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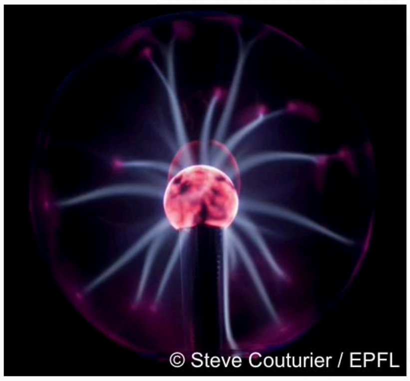
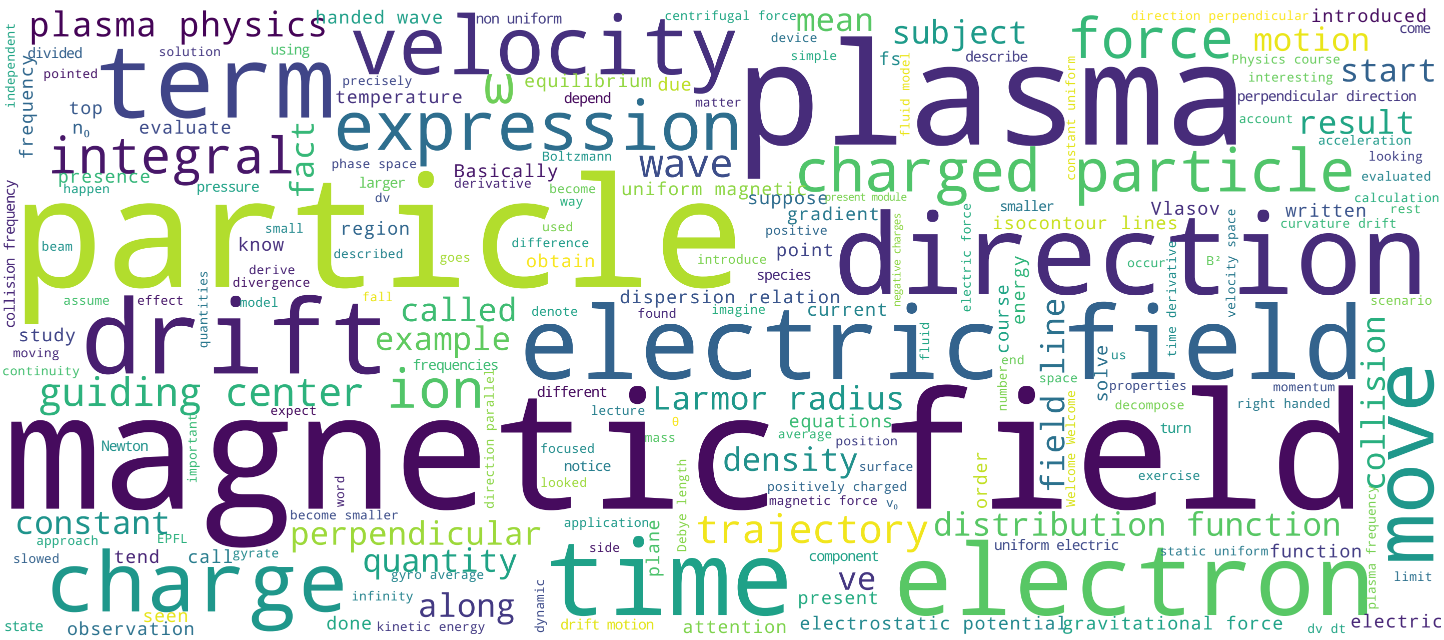


Particle motion in given electromagnetic fields: the drifts

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 1e

Paolo Ricci



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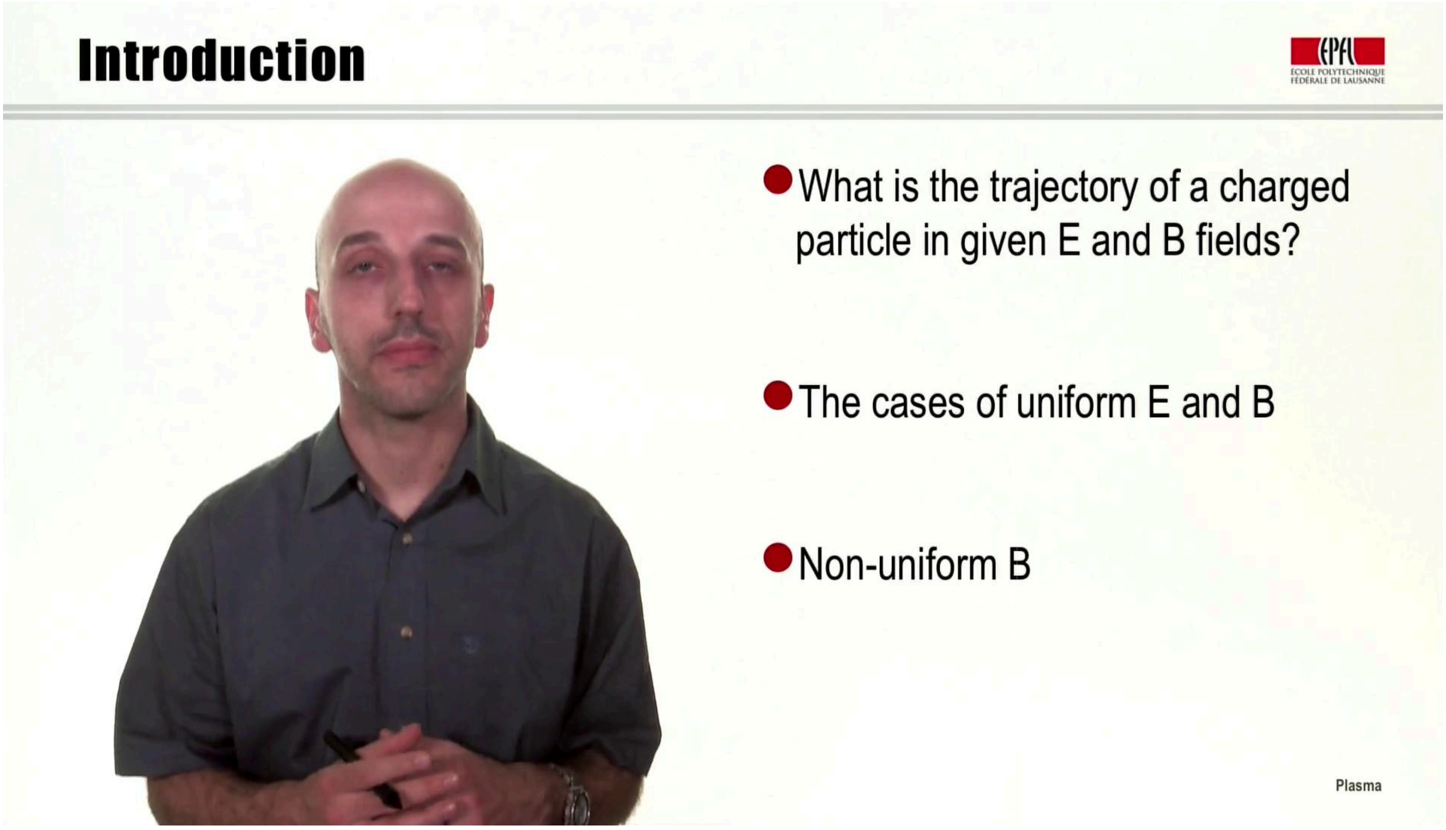


