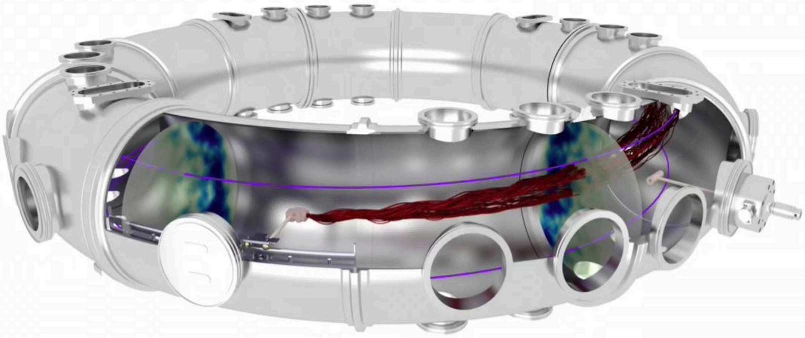

Coulomb collisions in plasmas

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 2b

Paolo Ricci

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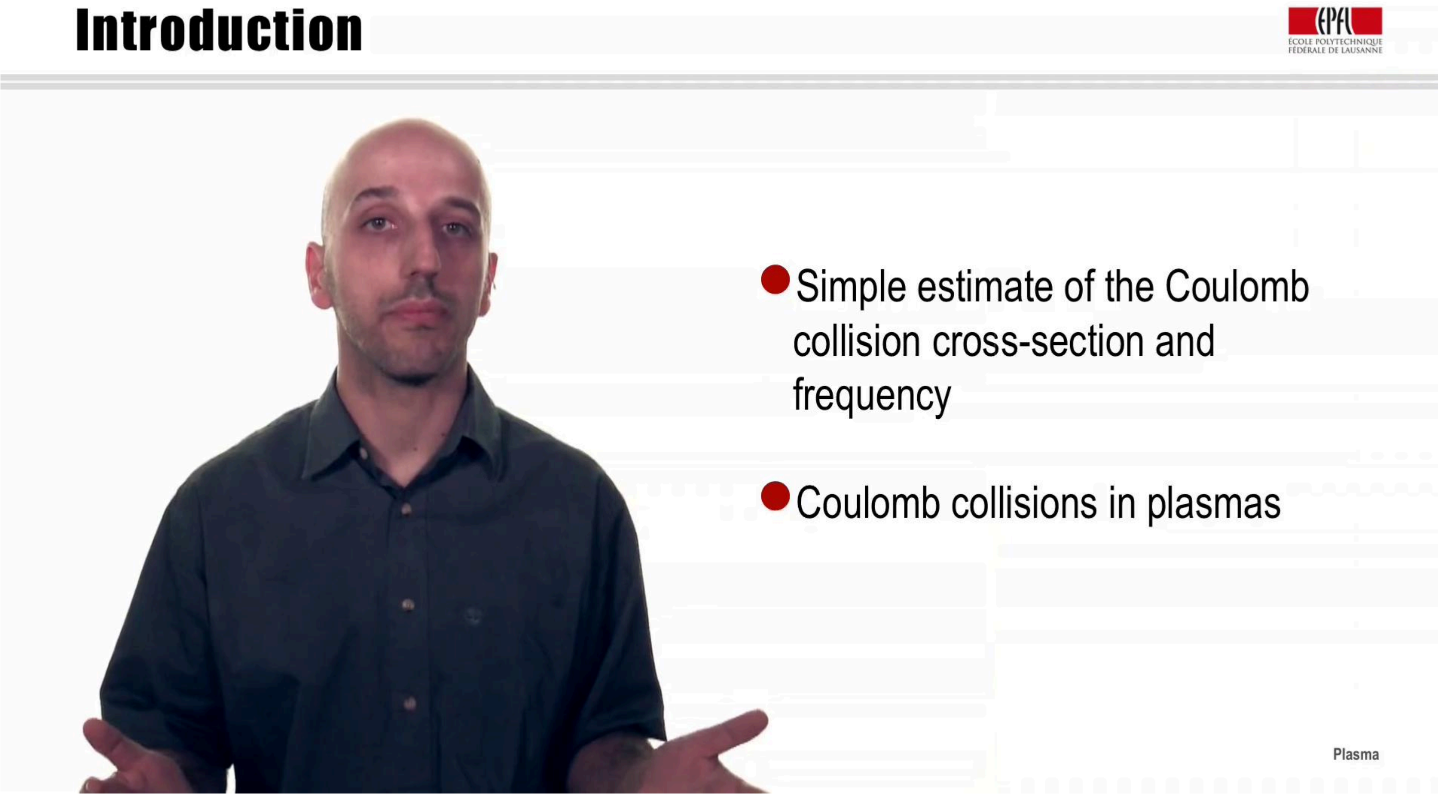
Video

