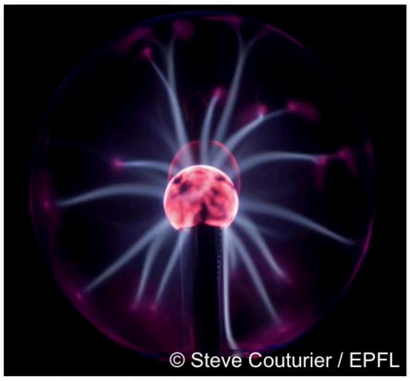
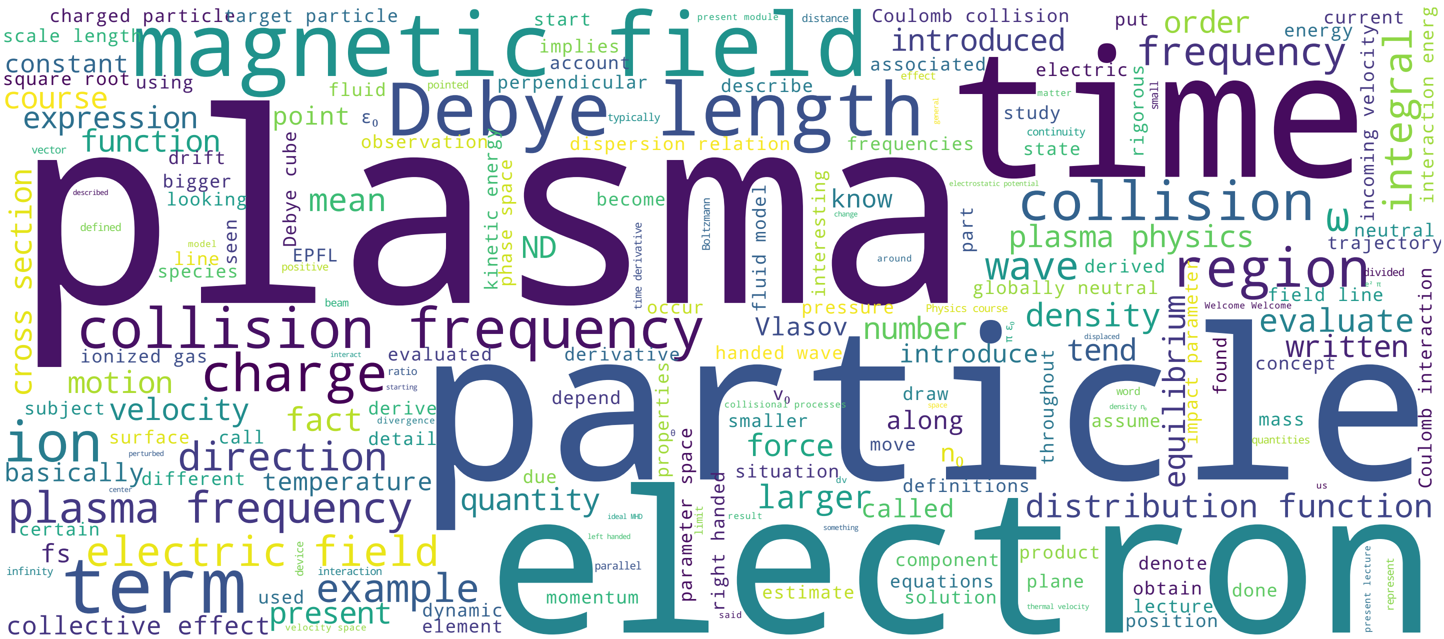