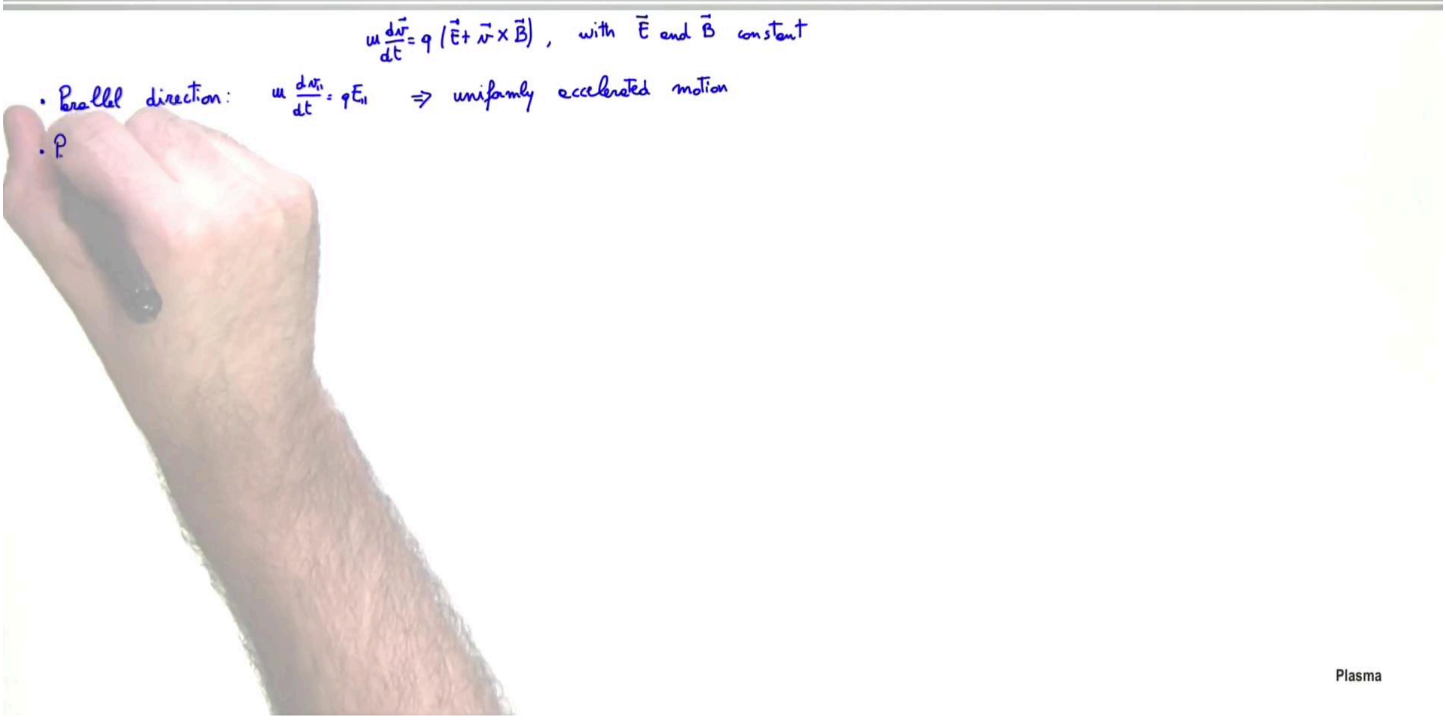
Welcome. Welcome to the Plasma Physics course of the EPFL. What is the trajectory a charged particle in given electric and magnetic fields? In the past module we focused our attention on the simplest possible scenario, that is the one of a constant, uniform magnetic field. We will now turn our attention to more complicated settings. We will first consider the one of a constant electric and magnetic field, and then we will turn our attention to the case of a non-uniform magnetic field.

