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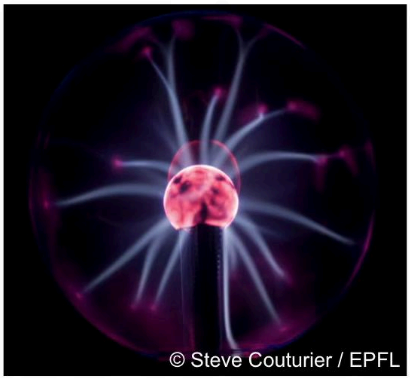
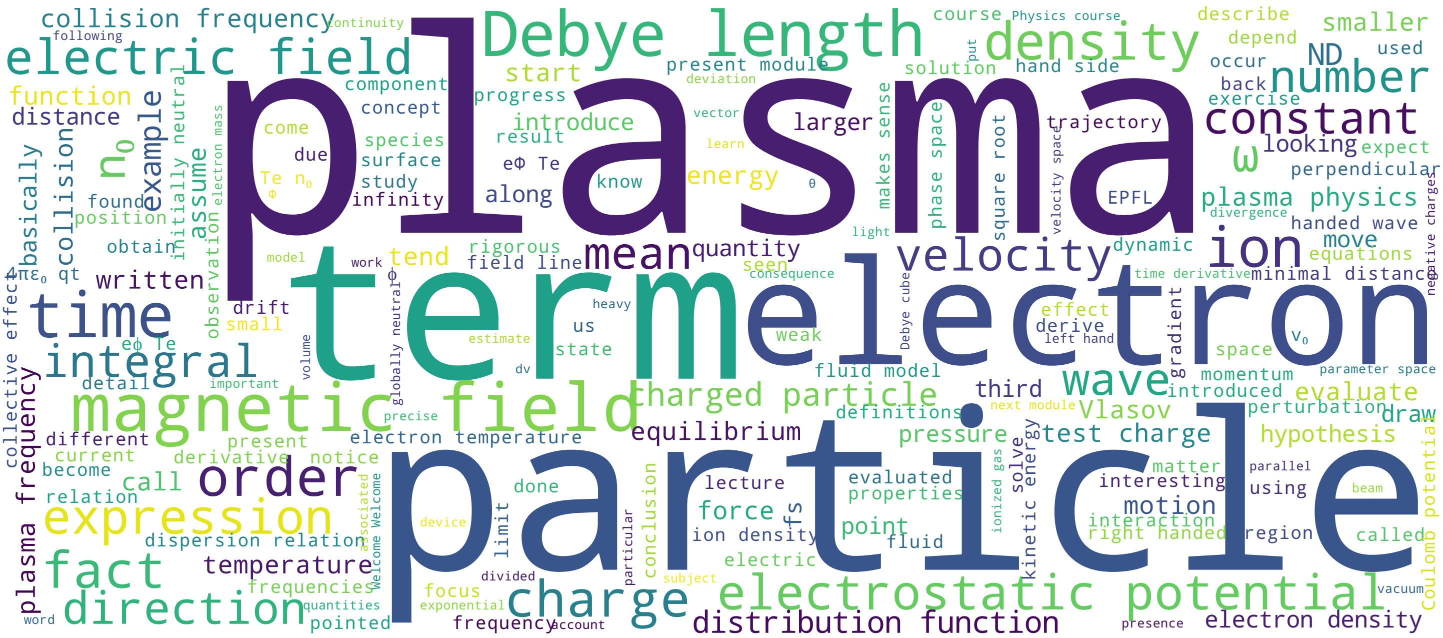


Rigorous definition of plasma: Debye length

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 1b

Paolo Ricci



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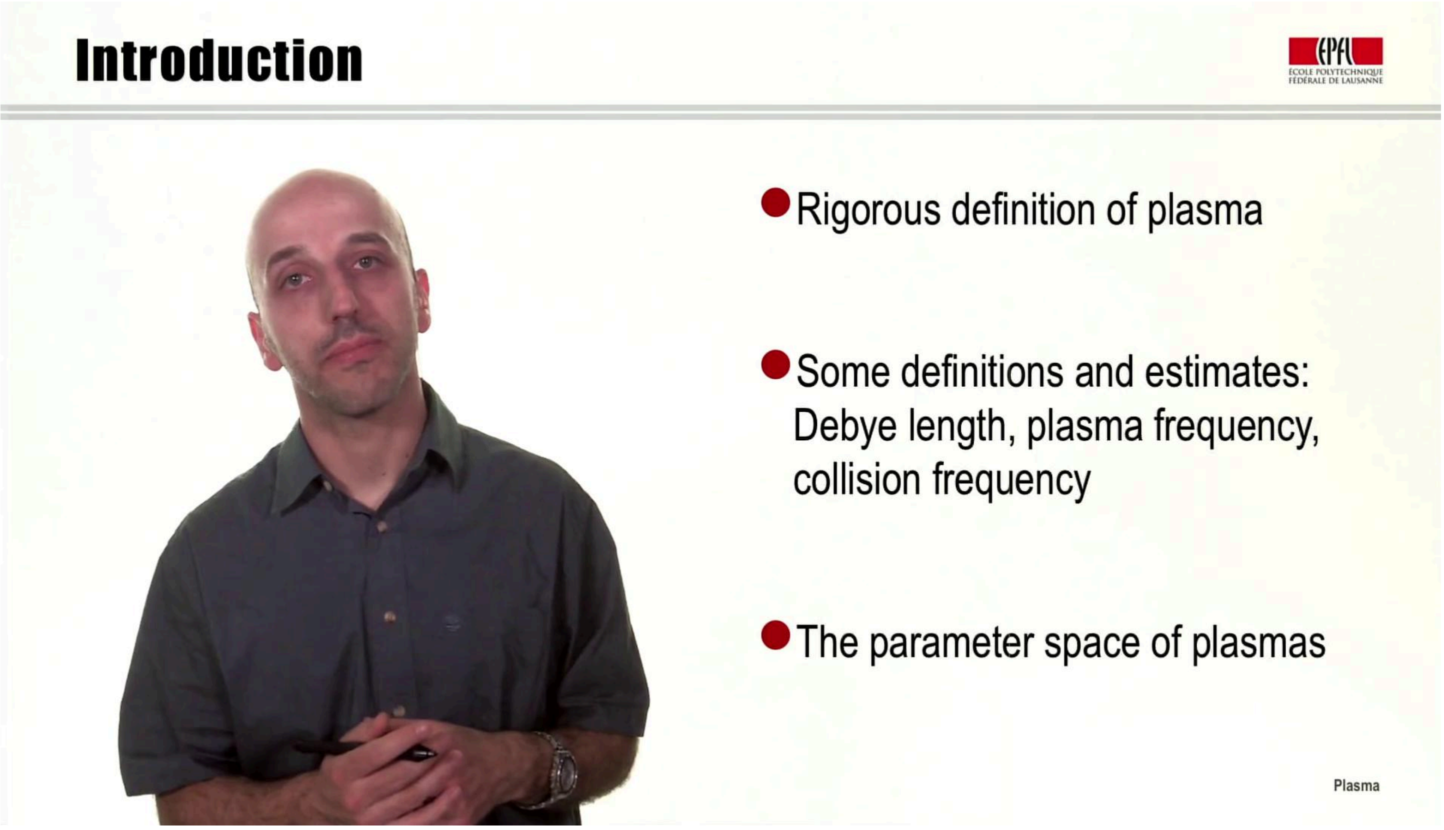


