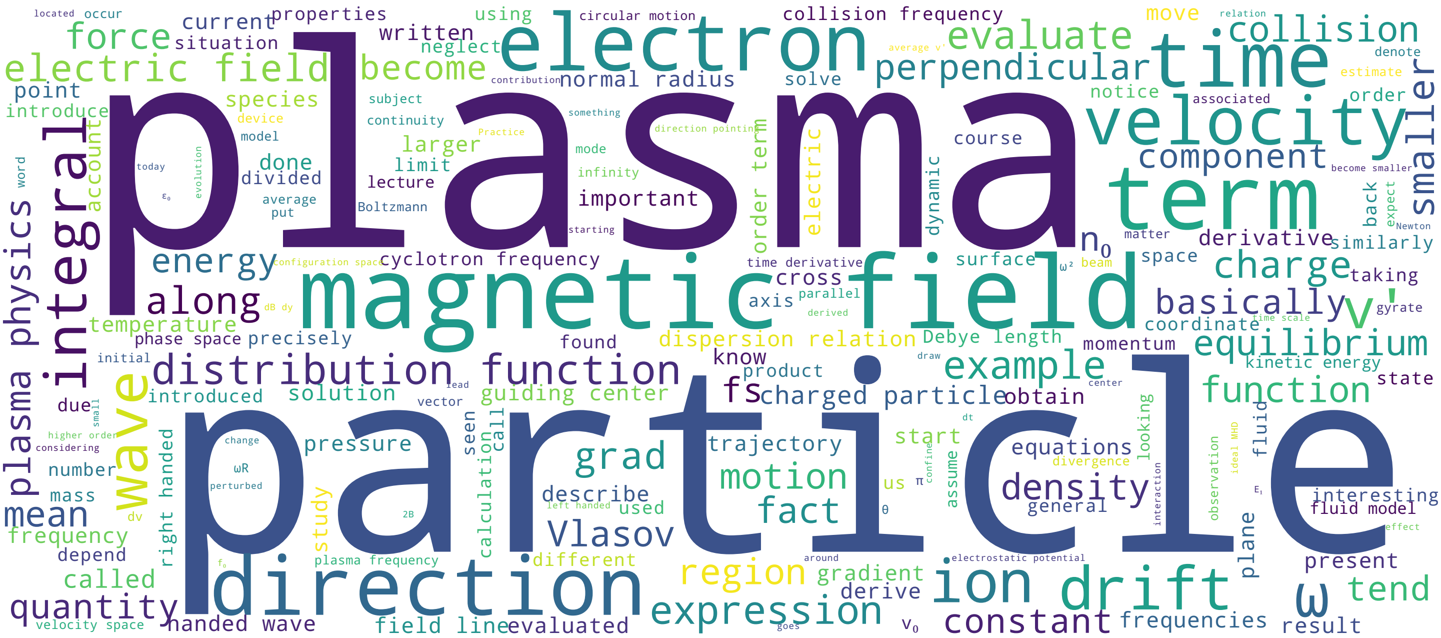
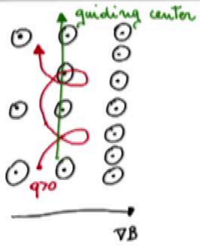


Search MOOC

Video

Gradient drift, $\nabla B \perp \mathbf{B}$



Plasma

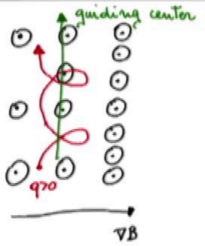
In this short appendix I will give you all the details necessary to derive the grad B drift. So the situation is the one of a magnetic field, The count goes out from the plane. And with a gradient in this direction. Perpendicular to B, it means that the fill lines will become more compressed by going this direction. What is the trajectory of a particle moving in this magnetic field? Let's consider a positively charged particle. Because of the vigorous B motion, the particle will tend to gyrate. Now, while it gyrates, it will find regions where the magnetic field is stronger, here. And with a stronger magnetic field, the normal radius will become smaller -- so, the normal radius, curvature radius of the trajectory will become smaller. The particle then will go back into a region where the magnetic field is weaker, and therefore the normal radius is larger. And then we'll go back into regions where the normal radius is smaller, we'll turn around and come back, etc. Practice what we will have also, in this case, well, we will have motion of a particle that is rather complicated but with the degrading center that moves upwards in the direction that is perpendicular to both grad B and B.