Notes

Summary

0m 05s

Motion in static and uniform E and B

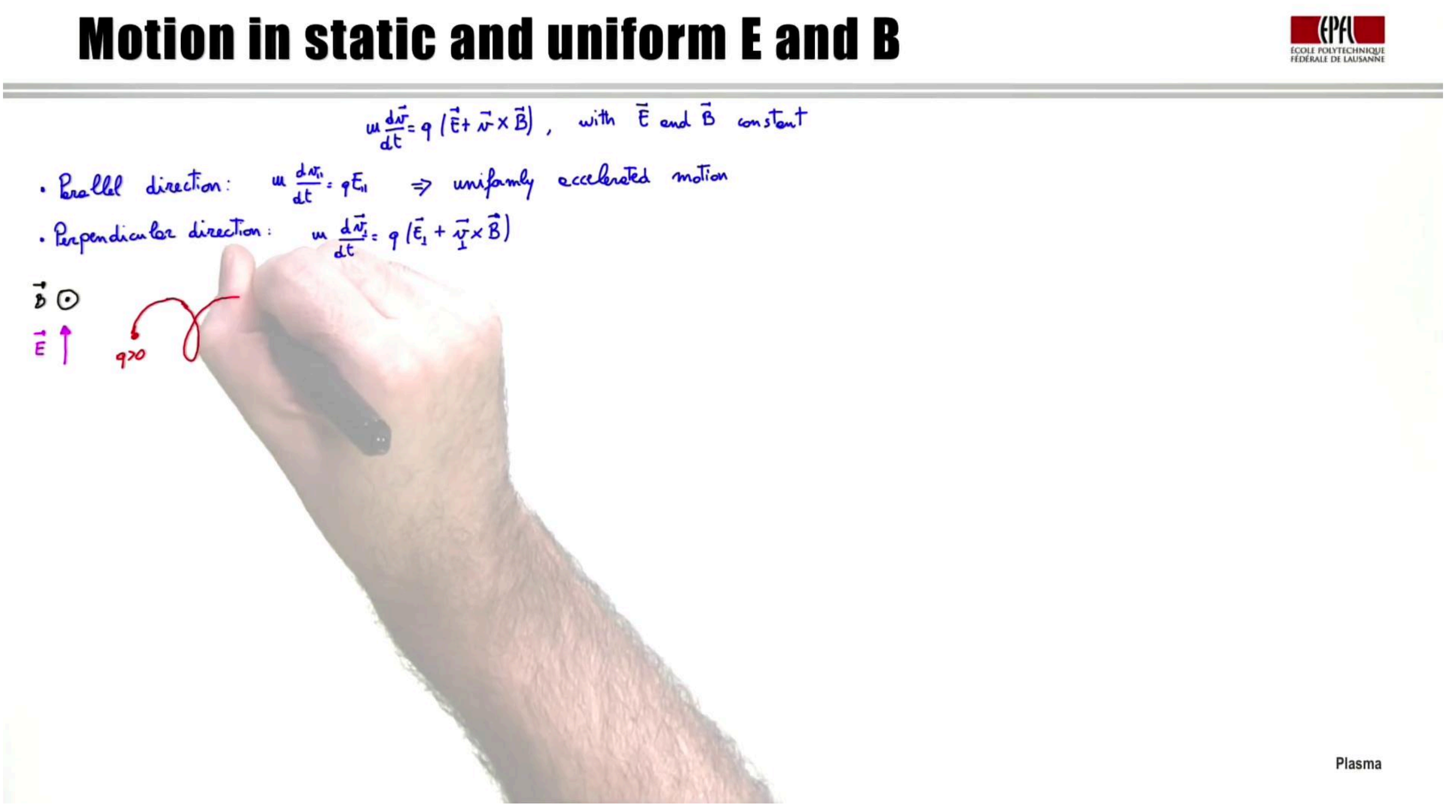


We've studied the trajectory of a charged particle in a static uniform magnetic field. Let's complicate a little bit the scenario. Let's add, on top of the static uniform magnetic field, a static uniform electric field. How will a particle move under the effect of these two forces? This is what we will see now, but let me already anticipate that the result will be quite surprising. So what we want to solve is a Newton equation of a particle with charge q, that is subject to an electric and a magnetic field, with the electric and magnetic fields that are constant. To approach this problem, we will decompose these equations in a direction parallel and perpendicular to the magnetic field. Let's start from the direction parallel to the magnetic field. Along the parallel direction, we will have the mass times the acceleration is equal to the parallel electric force, and of course the (v × B) force will have no component in the parallel direction. What does it mean? Well, this is a very simple equation to be solved, which gives us a uniformly accelerated motion. The perpendicular direction is, of course, more complicated.

Notes

Summary

0m 46s



In this direction, what we will have to solve is mass, times the acceleration in the perpendicular direction, equal to the electric and magnetic force taken in the perpendicular direction. So what is the motion induced by these forces? Well, let's start to consider a simple case. Let's imagine that we have a magnetic field that comes out of this plane, and an electric field that points upwards. So let's place a particle, a positively charged particle, q > 0 and let's study the trajectory of this particle, subject to these electric and magnetic fields. Okay, the particle is at rest. The first thing that it will do is that it will be accelerated by its electric field, upwards. Therefore the particle will start to move upwards. At the same time, it will be subject to the magnetic field, and therefore its trajectory will be curved. And then the particle will start to move down, and while moving downwards, the electric field will slow it down. When its electric field slows it down, then the velocity decreases, and when the velocity decreases, the Larmor radius of the particle decreases. Therefore the particle subject to the magnetic force will move and will turn this way, and then will be accelerated again in the direction of the electric field.

Notes

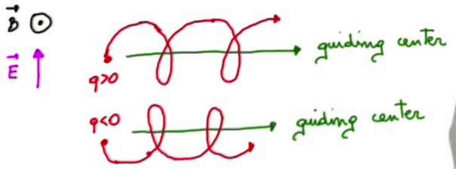
Summary

2m 14s

Motion in static and uniform E and B

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right), \text{ with } \vec{E} \text{ and } \vec{B} \text{ constant}$$

- Parallel direction: $m\frac{dv_{\parallel}}{dt} = qE_{\parallel} \Rightarrow$ uniformly accelerated motion
- Perpendicular direction: $m\frac{d\vec{v}_{\perp}}{dt} = q\left(\vec{E}_{\perp} + \vec{v}_{\perp}\times\vec{B}\right)$



Plasma

Again, larger Larmor radius, then the particle will move downwards, at the same time will be slowed down, and the Larmor radius will become smaller and then the particles will come up... This will be the trajectory. If you want, the guiding center of this particle will move in this direction, in a direction that is perpendicular to both the electric field, and the magnetic field. Let's take the case of negatively charged particles, that is initially at rest. Well, the particle will be accelerated by the electric field, this time downwards, because the particle is charged negatively. And then the particle will be subject to the magnetic field. It will gyrate with a certain Larmor radius, and then the particle because of the gyration will continue to move it in this direction, but while moving in this direction, the particle will be slowed down, and therefore its Larmor radius will become smaller, and then the particle will continue to move. Will be slowed down, etc. The result is that the particle's guiding center will still move in the same direction as the positively charged particle. So what do we expect from this image? Basically that there will be a periodic motion on top of which a secular drift is present.

