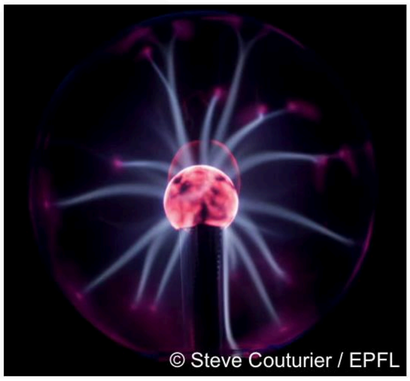
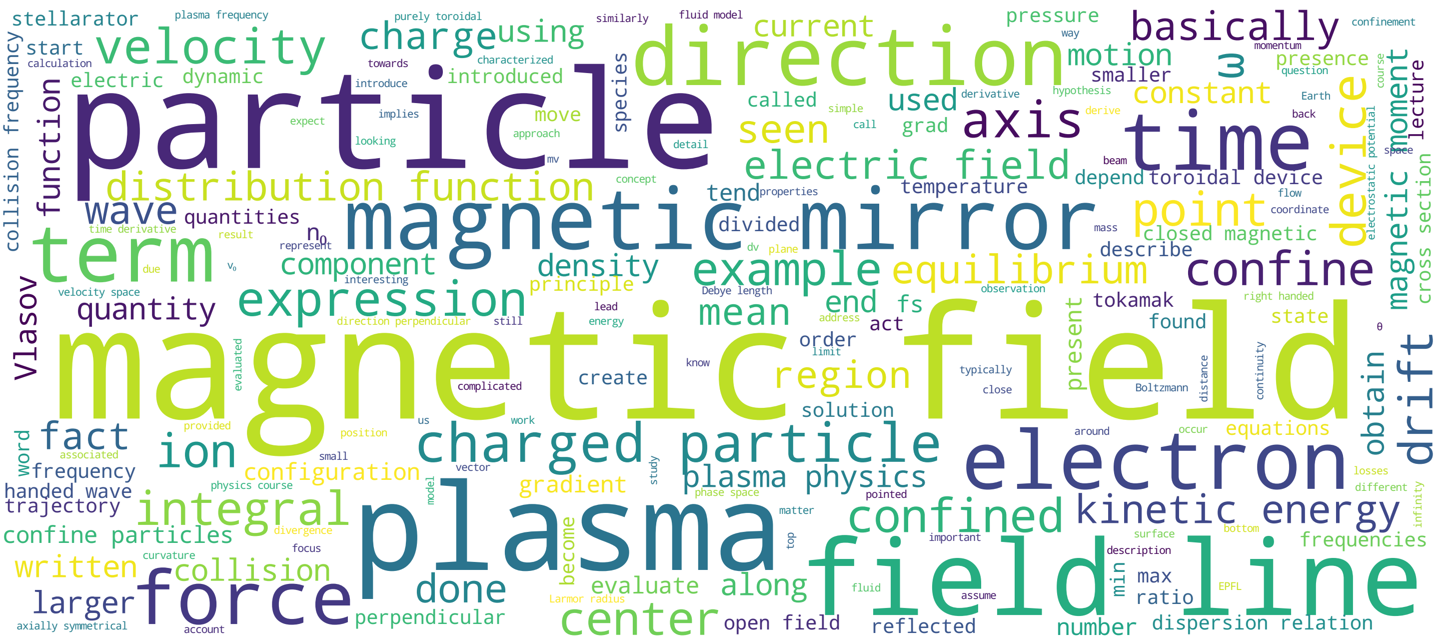


