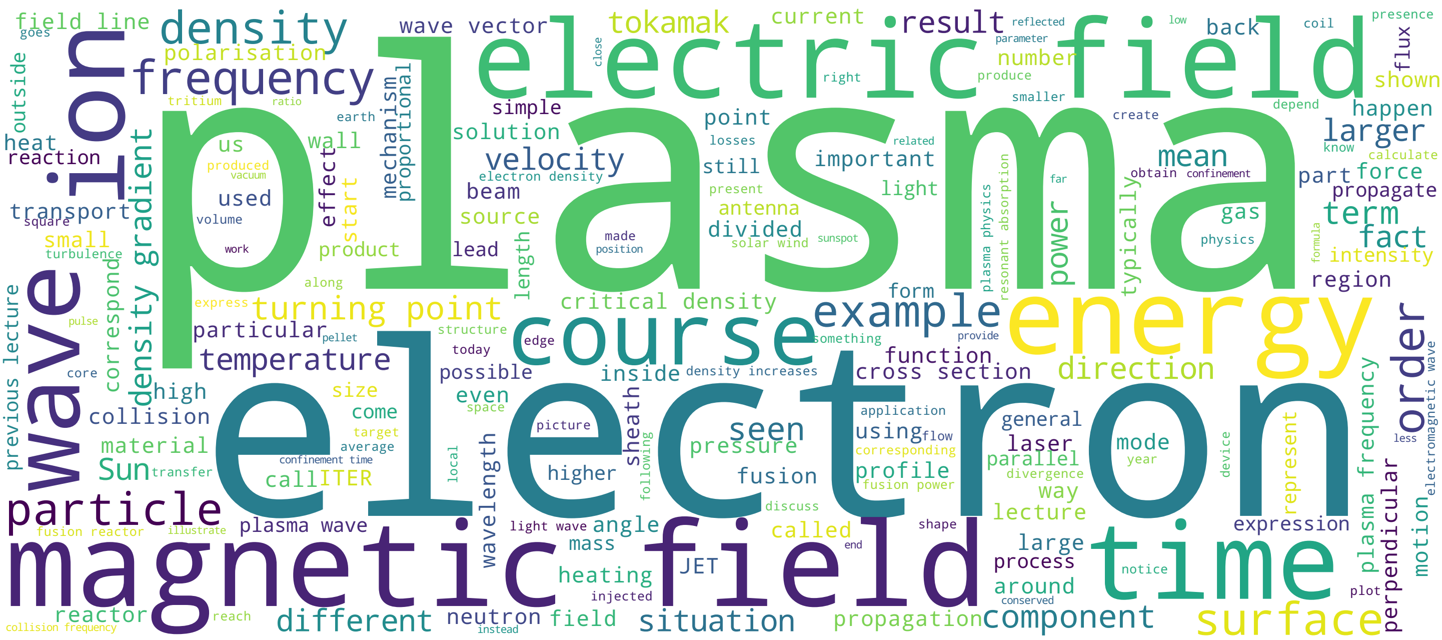
Propagation and absorption 2

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 8c

Caterina Riconda



Search MOOC

Video

Introduction

- Inverse Bremsstrahlung (absorption by collisions)
  - Wave propagation and absorbed power
- Light propagation in an inhomogenous plasma and resonant absorption
  - Incident light || to the density gradient
  - Light incident at an angle, S and P-polarization
  - Excitation of plasma waves and resonant absorption
- Collisionless absorption in a overdense plasma with a sharp density gradient

Plasma

Welcome. In this lecture, we are going to talk about different mechanisms for light absorption in a plasma. Thanks to these mechanisms, we can create a hot plasma starting from a gas or a solid target. The first that we will study is inverse bremsstrahlung or absorption by collisions. We will look at light propagation and absorbed power in a collisional plasma. The second mechanism is the so-called resonant absorption. This is a collisionless mechanism in the region close to the critical density. To study, we will first consider in some detail the propagation of a wave in an inhomogeneous plasma. The final one is again a collisionless absorption mechanism. It consists of absorption in an overdense plasma with a sharp density gradient. We remind that in such a plasma, the wave cannot propagate inside and all the interaction happens at the surface. So let's start with the lectures.

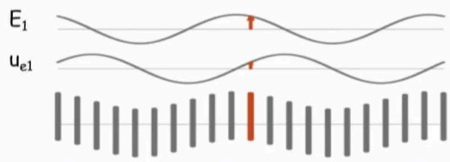
Notes

Summary

0m 05s

Inverse Bremsstrahlung

$$\nu_{ei} \propto \frac{Z n_e}{(k_B T_e)^{3/2}}$$



- Electrons orderly oscillating in the electric field direction 'participate' to the wave propagation
- Because of Coulomb collisions the ordered motion is randomized : plasma heating and wave absorption
- Inverse of the Bremsstrahlung process where free electrons are accelerated by the Coulomb collisions and emit light

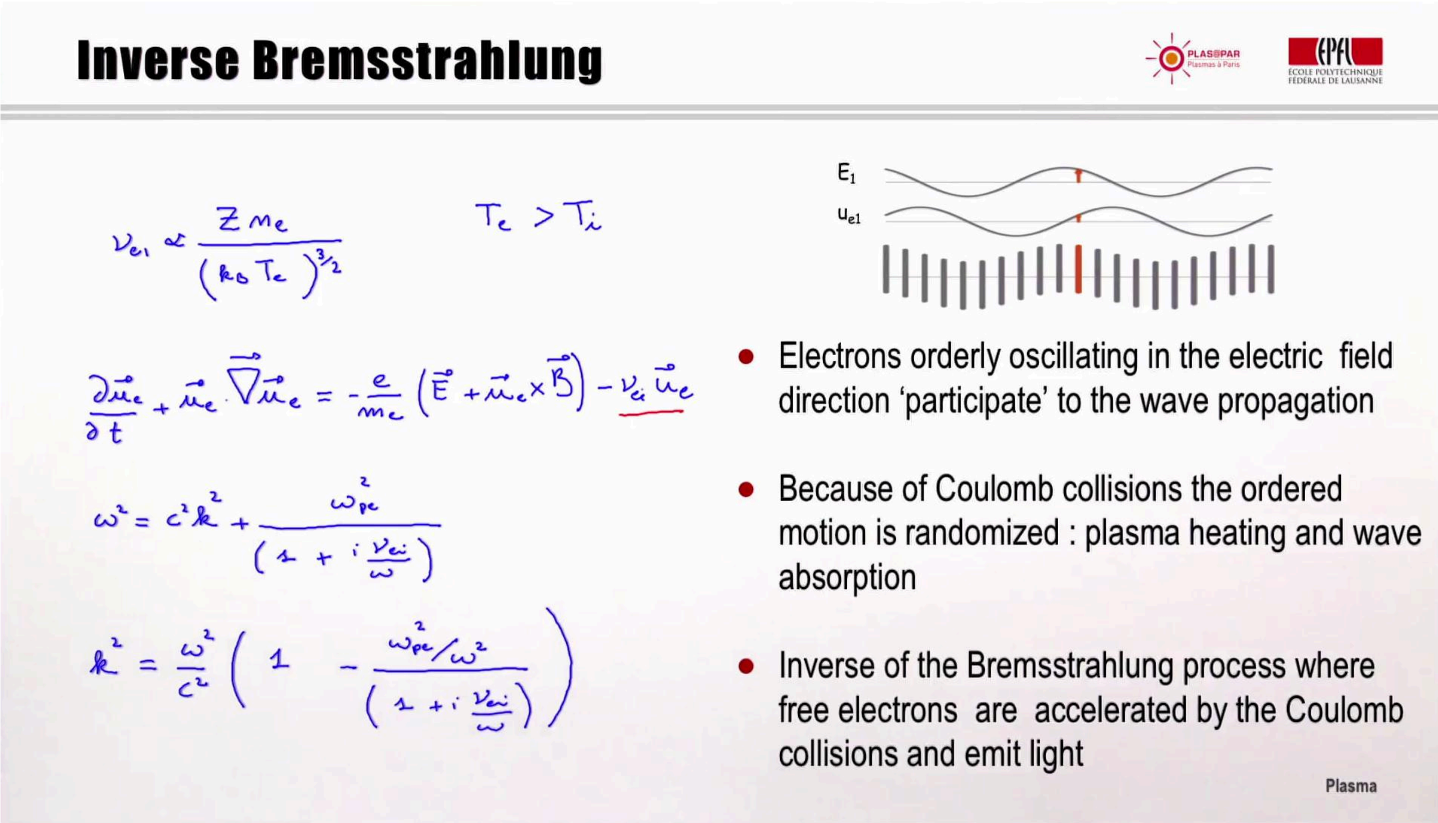
Plasma

In a previous lecture, we have seen that when a wave propagates, the electrons oscillate orderly in the electric field the direction, and by their motion, they participate to the wave propagation like it's shown here. While the ion stay as an immobile the ground as discussed by Professor Rich in previous lectures, however, the electrons can be subject to Coulomb collision with the background plasma, in particular with the ions. The collision frequency scales in the following way. So this is the ion density that I express as [inaudible 00:01:41] and this is electron temperature. Collisions will be more important for dense and cold plasma. Because of the collisions, part of the ordered motion of the electrons gets randomised. The energy of the electron is conserved, but the momentum changes. This results in electron heating. The random motion does not contribute any more to the wave propagation. The electric field is reduced and light is absorbed. This is called inverse bremsstrahlung because it's the inverse of the bremsstrahlung process, where three electrons are accelerated by the Coulomb collision and the emitted light, the ions will get heated as well.

Notes

Summary

1m 06s



But because of the large mass ratio between ions and electrons, it will be over much longer timescales. And it will take some time before the temperature of the ions is equal to the electron temperature. For this reason, we can talk about equilibrium in a plasma even if the temperature of the electron is different from the one of the ion, and in general is larger. This is really a quasi-equilibrium, but it can last for a very long time. The light propagation in a collisional plasma will be described by the same equations as in the last lecture, but we have to add a friction term in the momentum conservation equation. So this is the new term that we are adding to the equation. With the same procedure as in the previous lecture, we can now derive the disperse relation of light wave in a collisional plasma. This relation can be reversed to express the wave vector as a function of the frequency, as we have done before.