Notes

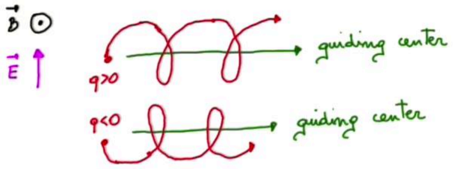
Summary

4m 01s

Motion in static and uniform E and B

$$m\frac{d\vec{v}}{dt} = q\,(\vec{E} + \vec{v}\times\vec{B})\,,\quad \text{with } \vec{E} \text{ and } \vec{B} \text{ constant}$$

- Parallel direction: $m\frac{dv_\parallel}{dt} = qE_\parallel \Rightarrow$ uniformly accelerated motion
- Perpendicular direction: $m\frac{d\vec{v}_\perp}{dt} = q\,(\vec{E}_\perp + \vec{v}_\perp\times\vec{B})$



gyroaveraging over one gyroperiod

$$\left\langle m\frac{d\vec{v}_\perp}{dt}\right\rangle = 0 = q\,\langle \vec{E}_\perp + \vec{v}_\perp\times\vec{B}\rangle \Rightarrow \langle\vec{v}_\perp\rangle\times\vec{B} = -\vec{E}_\perp$$

$$\langle\vec{v}_\perp\rangle\times\vec{B}\times\vec{B} = -\langle\vec{v}_\perp\rangle B^2 = -\vec{E}_\perp\times\vec{B} \Rightarrow \langle\vec{v}_\perp\rangle = \vec{v}_E = \frac{\vec{E}_\perp\times\vec{B}}{B^2}$$

Plasma

So let's try first to understand what is this drift, by averaging the trajectory over one gyro period. So we have to take this equation here, for perpendicular velocity, and average over one gyroperiod. The averaging I will denote with these brackets, and therefore what we will have to evaluate is $< m\, dv_\perp/dt >$. The average over a gyroperiod will be equal to zero, and the gyroaveraging, the right-hand side of this equation, we will have that this has to be equal to the gyro average of $E_\perp$, plus the gyro average $<v_\perp \times B>$. Now, as $E_\perp$ and B are constant quantities, we can write this equality as the gyro average $< v_\perp> \times B$, equal to minus $E_\perp$. And now we can extract the value of $< v_\perp>$ by crossing this expression with B. In fact, we will have this... which using standard vector formula is equal to - $< v_\perp> B^2$, and this has to be equal to - ($E_\perp \times B$), from which we can obtain immediately the gyro averaged value $< v_\perp>$, a quantity that we will call ve, and is equal to ve = $(E_\perp \times B)/B^2$. Now this will actually be the velocity of the guiding center that if you notice has, in the picture we have drawn, it's perpendicular to E, and to B, and does not depend on the charge, both positive and negative charges move in the same direction.

Notes

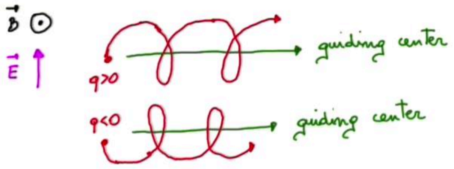
Summary

5m 44s

Motion in static and uniform E and B

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right), \quad \text{with } \vec{E} \text{ and } \vec{B} \text{ constant}$$

- Parallel direction: $m\frac{dv_\parallel}{dt} = qE_\parallel \Rightarrow$ uniformly accelerated motion
- Perpendicular direction: $m\frac{d\vec{v}_\perp}{dt} = q\left(\vec{E}_\perp + \vec{v}_\perp\times\vec{B}\right)$



gyroaveraging over one gyroperiod

$$\left\langle m\frac{d\vec{v}_\perp}{dt}\right\rangle = 0 = q\left\langle \vec{E}_\perp + \vec{v}_\perp\times\vec{B}\right\rangle \Rightarrow \langle\vec{v}_\perp\rangle\times\vec{B} = -\vec{E}_\perp$$

$$\langle\vec{v}_\perp\rangle\times\vec{B}\times\vec{B} = -\langle\vec{v}_\perp\rangle B^2 = -\vec{E}_\perp\times\vec{B} \Rightarrow \langle\vec{v}_\perp\rangle = \vec{v}_E = \frac{\vec{E}_\perp\times\vec{B}}{B^2}$$

We write $\vec{v}_\perp = \vec{v}_E + \vec{v}_\perp'$ $\Rightarrow$

$$m\frac{d(\vec{v}_E+\vec{v}_\perp')}{dt} = m\frac{d\vec{v}_\perp'}{dt} = q\vec{E}_\perp + q\vec{v}_E\times\vec{B} + q\vec{v}_\perp'\times\vec{B} = q\vec{E}_\perp + \underbrace{q\frac{\vec{E}_\perp\times\vec{B}}{B^2}\times\vec{B}}_{-q\vec{E}_\perp} + q\vec{v}_\perp'\times\vec{B} = q\vec{v}_\perp'\times\vec{B}$$

Plasma

Okay, so this is the averaged motion, how about the real motion? I mean the full motion, the difference between the, -if you want-, the actual motion of the particles and the drift. We can decompose v⊥ as the sum of the ve velocity, plus the 'perturbed' quantity, if you want to call it this way, that will give this oscillation that we see here. We will write therefore that v⊥ will be equal to ve plus the difference between v⊥ and ve, that is, a quantity that we will call v⊥'. [-> v⊥ = ve + v⊥'] Okay, what is the equation that v⊥' is subject to? Well, we can rewrite the Newton equation that we have written here, in terms of ve and v⊥', and this will be equal to m... the derivative of the perpendicular velocity that we have expressed in terms of these two quantities, with respect to time. And ve is a constant quantity, therefore this will be equal to m dv⊥'/dt, and this will be given by the forces that we have written here. Well, we have to decompose v⊥ as ve + v⊥'. And this is equal to q E⊥ +... - we can explicitely write the value of ve-, and then let's rewrite q v⊥' × B. Well, if you notice, this quantity is actually given by -q E⊥, and therefore will cancel out with this q E⊥, and what we will obtain is simply q v⊥' × B.

Notes

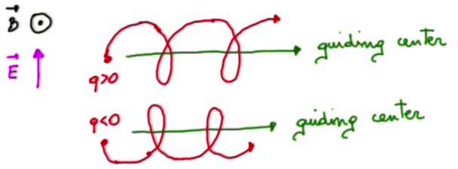
Summary

8m 18s

Motion in static and uniform E and B

$$m\frac{d\vec v}{dt} = q(\vec E + \vec v\times\vec B)\ ,\quad \text{with } \vec E \text{ and } \vec B \text{ constant}$$

- Parallel direction: $m\frac{dv_\parallel}{dt} = qE_\parallel \Rightarrow$ uniformly accelerated motion
- Perpendicular direction: $m\frac{d\vec v_\perp}{dt} = q(\vec E_\perp + \vec v_\perp\times\vec B)$



gyroaveraging over one gyroperiod

$$\langle m\frac{d\vec v_\perp}{dt}\rangle = 0 = q\langle\vec E_\perp + \vec v_\perp\times\vec B\rangle \Rightarrow \langle\vec v_\perp\rangle\times\vec B = -\vec E_\perp$$

$$\langle\vec v_\perp\rangle\times\vec B\times\vec B = -\langle\vec v_\perp\rangle B^2 = -\vec E_\perp\times\vec B \Rightarrow \langle\vec v_\perp\rangle = \vec v_E = \frac{\vec E_\perp\times\vec B}{B^2}$$