Plasma confinement based on single particle motion

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 1f

Paolo Ricci



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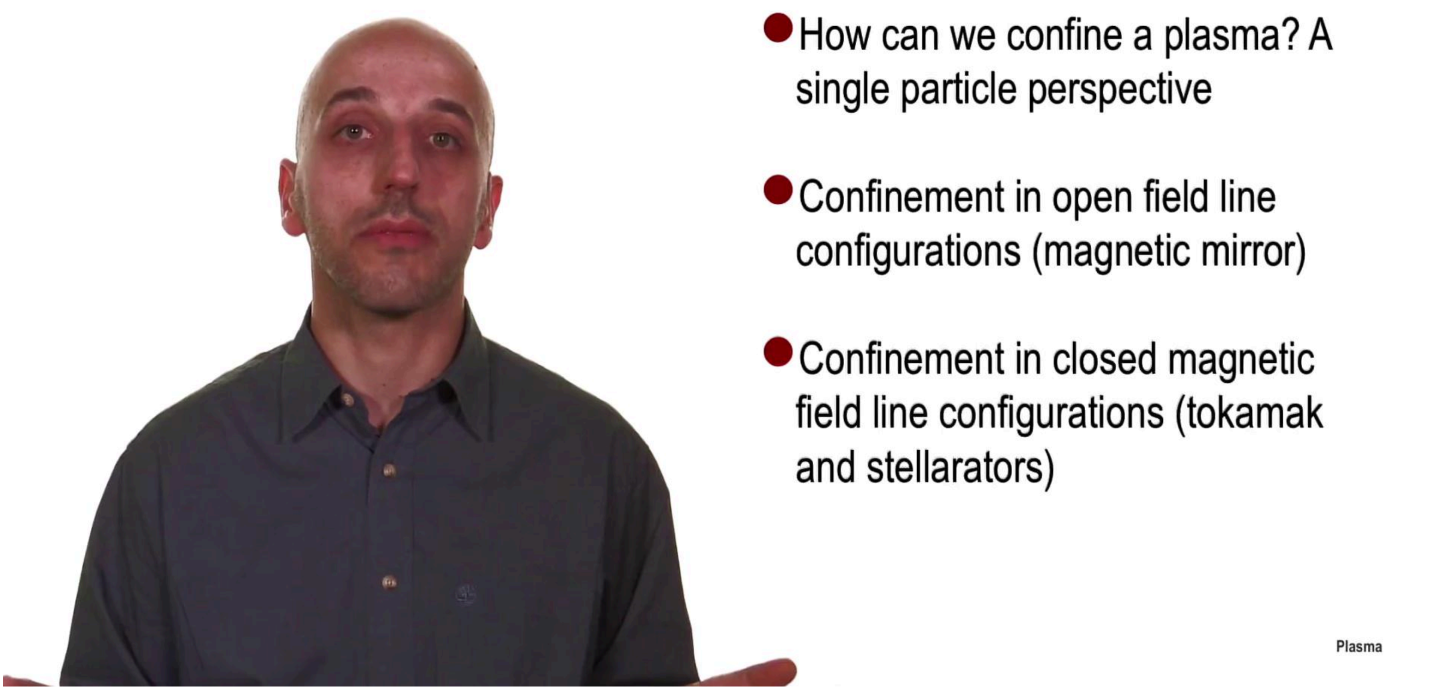
Video

Introduction



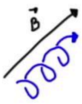
Welcome to the plasma physics course of the EPFL. How can we confine a plasma? Today we will address this question. We will use the concept that we have introduced into past lectures to describe single particle trajectories in a given electromagnetic field. We will see that particles can be confined in devices that are characterized by open field lines, the so-called magnetic mirrors, and also devices where magnetic field lines are closed, such as the stellarators and tokamaks.

Notes

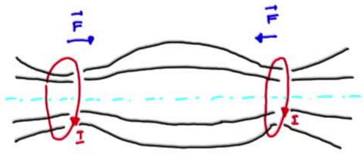
Summary

0m 05s

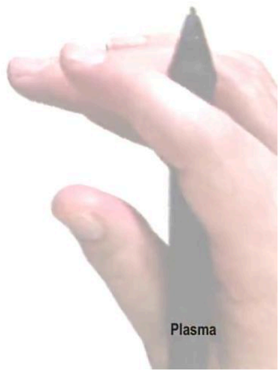
How can we confine a plasma?



A magnetic field provides confinement of charged particles perpendicularly to it, but not in the parallel direction.



① Open field lines, use $\vec{F} = -(\vec{\mu}\cdot\nabla)\vec{B}$



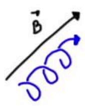
So how can we confine a plasma? We've seen that in the presence of a magnetic field particles follow a trajectory that is basically a helix. Therefore a magnetic field provides confinement in the direction perpendicular to it but not in the direction along it. So if we want to confine plasma we have to somehow limit the loss of charged particles that occur along the magnetic field. This can be done in two ways. The first one is still to use open field lines but reducing the losses along the magnetic field by noticing that there is a force, that acts on magnetic moments, that can in principle limit the flow of particles along the magnetic field lines. In fact, associated with each charged particle, there is a magnetic moment and there will be a force, one that I have written here, that actually acts on it and can limit the flow of particles. For example, if we have a two-ring current, we will create an axially symmetrical magnetic field, here I [drawed] axis of symmetry, which will be characterized by field lines that have the following shape. In this configuration, there is a gradient of the magnetic field and particles will feel a force that can push them towards this central region, confining the particles also in the direction parallel to the magnetic field.

Notes

Summary

0m 44s

How can we confine a plasma?



A magnetic field provides confinement of charged particles perpendicularly to it, but not in the parallel direction.