Welcome. Welcome to the Plasma Physics course of the EPFL. In the past module, we have given a very short introduction of the plasma state. It's now time to become a bit more precise and to give a rigorous definition of plasma. In order to do that, we need the three definitions: first one is the *Debye length*, the second one is the *plasma frequency*, and the third one is the *collision frequency*. In the present module, we will focus on the Debye length. Plasma frequency and collision frequency will the focus of the next module. By using these three concepts, we will see what is the parameter space of plasmas.

Notes

Summary

0m 05s

Rigorous definition of plasma

Plasma = ionized gas, globally neutral, that displays collective effects.

We need three definitions:

- Debye length
- Plasma frequency
- Collision frequency

Plasma

So let's start. Let's start by giving the definition, the rigorous definition of a plasma. Plasma is an ionized gas. This is what we have pointed out during the first lecture Now, let me be more rigorous about what a plasma is. It's an ionized gas that is globally neutral and that displays collective effects. Two key words: the first one is *globally neutral*, and the second one is *collective effects*. What do they exactly mean? Well, to address this, we need to give three definitions. The first one is the one of Debye length. The second, is the one of plasma frequency, and the third one is of collision frequency. Once we have given these three definitions, we will be able to see where plasmas can be found in nature. In the present module, we will focus on the Debye length. And in the next one, we will look at plasma frequency and collision frequency.

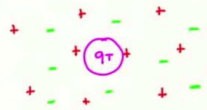
Notes

Summary

0m 56s

The Debye length 1/2

Plasma initially neutral, $n_e = n_i = n_0$ $(Z = 1)$



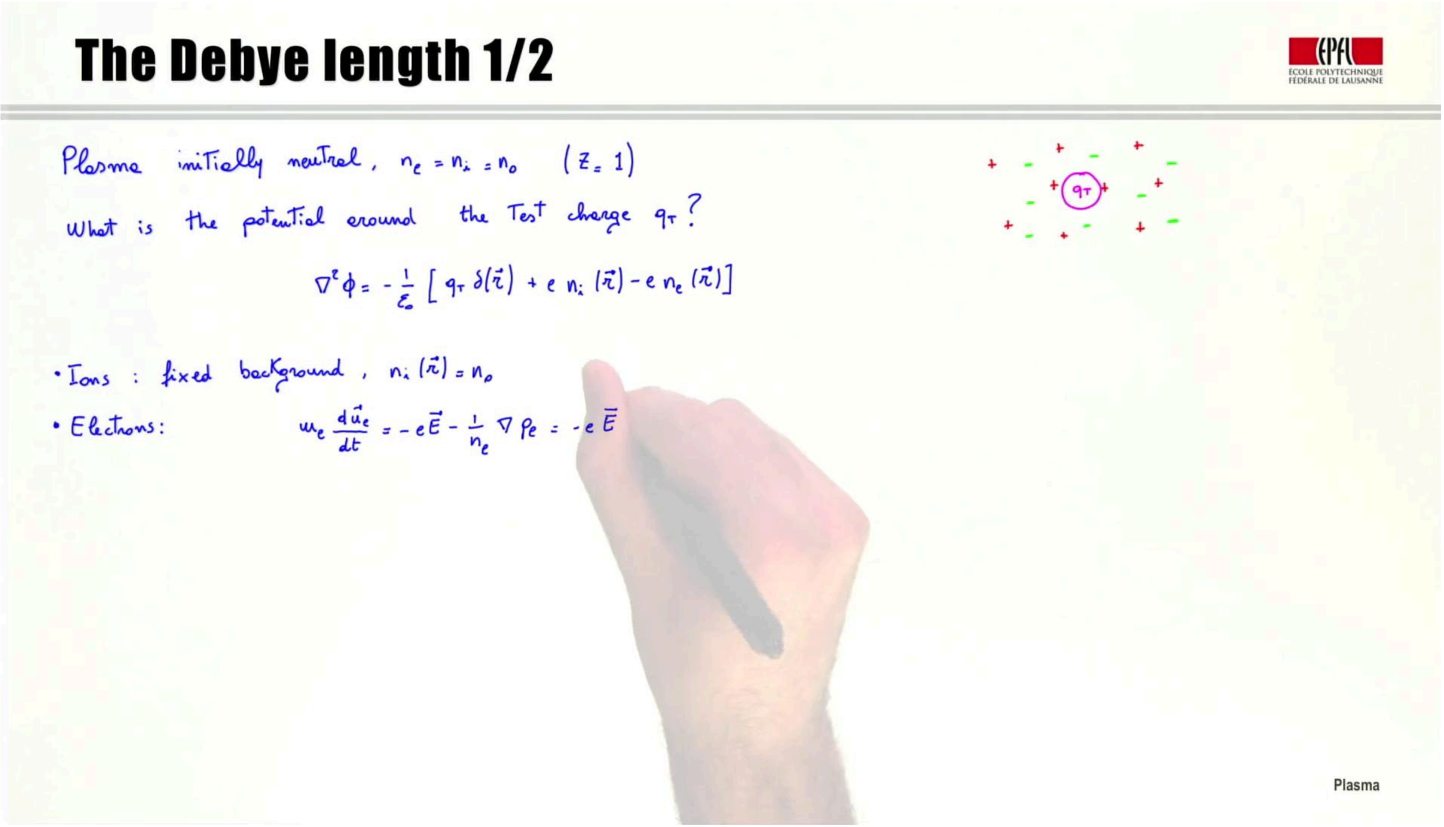
Plasma

Okay, let's start by giving the definition of Debye length. In order to do that, we will consider a simple scenario; the one of a plasma that is initially neutral. As a matter of fact, plasma comes from the ionization of matter. We expect in a plasma that the number of negative charges is equal to the number of positive charges. Now in this plasma that is initially neutral, we insert a charged particle, a test charge. And what we study is the electrostatic potential that forms around this test charge. As we will see in finding out what this electrostatic potential is, a length will appear which is the Debye length. So let's start by considering plasma that is initially neutral, and therefore, for which the electron density is equal to the ion density, and is equal to n_0. We will assume that we are dealing with singly ionized ions therefore that Z, the number of positive charges per ion, is equal to 1. We have therefore a system where there are a number of ions and of electrons, which have the same density. And what we do in this initially neutral plasma, is to introduce a charge, that we will call a *test charge*, with charge qt, and we want to learn about the potential that will form around this test charge.