We write $\vec v_\perp = \vec v_E + \vec v_\perp' \Rightarrow m\frac{d(\vec v_E+\vec v_\perp')}{dt} = m\frac{d\vec v_\perp'}{dt} = q\vec E_\perp + q\vec v_E\times\vec B + q\vec v_\perp'\times\vec B = q\vec E_\perp + \underbrace{q\frac{\vec E_\perp\times\vec B}{B^2}\times\vec B}_{-q\vec E_\perp} + q\vec v_\perp'\times\vec B = q\vec v_\perp'\times\vec B$

Conclusions

- Motion given by the superposition of gyromotion and drift motion of the guiding center, $\boxed{\vec v_E = \frac{\vec E_\perp\times\vec B}{B^2}}$ $\vec E\times\vec B$ drift
- $\vec E\times\vec B$ drift is independent of charge
- Guiding center motion along isocontour lines of

Plasma

So now look at the equation that we have obtained by comparing this term here, and this term here. This is exactly the equation that we have seen before that gives a circular motion. So what are the conclusions of our exercise? Well, that the motion, of a charged particle in a static and uniform electric field is given by the superposition of a gyromotion and a drift motion of the guiding center. The drift motion is the ve term that we have introduced here, and it's an important quantity in plasma physics that is called the *E × B drift*. The second observation that we want to do in this still relatively simple case is that the E × B drift is independent of charge. Here in this expression there is no charge that appears. This is why the guiding centers, both for the positive and negative particles are moving in the same direction. By extending the observation that we have just done, to the case of non-uniform electric field, we can observe that the particle is expected to move in a perpendicular direction to it, and therefore along the isocontour lines of the electrostatic potential. In other terms, the gyro center's motion is along the isocontour lines of the electrostatic potential.

Notes

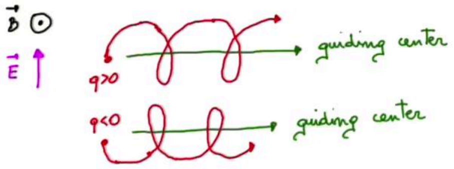
Summary

10m 52s

Motion in static and uniform E and B

$$m\frac{d\vec v}{dt} = q\left(\vec E + \vec v\times\vec B\right), \quad \text{with } \vec E \text{ and } \vec B \text{ constant}$$

- Parallel direction: $m\frac{dv_\parallel}{dt} = qE_\parallel \Rightarrow$ uniformly accelerated motion
- Perpendicular direction: $m\frac{d\vec v_\perp}{dt} = q\left(\vec E_\perp + \vec v_\perp\times\vec B\right)$



gyroaveraging over one gyroperiod

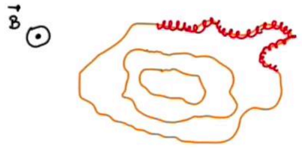
$$\left\langle m\frac{d\vec v_\perp}{dt}\right\rangle = 0 = q\left\langle \vec E_\perp + \vec v_\perp\times\vec B\right\rangle \Rightarrow \langle\vec v_\perp\rangle\times\vec B = -\vec E_\perp$$

$$\langle\vec v_\perp\rangle\times\vec B\times\vec B = -\langle\vec v_\perp\rangle B^2 = -\vec E_\perp\times\vec B \Rightarrow \langle\vec v_\perp\rangle = \vec v_E = \frac{\vec E_\perp\times\vec B}{B^2}$$

We write $\vec v_\perp = \vec v_E + \vec v_\perp' \Rightarrow m\frac{d(\vec v_E+\vec v_\perp')}{dt} = m\frac{d\vec v_\perp'}{dt} = q\vec E_\perp + q\vec v_E\times\vec B + q\vec v_\perp'\times\vec B = q\vec E_\perp + \underbrace{q\frac{\vec E_\perp\times\vec B}{B^2}\times\vec B}_{-q\vec E_\perp} + q\vec v_\perp'\times\vec B = q\vec v_\perp'\times\vec B$

Conclusions

- Motion given by the superposition of gyromotion and drift motion of the guiding center, $\boxed{\vec v_E = \frac{\vec E_\perp\times\vec B}{B^2}}$ $\vec E\times\vec B$ drift
- $\vec E\times\vec B$ drift is independent of charge
- Guiding center motion along isocontour lines of the electrostatic potential



Plasma

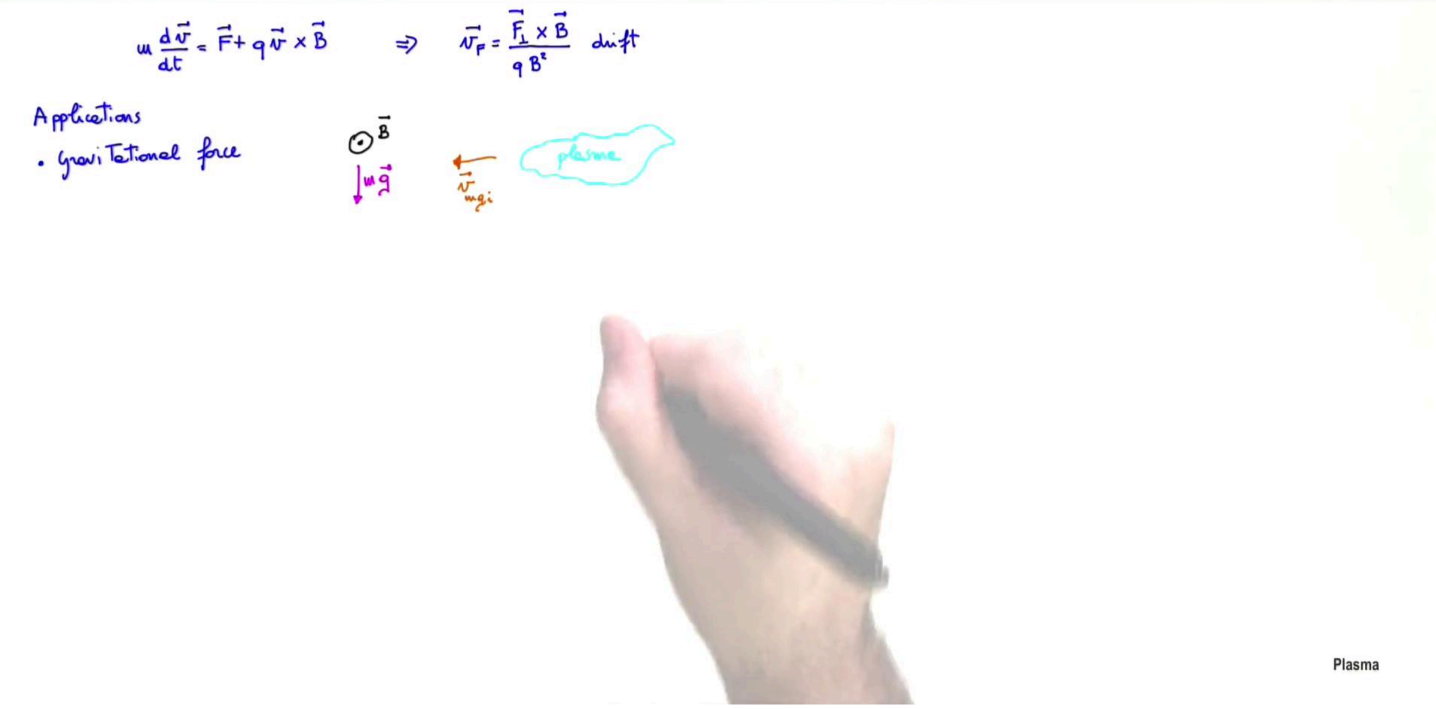
Let's in fact imagine we have a magnetic field pointing out of the page, and let's imagine we have an electrostatic potential on the plane perpendicular to B, with some isocontour lines. Now as the E × B velocity is in the direction perpendicular to E, and that E is perpendicular to the isocontour lines of the electrostatic potential, we will have that the particle, the charged particle, will move around, following the isocontour lines of the electrostatic potential.