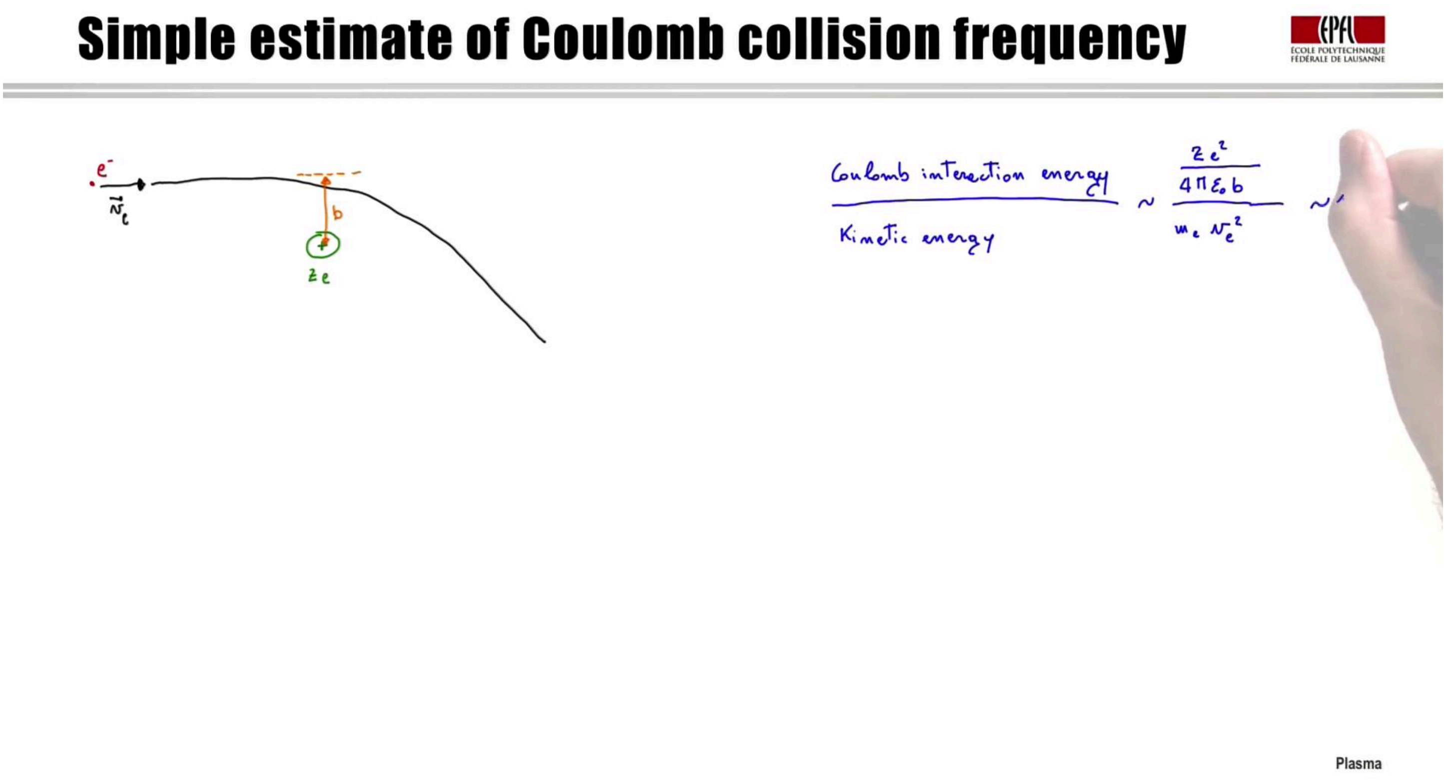


Welcome. Welcome to the Plasma Physics course of the EPFL. In the last module, we introduced the Boltzmann equation. When we have introduced this equation we have separated the long range forces that are associated to collective effects from the short range forces, the ones that are associated with the collisions. Now, let me first say that in a plasma, according to the definition of plasmas, long range collective effect have to dominate over collisions. However, despite this effect, collisions may still play an important role. And therefore it's worth, looking at those and studying the effect of collisions in a plasma. In the present module, we will give a basic introduction of collisions. Now, the particles present in a plasma can collide both with neutrals and with other charged particles. The most interesting collisions are the ones against other charged particles: *Coulomb collisions*, and this is what we will focus our attention on. We will therefore introduce the basic concept of *collision frequency cross-section* that is useful to analyze the Coulomb collisions in a plasma.

Notes

Summary

0m 05s

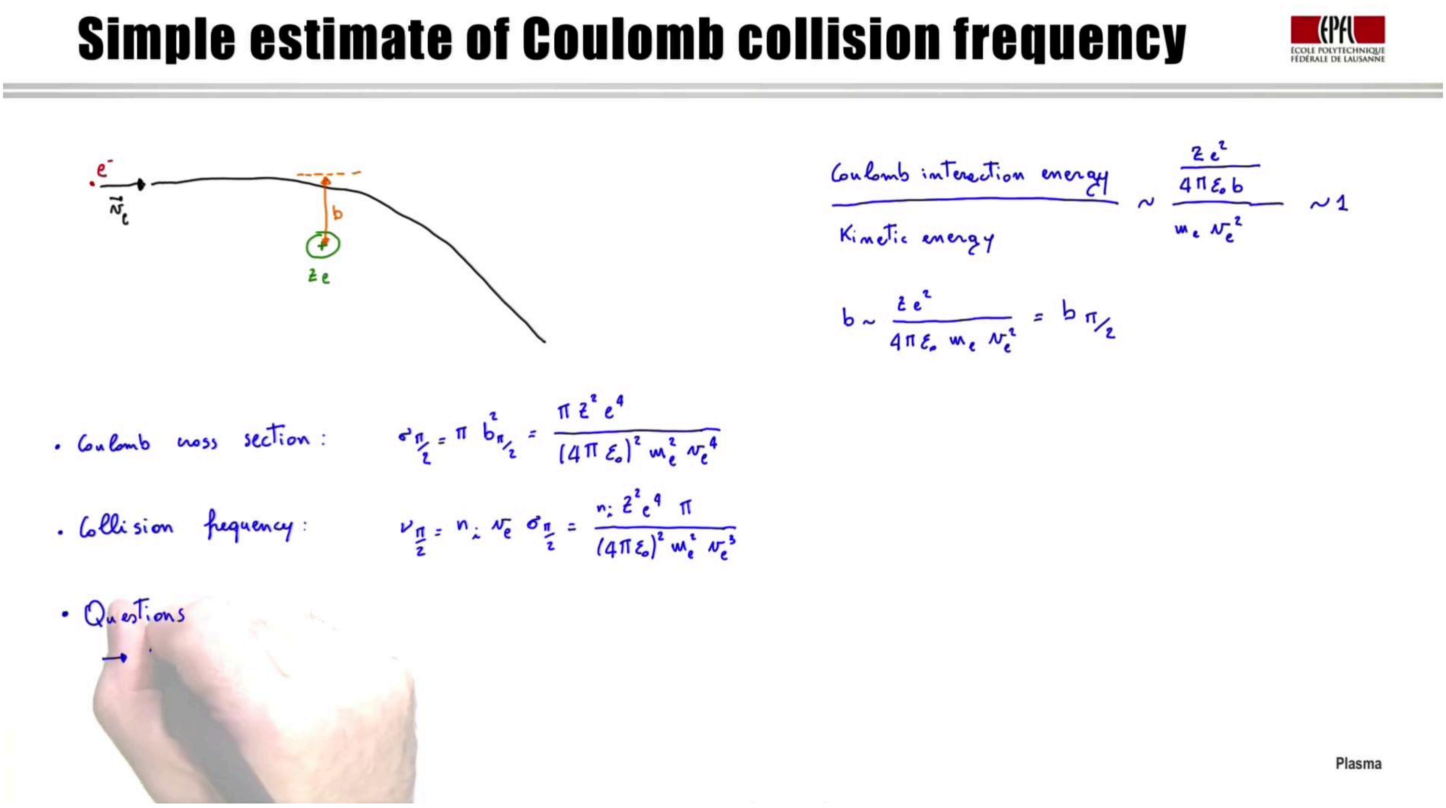


The first thing that we want to do is to give a simple estimate of the Coulomb collision frequency. It's something that we have already mentioned while giving a rigorous definition of a plasma. Let's do it now a bit more rigorously. Let's suppose that we have an electron that moves and goes across a heavy ion with a charge Z e. Because of the Coulomb interaction that exists between these two charged particles, then the trajectory of the electron which is initially moving in this direction, will be deviated. We will call ve to initial velocity of electrons. And we will denote with b the distance that I have pointed out here. Basically, the impact parameter is the shortest distance at which the electron and the ion would be if the electron's trajectory was not modified. Let's try to estimate when the deviation due to this Coulomb force starts to be important. Well, what we have to do is to evaluate ratio between the Coulomb interaction energy and the kinetic energy. That is for the kinetic energy me ve² and for Coulomb interaction energy this will be given by the product of the two charges of the two particles divided by ($4\ \pi\ \varepsilon_0$ b). The deviation of the trajectory will start to be important when this ratio is about 1.