Notes

Summary

2m 23s

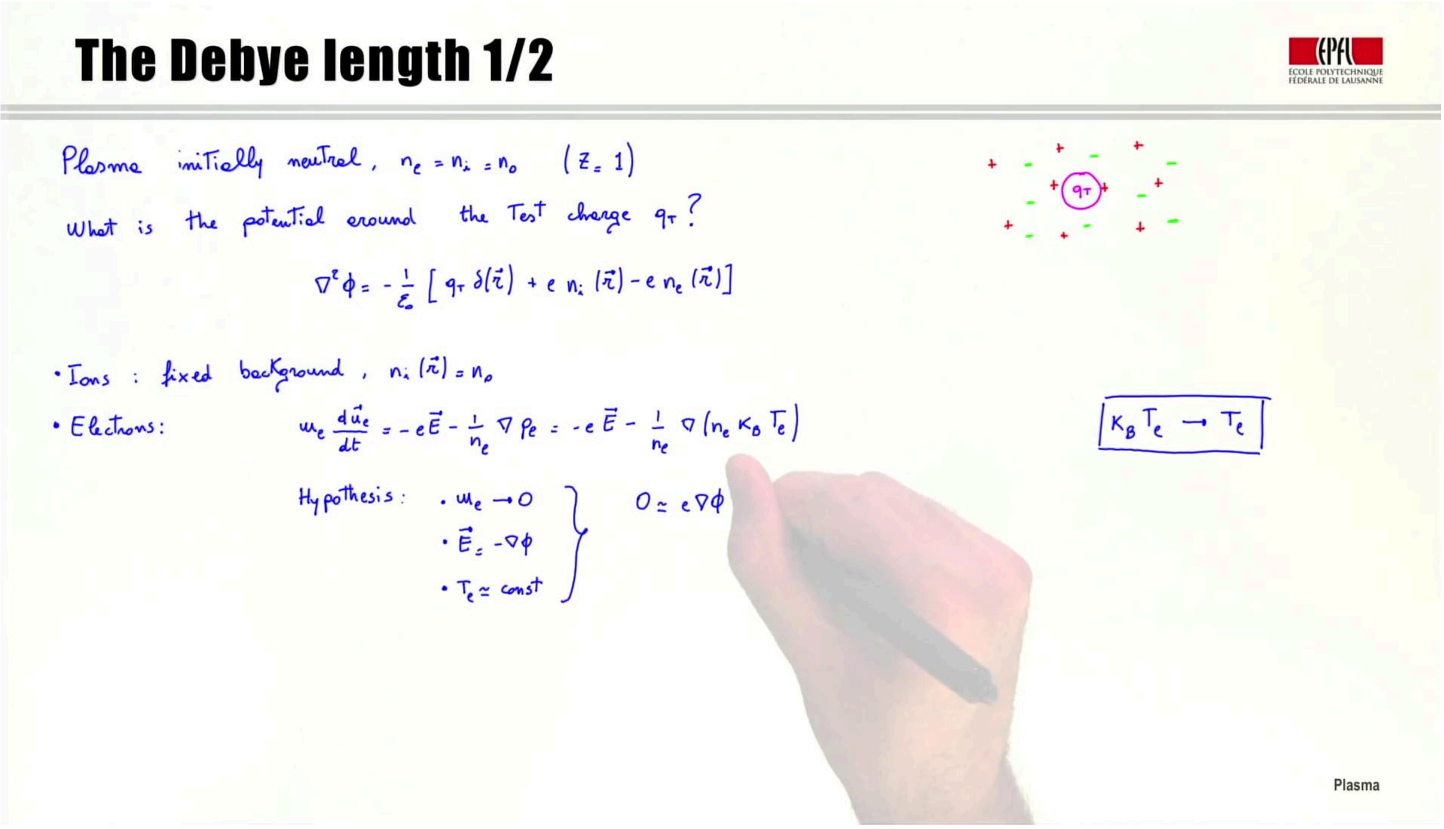


And we want to learn what about the potential that will form around this test charge. Well, to answer this question, what we have to do is to solve a Poisson's equation, where the laplacian square of Φ is equal to the charges present in the system that are given by the test charge that is located r = 0, plus the charges of the ions that will depend on r, and the the charges of electrons. Well, in order to make some progress, we have to somehow state what are the ion and the electron densities. For the ions, we say that they are heavy. When we introduce the test charge, they are so heavy that they don't move, so as a first approximation we can assume that they constitute a fixed positive background with therefore, an ion density that is constant and will be equal to n_0 How about the electrons? The electrons are light, and therefore they move. We have to write down an equation of motion if we want to study where they will go. This will be given by the electron mass times the derivative of the electron velocity, This will be equal to the forces acting on the electrons which are given by the electric field and the pressure.