Notes

Summary

12m 51s

The case of an arbitrary force and curvature drift



The drift that we have just seen can be easily generalized to whatever force, uniform constant force, that acts on the charged particles. Basically, if we have to solve the Newton equation with a constant force, on top of the Lorentz force, (the v × B force), well if we solve this equation exactly following what we have done earlier with the electric field, we will find that the particle moves with the gyrocenter, following, moving with the drift that is given by what we will call vf, and is given by the perpendicular component of the force [F⊥]... -perpendicular to the magnetic field,-... × B, divided by qB². So let's see a couple of applications of drifts, of these drifts that we have introduced here. The first one is the case of the gravitational force. Let's suppose we have a magnetic field pointing out of the plane, and then a gravitational force in this direction. Will the plasma fall? So, more precisely, let's suppose that we have a plasma here, and we are curious to know if, subject to the gravitational force, the plasma will fall. Well, this is not actually the case, because the particles will follow a drift, will move according to a drift that for the ions goes in -making the cross product between g and B,- goes in this direction, so this will be the drift of the ions.

Notes

Summary

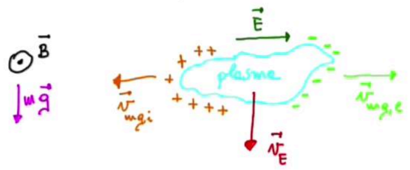
13m 34s

The case of an arbitrary force and curvature drift

$$m\frac{d\vec{v}}{dt} = \vec{F} + q\vec{v}\times\vec{B} \quad \Rightarrow \quad \vec{v}_F = \frac{\vec{F}_\perp \times \vec{B}}{qB^2} \text{ drift}$$

Applications

- gravitational force



- Curvature drift

Plasma

So the electrons will move in this direction. Let's not forget that there is a charge dependence on the expression of the drift associated with the gravitational force. This is different than what we have for the electric force. And therefore the electrons will move in this direction. So just because of the presence of the gravitational force, the plasma will not fall. However, if we are careful, what will happen because of the presence of these two drifts? Well, we will have that a sort of positive charge will accumulate on this side of the plasma, while negative charges will accumulate on his side of the plasma. And what will be the result of the accumulation of these charges? It will be an electric field, that points in this direction. And when we have, here, an electric field, a magnetic field, what is the result? it's an E × B drift, that is in this direction. The E × B drift is independent of the charge of the particles, therefore, at the end, the plasma will fall down, but through a much more complicated process that one could think of by looking just at the resulting effect of the gravitational force. The second application that I would like to discuss with you is the *curvature drift*.

Notes

Summary

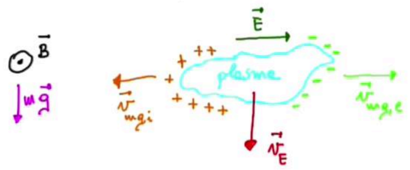
15m 44s

The case of an arbitrary force and curvature drift

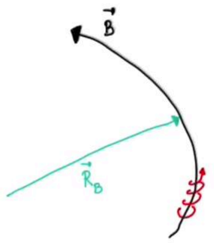
$$m\frac{d\vec{v}}{dt} = \vec{F} + q\vec{v}\times\vec{B} \quad \Rightarrow \quad \vec{v}_F = \frac{\vec{F}_\perp \times \vec{B}}{qB^2} \text{ drift}$$

Applications

• Gravitational force



• Curvature drift



Particle moving in curved $\vec{B}$ tries to follow the field line, as motion across $\vec{B}$ is resisted. It feels a centrifugal force

$$\vec{F}_c = \frac{m v_\parallel^2}{R_B^2}\vec{R}_B$$

and therefore a drift

$$\boxed{\vec{v}_d = \frac{\vec{F}_c \times \vec{B}}{qB^2} = \frac{m v_\parallel^2}{qB^2R_B^2}\left(\vec{R}_B \times \vec{B}\right)}$$ curvature drift