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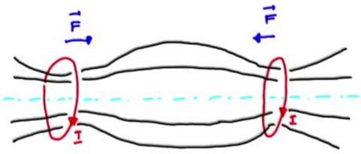
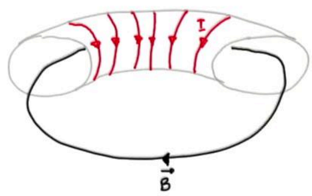


(magnetic mirrors)

② Closed the magnetic field lines



(Tokamaks and stellarators)

Plasma

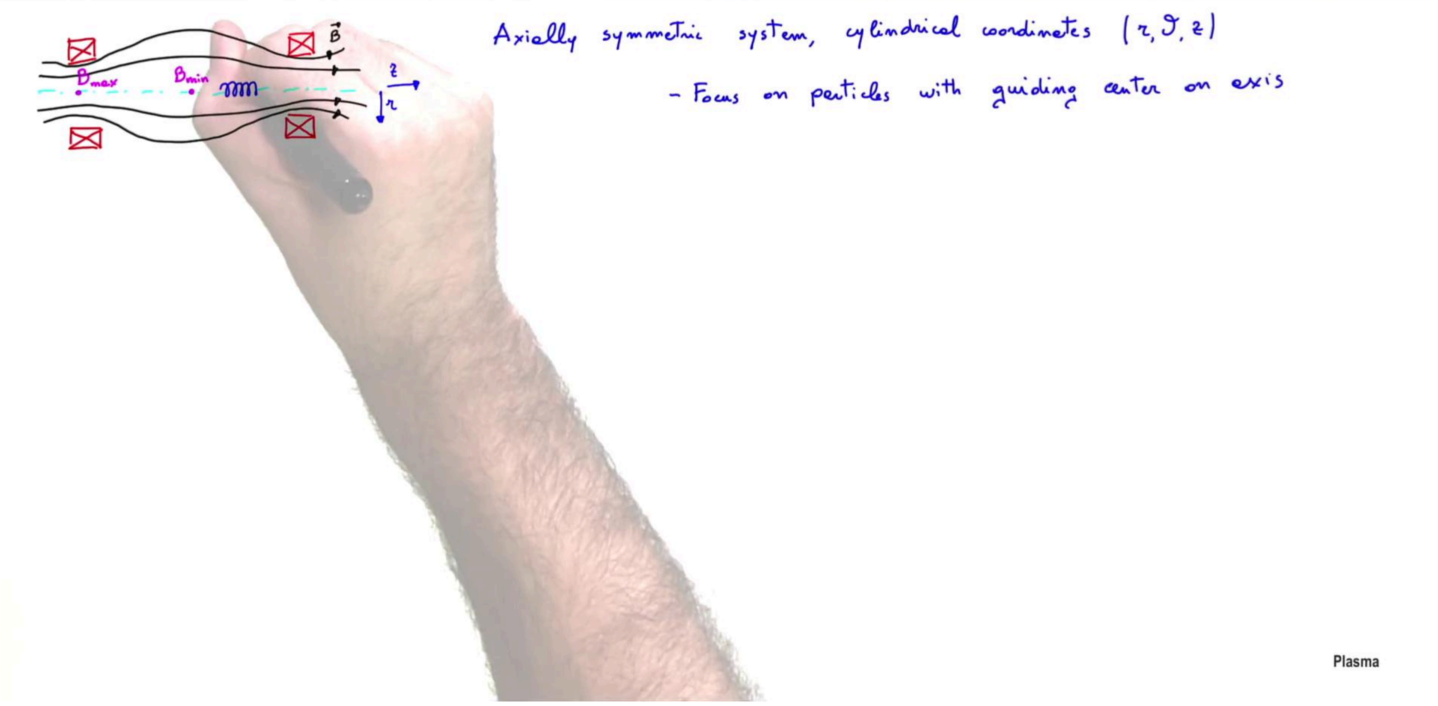
This is the principle that is used by magnetic mirrors. The second possibility is to use closed magnetic field lines. In fact, if we close the magnetic field lines, basically we avoid the losses that occur at the end of those. This configuration can be done, for example, if we create a toroidal device — that's a donut-shaped device — here I will just represent a section of it. So this device will extend over the full angle and here I am just drawing half of it. We will have a system of current that will run in this direction. By doing this, we will have magnetic field lines that will run throughout this toroidal device, this donut-shaped device, and will create closed magnetic field lines which in principle perfectly confine the charged particles, as the losses at the end of the magnetic field lines are avoided by closing the field lines. This is the principle that is used in devices such as tokamaks and stellarators. So let's give a look more into the details of how an open field line magnetic configuration can actually work and how can it confine particles, and similarly for the closed magnetic field line configuration.