Notes

Summary

4m 19s

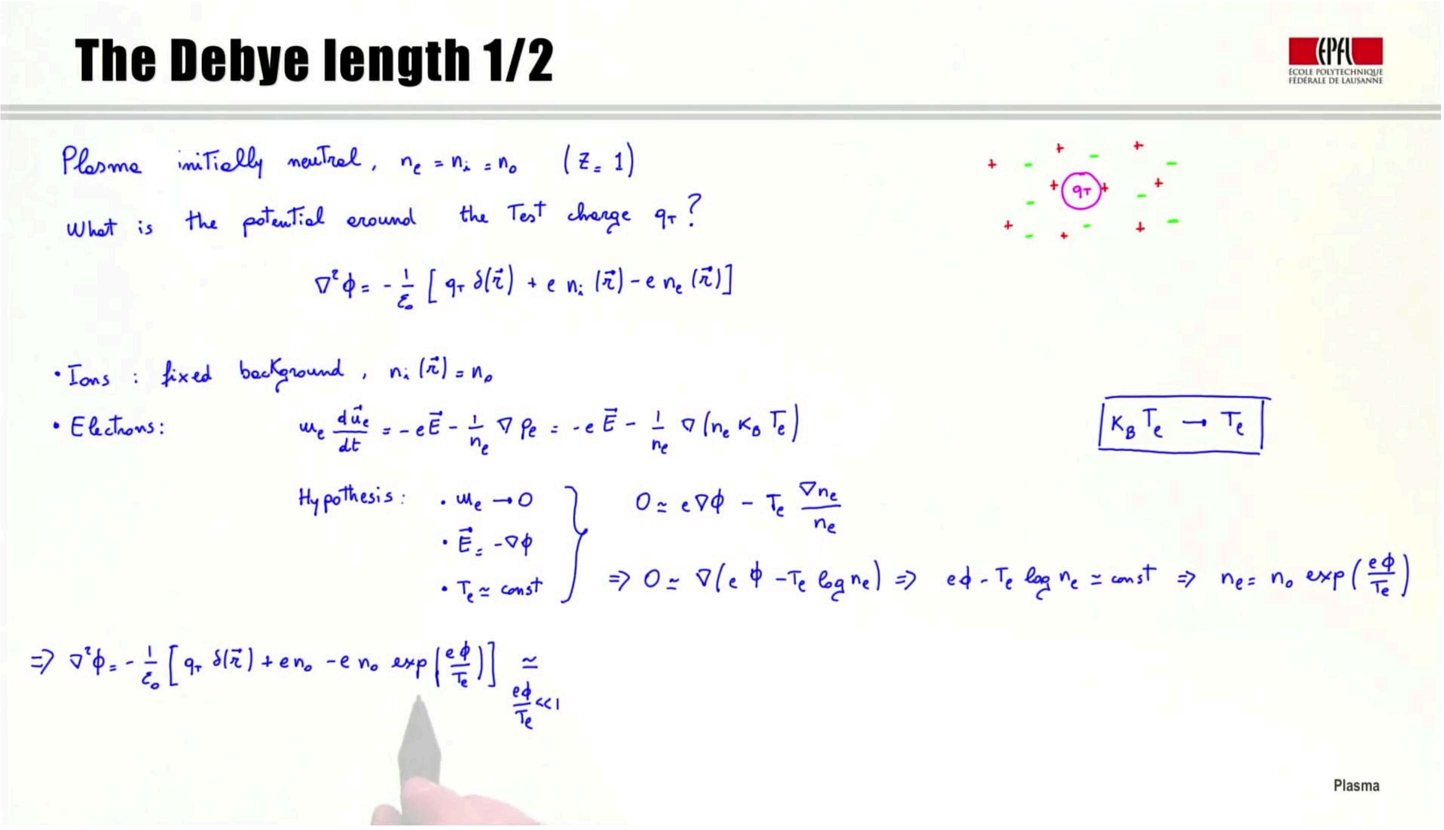


Now, pressure can actually be written in terms of temperature and density through the *Boltzmann constant* KB In order to simplify the notation in plasma physics, we tend to agglomerate the Boltzmann constant inside the temperature and therefore we'll redefine KB times the temperature T, simply as the temperature. KB T -> T. Therefore temperature will actually be measured in terms of energy, Joules [J] or electron volts [eV], for example. Now, to make some progress, this is already a fairly complicated equation, we make some hypotheses. The first one is that the electrons are very light. They basically respond immediately to the insertion of this charged particle into plasma, which corresponds basically to take the limit of the electron mass to go to zero. Second hypothesis: we will assume that the electric field is given by minus the gradient of the electrostatic potential and finally, we will assume that electrons are isothermal, therefore, the electron temperature is about constant. What is the consequence of this hypothesis? The left-hand side will be equal to zero. And this will be given by the gradient of the electrostatic potential, the electric field.

Notes

Summary

5m 57s



Then we will have the pressure term contribution where we can assume that the electron temperature Te is constant. We can bring everything under the same grad symbol and therefore we will have that e∇ϕ minus Te [times ∇ne/ne] and we will write that this ∇ne/ne as the gradient of the [natural] logarithm of ne. Well it is as to be equal to zero. Which means that (eϕ-Te log ne) has to be equal to a constant, which means that the electron density will be given by n_0 exp (eϕ/Te), where the constant has been determined by expecting that when there is no perturbation, --no electric field--, the electron density will be equal to the initial density, n_0. Well, what does it mean if we insert this expression for the ion density and electron density into the original Boltzmann equation that we were looking at ? We will have that the laplacian of ϕ will be equal to $1/\varepsilon_0$... we have to keep this charge, and then we can place the expression for the ion density and the one for the electron density. Now we can actually make an hypothesis that we will discuss later that eϕ/Te, this term here, is much less than 1, basically the perturbation is small. If we do this hypothesis, we can expand this exponential at the first order.

Notes

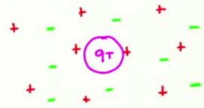
Summary

7m 47s

The Debye length 1/2

Plasma initially neutral, $n_e = n_i = n_0$ $(Z = 1)$

What is the potential around the Test charge q_T?



$$\nabla^2 \phi = -\frac{1}{\varepsilon_0}\left[q_T\, \delta(\vec{r}) + e\, n_i(\vec{r}) - e\, n_e(\vec{r}) \right]$$

- Ions : fixed background, $n_i(\vec{r}) = n_0$
- Electrons: $m_e \frac{d\vec{u}_e}{dt} = -e\vec{E} - \frac{1}{n_e}\nabla p_e = -e\vec{E} - \frac{1}{n_e}\nabla(n_e K_B T_e)$

$\boxed{K_B T_e \longrightarrow T_e}$

Hypothesis: $\cdot\ m_e \to 0$, $\cdot\ \vec{E} = -\nabla\phi$, $\cdot\ T_e \simeq const$

$$0 \simeq e\nabla\phi - T_e \frac{\nabla n_e}{n_e}$$

$$\Rightarrow 0 \simeq \nabla(e\phi - T_e \log n_e) \Rightarrow e\phi - T_e \log n_e = const \Rightarrow n_e = n_0 \exp\left(\frac{e\phi}{T_e}\right)$$

$$\Rightarrow \nabla^2\phi = -\frac{1}{\varepsilon_0}\left[q_T\, \delta(\vec{r}) + e n_0 - e n_0 \exp\left(\frac{e\phi}{T_e}\right)\right] \underset{\frac{e\phi}{T_e} \ll 1}{\simeq} -\frac{q_T}{\varepsilon_0}\delta(\vec{r}) + \frac{e^2 n_0}{\varepsilon_0 T_e}\phi = -\frac{q_T\,\delta(\vec{r})}{\varepsilon_0} + \frac{1}{\lambda_{De}^2}\phi$$

$$\boxed{\lambda_{De} = \sqrt{\frac{\varepsilon_0 T_e}{e^2 n_0}}}$$ electron Debye length