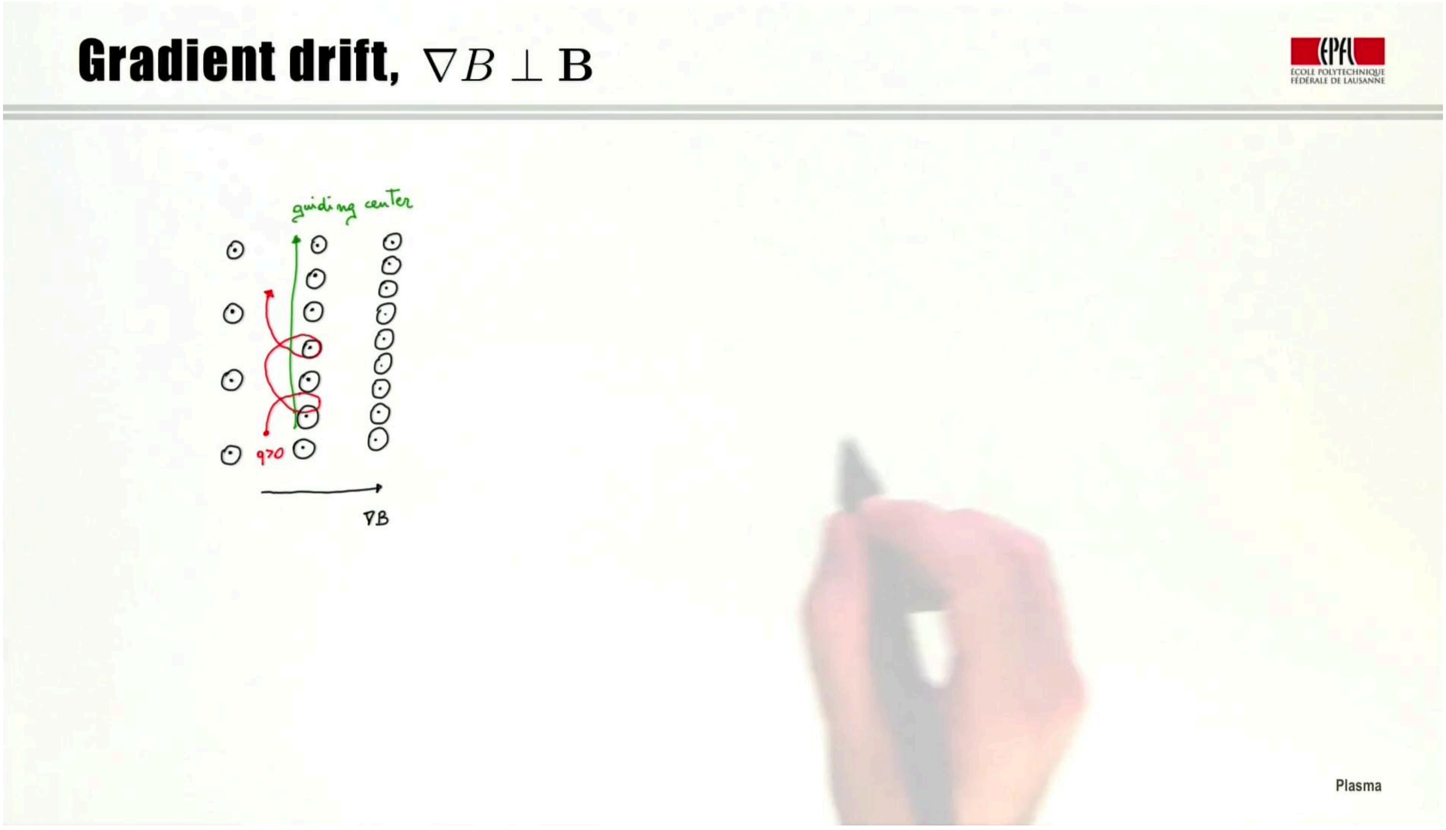
Plasma

Let's imagine we have magnetic field lines with some curavature. And we call this curvature, the curvature vector, RB. Now, a particle will tend to move along the magnetic field lines, and as the motion in the direction, perpendicular, across B is sort of resisted. Therefore, while moving along the field line, the particle will feel a centrifugal force that is the classic expression that we know. And therefore now, in the presence of a force and a magnetic field, we will have a drift that we can evaluate using the expression, and will be given by, we'll call it vd, curvature drift, which is the centrifugal force × B, and using the expression for centrifugal force, this will be given by the following expression. This, as I was saying, is what in plasma physics is called the curvature drift, and it's also an important quantity, in plasma physics.

