



## Introduction



- Self-consistent description of a plasma
- The distribution function
- Examples of distribution functions
- Conservation in phase space
- The Boltzmann equation

Plasma

Throughout lecture one, we have had the opportunity to familiarize ourselves with the most basic concepts of plasma physics. Plasma dynamics is a complex subject. And we have started to uncover it by studying the trajectory of charged particles in assigned electromagnetic fields. In reality, these fields result also from the position and motion of the charged particles that compose the plasma. Therefore their evaluation cannot really be separated from that of the particle trajectories. As a consequence, [inaudible] just simply describes the motion of the charged particles, and the generation of the electromagnetic fields, is a necessity to actually capture the plasma dynamics. A fairly complete model is given by the kinetic theory, the subject of this lecture which is given by Professor Paolo Ricci.

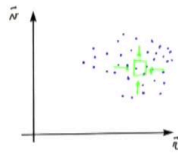
Notes

Summary



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## Conservation of particle number



If there are no sources nor sinks of particles

$$\frac{\partial f_0}{\partial t} = -\nabla \cdot (\vec{u} f_0) \quad (\text{particles are conserved})$$

$$\nabla_{\vec{r}} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3} \right) = \left( \frac{\partial}{\partial \vec{r}}, \frac{\partial}{\partial \vec{v}} \right)$$

$$\vec{u} = \left( \frac{d\vec{r}}{dt}, \frac{d\vec{v}}{dt} \right) = \left( \vec{v}, \frac{\vec{F}^{L\>}}{m_j} + \vec{F}^{S\>} \right)$$

L\> → long range

S\> → short range

$$\frac{\partial f_0}{\partial t} = -\frac{\partial}{\partial \vec{r}} \cdot (\vec{v} f_0) - \frac{\partial}{\partial \vec{v}} \cdot \left( \vec{F}^{L\>} + \vec{F}^{S\>} \right)$$

Plasma

Starting from the most complete model that describes the trajectories of all plasma particles, in a self-consistent electromagnetic fields, statistical matters will be applied to derive a kinetic model for a plasma.

Notes

Summary



## The Boltzmann equation



$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{r}} f + \frac{\partial}{\partial \vec{p}} \cdot \left[ \left( \frac{\vec{p}^{(1)} \cdot \vec{p}^{(2)}}{m_1} \right) \vec{f}_1 \right]$$

Observations:

•  $\vec{r}$  is independent of  $\vec{v}$ ,  $\frac{\partial}{\partial \vec{r}} (\vec{v} \cdot \vec{f}_1) = \vec{v} \cdot \frac{\partial \vec{f}_1}{\partial \vec{r}}$

•  $\vec{p}^{(1)}, \vec{p}^{(2)} \left( \vec{v}^{(1)} = \vec{v} \times \vec{v}^{(2)} \right)$   
independent of  $\vec{v}$   $\Rightarrow \vec{v} \times \vec{v}^{(1)} \perp \vec{v} \Rightarrow \frac{\partial}{\partial \vec{p}} (\vec{v}^{(1)} \cdot \vec{f}_1) = \vec{v}^{(1)} \cdot \frac{\partial \vec{f}_1}{\partial \vec{p}}$

$$\Rightarrow \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{r}} f = \frac{\vec{p}^{(1)} \cdot \vec{p}^{(2)}}{m_1} \cdot \frac{\partial}{\partial \vec{p}} \left( \frac{\vec{p}^{(1)} \cdot \vec{f}_1}{m_1} \right)$$

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{r}} f + \frac{\vec{p}^{(1)} \cdot \vec{p}^{(2)}}{m_1} \cdot \frac{\partial}{\partial \vec{p}} \left( \frac{\vec{p}^{(1)} \cdot \vec{f}_1}{m_1} \right) \Rightarrow \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{r}} f + \frac{q_1}{m_1} (\vec{v}^{(1)} \times \vec{v} \times \vec{v}^{(2)}) \cdot \frac{\partial \vec{f}_1}{\partial \vec{p}} = \left( \frac{\partial f}{\partial t} \right)$$

This model separates long-range electromagnetic interactions from one to one interactions, that is the core of collisions. Professor Ricci will first focus on the plasma collision processes. Despite being ultimately neglected in most commonly used kinetic model, the one based on the plasma equation, in fact collisions play an important role in the plasma dynamics, as well as in a number of applications.

Notes

Summary



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Then, the properties of the Vlasov equation will be described, as well as one of its simplest applications, the study of the dynamics of two counter-streaming beams. Finally, a brief introduction will be given to the numerical solution of the Vlasov equation.

- Notes

## Summary