Notes

Summary

3m 15s

Motion in a magnetic mirror field



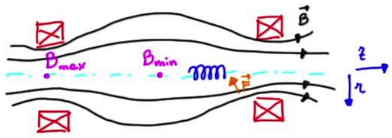
So let's start with the description of the particle confinement that is provided by a magnetic mirror. As we will point out, only a certain category of particles can actually be confined by this configuration. So let's look at a cross-section of the magnetic mirror. We will have the two coils that provide the current that generates the magnetic field. The magnetic axis will be here and the magnetic field lines will, as we have seen just before, have the following shape. Along the axis of symmetry there will be a point where the magnetic field is minimum. That is here, at the center of the magnetic mirror, and a point where it's maximum, that is here where the current rings are located. What we are dealing with is therefore an axially symmetrical system. For its description, as is typically the case for an axially symmetrical system, it is useful to use a cylindrical coordinate system with coordinates r, theta, and z. R is the distance from the axis, z is the coordinate along the axis, and theta will be the direction, the angle around the axis. We will focus on particles with the guiding center that falls on the axis. Basically, particles that gyrate along the magnetic axis.

Notes

Summary

5m 07s

Motion in a magnetic mirror field



Axially symmetric system, cylindrical coordinates (r, ϑ, z)

- Focus on particles with guiding center on axis

$F_z = q(\vec{v} \times \vec{B})_z = |q| v_\perp B_r$; since $\nabla \cdot \vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{\partial B_z}{\partial z} = 0$, for $B_z \gg B_r$ and $B_r(r=$

Plasma

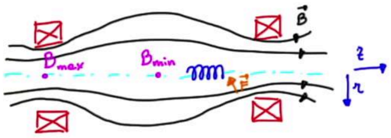
What we are going to do can be easily generalized to particles that are not exactly on the axis, but just move along the magnetic field lines that are around the axis. It's just that the algebra is much more complicated. Now, the [inaudible] B force is acting in the direction perpendicular to the magnetic field lines. And as the magnetic field lines are not straight, they are actually curved, we expect that a force will act in this direction, therefore with a component along the z direction that can actually confine the particles. Let's look at the z component of the force, which is the one that is actually able to confine the particles axially, and we will have this component F(z), which is the z component of the [recource b] force. If one does the calculation, it is found to be equal to q, the absolute value of the charge, times v(⊥) interradial magnetic field. Now since the magnetic field is divergence-free and here I have written to divergence in terms of the cylindrical coordinates we are using, then we have data with an approximation that are valid in this system that is B(z) much larger than B(r). B(r) on axis equal to zero.

Notes

Summary

7m 14s

Motion in a magnetic mirror field



Axially symmetric system, cylindrical coordinates (r, ϑ, z)

– Focus on particles with guiding center on axis

$F_z = q(\vec{v} \times \vec{B})_z = |q| v_\perp B_r$; since $\nabla \cdot \vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{\partial B_z}{\partial z} = 0$, for $B_z \gg B_r$ and $B_r(r=0) = 0$

$\Rightarrow B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z} \simeq -\frac{r}{2}|\nabla B| \Rightarrow F_z = -|q| v_\perp \frac{r}{2}|\nabla B| = -|q| v_\perp \frac{\rho}{2}|\nabla B| = -\frac{m v_\perp^2}{2B}|\nabla B| = -\mu|\nabla B|$

What are the particles confined by the magnetic mirror?

Plasma

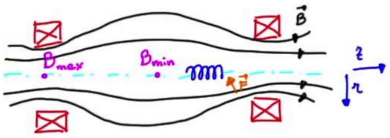
By integrating this expression with this hypothesis, we find that B(r) is equal to [(- r/2) (dB(z) / d(z))], which, as b(z) is much larger than b(r), can also be written as [-r/2 modulus of gradient of b]. This expression can be replaced here, used to replace b(r), and what we obtain is that f along the z component is equal to [-qv(⊥)B(r)] replaced with this expression. Now r is the distance of the particle from the axis and this is approximately the Larmor radius. If we replace the expression of the Larmor radius into this formula here, we have that this is equal to [-(mv⊥^2(2b))] times the gradient of b. Here we recognize the magnetic moment. We have basically retrieved the formula[ic] expression of the force acting on a charged particle which is associated to a magnetic moment that we have just drafted in the previous slide. So there is actually a force that basically, I think, to confines the particles in this central region of the magnetic mirror. What are the particles that are therefore actually confined by this configuration? All of them, or just some particular particles, categories of particles?

Notes

Summary

9m 08s

Motion in a magnetic mirror field



Axially symmetric system, cylindrical coordinates (r, ϑ, z)

– Focus on particles with guiding center on axis

$F_z = q(\vec{v}\times\vec{B})_z = |q|\, v_\perp B_r$; since $\nabla\cdot\vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{\partial B_z}{\partial z} = 0$, for $B_z \gg B_r$ and $B_r(r=0)=0$

$\Rightarrow B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z} \simeq -\frac{r}{2}|\nabla B| \Rightarrow F_z = -|q|\, v_\perp \frac{r}{2}|\nabla B| = -|q|\, v_\perp \frac{\rho}{2}|\nabla B| = -\frac{m v_\perp^2}{2B}|\nabla B| = -\mu|\nabla B|$

What are the particles confined by the magnetic mirror?