Plasma definition: frequencies and parameters

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 1c

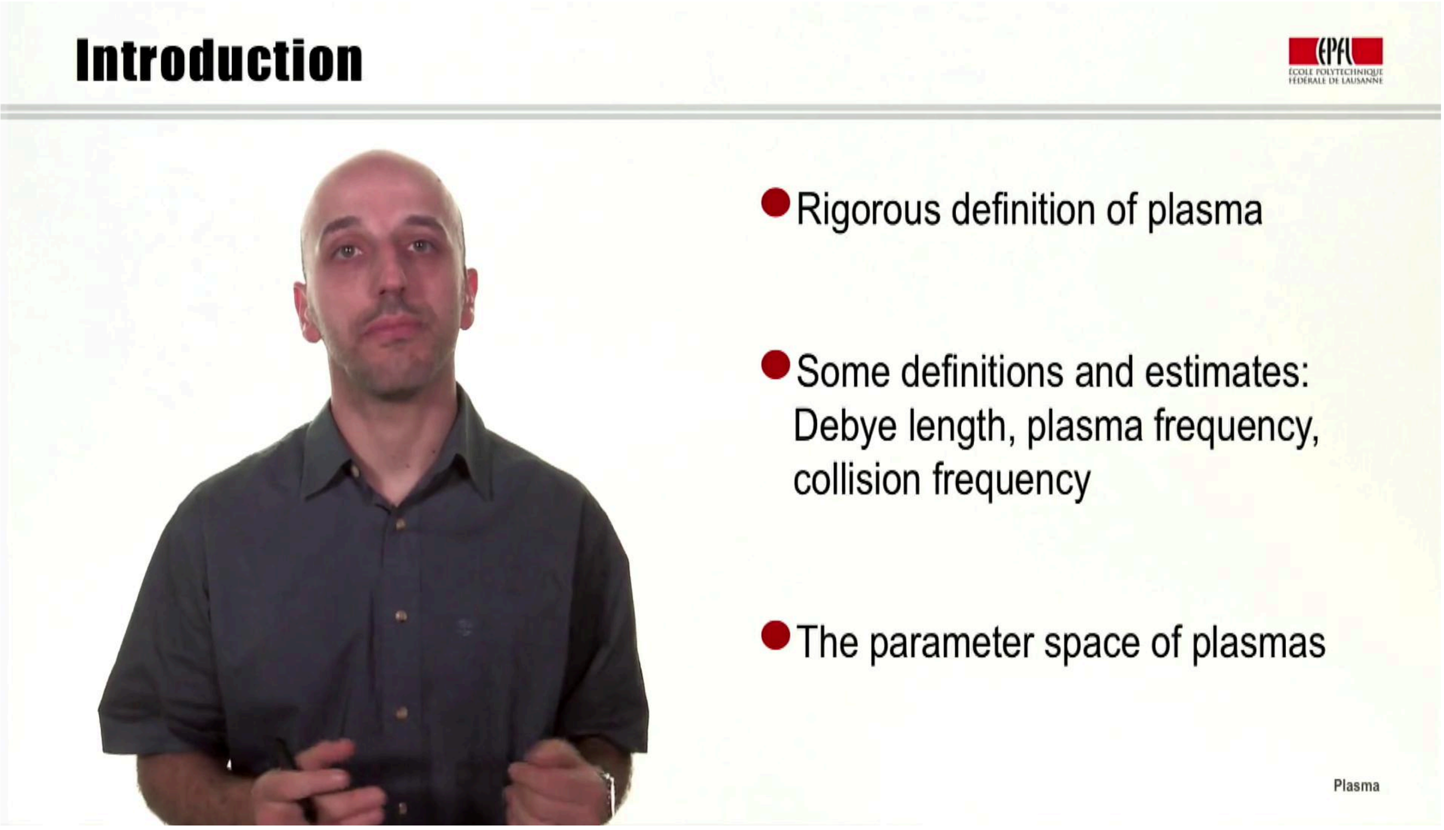
Paolo Ricci



Search MOOC

Video

EPFL



Welcome! Welcome to the Plasma Physics course of the EPFL. In the past module, we started to introduce the rigorous definition of a plasma. We have introduced the definition of the Debye length and now, today, with the present module it's time give a look at the last two definitions that we need, the one of *plasma frequency* and *collision frequency*. These two definitions will allow us to draw the conclusion and draw the parameter space of plasmas.

Notes

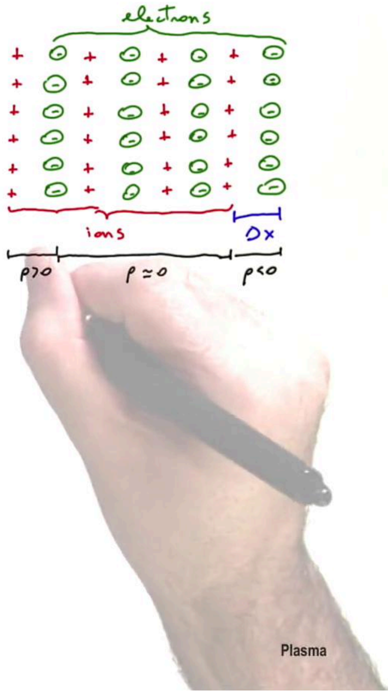
Summary

0m 05s

Plasma frequency

- A simple scenario
  - → Plasma, slab of electrons with number density n_0
  - → Ions, fixed background, same density
  - → Displace the electrons by Δx
  - → Frequency of the resulting oscillations?
- We evaluate the electric field