Notes

Summary

1m 43s



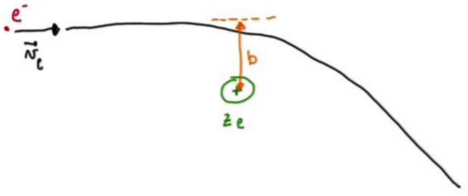
This means that the impact parameters at which Coulomb interaction becomes important can be estimated and will be equal to Ze²/(4 π ε_0 me ve²). And that this quantity for reasons that we will explain later has actually a name, it's called b_π/2. At this point, we can evaluate the Coloumb cross-section, which is the area effectively seen by the electron moving towards this ion and that can be estimated as π (b_π/2)². We will call this cross section -as it is associated to the b_π/2 parameter- as σ_π/2 and therefore using the expression that we have just given, it will be given by σ_π/2 = (π Z² e⁴) / {(4 π ε_0)² me² ve⁴} From these we can derive a collision frequency [**ν**_π/2] that is given by the product of the target density and the incoming electron velocity and the cross-section, which, by plugging in all the numbers we have put here will be given by **ν**_π/2 = (ni π Z² e⁴)/(4 π ε_0)² / (me² ve³) So we actually have some questions that have to be associated with this simple estimate. The first one is: Is this a correct estimate? It's so simple if you want.

Notes

Summary

3m 55s

Simple estimate of Coulomb collision frequency



$$\frac{\text{Coulomb interaction energy}}{\text{Kinetic energy}} \sim \frac{\frac{Ze^2}{4\pi\varepsilon_0 b}}{m_e v_e^2} \sim 1$$

$$b \sim \frac{Ze^2}{4\pi\varepsilon_0 m_e v_e^2} = b_{\pi/2}$$

- Coulomb cross section: $\sigma_{\frac{\pi}{2}} = \pi\, b_{\pi/2}^2 = \dfrac{\pi Z^2 e^4}{(4\pi\varepsilon_0)^2 m_e^2 v_e^4}$
- Collision frequency: $\nu_{\frac{\pi}{2}} = n_i\, v_e\, \sigma_{\frac{\pi}{2}} = \dfrac{n_i Z^2 e^4 \pi}{(4\pi\varepsilon_0)^2 m_e^2 v_e^3}$
- Questions
  - → Is this a correct estimate? Do small angle deflection matter in a plasma?
  - → How can we take into account properly the interaction with many particles?

Plasma

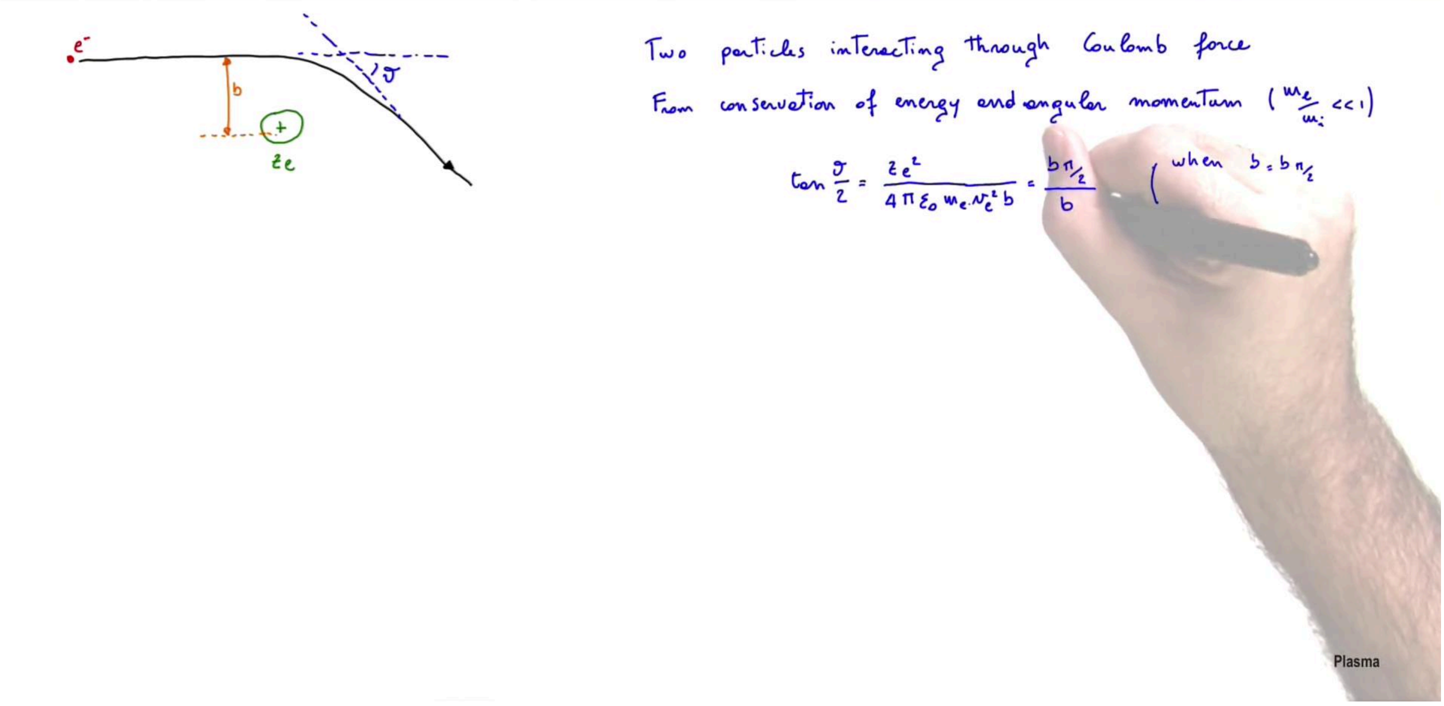
in particular, what we have supposed herer is that these are collisions where there is a large deflection of the electron's trajectory, so in general: do small angle deflections matter in a plasma, or shall we just account for these large angle deflections that we have described here? And second question: Here we have looked at the interaction of an electron with one single particle. Instead, as we know there are many charged particles in a plasma so, how can we take into account properly the interaction with many particles? These are two questions that we are going to address with a more rigorous estimate of the collision frequency.