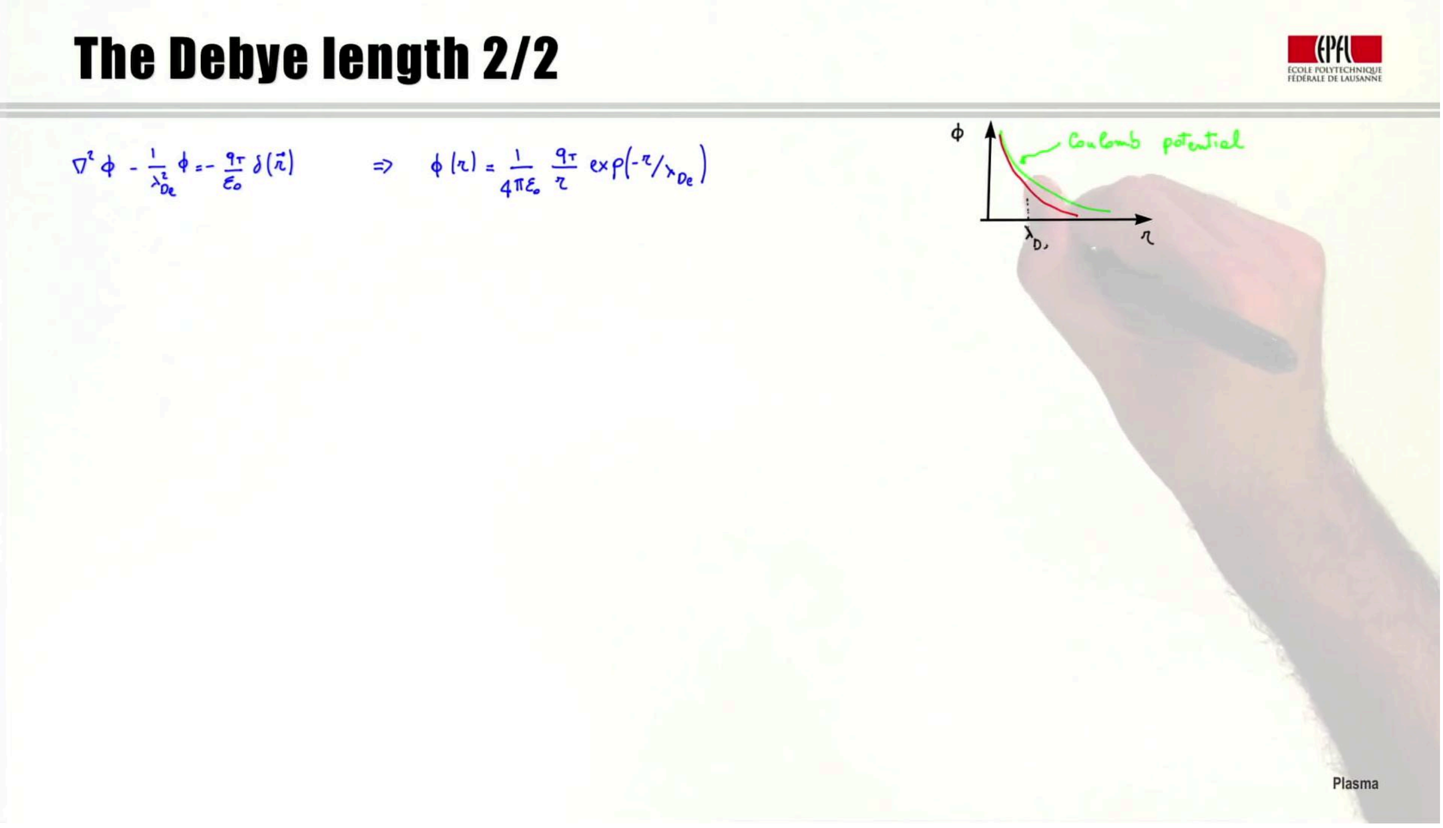
Plasma

At the zero-th order, it will give us 1 therefore, positive and negative charges will cancel and then we will have a term that is linear in ϕ. Basiscally, what we will find is -qt/ε_0 times the Dirac δ(r), plus e² n_0 ε_0 Te, the term that comes from the Taylor expansion of this term, times ϕ. This will be therefore equal -qt δ(r)/ε_0... If you look at this term here, you'll find that this is the inverse of a length square. We call this length the electron Debye length, λDe and therefore this equation becomes 1/λDe² ϕ where the Debye length is equal to the square root of (ε_0 Te)/(e² n_0) This is the electron Debye length, one of the most important quantities in plasma physics.