$$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}$$



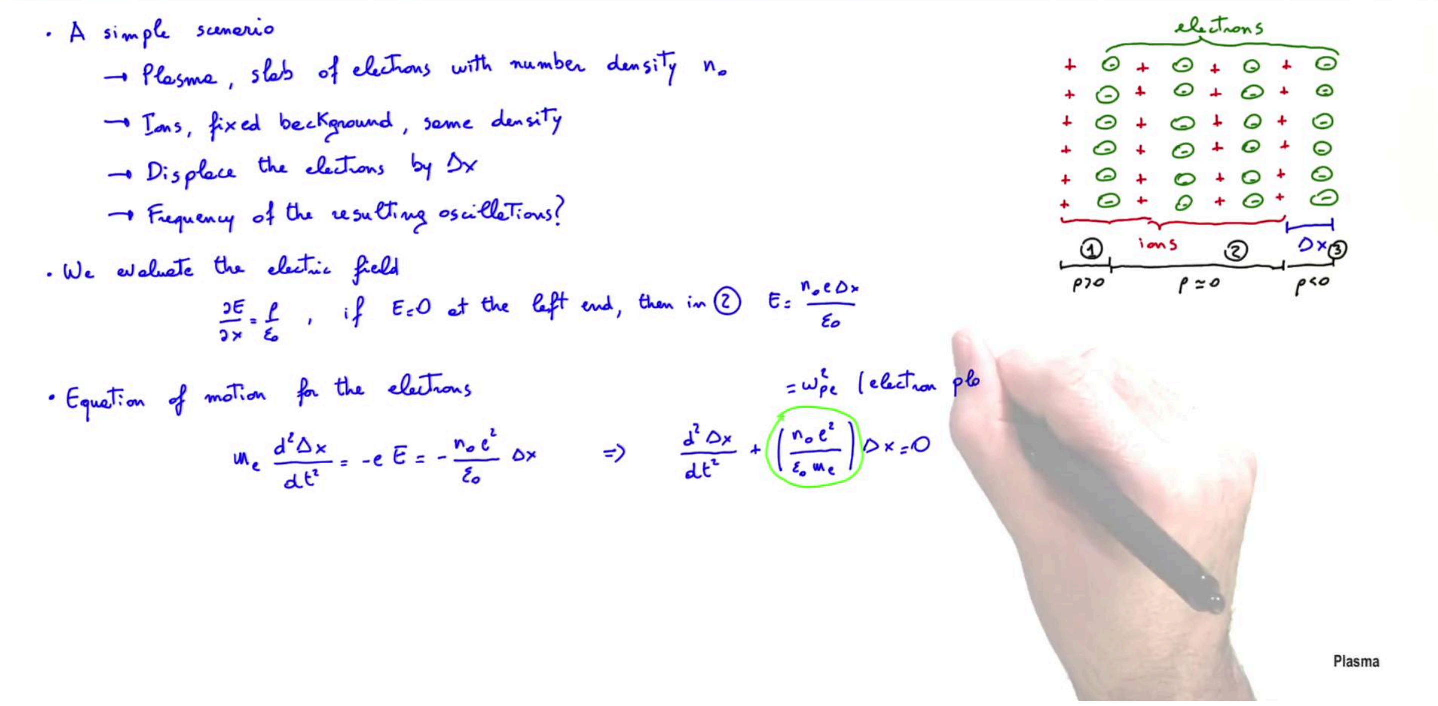
We start at the beginning, with the definition of plasma frequency. We consider a simple scenario, let's consider a plasma that is a slab of electrons with number density n_0. We will assume that ions are a fixed background with the same density as the electrons. And then we will displace the electrons with respect to the ions by a certain Δx. So what is the situation here? We have the ions and the electrons, and the electrons have been displaced with respect to the ions by a certain Δx. What we want to evaluate is the frequency of the resulting oscillations. The electrons have been displaced and therefore they will tend to be attracted back towards the ions. And these, as we will see, will cause an oscillatory motion of the electrons with respect to the background fixed ions. And what we want to evaluate is the frequency of this resulting oscillation. The first thing that we have to do is to evaluate the electric field. We do it by solving Gauss's law. Now, what is the charge that is present in our system? Well, we will have a region where the charge is positive, then a region where the charge is about zero, and then a region where ρ will be negative.

Notes

Summary

0m 40s

Plasma frequency



This will be the first region, second region and third region. What we are mostly interested in is the electric field in region 2. Now, if E is equal to zero at the left end, here, then in region 2, by integrating the Gauss Law we will have that E = n_0 e Δx / ε_0 I have integrated the charge that is present in region 1 because of the displacement of the electrons towards region 3. We can now write the equation of motion for the electrons as we know the electric field that they will be subject to and that the acceleration of the electrons times the mass will be equal to the force that is -e E. This will be equal to minus- with what we have just evaluated n_0 e^2 divided by ε_0, [times] Δx. Which implies that the second derivative with respect of time of displacement depends on a quantity that goes linearly with Δx. This is actually the equation of an oscillator, an oscillatory motion equation with a frequency that is given by this term, here. This is a frequency squared. It's an extremely important quantity in plasma physics that has a name. It's called the *electron plasma frequency* and it's typically denoted with ωpe.

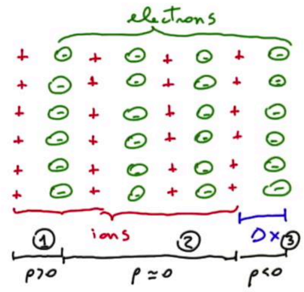
Notes

Summary

3m 09s

Plasma frequency

- A simple scenario
  - → Plasma, slab of electrons with number density n_0
  - → Ions, fixed background, same density
  - → Displace the electrons by Δx
  - → Frequency of the resulting oscillations?