Notes

Summary

6m 23s

Coulomb collisions in a plasma

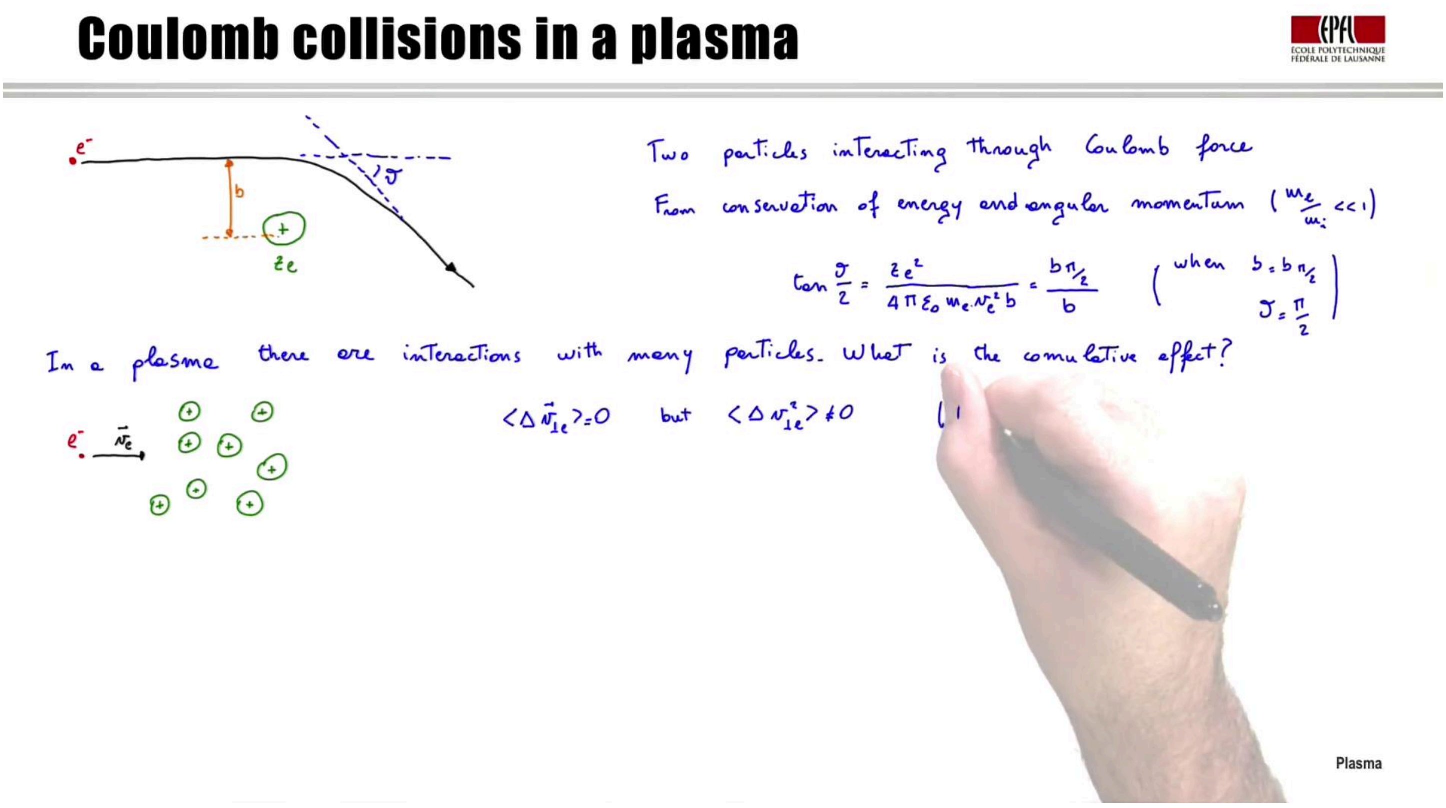


So what we are going to do is now more a detailed evaluation of how the trajectory of an electron impacting an ion is modified because of the Coulomb interaction and then we will take into account that the electron interacts at the same time with all the other particles present in the system. So let's go back to our electron impacting the ion. We will denote with θ the angle between the incoming velocity and the velocity after the collision more precisely we will denote with θ this angle. Now for two particles such as the electron and the ion that we are considering that are interacting through Coulomb force. One can show from the conservation of energy and angular momentum, -we won't do all the steps but this is a well known result from classical mechanics - [that] in the limit of me/mi << 1, from the conservation of the energy and the angular momentum, then one can show that tan (θ/2) is equal to Z e² / (4 π ε_0 me ve² b) Now the interesting thing is that this quantity, Z e² / (4 π ε_0 me ve²) is exactly what we have defined before as b_π/2. So this tangent can be written as tan (θ/2) = b_π/2 / b. And now the meaning of b_π/2 becomes clear. In fact, when b is equal to b_π/2, then it's found that tan (θ/2) = 1.

