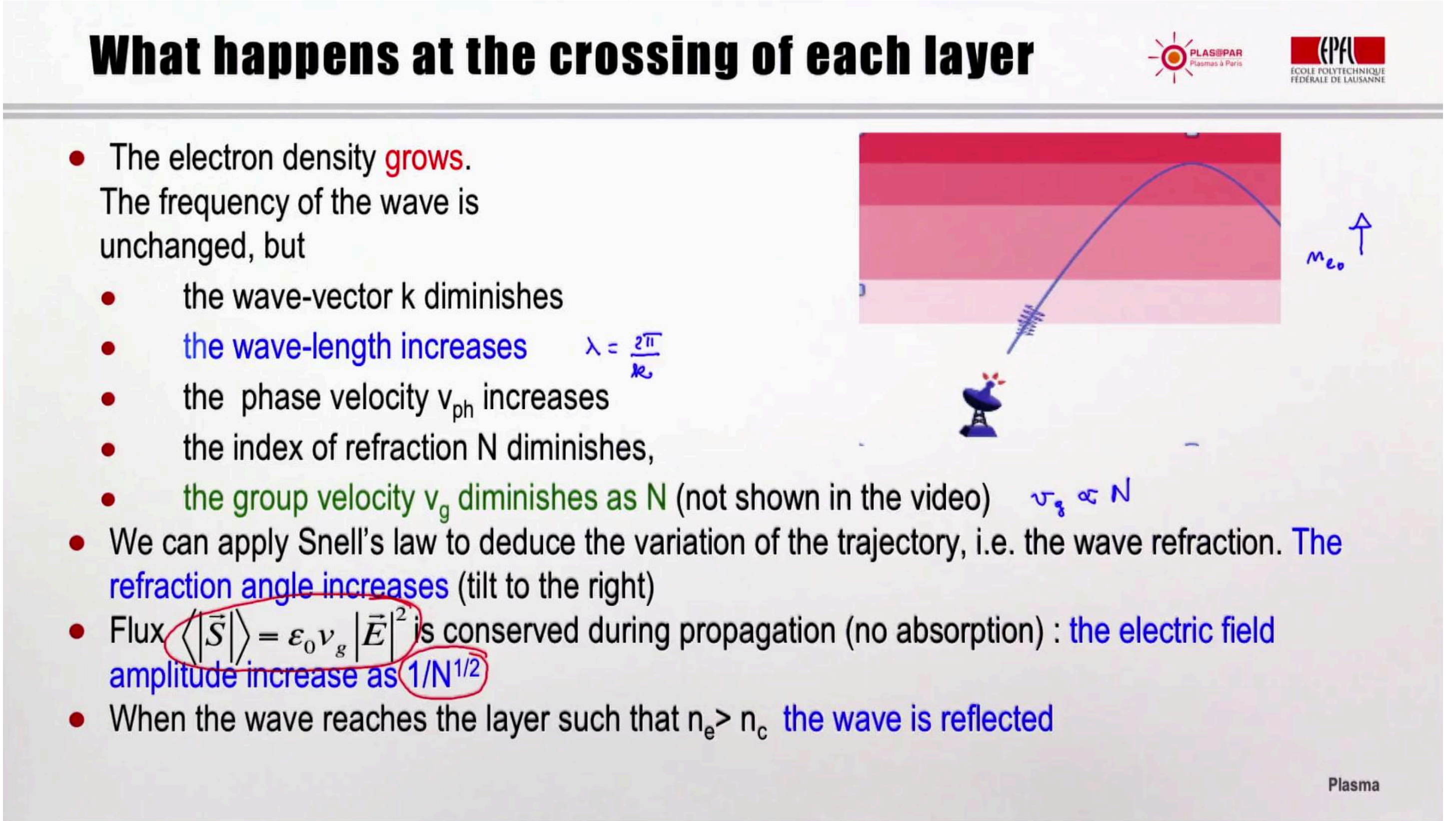


We can now discuss the video in more detail. So as we said, the electron density grows and the frequency of the wave is unchanged during propagation. However, because of the growth of the density. However, as you have seen in this lecture, the wave vector diminishes according to the dispersion relation of light in plasma. The wavelength that is equal to the inverse of the wave vector will increase and this is shown in the image. You see that there is an increase of the wavelength going from one layer to the next. The phase velocity increases and the index of refraction diminishes accordingly. The group velocity will also diminish as the density increases. Vg is proportional to N thus diminishes the index of refraction. This is not shown in the video, but in fact what will happen is that light will slow down as it reaches the higher level of the ionosphere. Then we can apply Snell's law to deduce the variation of the trajectory so the impinging angle is increased and this corresponds to a tilt to the right and gives this trajectory. And finally, in this process flux is conserved because the wave is just propagating with no absorption that is negligible in this low density plasma.

Notes

Summary

19m 53s



So since flux is conserved, this quantity is constant, and since this quantity is constant given that the group velocity goes with the index of refraction, it means that the electric field will increase as one over the index of refraction to the one half. And again, you see this in the video where the amplitude of the electric field increases. Finally, when the wave reaches the layer such that the density is above the critical density, the wave is reflected and the reverse trajectory is done by the wave.

Notes

Summary

21m 30s

Summary

- Light propagation in a plasma : two fluid model with immobile ions
- Light can propagate in a plasma only if its frequency is above the plasma frequency
- If the frequency is below the plasma frequency it will be reflected
- The index of refraction diminishes if the electron density increases.

Plasma

We can now summarize what we have learned in this lecture. In order to describe light propagation in a plasma, we can use the two fluids model, but treat the ions as a static neutralizing background. Light can only propagate in a plasma if its frequency is above the plasma frequency. In the opposite case, it will be reflected. If light propagates in an inhomogenous plasma, the index of refraction decreases as the density increases.

Notes

Summary

22m 11s