Notes

Summary

0m 00s

Gradient drift, $\nabla B \perp \mathbf{B}$



We consider the case $\rho \ll \frac{B_0}{|\nabla B|_0}$ ($\vec{B}_0$: $\vec{B}$ at particle guiding center)

$$\Rightarrow \vec{B} = \vec{B}_0 + \underbrace{(\vec{r}\cdot\nabla)\vec{B}\big|_0}_{\sim \rho\nabla B < B_0} + \dots$$

We want to solve $m\frac{d\vec{v}}{dt} = q(\vec{v}\times\vec{B}) = q\left\{\vec{v}\times\left[\vec{B}_0 + (\vec{r}\cdot\nabla)\vec{B}\big|_0\right]\right\}$

We decompose $\vec{v} = \vec{v}_0 + \vec{v}'$, $m\frac{d\vec{v}_0}{dt} = q\,\vec{v}_0\times\vec{B}_0$, $v' \ll v_0$

$\Rightarrow m\frac{d\vec{v}'}{}$

Plasma

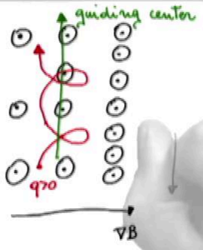
Okay, let's try to study this analytically. We will consider the limit of normal radius that is much smaller than the typical scale over which the magnetic field varies. Where with zero, for example, B(zero), I will denote the value of the magnetic field or of its gradient at the particle guiding center. What is the advantage of doing that? If we want to know what is the magnetic field acting on the particle, we can expand around the guiding center. And if we estimate this quantity here, this will be of the order of ρ, grad B, that is much smaller than B(zero), and therefore we will be able to neglect all the other terms of higher order. Therefore, what we want to solve is basically the Newton equation with the magnetic field that we can expand as here. Now, if we look at this equation, we notice that *this* term, because of this ordering, this first term will be much larger than the second one. Therefore what we do, we decompose the velocity as the sum of a velocity that satisfied medium order term of this equation plus a perterbation. Well, v(zero) is the leading order term to one that satisfied the first order term here of this equation. And also, v' will take into account this small correction.

Notes

Summary

1m 47s

Gradient drift, $\nabla B \perp \mathbf{B}$



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$$\Rightarrow m\frac{d\vec{v}'}{dt} = q\left[\vec{v}'\times\vec{B}_0 + \vec{v}_0\times(\vec{r}\cdot\nabla)\vec{B}\big|_0\right]$$ where we neglect $\vec{v}'\times(\vec{r}\cdot\nabla)\vec{B}\big|_0$

Averaging over a gyroperiod $0 = \langle\vec{v}'\times\vec{B}_0\rangle + \langle\vec{v}_0\times(\vec{r}\cdot\nabla)\vec{B}\big|_0\rangle \Rightarrow \langle\vec{v}'\rangle\times\vec{B}_0\times\vec{B}_0 = -\langle\vec{v}_0\times(\vec{r}\cdot\nabla)\vec{B}\big|_0\rangle\times\vec{B}_0$

As $\vec{v}'\perp\vec{B}_0 \Rightarrow \langle\vec{v}'\rangle = \frac{1}{B_0^2}\langle\vec{v}_0\times(\vec{r}\cdot\nabla)\vec{B}\big|_0\rangle\times\vec{B}_0$

Plasma

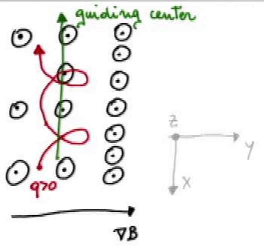
We expect that that v' will be much smaller than v(zero). Therefore v' will satisfy the equation where we can neglect the higher order term as this term is given by the product of two small quantities, v' and grad B. all grad B. Now, what we are interested in here is in solving this equation is getting the average v', the v' that actually represents motion -- steady state motion of the guiding center, the drift. So now averaging over one gyroperiod, this equation here, the first term will give zero, the second one will lead to- well, it's all proportional to the charge so we can drop the charge, we will have v' + B(zero) + the gyroaverage of the second term and we can actually solve this equation by crossing it with B(zero) in order to get v'. Now, as v' is perpendicular to B(zero) in this direction -- we have the average of v', this double-cross product can be solved as we've done for the case of B(cross) B(drift), and as I was saying, the average of v' would be equal to 1/B(zero)^2 -- B(zero) cross r.grad B(zero) crossed -- still crossed with B(zero). Now, to make things a bit easier, let's fix some axis.