Notes

Summary

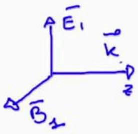
2m 25s

Inverse Bremsstrahlung

Propagation along z $\nu_{ei} \ll \omega_{pe} \longrightarrow k_i \ll k_r$

$$k_r \simeq \frac{\omega}{c}\sqrt{1-\frac{\omega_{pe}^2}{\omega^2}}$$

cutoff $\omega = \omega_{pe}$



$$k_i \simeq \frac{1}{2}\frac{\nu_{ei}}{c}\frac{\omega_{pe}^2}{\omega^2}\frac{1}{\sqrt{1-\frac{\omega_{pe}^2}{\omega}}}$$

$$E_1 = \tilde{E}_1 e^{-k_i z} e^{-i(\omega t - k_r z)}$$

$$B_1 = \tilde{B}_1 e^{-k_i z} e^{-i(\omega t - k_r z)}$$

Plasma

Let us consider that the propagation direction is along the Z-axis. Moreover, we can consider the collisions are weak with respect to the plasma frequency. This is typically the case in a plasma. If we solve for the K vector, we see that there will be an imaginary component. This condition implies that the K vector will have an imaginary part that is much less than the real part because the imaginary contribution is related to the presence of collisions. We can now solve the previous relation in order to find explicitly the value of the real and imaginary part of the wave vector. As we can see, the real part of the wave vector is the same as in the collisionless case, and we have the same cut of frequency for light waves, Omega equal to the plasma frequency. Moreover, we have the same period of orthogonal vectors for the wave vector in the fields. However, since now K has an imaginary part, the fields will be given for a wave propagating to the right by the following formula. So as we can see, the amplitude of the wave is decreasing with space while the wave propagates. And of course, in order to have the fields in real space, we have to take the real part of this quantity.

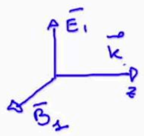
Notes

Summary

3m 44s

Inverse Bremsstrahlung

Propagation along z $\qquad \nu_{ei} \ll \omega_{pe} \longrightarrow k_i \ll k_r$



$$k_r \simeq \frac{\omega}{c}\sqrt{1-\frac{\omega_{pe}^2}{\omega^2}} \qquad \text{cutoff } \omega = \omega_{pe}$$

$$k_i \simeq \frac{1}{2}\frac{\nu_{ei}}{c}\frac{\omega_{pe}^2}{\omega^2}\frac{1}{\sqrt{1-\frac{\omega_{pe}^2}{\omega}}}$$

$$E_1 = \tilde{E}_1 e^{-k_i z} e^{-i(\omega t - k_r z)}$$

$$B_1 = \tilde{B}_1 e^{-k_i z} e^{-i(\omega t - k_r z)}$$

$$P_a = \langle \vec{j}_1 \cdot \vec{E}_1 \rangle = \nu_{ei}\frac{\omega_{pe}^2}{\omega^2 + \nu_{ei}^2}\varepsilon_0 \langle E_1^2 \rangle$$

Plasma

Finally, we can calculate the local absorbed power that will be proportional to the collision frequency. This can be calculated just as Joule heating, where the average is over the laser period. As you can see, the absorbed power is proportional to the collision frequency to the local value of the amplitude of the field square. And it also depends on the frequency of the wave in the denominator to the second power. This means that the absorption mechanism can be more or less efficient depending on the frequency of the wave.

Notes

Summary

5m 31s

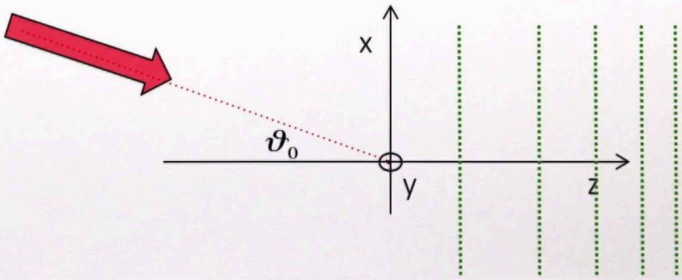
Light propagation in an inhomogenous plasma

Equilibrium $n_{e0}(r) = Z\, n_{i0}(r)$.

Ions : immobile background as the high frequency wave propagates.

1. The wave propagates parallel to the density gradient

1. The wave propagates at an angle with respect to the density gradient, S- and P-polarization



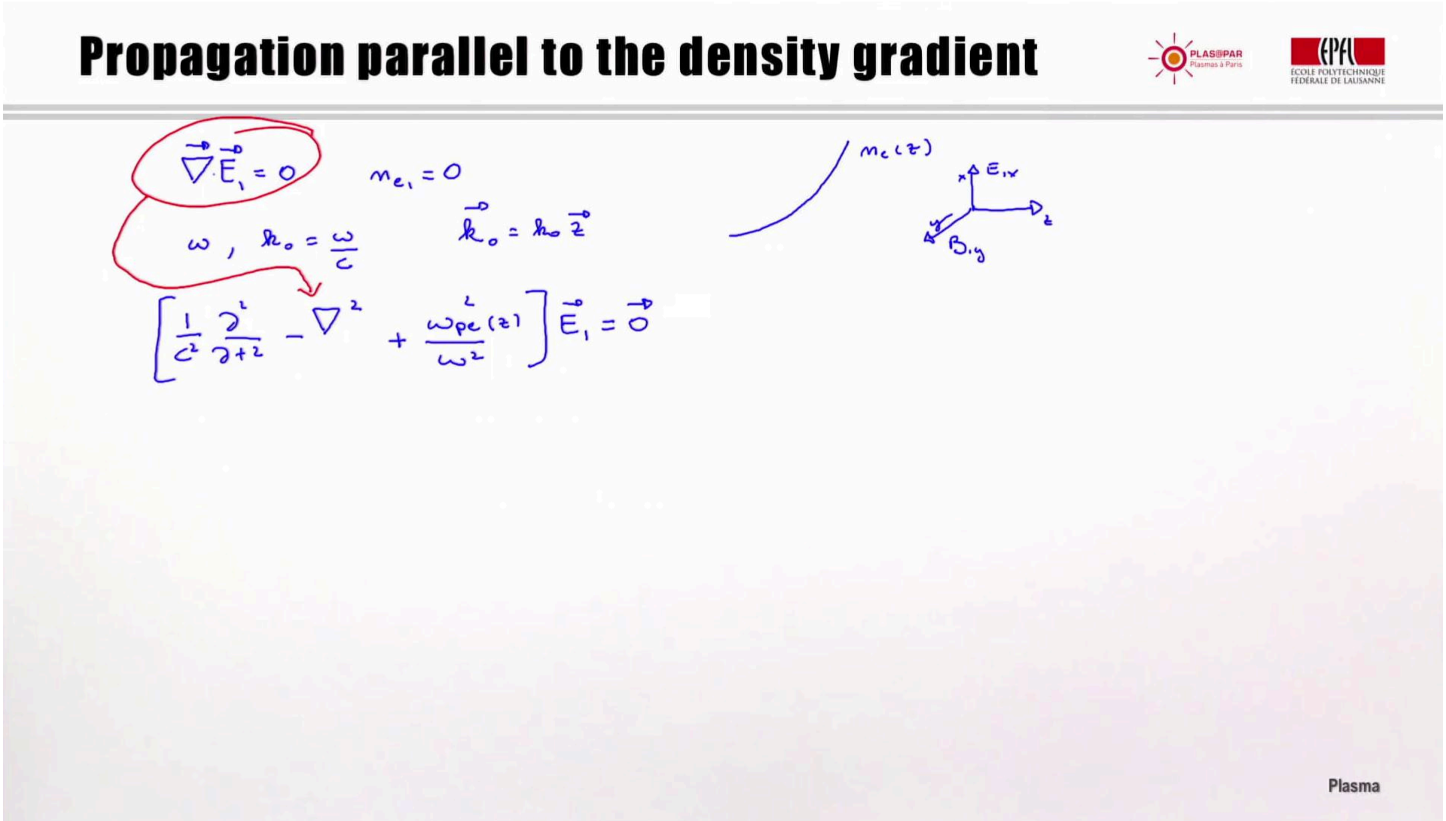
Plasma

We will consider separately two situations. The first one is the case of propagation along the density gradient. So in this case, I consider a density that is growing with Z, so the gradient is in this direction, and the laser propagates in the same direction. In the second situation, we will consider propagation at an angle with respect to the density gradient. Here, we see a top view of the same situation as before. So the density is growing with Z, but the laser is now impinging at an angle with respect to the direction of the density gradient. In this part of the lecture, we will neglect collisions. However, they can be present, and there can be some absorption in the plasma during the propagation of the wave.