Notes

Summary

17m 35s

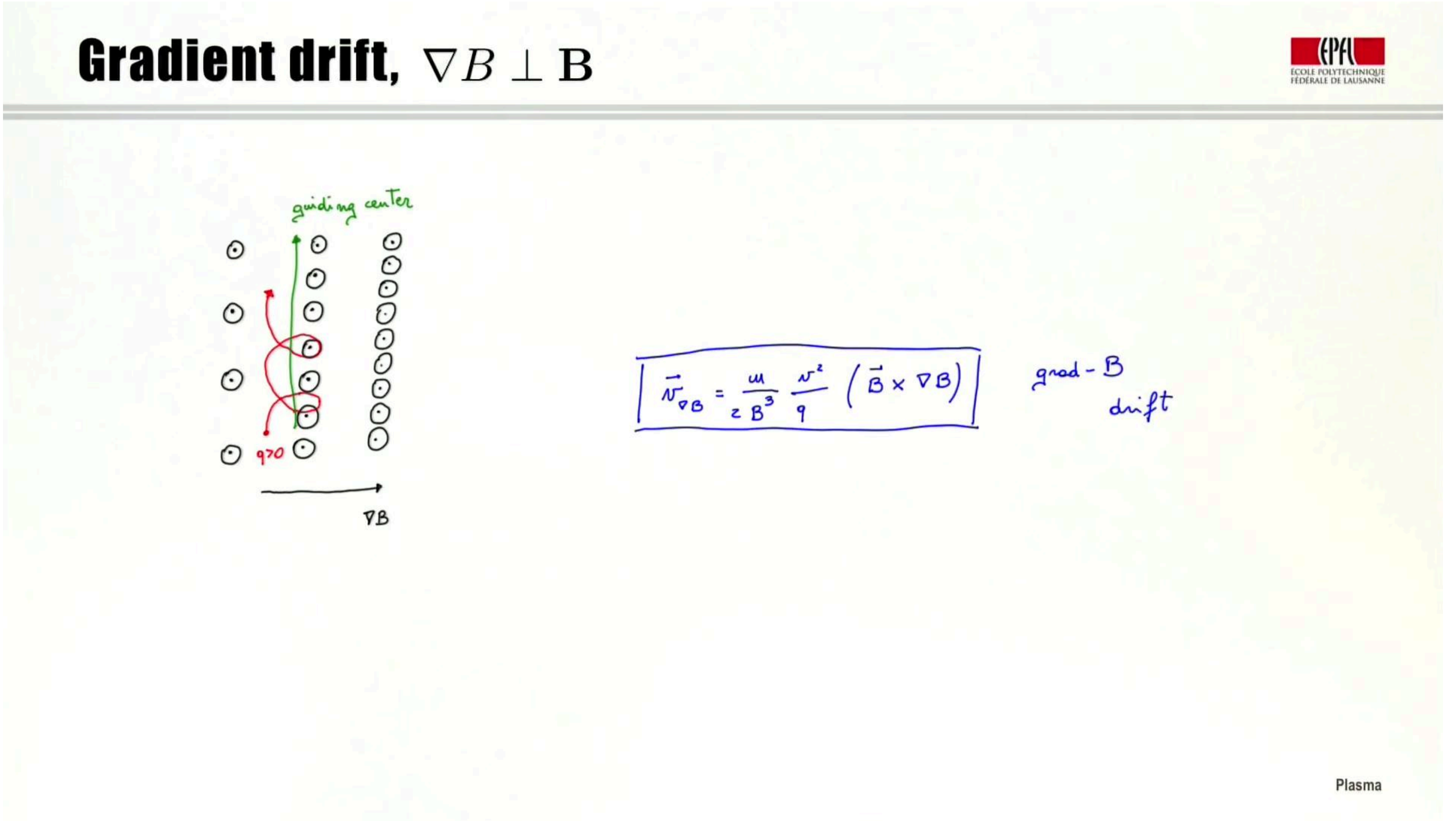


Let's now consider the case of a non-uniform magnetic field, one in which there is a gradient of the magnetic field perpendicular to the magnetic field itself. The scenario is therefore the following one, we have a magnetic field that points out of the plane, and which is subject to a gradient, in this direction, perpendicular to B therefore. So let's suppose that we have a charged particle. Let's suppose that is positively charged. Now this particle may move, and because of the v × B force will tend to gyrate, and while it starts to gyrate, it will experience regions where the magnetic field may become stronger, and when the magnetic field may become stronger, the Larmor radius will become smaller. And then when the particle goes back it will experience regions where the magnetic field is weaker, and therefore the Larmor radius is larger, and then again regions where the magnetic field is stronger, and therefore the Larmor radius smaller, etc. What is the result of such a motion? It is again, the one that can be described by the motion of the guiding center of the particle in this direction. Now, what is the velocity of the drift of this guiding center?

Notes

Summary

19m 02s



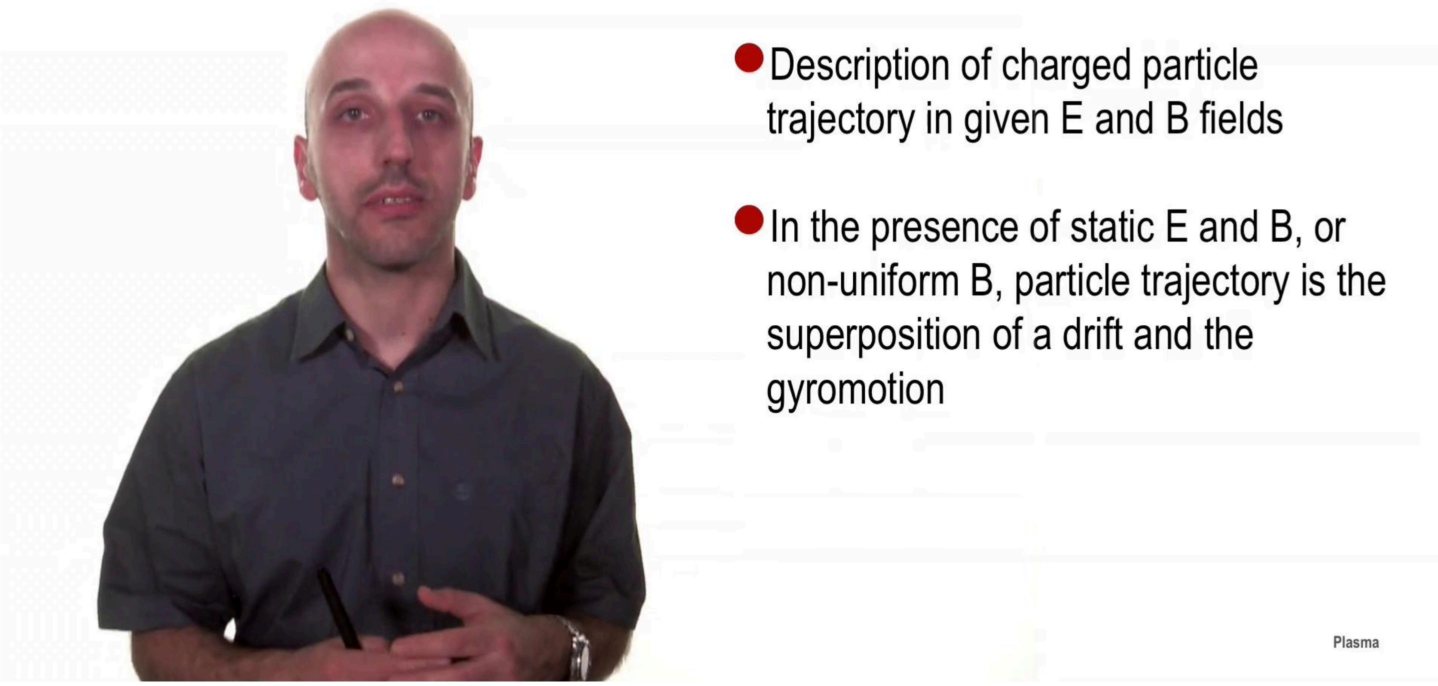
Well, the calculation is fairly complicated, and you can find it in an appendix to the present lecture. Here I will just give the result, it is what you actually will need in the future. That is the v due to the gradient of the magnetic field is equal to the mass of the particle divided by $2B^3$ v^2 of the particles, divided by the charge times the magnetic field cross the gradient of the magnetic field. As a matter of fact, as we see here, the guiding center with the drift, on a direction that is perpendicular to both ∇B and B. This is what is called the *grad-B drift*.

Notes

Summary

20m 49s

Summary



In the present module, we have focused our attention on the trajectory of a particle in a given electric and magnetic field. In the past lecture we had focused our attention on a quite simple case, the one of a uniform and constant magnetic field. Here we have considered slightly more complicated cases. First of all, we have looked at what happens when there is a constant uniform electric and magnetic field, and then we have looked at what is the particle trajectory when the magnetic field is non-uniform. We have pointed out that in all cases the particle trajectory can be described as the superposition of a guiding center drift motion plus a gyromotion, and we have given the expression of this drift motion in all the relevant cases that we have analyzed.

Notes

Summary

21m 51s