- We evaluate the electric field

$$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}, \quad \text{if } E=0 \text{ at the left end, then in ② } E = \frac{n_0 e \Delta x}{\varepsilon_0}$$

- Equation of motion for the electrons

$$m_e \frac{d^2 \Delta x}{dt^2} = -eE = -\frac{n_0 e^2}{\varepsilon_0}\Delta x \Rightarrow \frac{d^2 \Delta x}{dt^2} + \underbrace{\left(\frac{n_0 e^2}{\varepsilon_0 m_e}\right)}_{=\omega_{pe}^2 \text{ (electron plasma frequency)}} \Delta x = 0$$

- Observations
  - → $\omega_{pe} = \sqrt{\frac{n_0 e^2}{\varepsilon_0 m_e}} = \sqrt{\frac{n_0 e^2}{\varepsilon_0 T_e} \cdot \frac{T_e}{m_e}} = \sqrt{\frac{n_0 e^2}{\varepsilon_0 T_e}} \cdot \sqrt{\frac{T_e}{m_e}} = \frac{v_{th,e}}{\lambda_{De}}$
  - → $f_{pe} = 8980\sqrt{n_0}$ Hz, n_0 is electrons cm^{-3}

Plasma

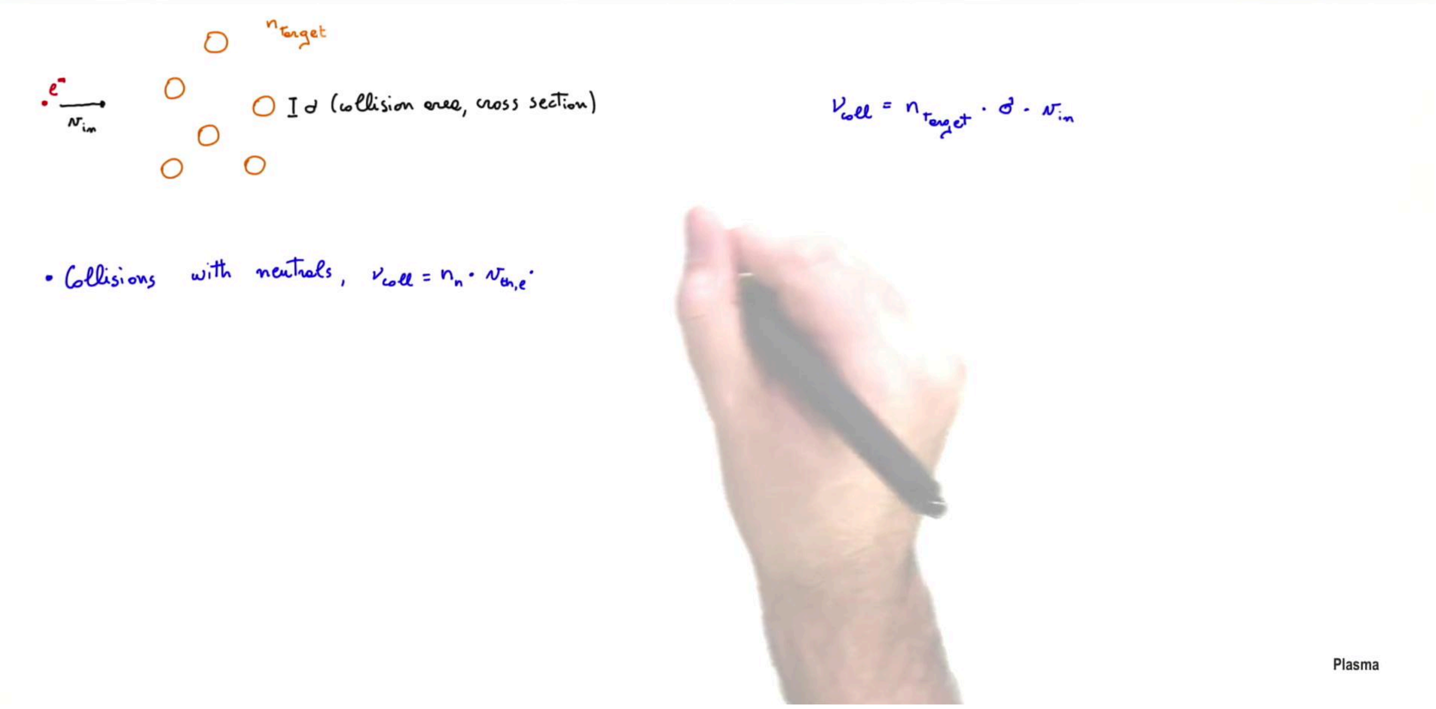
So, a couple of observations; The plasma frequency, which we have just introduced here is the square root of (n_0 e²)/(ε_0 me). It can be written as, --by dividing and multiplying by Te-- as the product of these two functions. And the first one, if you remember, is a quantity that we have just introduced and is exactly equal to one over the Debye length, the electron Debye length. While the second one is what we are used to seeing as v thermal of the electrons (vth). So there is a easy correlation between the plasma frequency and the Debye length that passes through the thermal velocity of the electrons. Second observation; how much is the plasma frequency? Well, in Hertz, the frequency of the plasma oscillation can be written as 8,980, square root of n_0, where, n_0 has to be expressed in particles per cm³.