Notes

Summary

9m 56s

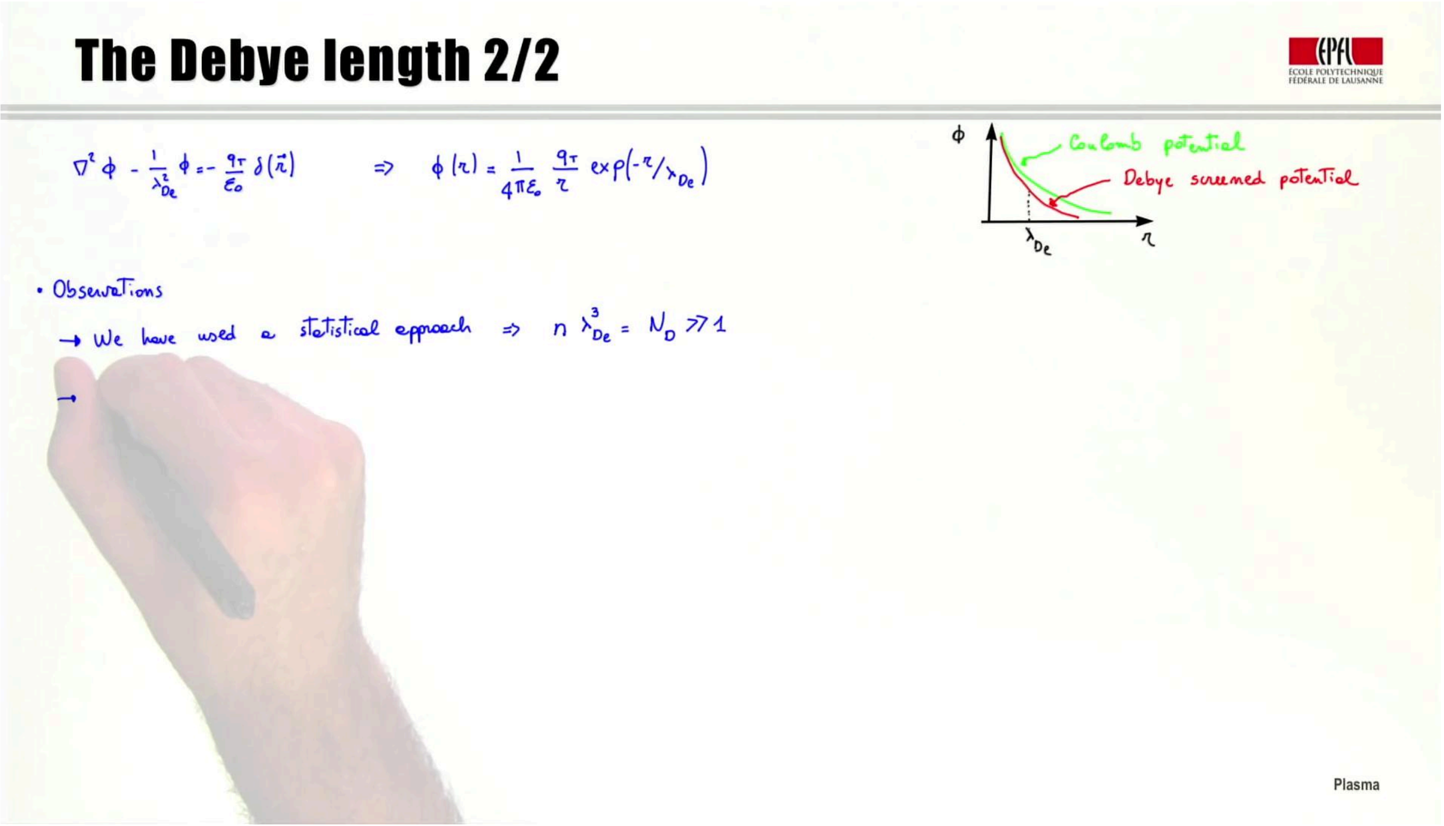


So we have reduced the problem of finding electrostatic potential around the test particle to the solution of the following equation, -- where I have brought on the left-hand side, all the terms in ϕ--: this equation can be solved. I spare you all the details. Well, let me just mention that one has to work in spherical coordinates, but without going into the details, let me just give the solution of this equation that tells you that the electrostatic potential will depend on the distance from the test particle r, as $1/(4\pi\varepsilon_0)$ qt/r times exp (-r / λDe). And here we notice something interesting: that the electrostatic potential that we would expect for a charge in a vacuum $1/(4\pi\varepsilon_0)$ qt/r is modified by the presence of the plasma that is this term here, by this exponential. So how does this electrostatic potential looks like if we try to draw it? Well, first of all let's plot that the standard Coulomb potential $1/(4\pi\varepsilon_0)$ qt/r, it will have a shape like this one. And if we plot the potential that we obtain in a plasma, we observe that there is an exponential factor here who's argument is always negative, which will provide screening. Now where this screening will become important, is when we are at distances r of the order of the Debye length.

Notes

Summary

11m 19s

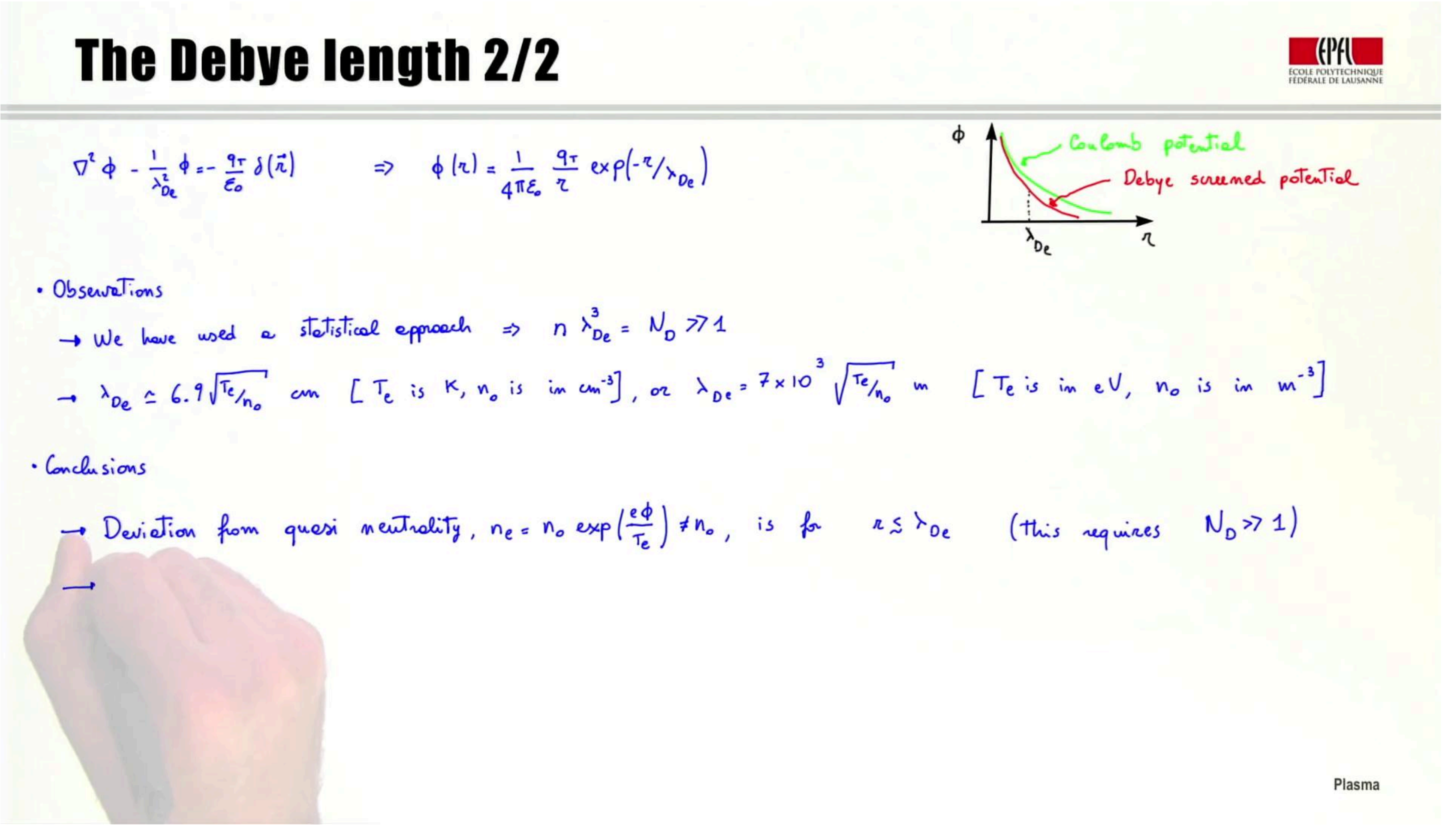


This is when the deviation from the Coulomb potential will become important. So, what is the effect of the plasma? It is actually to screen, the potential that the charge particle would have in vacuum. What do we have here which is the case of a plasma, is a screened potential that we will call *Debye screened potential*. So let me make a few observations. The first one is that in order to describe the plasma, we have used concepts to like density. If you want, we forgot about the fact that charged particles in a plasma are discrete. We have used a statistical approach that makes sense only if inside of the volume given by λDe^3, there is a high number of electrons or ions in other terms, if n, the plasma density times λDe^3, this is what we will call *ND*, number of charged particles in a Debye cube, is much greater than one. (ND >> 1) Otherwise if this is not respected, then the treatment that has lead to say that the Coulomb potential is screened with respect to in vacuum, is not valid anymore. We would have to look at the single, discrete nature of the particles. Second observation.