Notes

Summary

6m 11s

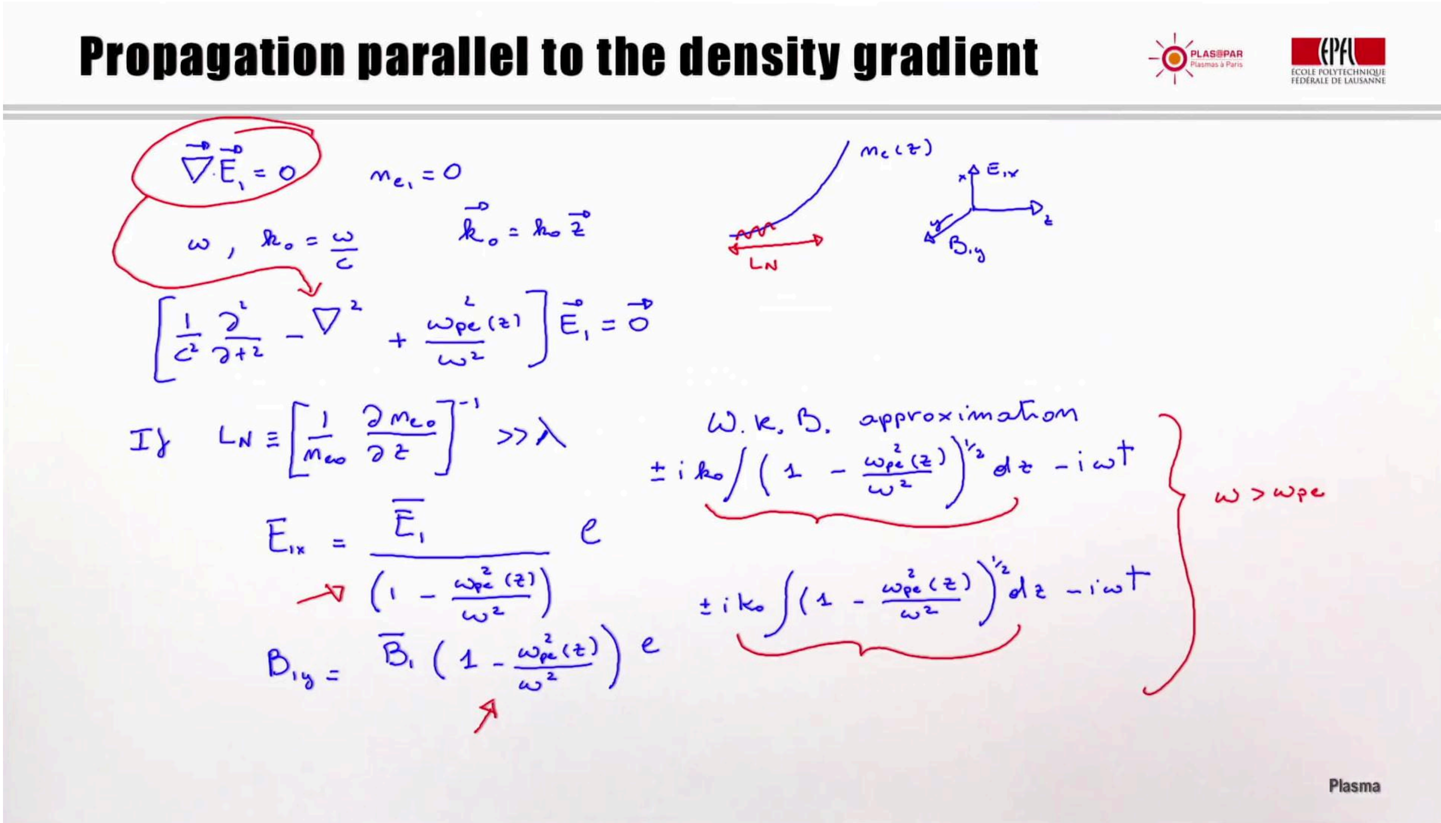


Let us consider the first situation. The electromagnetic wave solution in this case is still a purely transverse wave all the time. So we can write the divergence of the electric field is equal to 0, and consistently, there is no perturbation of the electron density. However, since now the density depends on space, we do not have any more simple wave solution with fixed frequency and wave vector. And in this case this is not convenient to do the Fourier transform. We can define the frequency and wave vector in vacuum before the wave enters in the plasma. As we have seen, the wave propagates along the density gradient that we have fixed along Z. So I just remind you the shape of the density gradient that is increasing towards the right, and we have the same geometry as before. So the wave it propagates along Z and the electric field is, for example, along X and the magnetic field along Y. By combining Maxwell's equation and the momentum conservation for the electrodes in real space, we obtain the following equation from the evolution of the electric field, where in order to write the equation in this form, I have used the divergence of E is equal to 0 so that the curl, curl operator is simplified to this form.

Notes

Summary

7m 07s

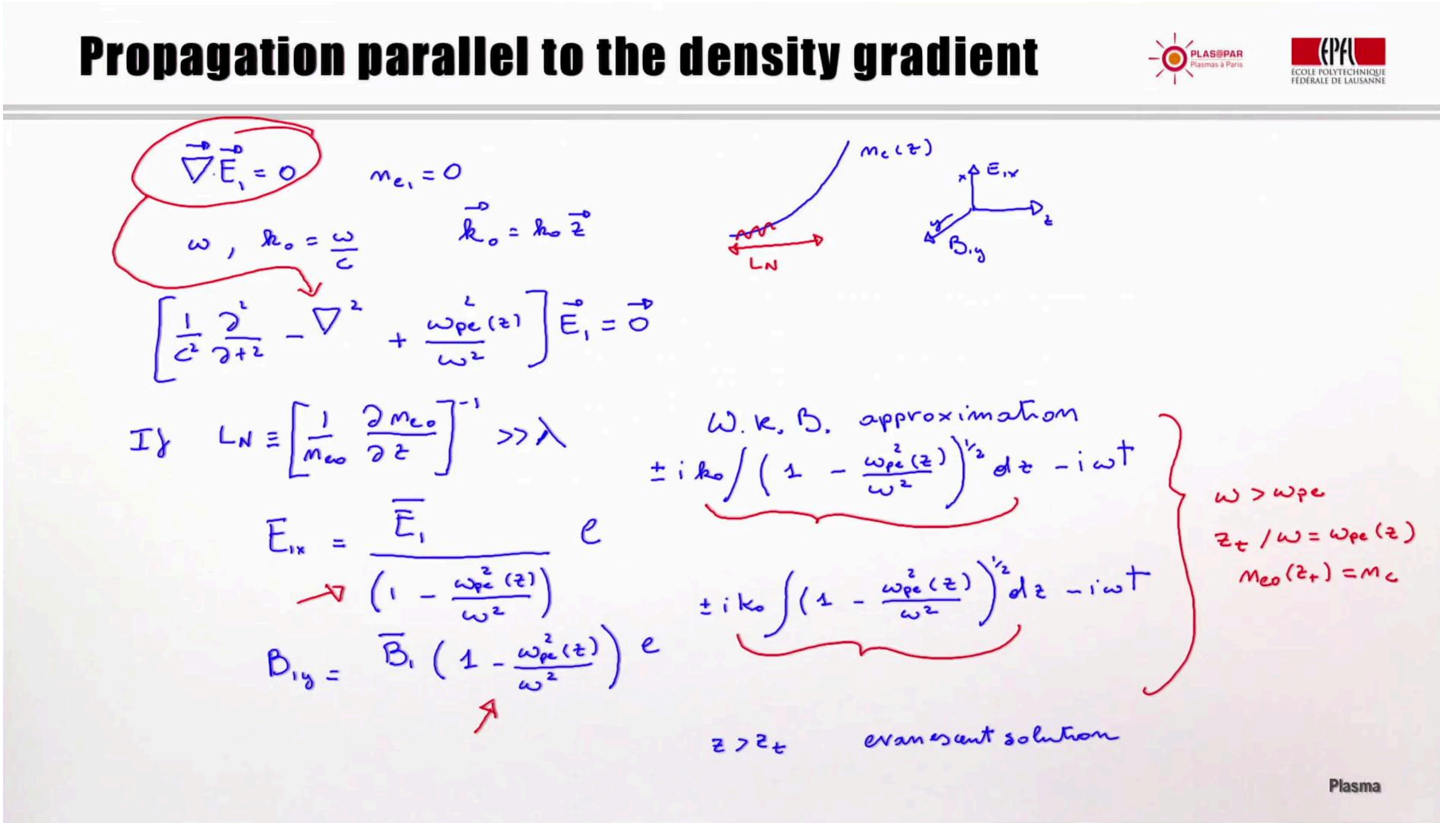


For a given density gradient, we can associate a typical length of variation of the density that is, the length over which the density changes significantly. We can compare this length to the wavelength of the wave propagating inside the plasma. And in many situation, the wavelength will be very small with respect to this length as I'm showing here. When this is the case, we can find easily solutions for our problem. So this is definition of the typical length of evolution of the density. And if this is very big with respect to the wavelength, we are in the so called WKB approximation, and we can write the solution for the electric and magnetic field. This solution is a propagating wave with a wave vector or wavelength that slowly adapts as the local value of the density changes, so is this quantity here. Moreover, the amplitude of the electric field increases as the density increases, is this term. And this is analogous to what we have seen in the previous lecture, and instead the amplitude of the magnetic field decreases as the density increases so that the flux associated to the wave propagating is conserved. This solution is valid as long as the frequency is larger than the plasma frequency.

Notes

Summary

8m 52s

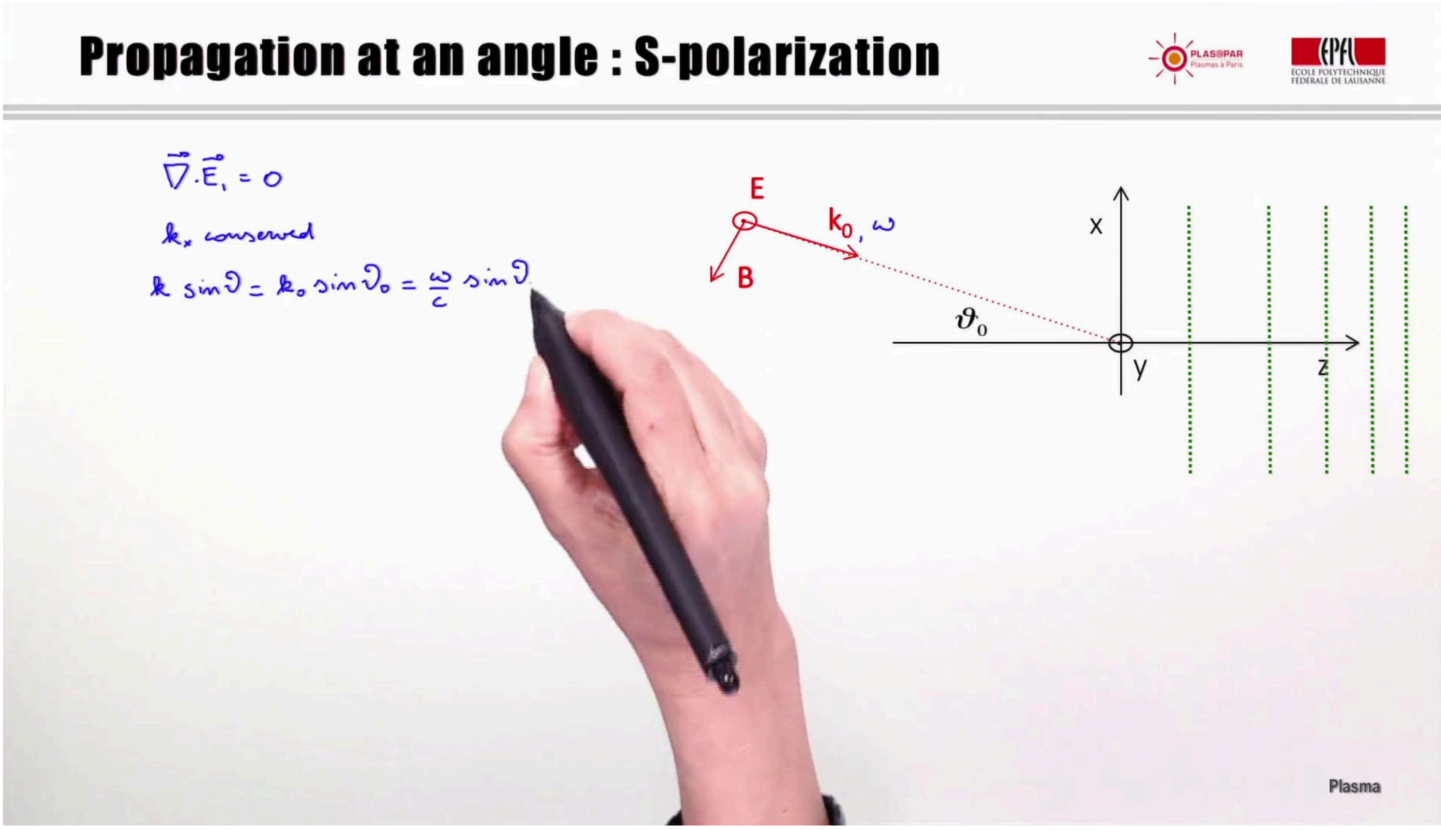


But as the density increases at some point, we can have a situation, where the frequency of the wave is equal to the plasma frequency. We can then define the turning point as the position space, where the wave frequency is equal to the plasma frequency. At the turning point, the wave will be reflected. Notice also that in this case the density at the turning point is equal to the critical density. After the turning point, that is, for Z larger than the turning point, the electric field and magnetic field are evanescent. And asymptotically, they will just decrease exponentially.