Notes

Summary

7m 36s



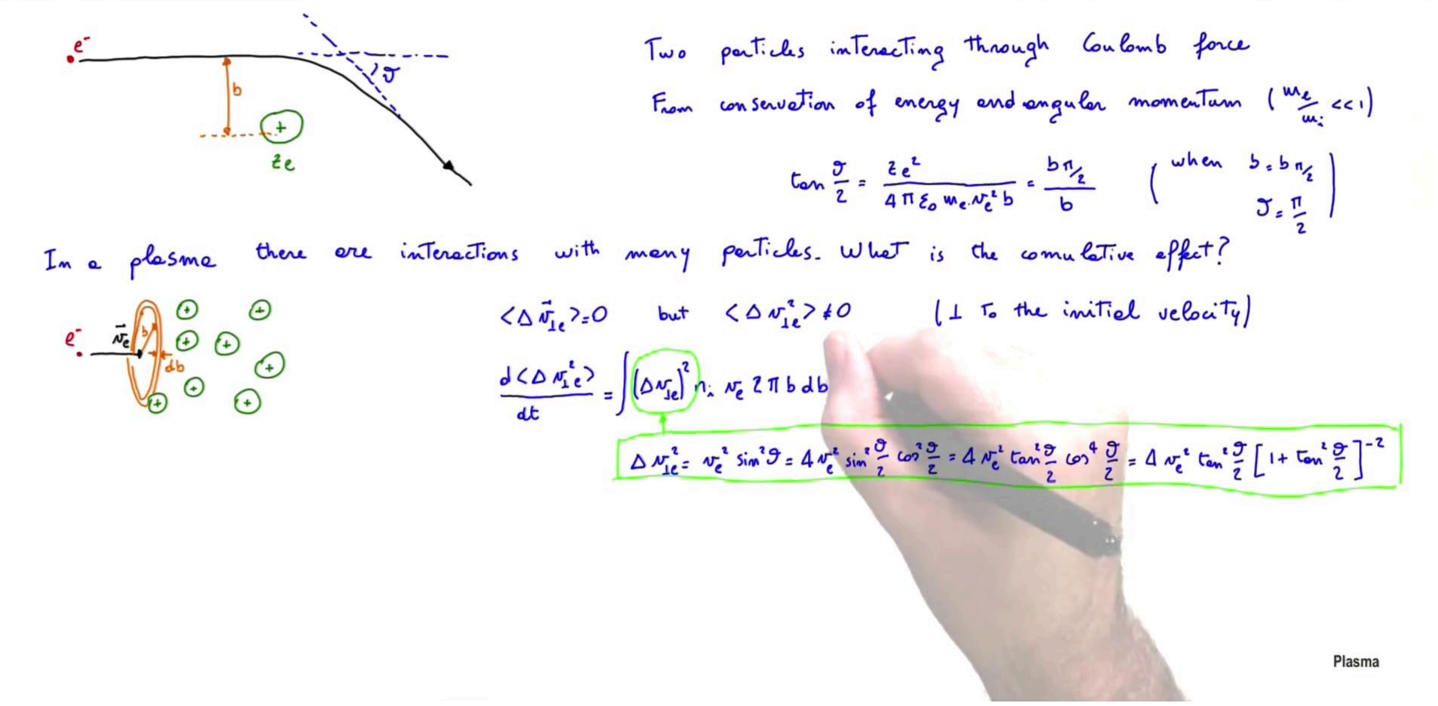
Therefore, θ = π/2. So b_π/2 corresponds to the impact parameter at which the electron is deflected by 90 degrees. Now, taking this into account, the deflection due to each collision, we notice that in a plasma there are interactions with many particles, so what we want to address is: What is the cumulative effect of these collisions? So more precisely, what we have is for example an electron which is going towards a plasma. And what we want to know -to learn about- is how much this velocity will be deflected because of the interaction of this electron with all the ions present in the plasma. In general, we will have that, because of the symmetry of these one-to-one collisions, that the [average] deviation of the perpendicular electron velocity will be equal to zero. [$< \Delta\mathbf{v}_{\perp e} > = 0$] However, the spreading of the perpendicular electron velocity will not be equal to zero. [$< (\Delta v_{\perp e})^2 > \neq 0$] Here with perpendicular [⊥] and parallel [∥], I mean perp. and para. to the initial velocity of the electron.

Notes

Summary

10m 06s

Coulomb collisions in a plasma



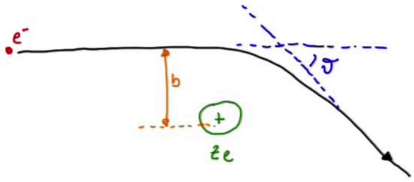
Now, if we want to evaluate what is the spreading of the perpendicular velocity as a function of time, well this derivative will be due to the interaction of the electrons with the particles that are all located at different impact parameters, and more precisely, we will count particles that are in the differential cross-section that goes from b to b+db and the number of these collisions will be equal to the target density times ve, the initial velocity times the differential cross-section that is this area among the two orange lines and is given by 2π b db and then each collision will give a certain $(\Delta v_{\perp e})^2$ and then we will have to integrate over all possible impact parameters. Now, to make some progress, let me point out that this quantity can be expressed by knowing what is the deviation due to the Coulomb force, and in fact, $(\Delta v_{\perp e})^2$ is equal to $v_e^2 \sin^2\theta$. And now we can use one of the properties of the sinus function that is $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$, and then, dividing and multiplying by $\cos^2(\theta/2)$, this is equal to $4 v_e^2 \tan^2(\theta/2)\cos^4(\theta/2)$ This again, using trigonometric identities can be written in terms of $\tan^2(\theta/2)$. The next thing of this expression is that $\tan^2(\theta/2)$ can be expressed in terms of b.

Notes

Summary

12m 01s

Coulomb collisions in a plasma

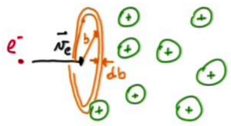


Two particles interacting through Coulomb force

From conservation of energy and angular momentum $\left(\frac{m_e}{m_i} \ll 1\right)$

$$\tan\frac{\vartheta}{2} = \frac{Ze^2}{4\pi\varepsilon_0 m_e v_e^2 b} = \frac{b_{\pi/2}}{b} \qquad \left(\text{when } b = b_{\pi/2},\ \vartheta = \frac{\pi}{2}\right)$$

In a plasma there are interactions with many particles. What is the cumulative effect?



$\langle \Delta \vec{v}_{\perp e}\rangle = 0$ but $\langle \Delta v_{\perp e}^2\rangle \neq 0$ (⊥ to the initial velocity)

$$\frac{d\langle \Delta v_{\perp e}^2\rangle}{dt} = \int (\Delta v_{\perp e})^2 n_i v_e 2\pi b\, db = 8\pi n_i v_e^3 \int_0^{\lambda_D} \frac{(b_{\pi/2}/b)^2\, b\, db}{[1+(b_{\pi/2}/b)^2]^2} \underset{\lambda_D \gg b_{\pi/2}}{\simeq} 8\pi n_i v_e$$

$$\boxed{\Delta v_{\perp e}^2 = v_e^2 \sin^2\vartheta = 4 v_e^2 \sin^2\frac{\vartheta}{2}\cos^2\frac{\vartheta}{2} = 4 v_e^2 \tan^2\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2} = 4 v_e^2 \tan^2\frac{\vartheta}{2}\left[1+\tan^2\frac{\vartheta}{2}\right]^{-2}}$$