$\mu = const = \frac{1}{2}\frac{m v_{\perp c}^2}{B_{min}} = \frac{1}{2}\frac{m v_{\perp e}^2}{B_{max}}$ ($v_{\perp c}$ and $v_{\perp e}$, $\perp$ velocities at B_{min} and B_{max})

$E_{kin} = const = \frac{1}{2} m (v_{\perp c}^2 + v_{\parallel c}^2) = \frac{1}{2} m (v_{\perp e}^2 + v_{\parallel e}^2)$

A particle is reflected if, while moving towards B_{max}, $v_\parallel$ vanishes, $E_{kin} = \frac{1}{2} m v_\perp^2$

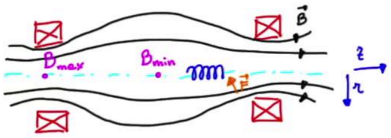
Plasma

To address this question, the first thing that we point out is that as we have seen in the previous lecture, the magnetic moment is a constant of the particle motion. [Therefore 1/2] (mv(⊥)^2)]. At the center of the magnetic mirror, at the location of B(min) — minimum magnetic field — divided by the magnetic field, B(min), this will be equal to the same quantity evaluated where the magnetic field is maximum. Here I am denoting with c and e the particle velocity at the center of the magnetic mirror and at the edge. So with v(⊥)c and v(⊥)e, two perpendicular velocities at B(min) and at B(max). We will use a similar notation for the parallel velocities. There is another quantity that is conserved. That is kinetic energy. Therefore the kinetic energy at the center of the magnetic mirror — of the particles located at the center of the magnetic mirror will be equal to the same quantities evaluated when the particles is at the edge of the magnetic mirror. So when is a particle confined? A particle is confined if it is reflected while moving towards the end of the magnetic mirror. The particle is reflected if, while moving towards B(max), therefore towards the end of the magnetic mirror, [B(‖)] vanishes — that is, if the kinetic energy becomes only equal to the perpendicular kinetic energy.

Notes

Summary

11m 31s



Axially symmetric system, cylindrical coordinates (r, ϑ, z)

- Focus on particles with guiding center on axis

$F_z = q(\vec{v}\times\vec{B})_z = |q|\, v_\perp B_r$; since $\nabla\cdot\vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{\partial B_z}{\partial z} = 0$, for $B_z \gg B_r$ and $B_r(r=0)=0$

$$\Rightarrow B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z} \simeq -\frac{r}{2}|\nabla B| \quad \Rightarrow \quad F_z = -|q|\, v_\perp \frac{r}{2}|\nabla B| = -|q|\, v_\perp \frac{\rho}{2}|\nabla B| = -\frac{m v_\perp^2}{2B}|\nabla B| = -\mu|\nabla B|$$

What are the particles confined by the magnetic mirror?

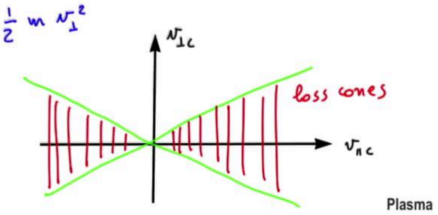
$\mu = const = \frac{1}{2}\frac{m v_{\perp c}^2}{B_{min}} = \frac{1}{2}\frac{m v_{\perp e}^2}{B_{max}}$ ($v_{\perp c}$ and $v_{\perp e}$, $\perp$ velocities at B_{min} and B_{max})

$E_{Kin} = const = \frac{1}{2} m\,(v_{\perp c}^2 + v_{\parallel c}^2) = \frac{1}{2} m\,(v_{\perp e}^2 + v_{\parallel e}^2)$

A particle is reflected if, while moving towards B_{max}, $v_\parallel$ vanishes, $E_{Kin} = \frac{1}{2} m v_\perp^2$

$$\Rightarrow \mu = \frac{1}{2}\frac{m v_{\perp c}^2}{B_{min}} = \frac{1}{2}\frac{m v_{\perp e}^2}{B_{max}} \geq \frac{m(v_{\perp e}^2 + v_{\parallel e}^2)}{2B_{max}} = \frac{E_{Kin}}{B_{max}} = \frac{1}{2}\frac{m(v_{\perp c}^2 + v_{\parallel c}^2)}{B_{max}}$$

$$\Rightarrow \frac{v_{\perp c}^2}{v_{\perp c}^2 + v_{\parallel c}^2} > \frac{B_{min}}{B_{max}}$$