Notes

Summary

10m 46s



We now have a top view of the electron density increasing along Z. We consider a light wave that is propagating in the XZ plane at an angle with respect to the normal to the surface. The initial value of the wave vector is K 0, the frequency is Omega and stays constant, and the initial angle is Theta 0. The wave here is linearly polarised and has S-polarisation. This means that the electric field is along the Y direction and is orthogonal to the gradient of the density. The light wave is linearly polarised and it has S-polarisation. This means that the electric field is parallel to the Y direction and is perpendicular to the gradient of the density. In this situation, again, we can have a solution that is a purely transverse electromagnetic wave all the time. And this means that the divergence of the electric field will be zero at all times. Moreover, the component of the wave vector that is perpendicular to the density gradient, in this case the X component, will be conserved at all times, and this is the analogous of [inaudible 00:13:01]. Explicitly, I can write this as K sine of Theta equal to its initial value that I can express in terms of the frequency.

Notes

Summary

11m 37s

Propagation at an angle : S-polarization

$\vec{\nabla}\cdot\vec{E}_1 = 0$

k_x conserved

$k \sin\vartheta = k_0 \sin\vartheta_0 = \frac{\omega}{c}\sin\vartheta_0$

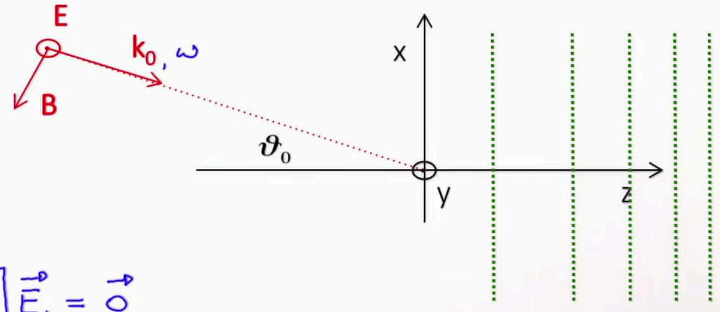
$\vec{E}_1 = \vec{E}_1(z)\, e^{i\frac{\omega}{c}\sin\vartheta_0 x - i\omega t}$

$\vec{B}_1 = \vec{B}_1(z)\, e^{i\frac{\omega}{c}\sin\vartheta_0 x - i\omega t}$

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} + \underbrace{\frac{\omega^2}{c^2}\sin^2\vartheta_0 + \frac{\omega_{pe}^2(z)}{c^2}}_{\frac{\omega'^2_{pe}(z)}{c^2}}\right]\vec{E}_1 = \vec{0}$$

z_t $\quad \omega^2 = \omega'^2_{pe}(z_t) = \omega^2\sin^2\vartheta_0 + \omega_{pe}^2(z_t) \longrightarrow \omega_{pe}^2(z_t) = \omega^2\cos^2\vartheta_0$

$n_{e0}(z_t) = n_c \cos^2\vartheta_0$



Plasma

By using this, we can take the following form for the electric and magnetic field is equal a function of Z that we need to calculate while we can take a simple wave solution for the other component, and same thing for the magnetic field. We can now plug in this solution in the wave equation in order to have an equation for the function of Z. Formally, this equation is the same one as we already had before, except that now, instead of having just this term that gives the function of Z, I can say that this whole term is a function of Z, and I can define an effective plasma frequency so that formally, I have the same equation as before. The solution of this equation in the WKB approximation will be the same as before. However, the turning point is now given by a different condition. In fact, the turning point is given by the condition that the frequency square is equal to Omega P Prime square at the turning point, Omega Pe square at the turning point. This equation can be simplified by bringing this term to the other side. This condition means that the turning point will not be at the critical density but at a lower density value at an earlier position. In fact, we can rewrite this equation in the form so that the turning point is for a lower density.

Notes

Summary

13m 19s

Electric field in an inhomogeneous plasma

$n_{eo}(z)\uparrow$ "k" $\downarrow$ $E_1\uparrow$ $B_1\downarrow$

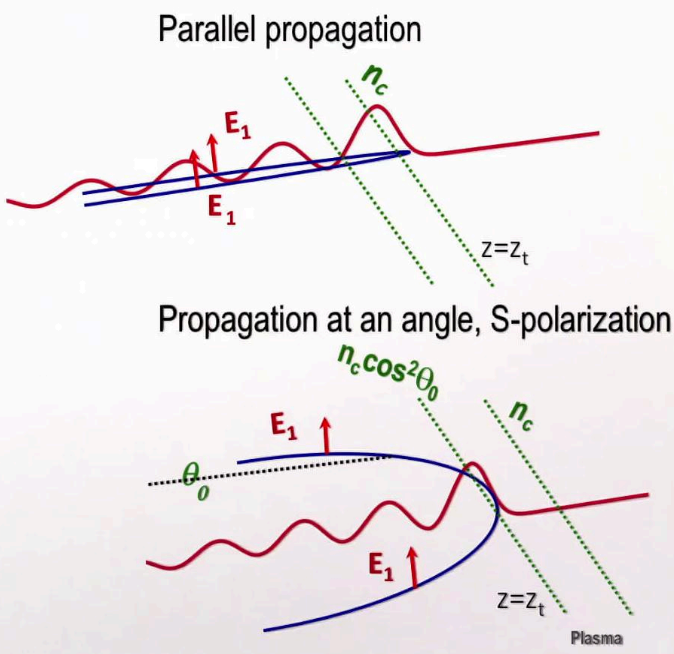
z_t / $\omega_{pe}^2(z_t)=\omega^2$

$n_{eo}(z_t)=n_c$

$k\sin\vartheta=\frac{\omega}{c}\sin\vartheta_0$

z_t / $\omega_{pe}^2(z_t)=\omega^2\cos^2\vartheta_0$

$n_{eo}(z_t)=n_c\cos^2\vartheta_0$

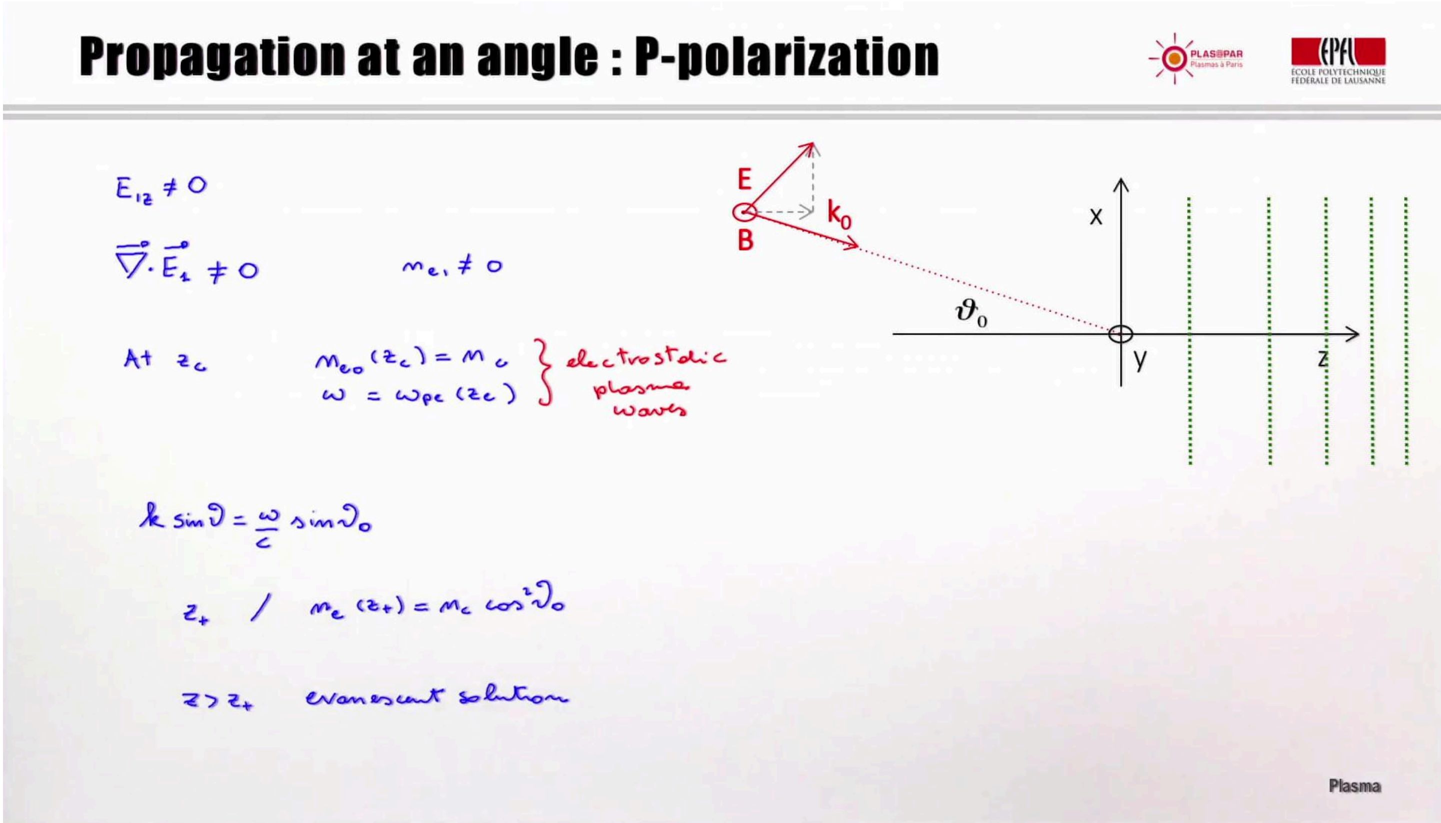


In this slide, we summarise what we have just learned. In case one, propagation parallel to the density gradient. As the density increases, the effective wave vector decreases so adapts to the local density value. The electric field amplitude increases and the magnetic field amplitude decreases. Propagation will happen up to the turning point and the turning point is at the critical density. In the second situation, when we have propagation at an angle in the S-polarisation, the component of the wave vector perpendicular to the density gradient is conserved while the other component slowly adapts to the local value of the density and electric field increases as in parallel propagation. However, now the turning point is at a lower density, and this is illustrated by this figure, where you can see that the wave is reflected earlier.

Notes

Summary

15m 38s



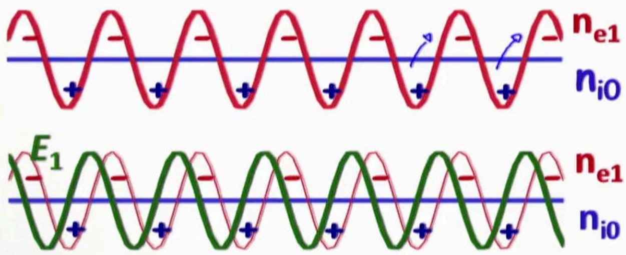
We will now consider propagation at an angle for a P-polarised wave. In P-polarisation, the electric field is in the X Y Z plane, so it's in this plane, and then it has a component that is parallel to the density gradient that is the Z component, E1Z is different from 0. In these situations, since the fields depend on the Z variable, we cannot have a purely transverse wave, but in our solution we will have that divergence of E1 is different from 0. By go slow, this implies that in the plasma, there will be a density perturbation that is not zero. As you will see in the next slide, the EZ component of the electric field can act as a force in term at the critical density and excite resonantly electrostatic plasma wave oscillating at the plasma frequency. So ZC is the value of Z at the critical density. And here the frequency is equal to the plasma frequency and can excite electrostatic waves. Apart from this new behaviour, the solution will be similar to the S-polarised case the way we'll propagates by conserving the component of the K vector perpendicular to the density gradient. The wave will propagate up to the turning point, where it will be reflected and as before, the turning point is a densities that are lower than the critical densities. For Z larger than the turning point, the solution will be evanescent.