Plasma

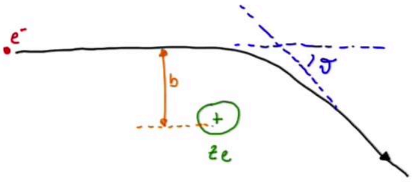
Therefore this integral becomes the integral... the integral of this function that depends, as we have seen here, only from $\tan^2(\theta/2)$ that we have expressed in terms of ($b_{\pi/2}$ / b). Now what are the limits of integration of this function? Well, there are no issues for b going to zero unless quantum effects become important but we have excluded to take those into account in our system so the integral will go from zero to... if you look here, for b that goes to infinity, what you actually have is an integral that will diverge. Why is that? Well, it's because we have assumed that the Coulomb interaction will be valid for all distances but as we know, in a plasma the Coulomb potential is screened at a distance that is compatible to the Debye length [λ_D] and therefore there are no 1-to-1 particles [interactions] at impact parameters that are greater than λ_D. More precise calculations would have required to evaluate this expression by taking to account a screened Coulomb force which we did not do, so we can replace this by basically limiting our integral to the Debye length of our plasma. And this integral can actually be solved analytically, and in the limit of $\lambda_D \gg b_{\pi/2}$ is found to be equal to...

Notes

Summary

15m 00s

Coulomb collisions in a plasma

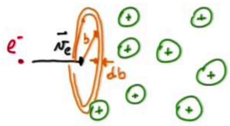


Two particles interacting through Coulomb force

From conservation of energy and angular momentum $\left(\frac{m_e}{m_i} \ll 1\right)$

$$\tan\frac{\vartheta}{2} = \frac{Ze^2}{4\pi\varepsilon_0 m_e v_e^2 b} = \frac{b_{\pi/2}}{b} \qquad \left(\text{when } b = b_{\pi/2},\ \vartheta = \frac{\pi}{2}\right)$$

In a plasma there are interactions with many particles. What is the cumulative effect?



$\langle \Delta \vec{v}_{\perp e}\rangle = 0$ but $\langle \Delta v_{\perp e}^2\rangle \neq 0$ (⊥ to the initial velocity)

$$\frac{d\langle \Delta v_{\perp e}^2\rangle}{dt} = \int (\Delta v_{\perp e})^2 n_i v_e 2\pi b\, db = 8\pi n_i v_e^3 \int_0^{\lambda_D} \frac{(b_{\pi/2}/b)^2\, b\, db}{[1+(b_{\pi/2}/b)^2]^2} \underset{\lambda_D \gg b_{\pi/2}}{\simeq} 8\pi n_i v_e^3 b_{\pi/2}^2 \ln\frac{\lambda_D}{b_{\pi/2}}$$

$$\boxed{\Delta v_{\perp e}^2 = v_e^2 \sin^2\vartheta = 4 v_e^2 \sin^2\frac{\vartheta}{2}\cos^2\frac{\vartheta}{2} = 4 v_e^2 \tan^2\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2} = 4 v_e^2 \tan^2\frac{\vartheta}{2}\left[1+\tan^2\frac{\vartheta}{2}\right]^{-2}}$$

Observations

- Electrons do not lose much energy ($m_e \ll m_i$)