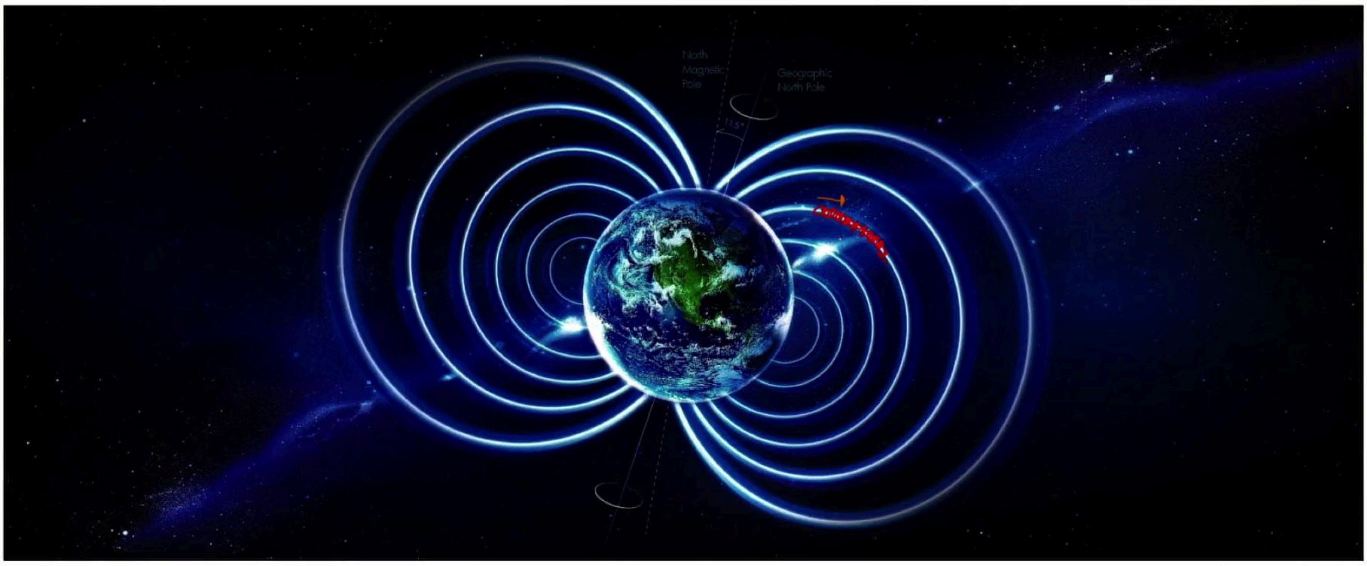
What does this mean in practice? For particles to be reflected, the magnetic moment that is equal to 1/2((Mv⊥c^2) divided by B(min)), which, as we have seen here, is equal to... 1/2((Mv(⊥)e^2)/B(max)), this has to be greater than the same quantities where we have actually added the parallel velocity. This quantity here is actually the kinetic energy divided by the maximum magnetic field. The kinetic energy can be expressed here by using the quantities at the the center of the magnetic mirror. Now we can compare [this quantity] with this one and what we obtain is that a particle is reflected if the ratio between the perpendicular velocity at the center of the magnetic mirror and the total velocity is larger than the ratio between B(min) and B(max). In other words,which are the particles that are confined? We can represent the particles as they are at the center of the magnetic mirror, drawing the v(⊥)c and [v(||)c] axes. This equation here will point out that there are two lines that will define the region in [inaudible] space where the particles are lost along the magnetic field lines, that is, here — these are the so-called loss cones. The regions in [faith] space which are the ones that I am pointing out where particles are actually confined while moving along the axis of the magnetic mirror.

Notes

Summary

14m 08s

Earth's dipolar field: an example of magnetic mirror



Source: NASA

Plasma

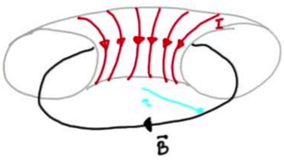
An example of a magnetic field mirror is provided in nature by the Earth — the dipolar field of the earth. Earth is surrounded by a dipolar field and there are charged particles that move, subject to the presence of this magnetic field. Particles therefore stream along the magnetic field lines because of the Lorentz force and once they approach the magnetic poles the feel a stronger magnetic field which actually pushes them back. This of course leads to an acculumation of charged particles in this region of the Earth's magnetic field.

Notes

Summary

17m 15s

Closed magnetic field line confinement



$B = \frac{\mu_0 N_c}{2\pi}\frac{I}{r}$ ⇒ B is not homogeneous and it is curved: drifts are present!

$$\vec{v}_d = \frac{m v_\parallel^2}{q B^2 R_B^3}(\vec{R}_B \times \vec{B}), \quad \vec{v}_{\nabla B} = \frac{m v_\perp^2}{2 B^3 q}(\vec{B} \times \nabla B)$$

