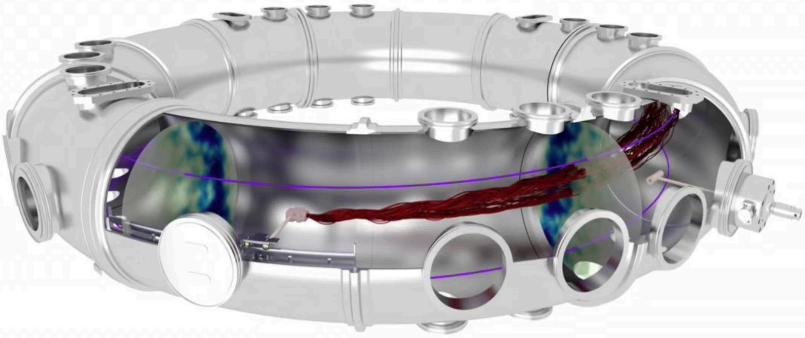
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

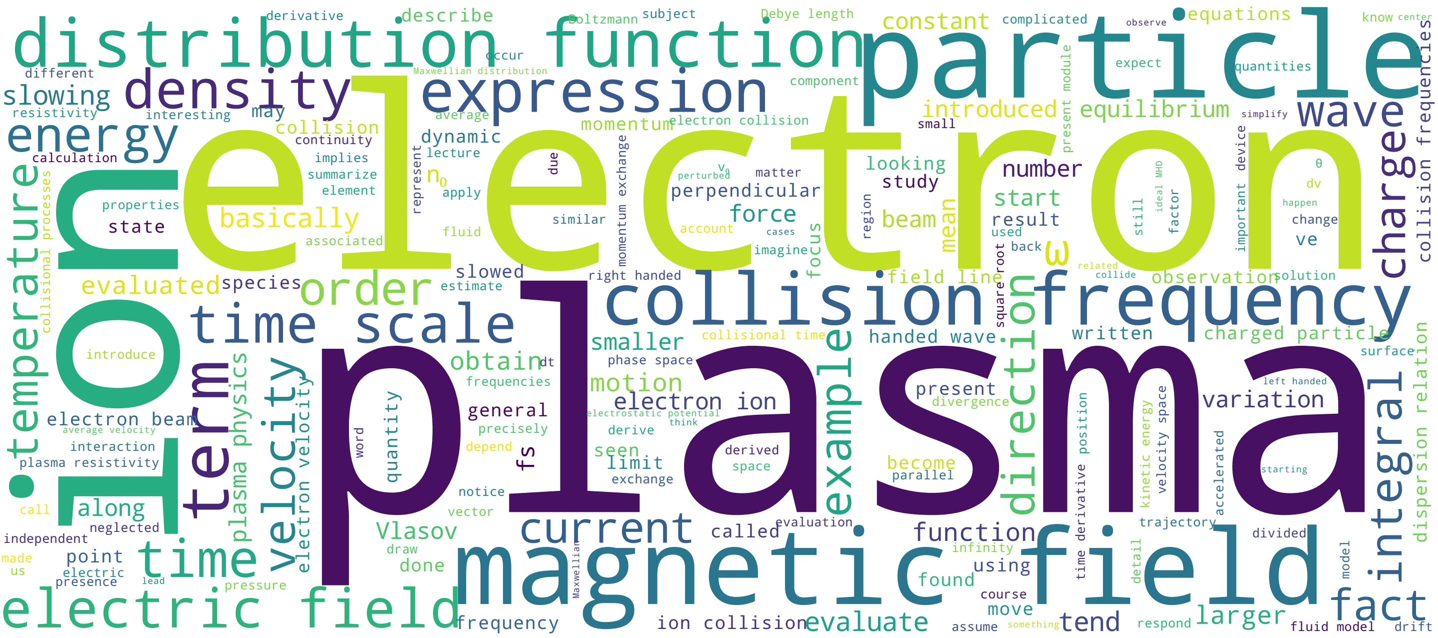


Collisional processes in plasmas

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

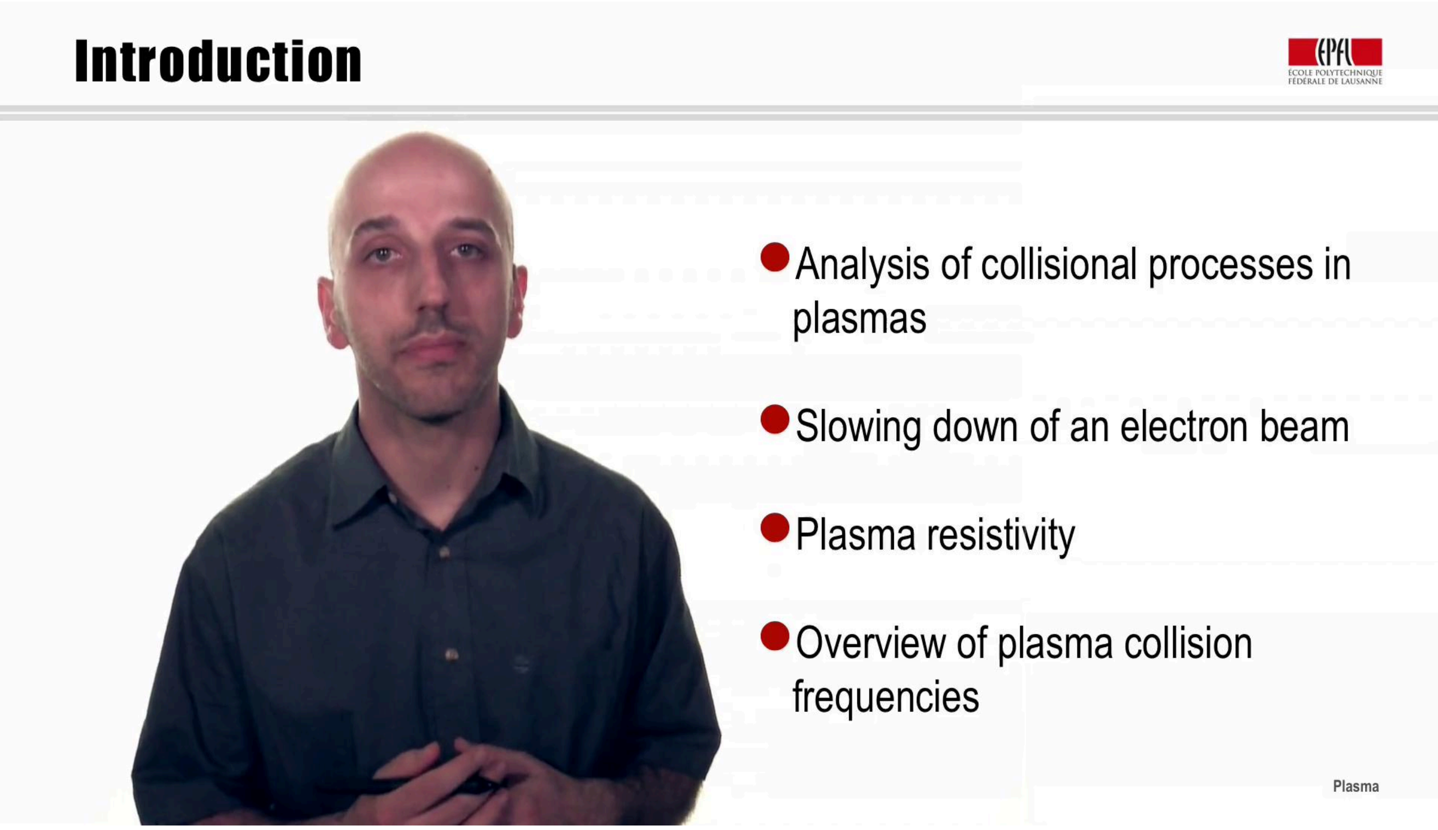
Lecture 2c

Paolo Ricci



Search MOOC

Video



In the past module we have introduced the basic properties of collisions, and Coulomb collisions in a plasma. We have now all the elements to study, to analyze the collisional processes in plasma, and we will focus on three of those. The first one is the slowing down of a beam that passes through a plasma. The second collisional process that we will look at is the plasma resistivity, and then we will order the collisional time scales present in a plasma.