Notes

Summary

13m 26s

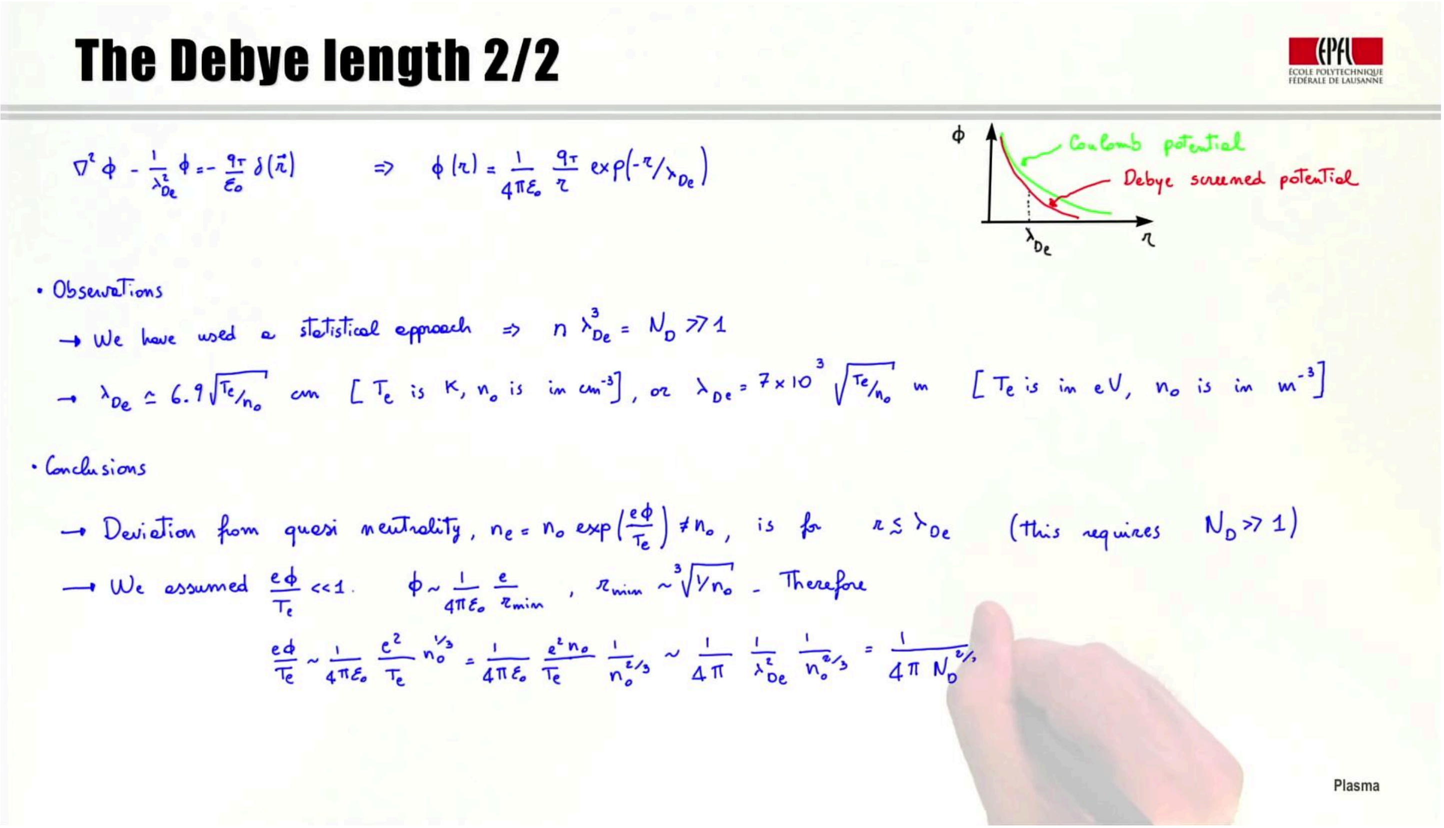


If we want to give an estimate of the Debye length, this will depend basically on temperature and density and will be given by 6.9 square root of (Te/n_0) in cm If you express the electron temperature in Kelvin, and the density in terms of particles per centimeter cube, or you can also express λDe^3 as 7 x 10^{-2} $\sqrt{}$(Te/n_0) in meters. Erratum: it should be 7 x 10^{-3} $\sqrt{}$(Te/n_0) But in this case, you have to express the electron temperature in eV and the density in particles per m^3. So what is the conclusion? What have we known by doing this exercise? The first conclusion that we can draw is that in order to have a deviation from neutrality or to be more precise from quasi-neutrality which is the initial state of system, well, we need to have the electron density which is given by n_0 exp (eΦ/Te) becomes different from n_0. This can occur only for distances that are smaller than the Debye length. In fact, at distances that are larger than the Debye length, the perturbed potential will tend to zero, and therefore ne will tend to be equal to n_0. Now let me point out that this procedure requires to have ND >> 1. There is a second conclusion that we can draw, which is quite important.

Notes

Summary

15m 26s



So let's go back to the hypothesis that we have done in the previous slide, where we have assumed that eΦ/Te << 1. Well, does it make sense? After all Φ goes to infinity for r going to zero. But actually it doesn't make much sense physically to go to r=0. There is minimal distance up to which looking at this equation makes sense. In fact, what we have to do is to evaluate the electrostatic potential at the minimal distance that makes sense to look at, where therefore the Debye screening will not have not intervened yet. And this minimal distance will be given by the typical distance between particles in plasma, which will be given by the cubic root of $1/n_0$. Therefore, if we want to evaluate the eΦ/Te, we can write this as -- use this expression for eΦ with the minimal distance given by this expression and then we can multiply and divide it by n_0 to the two-thirds, we will get therefore the numerator n_0 and now we can recognize a quantity that is the Debye length here, in fact this will be given by 1/4π one over the Debye length square, $1/n_0$ to the two-thirds. And here with this, this can be put in relation with ND, and this is equal to 1/4πND to the two thirds.