Plasma

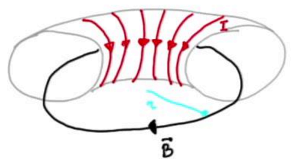
So we have seen that by using an open field line configuration — the magnetic mirror — one can confine a certain class of particles. If we were able to close the magnetic field lines, which in principle is not extremely complicated, then in principle we would be able to completely eliminate the losses at the ends of the magnetic field lines. But unfortunately, as we will see here, things are not so simple and it is not so easy to confine particles by using closed magnetic field lines. To see that, let's consider the toroidal device that we were sketching previously where a set of coils carrying a current create the magnetic field lines that are closed. According to Ampère's law, the intensity of the magnetic field that is created by these current-carrying loops is given by [inaudible][zero] — the number of current-carrying coils — divided by 2πi, the current carried by each of the coils, divided by r, where r is basically the radius of curvature of the magnetic field lines. This implies that the magnetic field is not homogeneous, and it's curved. Drifts are therefore present. Which drift? The curvature drift, first of all... and the [grad B] drift. So what is the dynamics of a particle subject to these drifts?

Notes

Summary

18m 06s

Closed magnetic field line confinement



$B = \frac{\mu_0 N_c}{2\pi} \frac{I}{r}$ $\Rightarrow$ B is not homogeneous and it is curved: drifts are present!

$$\vec{v}_d = \frac{m v_{\parallel}^2}{q B^2 R_B^3} (\vec{R}_B \times \vec{B}), \quad \vec{v}_{\nabla B} = \frac{m v_{\perp}^2}{2 B^3 q} (\vec{B} \times \nabla B)$$