Notes

Summary

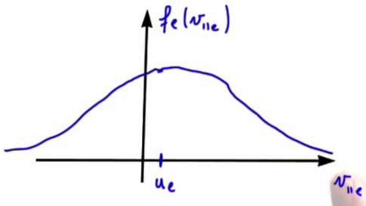
0m 05s

Slowing down of an electron beam in a plasma

We have evaluated $\frac{d}{dt}\langle \Delta v_{\parallel e} \rangle = -\nu_{ei}\, v_e = -4\pi n_i v_e b_{\pi/2}^2 \ln\Lambda = -\frac{n_i Z^2 e^4 \ln\Lambda}{4\pi \varepsilon_0^2 m_e^2 v_e^3}$

This allows to evaluate the slowing down of an electron beam in a plasma.

Focus on a Maxwellian beam $f_e(v_{\parallel e}) = n_0 \left(\frac{m_e}{2\pi v_{th,e}}\right)^{1/2} \exp\left[-\frac{m_e (v_{\parallel e}-u_e)^2}{2 v_{th,e}^2}\right]$



$u_e << v_{th}$

Plasma

More precisely, what we have evaluated was the slowing down of one single particle with the ve velocity, and this [d/dt (Δv∥e)], which is the variation of the parallel electron velocity is equal to the electron-ion collision frequency [ν_{ei}] times the electron velocity [ve]. This is equal to, we have found, -4 π ni ve² (b_π/2)² ln(Λ) And inserting now the expression we have for b_π/2 this will be equal to this expression here. This formula here actually allows us to evaluate the slowing down of an electron beam in a plasma. We will focus on a beam that is characterized by having a Maxwellian distribution function that is in the direction parallel to the incoming beam, to the velocity of the incoming beam. The distribution function will be given by the density times a normalization factor times this exponential where ue is the collective velocity of the beam, the average velocity of the beam. We can draw this distribution function. It's a function of v∥e, u* will be localized here, and the Maxwellian distribution function will develop around ue. We will make the assumption that ue is much smaller than v thermal e [vthe].

Notes

Summary

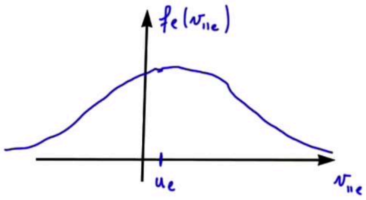
0m 38s

Slowing down of an electron beam in a plasma

We have evaluated $\frac{d}{dt}\langle \Delta v_{\parallel e}\rangle = -\nu_{ei}\, v_e = -4\pi n_i v_e b_{\pi/2}^2 \ln\Lambda = -\frac{n_i Z^2 e^4 \ln\Lambda}{4\pi\varepsilon_0^2 m_e^2 v_e^3}$

This allows to evaluate the slowing down of an electron beam in a plasma.