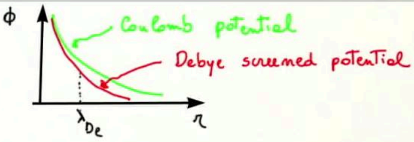
Notes

Summary

17m 35s

The Debye length 2/2

$$\nabla^2 \phi - \frac{1}{\lambda_{De}^2}\phi = -\frac{q_T}{\varepsilon_0}\delta(\vec{r}) \quad \Rightarrow \quad \phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_T}{r}\exp(-r/\lambda_{De})$$



- Observations
  - → We have used a statistical approach $\Rightarrow$ $n\lambda_{De}^3 = N_D \gg 1$
  - → $\lambda_{De} \simeq 6.9\sqrt{T_e/n_0}$ cm [T_e is K, n_0 is in cm^{-3}], or $\lambda_{De} = 7\times10^3\sqrt{T_e/n_0}$ m [T_e is in eV, n_0 is in m^{-3}]

- Conclusions
  - → Deviation from quasi neutrality, $n_e = n_0 \exp\left(\frac{e\phi}{T_e}\right) \neq n_0$, is for $r \lesssim \lambda_{De}$ (this requires $N_D \gg 1$)
  - → We assumed $\frac{e\phi}{T_e} \ll 1$. $\phi \sim \frac{1}{4\pi\varepsilon_0}\frac{e}{r_{min}}$, $r_{min} \sim \sqrt[3]{1/n_0}$. Therefore

$$\frac{e\phi}{T_e} \sim \frac{1}{4\pi\varepsilon_0}\frac{e^2}{T_e}n_0^{1/3} = \frac{1}{4\pi\varepsilon_0}\frac{e^2 n_0}{T_e}\frac{1}{n_0^{2/3}} \sim \frac{1}{4\pi}\frac{1}{\lambda_{De}^2}\frac{1}{n_0^{2/3}} = \frac{1}{4\pi N_D^{2/3}} \ll 1$$

One-to-one interactions between particles is weak if $\frac{e\phi}{T_e} \ll 1$, that is for $N_D \gg 1$

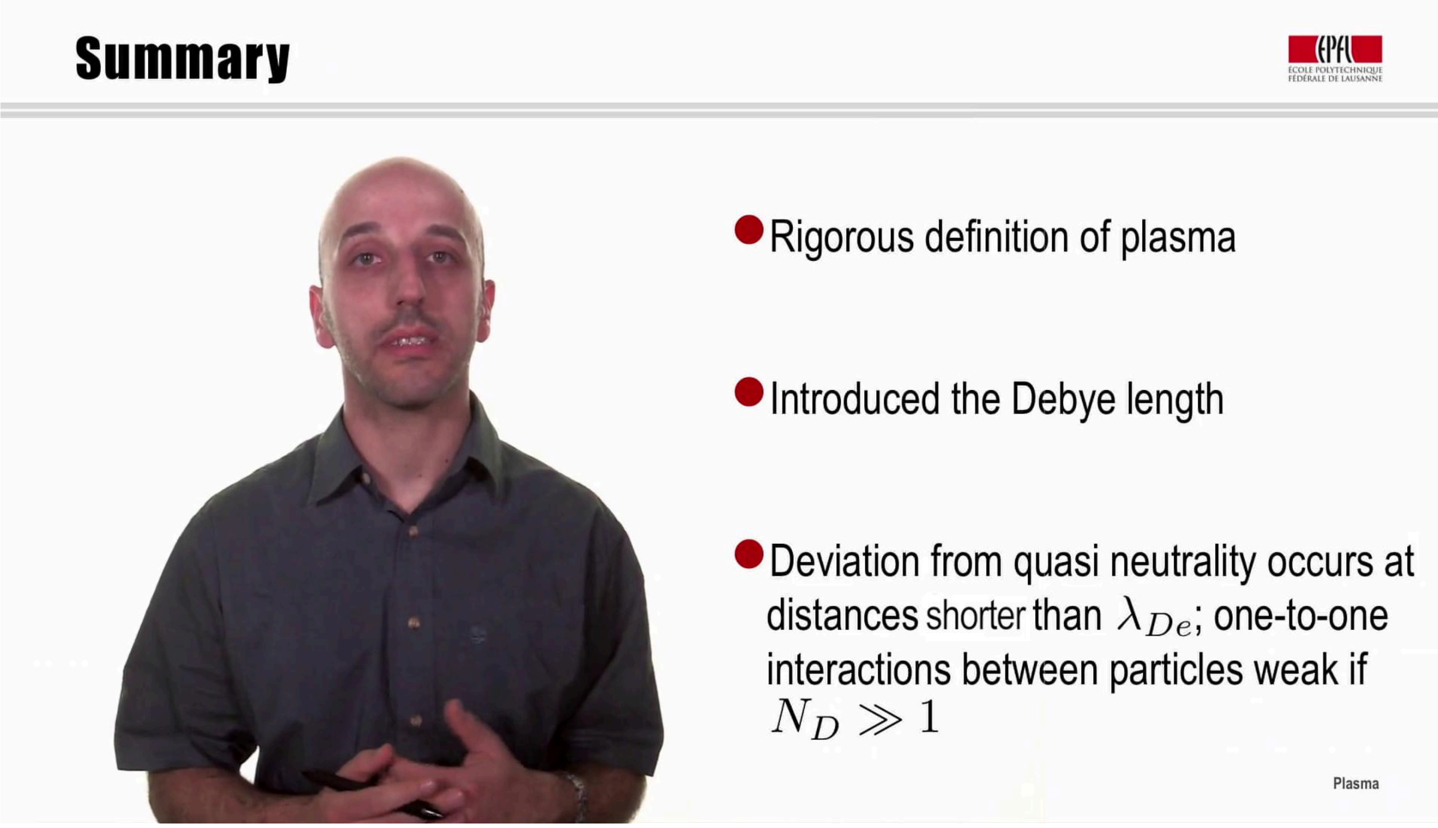
Plasma

And this is okay because we have already required that ND is much bigger than 1. The result will be much smaller than 1 as we've asked. So what is the conclusion? Actually we can draw one consequence from what we have just written here. And this will be much, much smaller than 1 because we have already required that ND is much larger than 1. Well, actually by looking at this, we can notice one thing: it is that one to one interaction between particles is weak if eΦ/Te << 1, which means for ND >> 1. In fact if ND >> 1 then the electrostatic potential that acts between two charged particles will be much smaller than the electron temperature, and therefore, the one-to-one interaction between particles will be weak.

Notes

Summary

19m 47s



In the present module, we have started to introduce the fundamental concepts that are required, in order to give a precise rigorous definition of a plasma. In particular, we have introduced the concept of the Debye length. We have see the quasi-neutrality of a plasma is a concept that makes sense at distances that are beyond the Debye length. Second thing, we have pointed out that one-to-one particle interactions are weak, if the number of particles in a Debye cube ND is much larger than 1. In the next module, we will look at the definition of plasma frequency and collision frequency, and through these we will fully understand the rigorous definition of plasma that we have given here in this module and we will look at what is the typical parameter space of plasma.

Notes

Summary

21m 09s