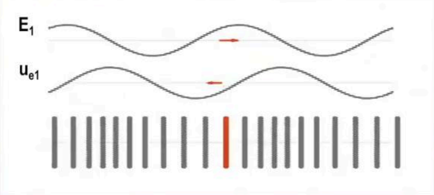
Notes

Summary

17m 00s

Electrostatic plasma waves (reminder)





- A plasma electron wave is a natural electrostatic mode at frequency ω_p
- It is a longitudinal oscillation $\vec{E}_1 \parallel \vec{u}_{e1} \parallel \vec{k}$
- It can be excited by creating a space charge field, resulting in a restoring force
- The electric field of the electromagnetic wave can be a forcing term for this oscillation

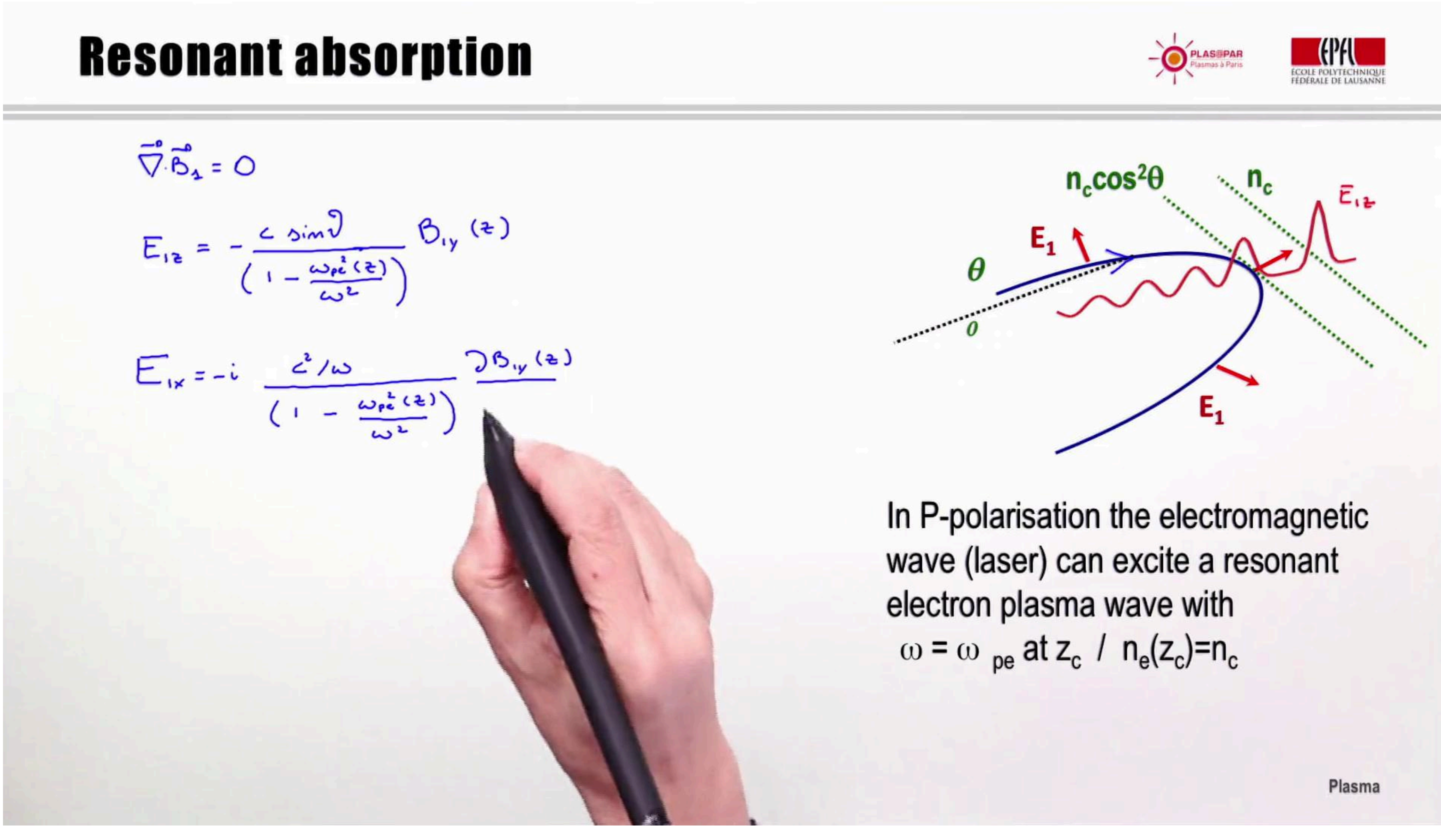
Plasma

As we have mentioned in P-polarization, there can be excitation of electrostatic plasma wave. So here just a brief reminder of this wave, a plasma electron wave is a natural electrostatic mode that oscillates at frequency Omega P, the plasma frequency. It is a longitudinal oscillation, meaning that the electric field, will be parallel to the velocity of the electron and both will be parallel to the K vector to the propagation of the wave. And there is no magnetic field. This wave can be excited by creating a space charge field that results in a restoring force. This is what is shown here. The ions can be taken as immobile because again, it's a high frequency mode. By some initial perturbation, the electrons are displaced from their position, so they create a difference in charge. This is a space charge field that results in the electric field shown here. And the direction of the electric field will be such to pull back the electrons that will then oscillate in the opposite direction and so forth. And this is shown in this animation here.

Notes

Summary

19m 13s

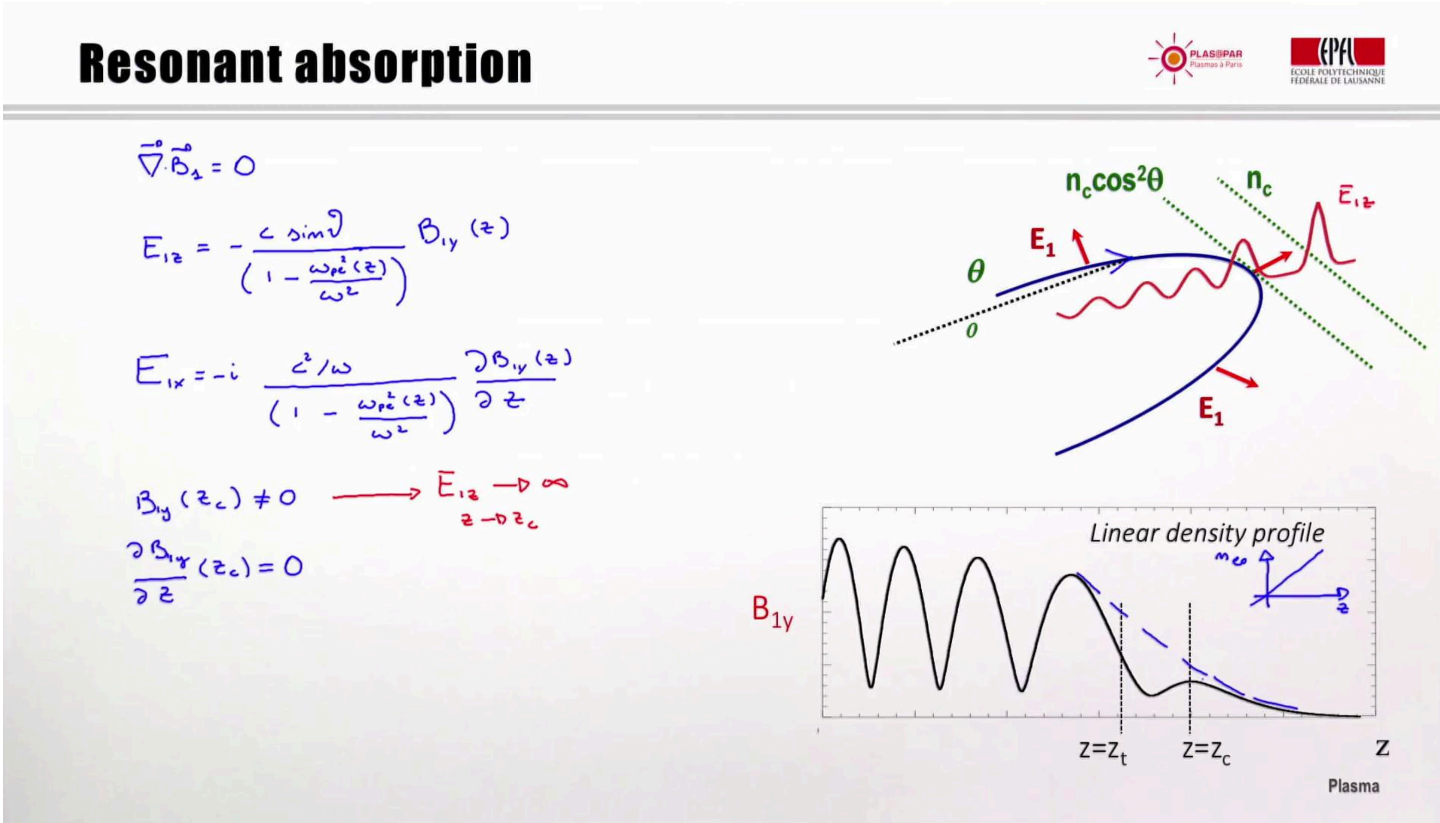


The discussion of the previous slide is summarised here. In P-polarisation, the electromagnetic field that is incoming from this direction at an angle will be reflected in the turning point. However, the evanescent part of the field will be able to penetrate up to the critical density, where it can excite a resonant electron plasma wave. It's very large amplitude. So what I'm showing here, of course, is the Z component of the electric field, and this is the plasma wave. We can give them mathematical solution for the electric field. However, since the divergence of B is always 0 and in P-polarisation, we only have a component for the magnetic field, it is more convenient to solve for the magnetic field and then express the electric field in terms of the magnetic field by far the equation.