Notes

Summary

5m 41s

Collision frequency



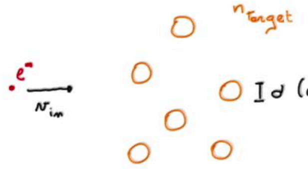
Before going to the details of the collision frequency in plasmas let's try to remind us how collisional processes can be described. Let's imagine that we have a certain density of target particles, we will call it ntarget, and then an incoming particle, for example, an electron that enters, moves towards these target particles with a certain velocity, vin. Now, each part of this particle as seen from the electron, coming in, will have a collision area that we will denote with σ. This is also known as a cross-section. Now, the frequency of collisions will be proportional to the density of the target times the collisional area of each target particle, the cross section, σ times the incoming velocity. The faster the electron goes through this target, the more particles it will effectively see in the unit of time. Therefore, the higher will be the collision frequency. Now, an electron moving throughout a plasma, what kind of collision can it have? Well, it can have collision with neutrals, and the collision frequency associated to the collisions with neutrals [vcoll] will be given by the density of the target particles, - it will be the neutral density [nn] - times the incoming velocity which can be approximated as the thermal electron velocity [vth,e] times the cross section [σ].

Notes

Summary

7m 20s

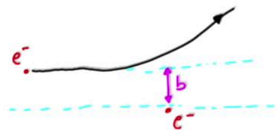
Collision frequency



σ (collision area, cross section)

$$\nu_{coll} = n_{target} \cdot \sigma \cdot v_{in}$$

- Collisions with neutrals, $\nu_{coll} = n_n \cdot v_{th,e} \cdot \pi a_0^2$, $\pi a_0^2 \sim 10^{-20}\ m^2$
- Coulomb collisions



$$\frac{\text{Coulomb interaction energy}}{\text{Kinetic energy}} \sim \frac{\dfrac{e^2}{4\pi\varepsilon_0 b}}{m_e v_{th,e}^2} \sim 1$$

$$\Rightarrow b \sim \frac{e^2}{4\pi\varepsilon_0 m_e v_{th,e}^2} = b_{\pi/2}$$

Plasma

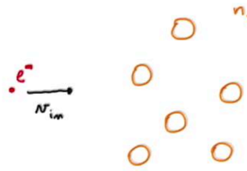
For neutral collisions the cross-section can be very close to the Bohr size of the atom. More precisely, πa_0^2 where a_0, the Bohr radius, is the order of 1×10^{-20} m^2. But the electrons can also do Coulomb collisions, collisions with other charged particles. And for example, an electron can impact against another electron while moving, and its trajectory can be deviated because of the Coulomb force. This interaction, this deviation, becomes important when the Coulomb interaction energy is comparable to the kinetic energy. And how can we estimate the Coulomb interaction energy? Well, we can introduce the impact parameter. Basically the distance of closest approach that we will denote with b. [b = closest approach] if the particle would continue along its unperturbed trajectory, and therefore the Coulomb interaction energy will be given by $e^2 / (4 \pi \varepsilon_0 b)$ and kinetic energy will be of the order of me $(vth,e)^2$ from which we obtain that the deflection the electron impacting against another electron will become important when this ratio is about one, which means when the impact parameter is about $e^2 / (4 \pi \varepsilon_0$ me $vth,e^2)$ And this, for reasons that will become clear later, it's what we tend to call b_π/2, - b 90 degrees- parameter And now we have all the elements to estimate the collision frequency due to Coulomb collisions.

Notes

Summary

9m 23s

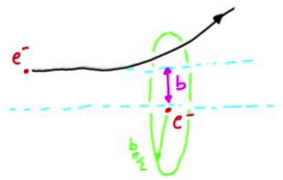
Collision frequency



I σ (collision area, cross section)

$$\nu_{coll} = n_{Target} \cdot \sigma \cdot v_{in}$$

- Collisions with neutrals, $\nu_{coll} = n_n \cdot v_{th,e} \cdot \pi a_0^2$, $\pi a_0^2 \sim 10^{-20}\,m^2$
- Coulomb collisions



$$\frac{\text{Coulomb interaction energy}}{\text{Kinetic energy}} \sim \frac{\frac{e^2}{4\pi \varepsilon_0 b}}{m_e v_{th,e}^2} \sim 1$$

$$\Rightarrow b \sim \frac{e^2}{4\pi \varepsilon_0 m_e v_{th,e}^2} = b_{\pi/2}$$

$$\nu_{coll} = n_e v_{th,e} \pi b_{\pi/2}^2 = \frac{\pi n_0 v_{th,e} e^4}{16\pi^2 \varepsilon_0^2 m_e^2 v_{th,e}^4} = \frac{n_0 e^4}{16 \pi \varepsilon_0^2 m_e^2 v_{th,e}^3}$$