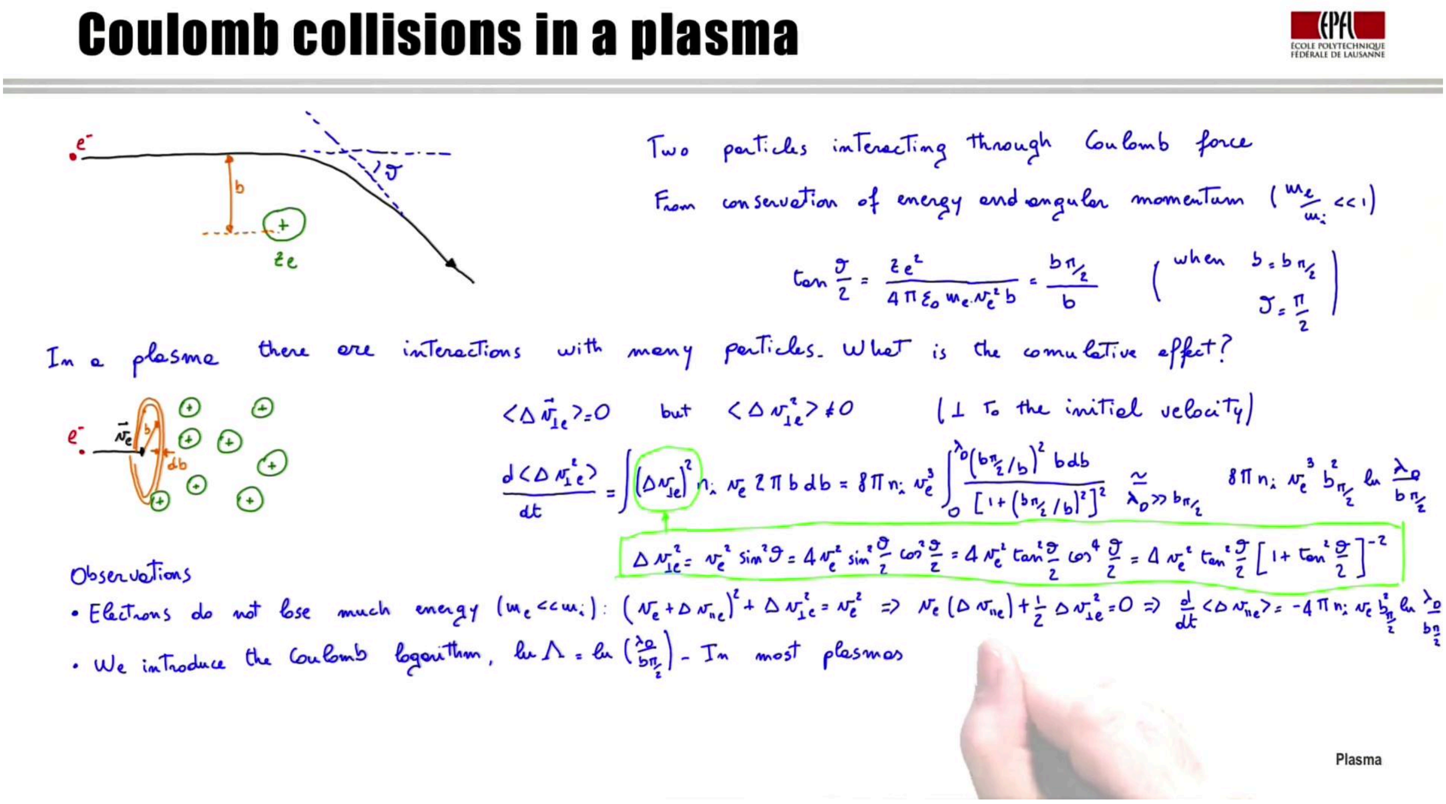
Plasma

to this quantity which depends on ion density, the electron velocity the b_π/2 parameter and the logarithm of the Debye length divided by b_π/2, (the π/2 impact parameter). Well, this is a key result that allows us to make three observations. The first one is related to the evaluation of what is the change in the parallel velocity of the electron beam due to these collisions, to this ensemble of collisions. This can be done by noticing that the electrons do not lose much energy. So we can actually make three observations. The first one leads to the estimate of what is the change of the parallel electron velocity because of the multiple interactions with the ion present in a plasma. Why is that? Well, this is because the electrons impacting such heavy particles, can be assimilated with the collision between for example, of a ball against a wall. When you throw a ball against a wall, the ball changes considerably it's velocity, but if the collision is elastic, then the ball is reflected with the module of the velocity that is similar to the incoming one, before the collision.

Notes

Summary

17m 18s



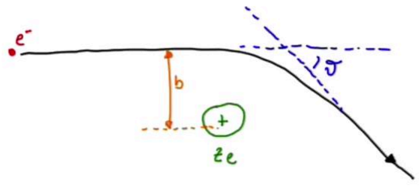
Therefore we have that because of conservation of the energy, of the electron energy, that the kinetic energy, after the collision, that is given by the sum of the parallel energy which has been deviated by Δv‖e, and the perpendicular energy [(Δv⊥e)²] is equal to the initial kinetic energy. From which by developing the square, and eliminating ve² and ve², we get that... ve (Δv‖e)+ 1/2 (Δv⊥e)² = 0. In developing this expression, we have neglected (Δv‖e)²; this is valid for collisions leading to small deviations. As we will see in a few moments, it's a reasonable assumption for most of the collisions that occur in a plasma. From which, by inserting into this expression, we obtain that d/dt(Δv‖e) = -4 π ni ve²(b_π/2)²... and this logarithm:... ln(λD/b_π/2) As a matter of fact, ln(λD/b_π/2) is coming back and it has a name, we give it a name in plasma physics. We introduced this so called *Coulomb logarithm* that we will denote as ln Λ capital lambda, and it is equal to ln(λD/b_π/2) And while λD can change a lot, b_π/2 can change a lot. However, the logarithm of the ratio is fairly constant throughout all the plasmas that we have in the universe. In fact, in most plasmas ln Λ is between 15 and 25.

Notes

Summary

19m 01s

Coulomb collisions in a plasma

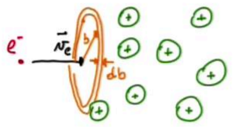


Two particles interacting through Coulomb force

From conservation of energy and angular momentum ($\frac{m_e}{m_i} \ll 1$)

$$\tan\frac{\vartheta}{2} = \frac{Z e^2}{4\pi\varepsilon_0 m_e v_e^2 b} = \frac{b_{\pi/2}}{b} \qquad \left(\text{when } b = b_{\pi/2},\ \vartheta = \frac{\pi}{2}\right)$$

In a plasma there are interactions with many particles. What is the comulative effect?



$\langle \Delta \vec{v}_{\perp e} \rangle = 0$ but $\langle \Delta v_{\perp e}^2 \rangle \neq 0$ ($\perp$ to the initial velocity)

$$\frac{d\langle \Delta v_{\perp e}^2 \rangle}{dt} = \int (\Delta v_{\perp e})^2 n_i v_e 2\pi b\, db = 8\pi n_i v_e^3 \int_0^{\lambda_D} \frac{(b_{\pi/2}/b)^2\, b\, db}{[1+(b_{\pi/2}/b)^2]^2} \underset{\lambda_D \gg b_{\pi/2}}{\simeq} 8\pi n_i v_e^3 b_{\pi/2}^2 \ln\frac{\lambda_D}{b_{\pi/2}}$$

$$\boxed{\Delta v_{\perp e}^2 = v_e^2 \sin^2\vartheta = 4 v_e^2 \sin^2\frac{\vartheta}{2}\cos^2\frac{\vartheta}{2} = 4 v_e^2 \tan^2\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2} = 4 v_e^2 \tan^2\frac{\vartheta}{2}\left[1+\tan^2\frac{\vartheta}{2}\right]^{-2}}$$

Observations

- Electrons do not lose much energy ($m_e \ll m_i$): $(v_e + \Delta v_{\parallel e})^2 + \Delta v_{\perp e}^2 = v_e^2 \Rightarrow v_e(\Delta v_{\parallel e}) + \frac{1}{2}\Delta v_{\perp e}^2 = 0 \Rightarrow \frac{d}{dt}\langle \Delta v_{\parallel e}\rangle = -4\pi n_i v_e^2 b_{\frac{\pi}{2}}^2 \ln\frac{\lambda_D}{b_{\frac{\pi}{2}}}$
- We introduce the Coulomb logarithm, $\ln\Lambda = \ln\left(\frac{\lambda_D}{b_{\frac{\pi}{2}}}\right)$. In most plasmas $15 < \ln\Lambda < 25$
- $\frac{d}{dt}\langle \Delta v_{\parallel e}\rangle = -\nu_{ei} v_e$, $\nu_{ei} = 4\pi n_i b_{\frac{\pi}{2}}^2 v_e \ln\Lambda$. As $\nu_{ei} = n_i \sigma_{ei} v_e$, $\sigma_{ei} = 4\pi b_{\frac{\pi}{2}}^2 \ln\Lambda$. We have $\frac{\sigma_{\frac{\pi}{2}}}{\sigma_{ei}} = \frac{\pi b_{\pi/2}^2}{4\pi b_{\frac{\pi}{2}}^2 \ln\Lambda} \ll 1$

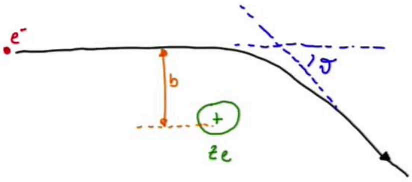
Plasma

The third observation is relative to what we have just derived now. The variation of the parallel energy can be expressed in terms of a frequency. That is the collision frequency of the electrons against the ions [$\boldsymbol{\nu}_{ei}$] times the incoming velocity. Where this collision frequency of the electrons against the ions is given by the expression that we can derive from here. And I realize that I forgot a squared here. Now the collision frequency is associated with a cross-section through the formula that follows: [$\boldsymbol{\nu}_{ei}$ = ni σ_{ei} ve] We have that the collision [cross-section], the effective collision [cross-section] for the incoming electrons impacting against the ions is equal to $\sigma_{ei} = 4\pi\ (b_{\pi/2})^2 \ln \Lambda$ And now it's interesting to compare this cross-section with the one that we had evaluated just before. It was the cross-section relative to short impact parameters $\pi\ (b_{\pi/2})^2$. This ratio will be equal to... $\pi\ (b_{\pi/2})^2$ divided by what we have just found here. And this is an easy ratio to evaluate because it will be given by $1/(4 \ln \Lambda)$. The Coulomb logarithm is of the order of 20, times 4 it will be of the order of 10^{-2}. It's much, much smaller than one. Conclusion?