Notes

Summary

3m 51s

Gradient drift, $\nabla B \perp \mathbf{B}$



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As $\vec{v}'\perp\vec{B}_0 \Rightarrow \langle\vec{v}'\rangle = \frac{1}{B_0^2}\langle\vec{v}_0\times(\vec{r}\cdot\nabla)\vec{B}|_0\rangle\times\vec{B}_0$

If $\vec{B}_0 = B_0\vec{e}_z$ and $(r\cdot\nabla)B|_0 = (y-y_0)\frac{\partial B}{\partial y}\vec{e}_z$, $\langle\vec{v}_0\times(y-y_0)\frac{\partial B}{\partial y}\vec{e}_z\rangle = \langle v_{0y}(y-y_0)\frac{\partial B}{\partial y}\vec{e}_x - v_{0x}(y-y_0)\frac{\partial B}{\partial y}\vec{e}_y\rangle$

Plasma

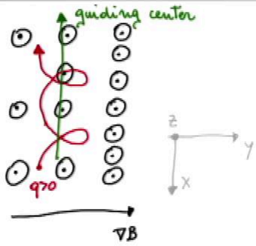
We will put a system of coordinate with the x-direction pointing downwards, the y-direction here and the z-direction pointing in the direction of the B field outwards from this plane. Therefore, if B(zero) = B(zero)ez, as it is in the z-direction, and similarly, this term here, v(cross) r.grad B is in the z-direction, more precisely, so if B is in the direction of z, therefore if B(zero) = B(zero)z and this term here can be evaluated and will still be in the direction of z, more precisely, it will be given by, taking into account that the derivative is only in the y-direction, but the derivative of B along the y-direction, that's the difference of particle position along y with respect to the guiding center which is located at y(zero). Well, then, if we try to evaluate this term here, what we will have is that we will have to evaluate v(zero) cross y - y(zero) dB/dy, well, here, you can now develop v(zero) along its y- and x-component, and therefore this will be equal to the y-component of v(zero) -- this will result into the x-component plus the component due to v(zero)x. Now, if we are careful, we actually know what is v(zero)y and v(zero)x, as these are solutions of the equation here that is the solution of an equation with uniform static magnetic field.

Notes

Summary

6m 03s

Gradient drift, $\nabla B \perp \mathbf{B}$



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We have $v_{0x} = v_0\sin(\omega_c t+\varphi)$, $v_{0y} = v_0\cos(\omega_c t+\varphi)$, $y-y_0 = \frac{v_0}{\omega_c}\sin(\omega_c t+\varphi)$

$\Rightarrow \langle\vec{v}'\rangle = -\frac{m}{2B_0^2}\frac{v_0^2}{q}\frac{\partial B}{\partial y}\vec{e}_x$, in the most general case $\boxed{\vec{v}_{\nabla B} = \frac{m}{2B_0^3}\frac{v_0^2}{q}\left(\vec{B}_0\times\nabla B\big|_0\right)}$ grad-B drift

Therefore we have that the particles in circular motion, v(zero) does a circular motion and v(zero)x will be given by... while v(zero)y will be given by oscillation with the same frequency just shifted by π/2. And the y-y(zero) can also be evaluated and will be equal to v(zero) divided by the omega cyclotron frequency sin of omerga ct + φ cyclotron frequency -- time + φ -- φ here is a phase shift that will be determined by the initial condition of the particle. Now we can plug this expression here of v(zero)x v(zero)y and y-y(zero) into here, evaluate the gyroaverage, cross it with B, and what do we obtain? v' And if you do all the calculation, what you get is gyro average v' with B = m, the mass of the particles, divided by 2B(zero)^2, v(zero) of the particle/charge dB/dy ex. This is the result in this specific case in this geometry. If we look at the most general case, what we have is a drift that is given always by the same coefficient in front m 2B(zero)^3 B(zero)^2q B(zero) cross the gradient of B(zero). This drift, which is also very important in plasma physics is the so-called *grad B drift*.

Notes

Summary

8m 14s