FOCUS

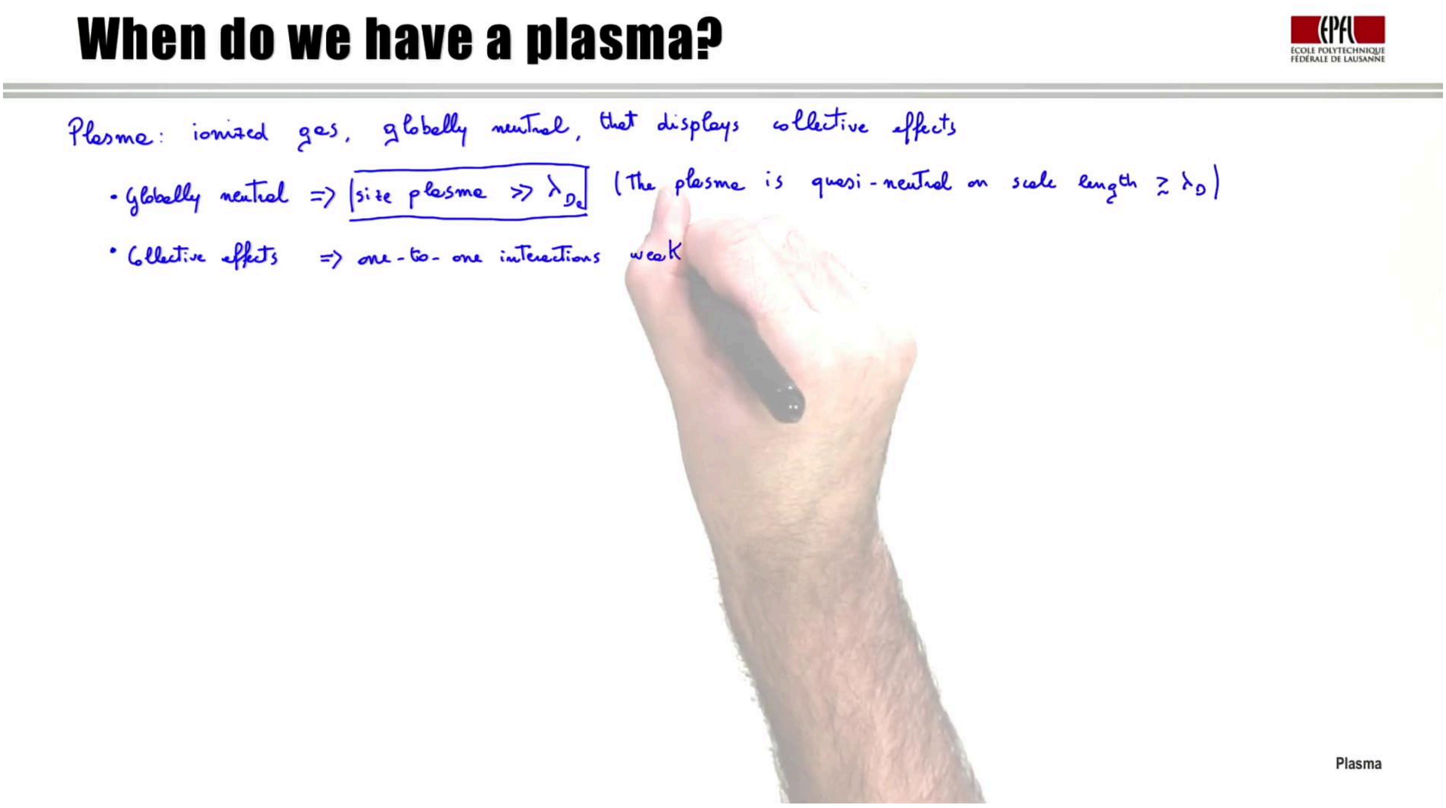
Plasma

It's a very rough estimate. We will look more into the details of the collision frequency, of Coulomb collision frequency later in the course, but basically each particle, each electron can be associated with an area, with a radius b_π/2 and the collision frequency will be given by the electron density [n_0] times the incoming velocity of the particles - which can be assumed to be the thermal velocity [vth,e] - times the cross section that is π b_π/2². And if we plug in the numbers that we have just evaluated here this will be equal to (π n_0 vth,e e⁴) / (16 π² ε_0^2 me² vth,e⁴) which is also equal to (n_0 e⁴) / (16 π ε_0^2 me² vth,e³) Now in the rest of the present lecture we will actually neglect the collisions with neutrals and we will take into account only Coulomb collisions, which are the most interesting collisions you can look at in a plasma.

Notes

Summary

12m 18s



It is now time to put everything together; the three definitions that we have just given, Debye length, plasma frequency, collision frequency, to find out where we actually find plasma in nature, in which parameter regime plasmas can be found. I'll briefly recall the definition that I have given. Plasmas are ionized gases, globally neutral, that display collective effects. The two key concepts, we had said, were *globally neutral* and *collective effects*, so let's review them in view of what we have just introduced. Globally neutral: that means that the size of the plasma has to be much larger than the Debye length. This is, in fact, the Debye length, the scale length at which quasi-neutrality can be violated. And this will also tell us that the plasmas are, not really neutral, but what we typically say are quasi-neutral on scale lengths that are larger or of the order of the Debye length. This is in fact the scale length at which any charge is screened and therefore its effect is not felt farther away. Second concept is the one of collective effects. And this implies two things. On the one hand we need to have that one-to-one interactions are weak.

Notes

Summary

14m 17s

When do we have a plasma?