Notes

Summary

20m 37s

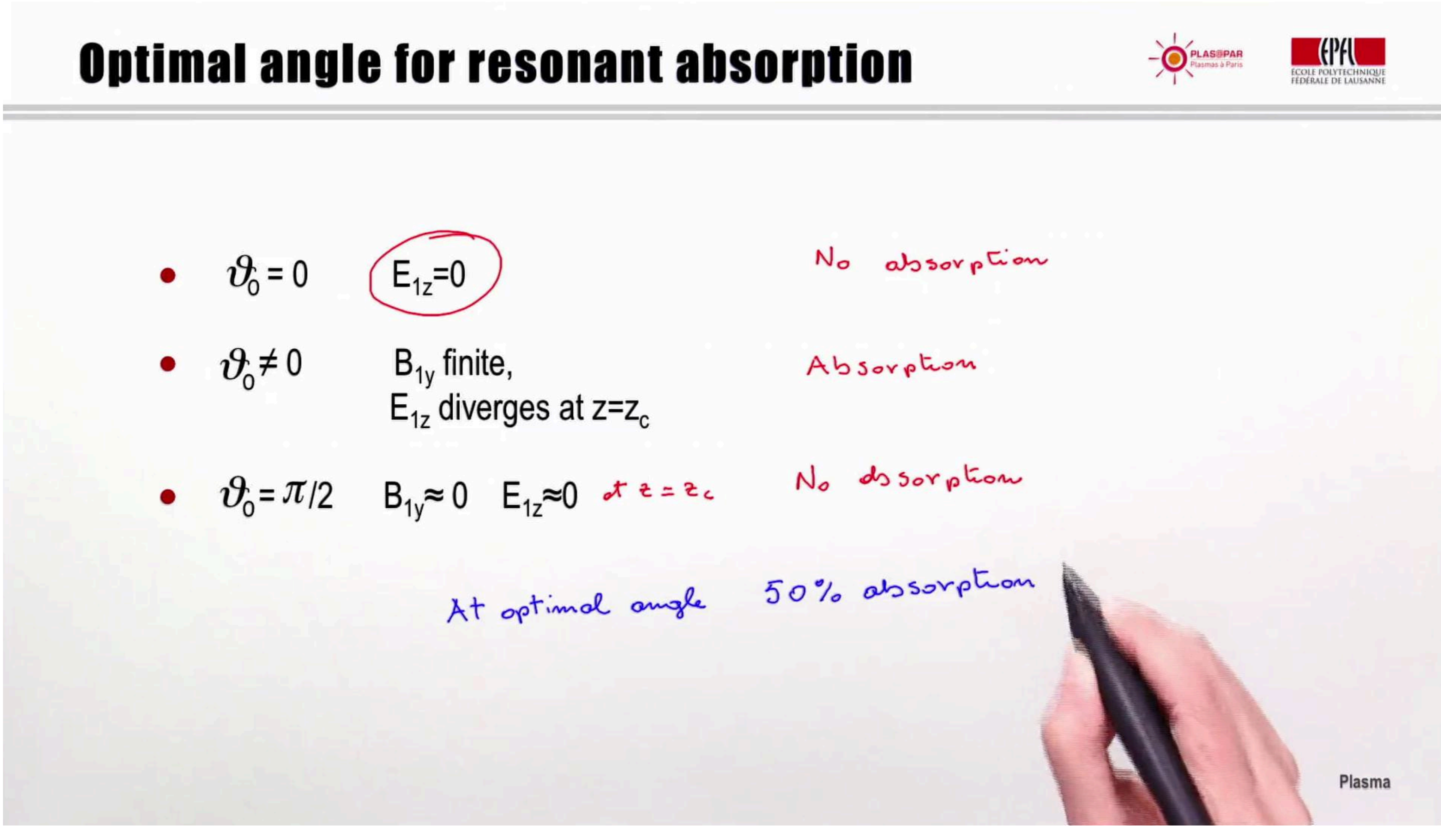


As an example, I have plotted here the solution for the magnetic field for a linear density profile, that is profile that grows linearly with Z. As you can see, the field becomes evanescent at the turning point. However, at the critical density the magnetic field is non-zero, but the derivative is zero. Because of this condition, we can see that the Z component of the electric field will diverge at the critical density, while there will be no problem for the X component because this term goes to zero. So we can see that it diverges because this term is finite while the denominator will go to zero at the critical density. In reality, the Z component of the electric field will not diverge. In fact, some mechanisms on much shorter spatial scales will become important and lead to dissipation of the wave. By this mechanism, the electromagnetic wave is converted to an electrostatic mode and is absorbed.

Notes

Summary

21m 55s

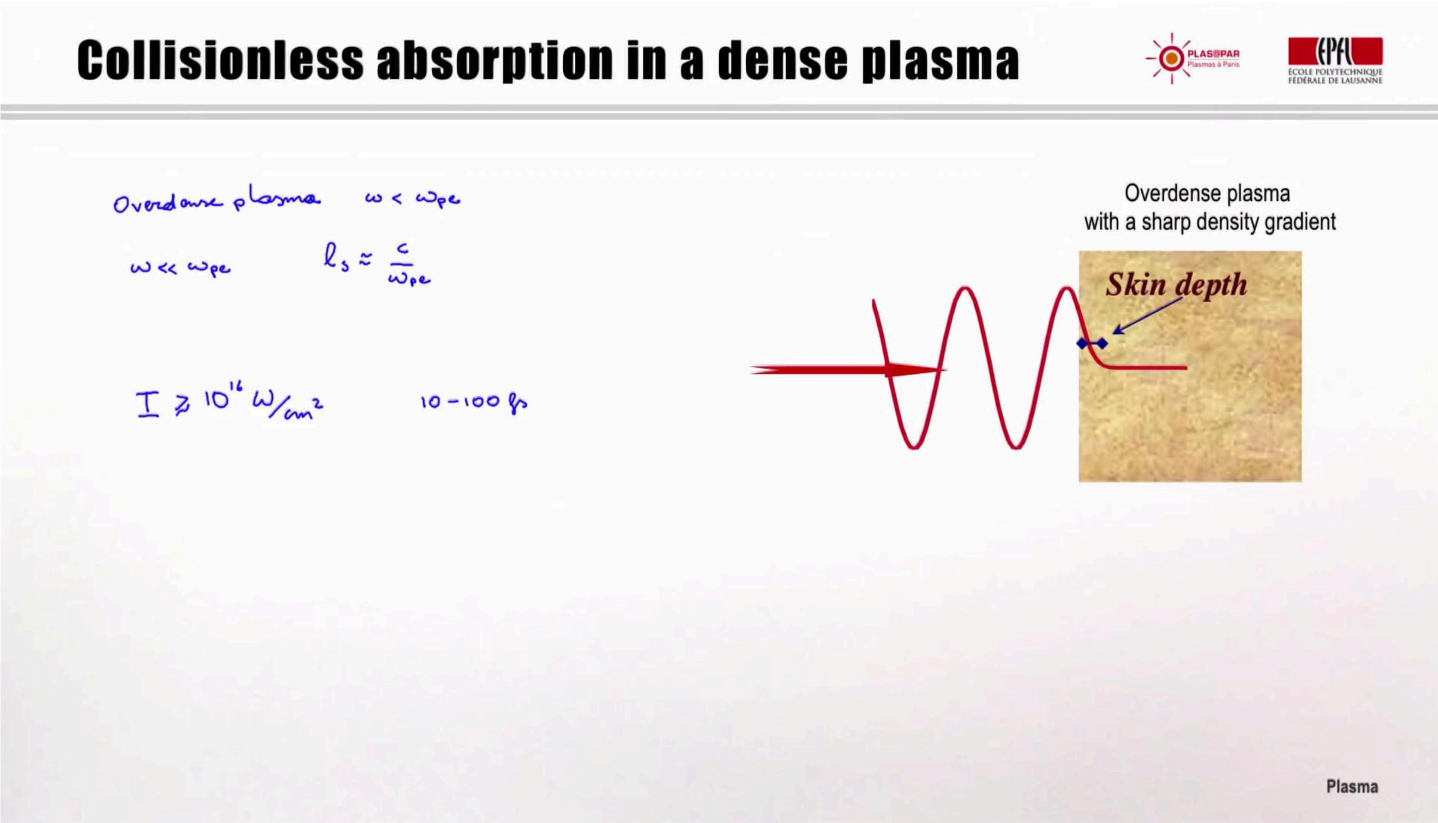


Notice that there is an optimal angle for resonant absorption. If the angle of incidence is zero, we can see from the formula written in the previous slide that the component of the electric field is zero, and then we cannot excite a plasma wave. This is the same situation of a way propagating parallel to the density gradient. When people arise light is incident at an angle instead as we have seen. At the critical density, the magnetic field is finite and the component of the electric field along the density gradient diverges. In this case, an electron plasma wave is resonantly excited and there is transfer of energy to the electrostatic mode and electrons are heated. If this angle is very large and in the limiting case, if Theta 0 is equal to Pi over 2, the turning point is very close to the plasma entrance in the low density region. Since the field are evanescent after the turning point, in this case, when they reach the critical density, they will be almost zero. And again, we have no absorption. As we can expect from these results, there is an optimal angle for resonant absorption that will be between 0 and Pi over two and depends on the density gradient and on the wavelength of the light that we are considering. At this optimal angle, resonant absorption can be very large up to 50 percent.