Notes

Summary

21m 48s



Two particles interacting through Coulomb force

From conservation of energy and angular momentum ($\frac{m_e}{m_i} \ll 1$)

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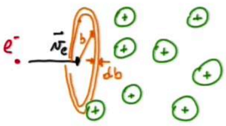

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Observations

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- We introduce the Coulomb logarithm, $\ln\Lambda = \ln\left(\frac{\lambda_D}{b_{\pi/2}}\right)$. In most plasmas $15 < \ln\Lambda < 25$
- $\frac{d}{dt}\langle \Delta v_{\parallel e}\rangle = -\nu_{ei} v_e$, $\nu_{ei} = 4\pi n_i b_{\pi/2}^2 v_e \ln\Lambda$. As $\nu_{ei} = n_i \sigma_{ei} v_e$, $\sigma_{ei} = 4\pi b_{\pi/2}^2 \ln\Lambda$. We have $\frac{\sigma_{\pi/2}}{\sigma_{ei}} = \frac{\pi b_{\pi/2}^2}{4\pi b_{\pi/2}^2 \ln\Lambda} \ll 1$

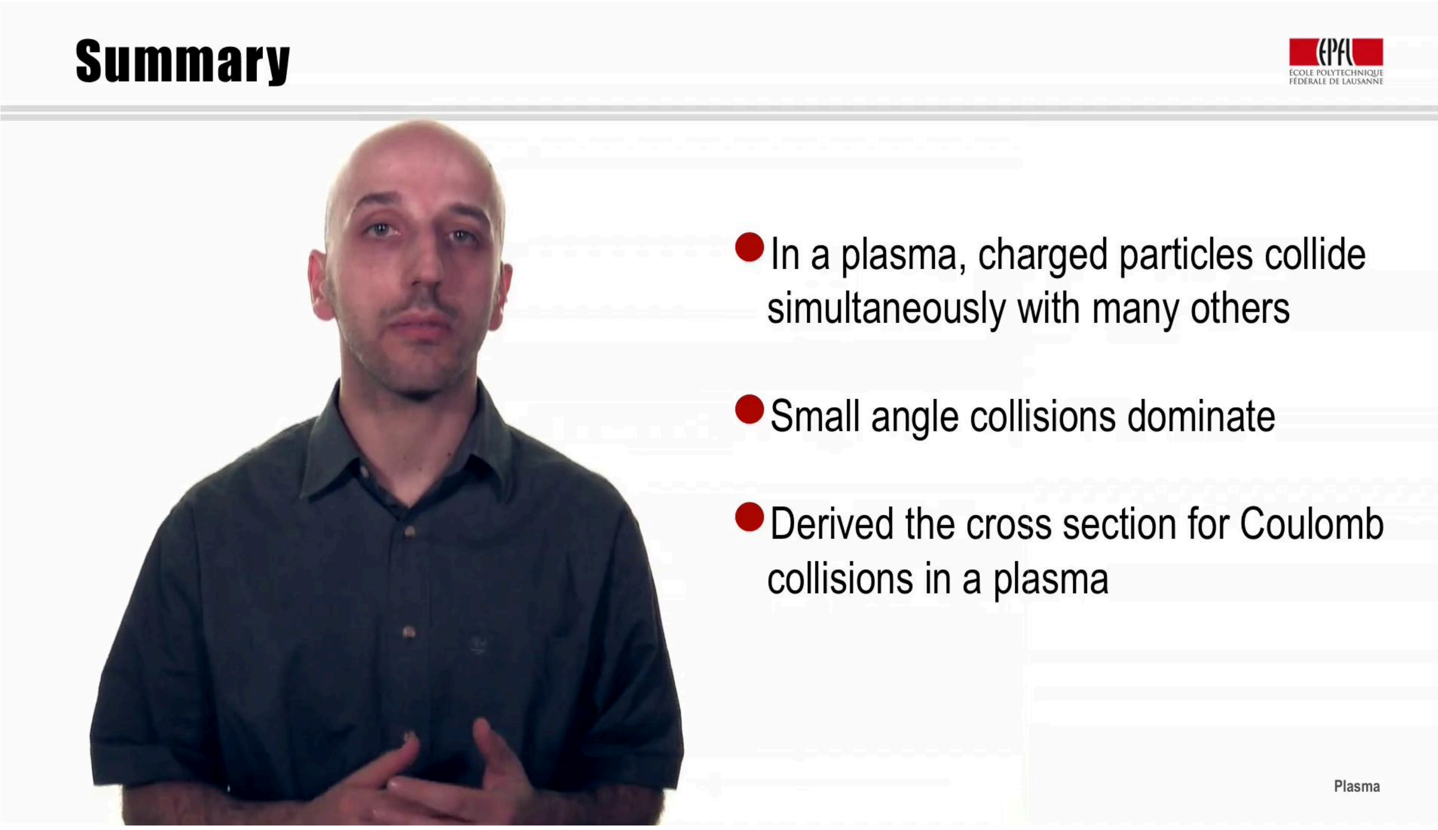
Plasma

The conclusion is that small angle deflections, -small angle collisions- play a dominant role with respect to large deflections in a plasma, as this ratio is much smaller than one. This is consistent with what we have done earlier when we have neglected (Δv∥e)².

Notes

Summary

24m 06s



To conclude the present module, let me say that what we have done was to introduce the main basic properties of Coulomb collisions in a plasma. We have started from a simple order of magnitude estimate and then we went more precise to study what the cross-sections are and frequency of Coulomb collisions in a plasma. We are now ready, -this is what we will do in the next module- to look at what the consequences of collisions are in a plasma.

Notes

Summary

24m 32s