Plasma: ionized gas, globally neutral, that displays collective effects

- Globally neutral $\Rightarrow$ $\boxed{\text{size plasma} \gg \lambda_{De}}$ (The plasma is quasi-neutral on scale length $\gtrsim \lambda_D$)
- Collective effects $\Rightarrow$ one-to-one interactions weak, $N_D \gg 1$

$$\Rightarrow \frac{\omega_{pe}}{\nu_{coll}} \gg 1 \Rightarrow \sqrt{\frac{n_0 e^2}{\varepsilon_0 m_e}} \; \frac{16\pi \varepsilon_0^2 m_e^2 v_{th,e}^3}{n_0 e^4} = 16\pi \sqrt{\frac{T_e^3 \varepsilon_0^3 n_0^2}{e^6 n_0^3}} = 16\pi \lambda_{De}^3 n_0 = 16\pi \; N_D \gg 1 \quad \Big\} \; N_D \gg$$

Plasma

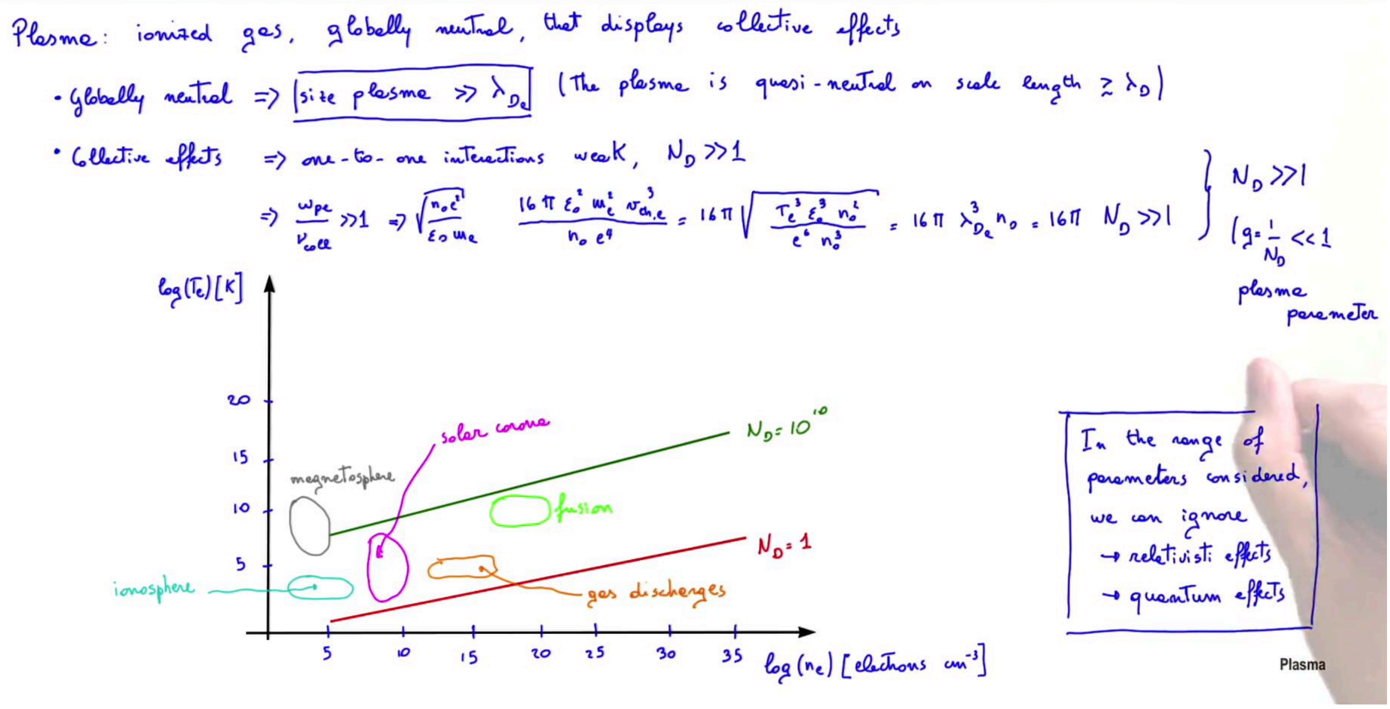
We want the plasma to interact through collective electric and magnetic fields that are generated collectively, as I was saying, and NOT that the interaction pass through collisions due to one-to-one particle interaction. And this, as we have seen occurs when the number of particles in a Debye cube is much larger than one. But it also implies one other thing, that the frequency, collective motion of the plasma, is larger than the frequency of collisional processes. And using the definition that we have just introduced of the plasma frequency and the collision frequency we have that this is given by the plasma frequency times [the inverse of] the collision frequency that we have just derived. And if we put these under the same square root we find that-- which can be written as-- introducing the Debye length, as sixteen pi. The Debye length cubed, this is actually the Debye length cubed-- times n_0. And this is equal to 16 π, this is the number that we have seen already seen many times today, ND and this has to be much bigger than one. In other terms, both these conditions lead us to say that plasma is an ionized gas for which *ND* is much bigger than one.

Notes

Summary

16m 16s

When do we have a plasma?