Particle dynamics in a purely Toroidal field

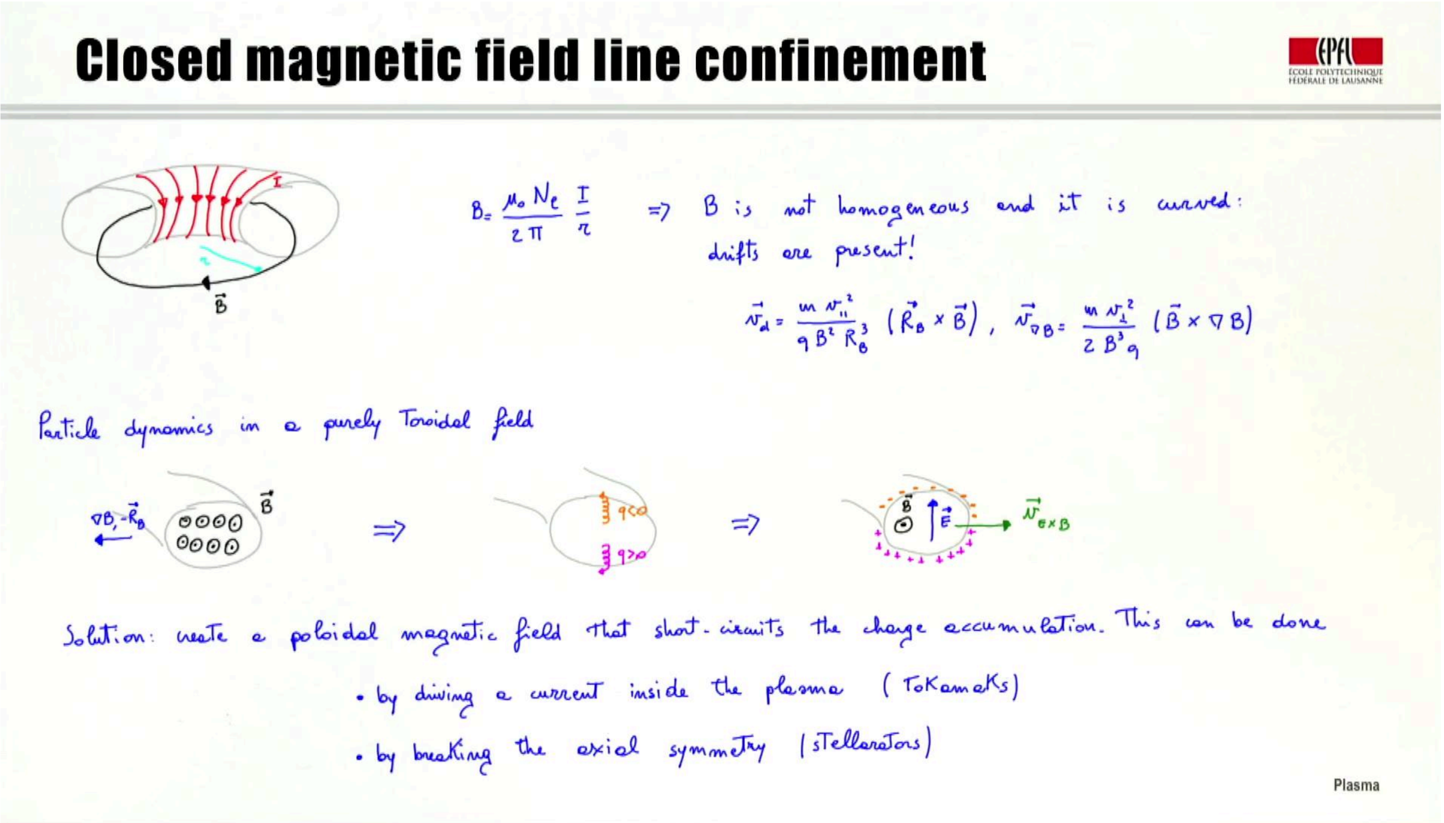
Plasma

In other words, what is the particle dynamics in a purely toroidal field such as the one that I have drawn here? Let's take a cross-section of this toroidal device that I am drawing here with the magnetic field lines that are coming out from the plane. What we have is that the [grad B] will be in this direction and also -rb, the curvature of the magnetic field lines will be in this direction. Therefore, if we evalute the curvature drift and the [grad B] drift, we will see that they all, both of them, are going the same direction. What does it mean? Well, the two drifts [will] sum up and they will induce motion of the particles that will depend on the charge of the particles. Particularly, what we will observe is that charged particles — a positively charged particle will drift towards the bottom of the device, while negatively charged particles will drift towards the top of the device. This will imply, as charge will tend to accumulate, that a region with negative charges will be created on the top of the device, and a region with positive charges at the the bottom of the device. This will actually generate an electric field in the vertical direction.

Notes

Summary

20m 35s



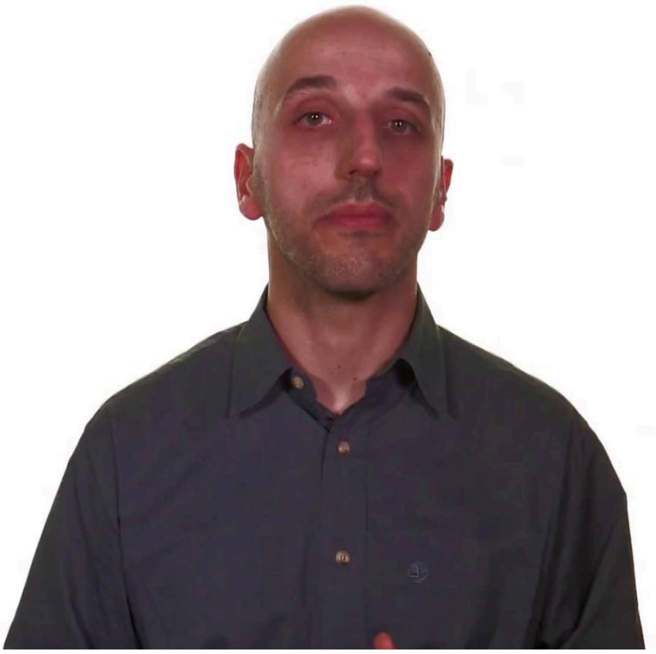
In the presence of a magnetic field, this will give [origin] to our [Icarus B] drift. That will lead to a loss of confinement of the particles from the device. In other words, a purely toroidal magnetic field is not able to confine particles, charged particles. An [Icarus B] drift will be generated, which will lead to a complete loss of particles from the device. So what is the solution to this problem? It is to create a poloidal magnetic field, a magnetic field in the poloidal plane, if you wanted the plane perpendicular to toroidal B, and the goal of this poloidal magnetic field is to short-circuit the charge accumulation. This can be done — first possibility — by driving a current inside the plasma. This is what is done in tokamaks, as you will see in future lectures, or by breaking the axial symmetry, which is what is done in stellarators.

Notes

Summary

22m 46s

Summary



- Confinement of plasma provided by open and closed magnetic field line configurations
- Magnetic mirror confine particles depending on the ratio of perpendicular and parallel velocities
- A toroidal magnetic field alone is not sufficient to confine the plasma

Plasma

How can we confine a plasma? This is the question that we have addressed in today's lectures by using the concept of single-particle trajectory that we have learned in the past lecture. A magnetic field can confine particles in the direction perpendicular to it. But the particles flow very rapidly along the magnetic field lines, and therefore in this direction cannot be confined. We've pointed out two strategies to avoid this problem. The first one is to create a gradient of the magnetic field and this confines particles, is actually able to confine particles, and is what is used in magnetic mirrors. However, only a certain class of particles depending on the ratio of perpendicular and parallel velocity, can be confined. The second approach is to close the magnetic field lines. In this way we avoid losses in the parallel direction. However, we have pointed out that a [purely] toroidal magnetic field is not sufficient to confine particles. More complicated configurations, such as the one realized in stellarators and tokamaks, are necessary. We have just given a brief introduction to the confinement of plasma in this lecture, and there will be a number of lectures in the future that will be devoted to this specific aspect of plasma physics.

Notes

Summary

24m 13s