Notes

Summary

23m 17s

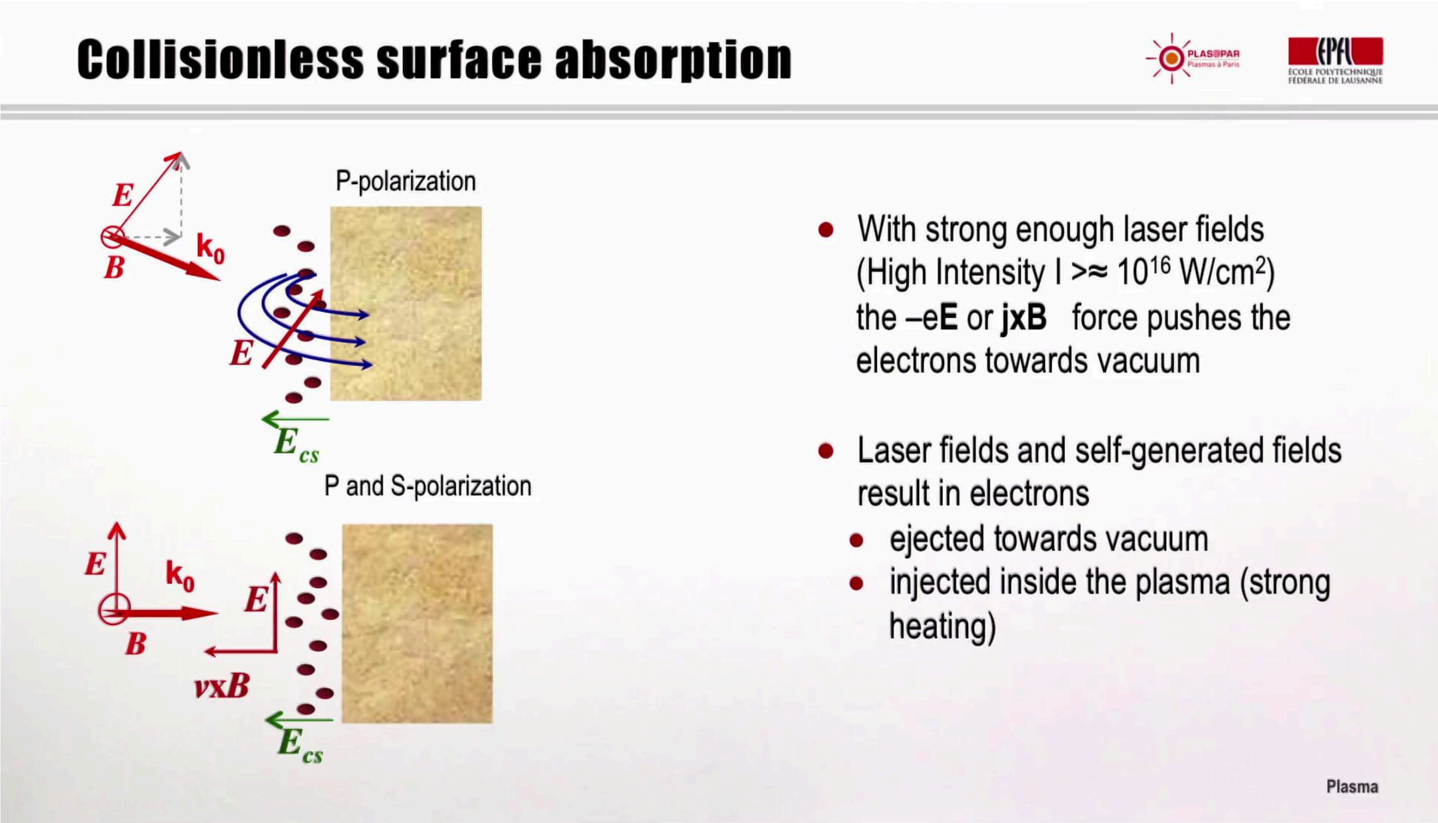


We will now discuss the last absorption mechanism treated in today's lecture. This is collisionless absorption in an overdense plasma with a sharp density gradient. For this part, we will just have a qualitative approach. An overdense plasma is the plasma at very high density such that the plasma frequency is larger than the frequency of the incoming light. In this situation, as we have seen in the previous lecture, the wave does not propagate but only penetrates and becomes evanascent inside the skin depth. If the plasma is very dense, for example, in a plasma that is created starting from a solid target, the frequency of typical laser light that we can consider will be much less than the plasma frequency. And in this case, the skin depth has the simple form already introduced the T's of the order of 100 Armstrong. In this condition, as we have seen previously, light will be reflected by the plasma or in the case of a laser pulse, if the pulse is long enough from 10 picoseconds to some nanoseconds, there can be some absorption, heating, and plasma expansion. However, if the pulse is very intense, that is, the intensity is above 10 to the 16 Watts per centimetre square, and if the pulse is very short, 10 to 100 femtosecond, there can be a strong collisionless absorption that will happen at the surface of the plasma, as discussed in the next slides.

Notes

Summary

25m 01s

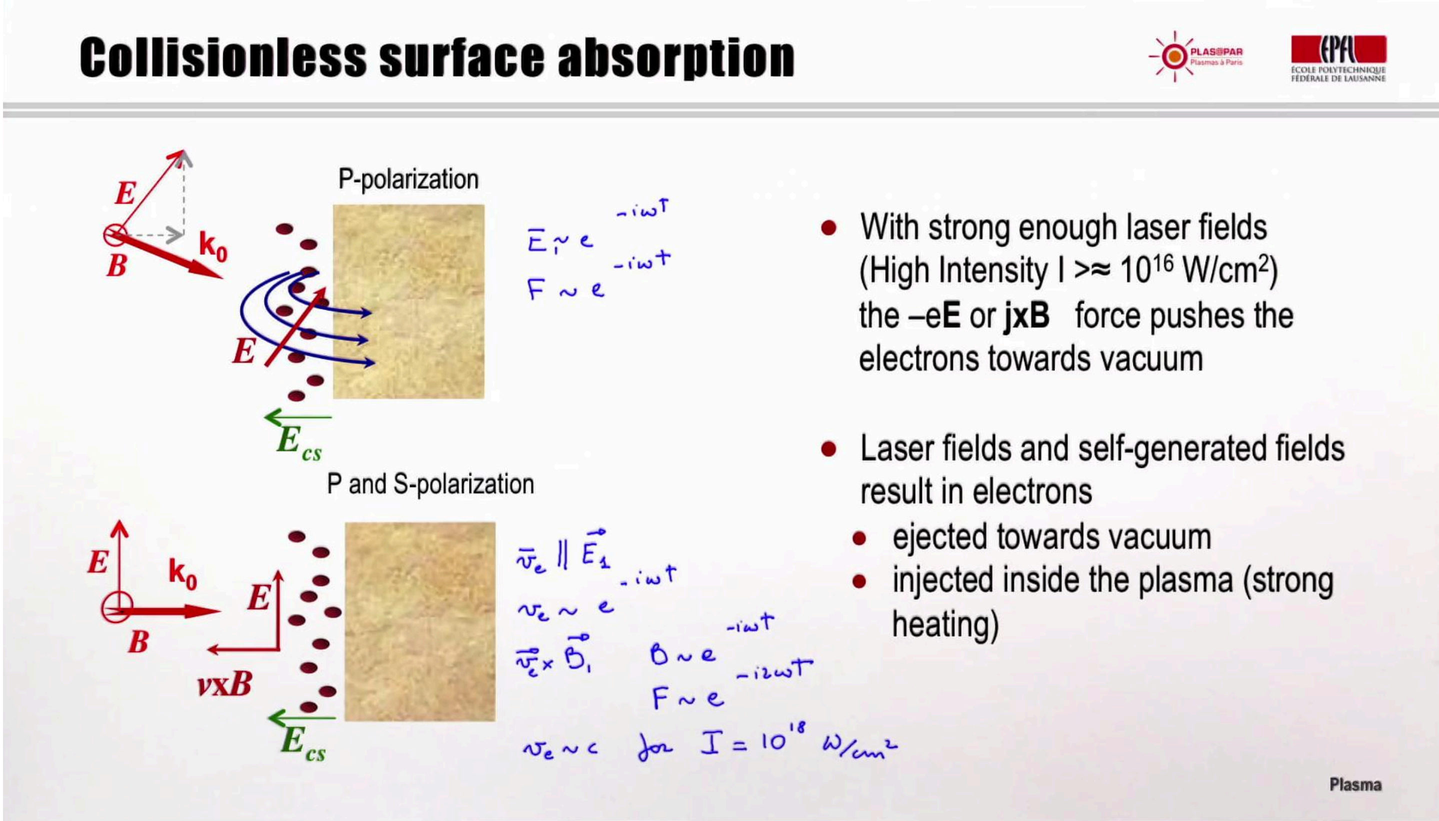


Let us consider an electromagnetic wave impinging on a very dense plasma. The way we penetrate a little bit, the electrons are pushed and pulled by the electric field. In P-polarisation, the electric field has a component parallel to the normal to the surface. For strong fields, this component can push the electron of the plasma outside and then back again. When they're pushed outside, they leave ions behind because they're much heavier and this results in a charge space field, I'm drawing here, that will give a further push to the electron. Once the electron are back in the plasma, since the plasma inside is roughly neutral, they will keep their motion and heat the whole plasma. The same mechanism can apply for the motion induced by the laser magnetic field, and this will hold for both P and S-polarisation. First of all, the electron, they move along the E vector as usual. But then their velocity in this direction crossed with the magnetic field that is out of the board here will give a component of the force of the Lawrence force that is perpendicular to the surface. This component will act in the same way as the Z component of the electromagnetic field pull them out and in when they're out, there is the space charge field that puts them back in.

Notes

Summary

26m 52s



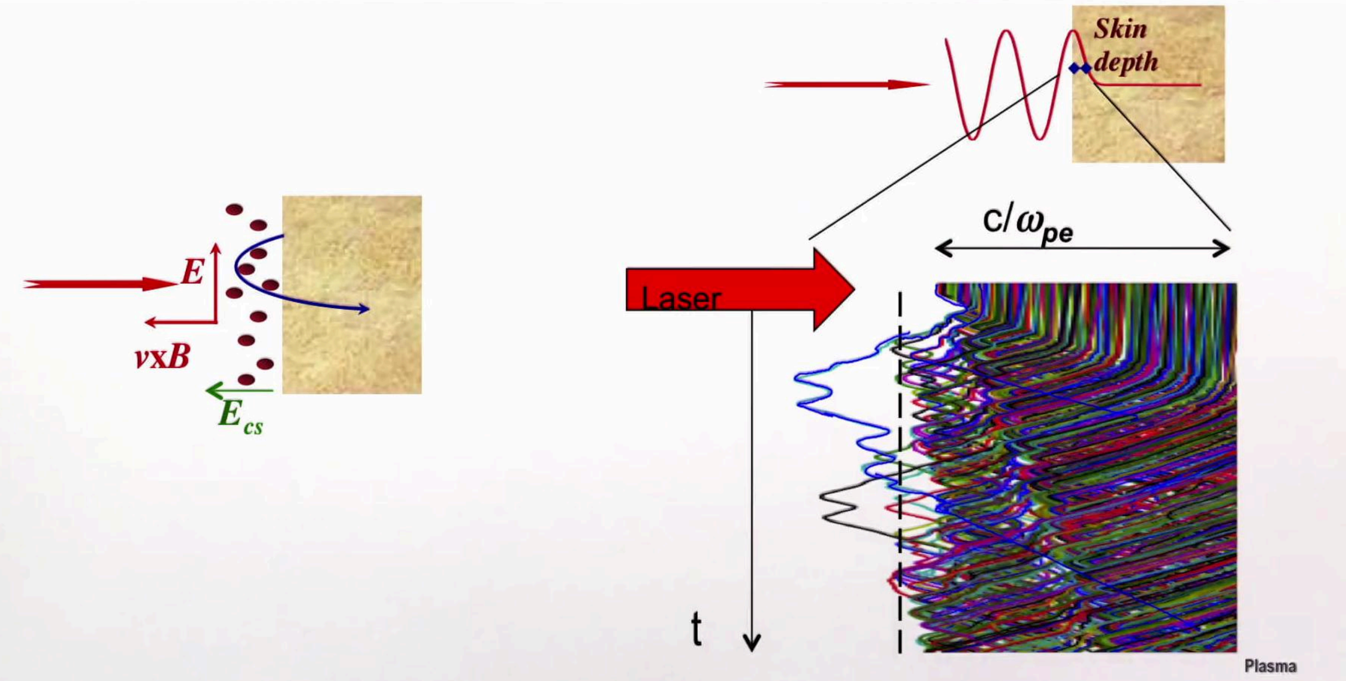
And again, we have a strong heating. In the first case, the movement of the electrons is related to the Z component of the electric field that is proportional to the exponential of minus I Omega T. This means that also the force that is felt by the electron will have frequency equal to the laser frequency. In the second case, the particles have first velocity along the electric field, and then their velocity is also oscillating at frequency equal to the laser frequency. And then they fill a force that is perpendicular to the surface that is given by the V cross B product, the Lawrence force. Since B also oscillates at frequency Omega of the laser, this means that the force will be oscillating at twice this value. Notice that the force being equal to the product v over B, this mechanism is normally less effective than this one because the magnetic field is less strong than the electric field. However, for very large intensity the velocity of the electron becomes relativistic and the product V cross B becomes comparable to the value of the electric field. To summarise these results, we can conclude by saying that laser field and self-generated field results in electron ejected towards vacuum and injected inside the plasma. Some of them will keep propagating in vacuum, but most of them will be called back in the plasma, and this will result in strong heating.