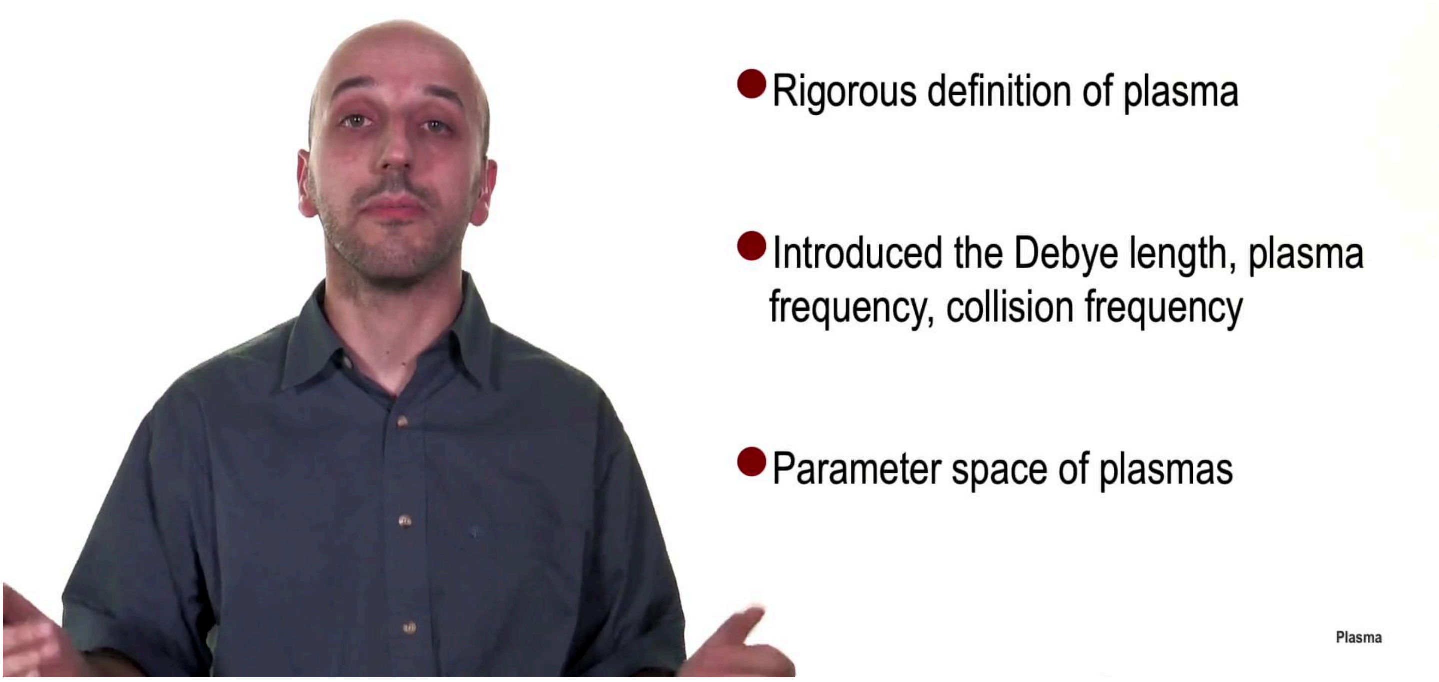
We introduce also another parameter, g, that is defined as one over 1/ND and this has to be much smaller than one and this is the so-called plasma parameter. So, when do we have a plasma? We have a plasma when ND is much bigger than one. Let's see on a plot that represents temperature and density when this actually occurs. We can represent the temperature in Kelvin and the density in electrons times centimeters to the minus three. And we will have-- let me put some scale. The first line that I can draw represents when *ND* is equal to one. The second line I draw is for ND = 1. x 10^{10}. Therefore plasma will lie above this part of the plane, they need to have ND >> 1 and in this part of the plane we can find all the plasma we know in the universe. The solar corona, for example, lies in this region. Ionosphere, plasmas in the ionosphere are located here in the parameter space. Magnetospheric plasmas are here. Plasmas used for fusion application lie in this region. And then, for example, gas discharges, they are here. These are the plasmas that we will actually consider throughout our course. Let me point out one thing that is important for the remainder of the course, it's that in the range of parameters considered we can ignore, first, relativistic effects, second, quantum effects.

Notes

Summary

18m 26s

Summary



The goal of the present lecture was to give a rigorous definition of a plasma. We have said that a plasma-- a plasma is a ionized gas which is globally neutral, and that displays collective effect. What does it mean? Well, to answer this question we have introduced three definitions. The first one is the Debye length, plasma frequency and collision frequency. In working around those we have pointed out that the plasma is globally neutral when its size is larger than the Debye length, on the one hand, and on the other hand, which displays collective effect when there are a number of particles in a Debye cube that is much larger than one. Under this condition, particles in an ionized gas do not interact through one-to-one interaction, through one-to-one collision, but rather through the electric and magnetic field that they have collectively generated. Based on this observation, we have drawn in a parameter space represented by density and temperature, what ND (number of particles in a Debye cube) equal to one line falls. And we have made sure that the plasmas we know in nature fall in a region above this line, and this is actually the case. And we have localized in this parameter space where the plasmas that we will use and we will look throughout this course are.

Notes

Summary

21m 13s