Notes

Summary

28m 31s

Heating by electrons injected in the solid target

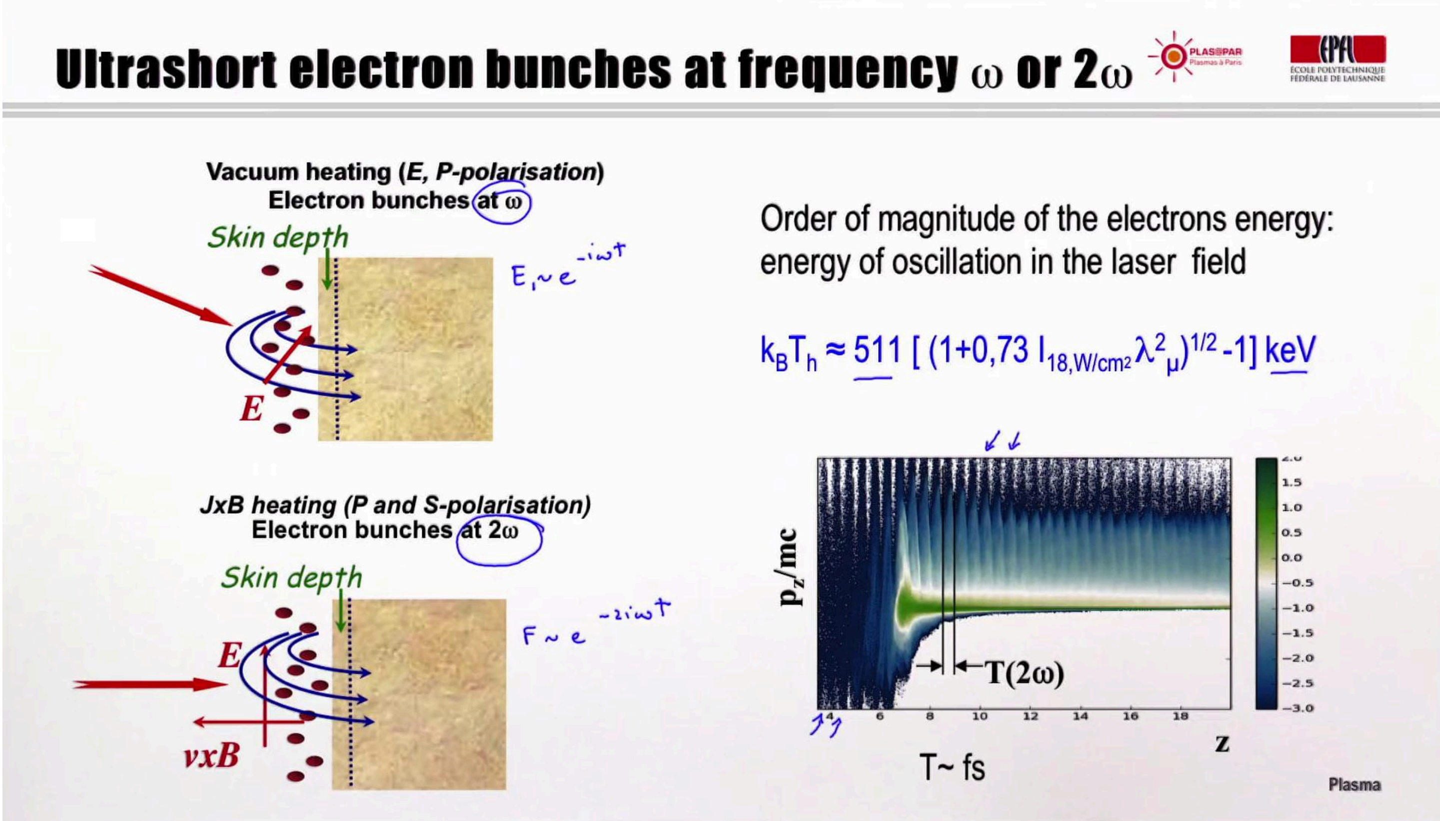


So in this slide, we'll illustrate the heating of electron by this mechanism, in particular, I chose the V cross B heating with some test particle simulations. So this is the scheme and this represents a particle that is push and pull back in the plasma. Instead, here, it's a real simulation. Every line here represents a particle. This is inside skin depth, so this is a zoom of the area where the interaction happens. The laser is arriving from the left and time is increasing in this direction. When the laser arrives, I can see that the particle initially at this position with time will go inside, then outside because the field changes sign. We'll oscillate back and forth in the fields at the surface, and eventually get inside the plasma and heat the plasma. Other particles that are reached by the laser at a later time will only do one oscillation and then heat the plasma.

Notes

Summary

30m 43s

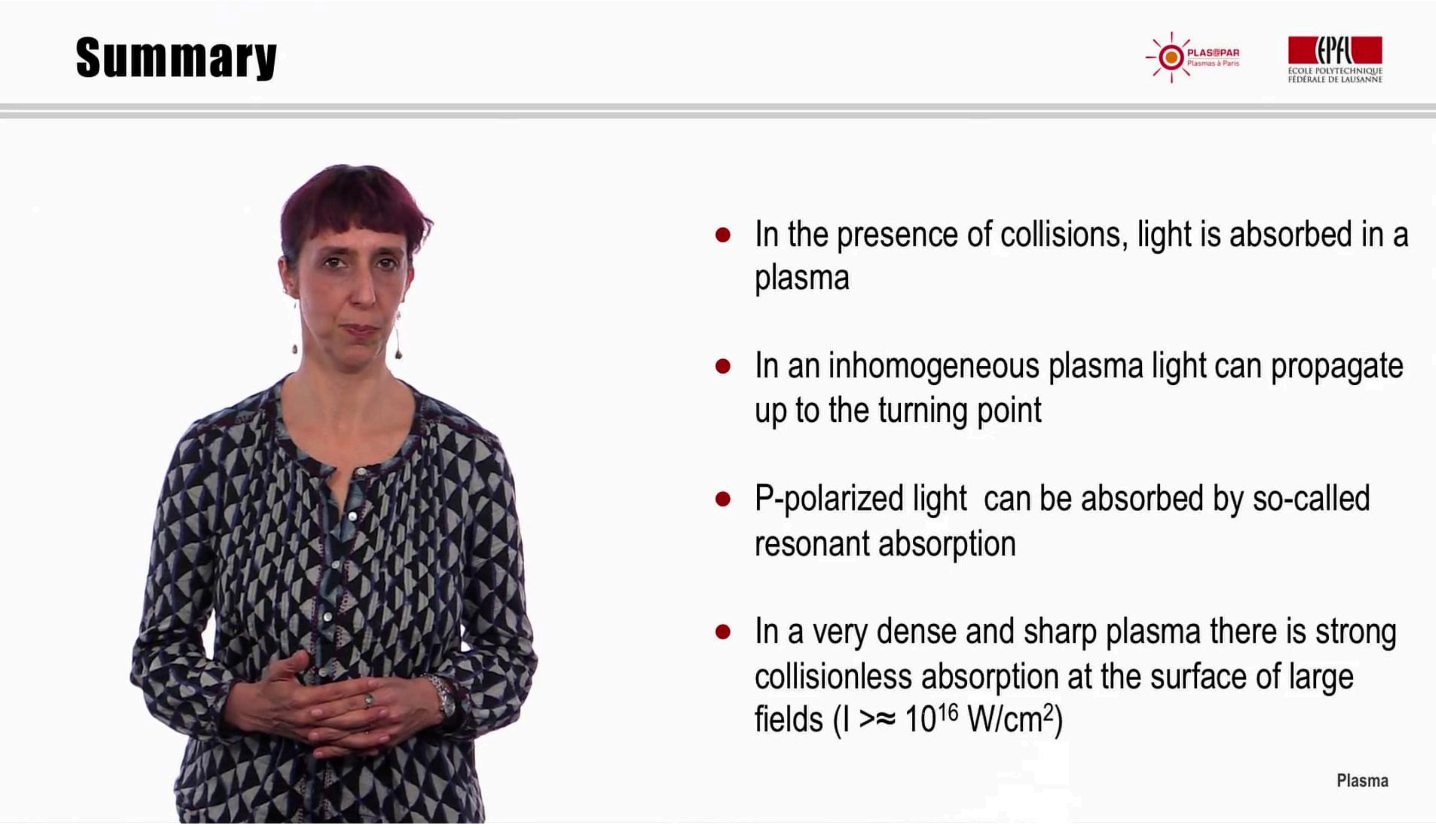


The pull, push motion of the particle under the effect of the laser field and the self-consistent field with results in particle that are injected inside the plasma at regular frequency. In the first case, since laser frequency Omega, particles will be injected by bunches at the laser frequency. In the second case, since the push and pull force is proportional to the exponential of minus 2i Omega T, the particles will be injected at twice this frequency. What is shown here is the momentum of the electrons in the direction normal to the surface. As a function of space, the plasma starts here, and as you can see, separate bunches of electrons are injected in the plasma. Bunches of electron are also injected towards vacuum. And the particles that are ejected towards vacuum can be used for various applications. Typical duration of an electric bunch produced in this way will be of the order of a femtosecond with today's laser. Finally, the order of magnitude of the energy gained by the electrons via this mechanism is roughly equal to the energy of oscillation of the electrons in the laser field and is given by this formula. The energy of the hot electron for a laser at one micron and an intensity of 10 to 18 Watt per centimetre square is roughly equal to half MeV, that is the electron [inaudible 00:34:07] mass.

Notes

Summary

32m 08s



We can now summarise what we have learned in today's lecture. First of all, in the presence of collisions, a propagating light wave is absorbed and heats the plasma. We have then examined different situations of light propagation in homogeneous plasma. And we have seen that if the density increases, light can propagate up to the turning point. We have seen that there is a collisionless absorption process for people arise light at the critical density. This is called resonant absorption. Finally, we have seen that if the electromagnetic fields are very large, light can be partially or totally absorbed by collision as mechanism at the plasma surface in an overdense plasma with a sharp density gradient.

Notes

Summary

34m 09s