Focus on a Maxwellian beam $f_e(v_{\parallel e}) = n_0\left(\frac{m_e}{2\pi v_{th,e}}\right)^{1/2}\exp\left[-\frac{m_e(v_{\parallel e}-u_e)^2}{2v_{th,e}^2}\right]$



$u_e \ll v_{th,e}$

$$\frac{du_e}{dt} = -\langle \nu_{ei} v_{\parallel e}\rangle = \frac{-1}{n_0}\int \nu_{ei} v_{\parallel e} f_e(v_{\parallel e})\, dv_{\parallel e} \underset{u_e \ll v_{th,e}}{\simeq} -\langle \nu_{ei}\rangle u_e$$

where $\langle \nu_{ei}\rangle = \frac{\sqrt{Z}}{12\pi^{3/2}}\frac{n_i Z^2 e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}$

There are also collisions with plasma electrons. Scattering particles are no longer fixed, t

Plasma

Therefore, if you want, the spreading of this Maxwellian is much larger than the average electron velocity, Now in order to evaluate the slowing down of the beam, what we have to estimate is d/dt ue, the variation of the average velocity of the electrons that constitute our beam, and this will be equal to the frequency that we have just introduced here, [$\boldsymbol{\nu}_{ei}$] the electron-ion collision [frequency] times the parallel electron velocity averaged over the distribution function of the beam. In other words, this will be equal to -1/n_0 and then this quantity averaged over the distribution function, and in the case that we are looking at here, of ue << vthe, this is approximately equal to -< $\boldsymbol{\nu}_{ei}$ > ue, where... ... where this is the result of the averaged collision frequency that you find if you carry out this integral in the limit of ue << vthe, as we have given here. Okay? This collision frequency gives the slowing down of the electron beam due to the interaction with the ions. There are, however, also collisions of this electron beam with the electrons that are present in a plasma. So in general, in this case of electrons scattering against other electrons, one cannot assume any more that the target particles are fixed, and the calculation of the scattering angle has to be done again.

Notes

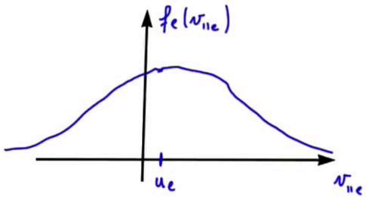
Summary

3m 07s

Slowing down of an electron beam in a plasma

We have evaluated $\frac{d}{dt}\langle \Delta v_{\parallel e}\rangle = -\nu_{ei}\, v_e = -4\pi n_i v_e b_{\pi/2}^2 \ln\Lambda = -\frac{n_i Z^2 e^4 \ln\Lambda}{4\pi \varepsilon_0^2 m_e^2 v_e^3}$

This allows to evaluate the slowing down of an electron beam in a plasma.



Focus on a Maxwellian beam $\quad f_e(v_{\parallel e}) = n_0 \left(\frac{m_e}{2\pi v_{th,e}}\right)^{1/2} \exp\left[-\frac{m_e (v_{\parallel e}-u_e)^2}{2 v_{th,e}^2}\right]$

$$\frac{du_e}{dt} = -\langle \nu_{ei} v_{\parallel e}\rangle = \frac{-1}{n_0}\int \nu_{ei} v_{\parallel e} f_e(v_{\parallel e})\, dv_{\parallel e} \underset{u_e \ll v_{th,e}}{\simeq} -\langle \nu_{ei}\rangle u_e$$

$u_e \ll v_{the}$

where $\quad \langle \nu_{ei}\rangle = \frac{\sqrt{Z}}{12\pi^{3/2}} \frac{n_i Z^2 e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}$

There are also collisions with plasma electrons. Scattering particles are no longer fixed, the analysis is more complicated, however the deflection is similar

$$\langle \nu_{ee}\rangle \sim \frac{n_e e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}} \sim \frac{\langle \nu_{ei}\rangle}{n_i Z^2/n_e}$$

Plasma

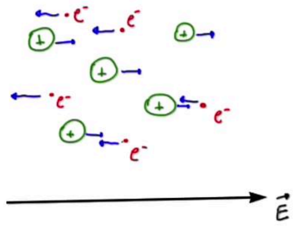
In general the analysis is more complicated. However, we expect that the deflection of the electrons impacting against electrons will be similar to the one of an the electron impacting against an ion. And without carrying out all the detailed calculation, we can write that the electron-electron collision frequency [$\boldsymbol{\nu}_{ee}$] can be estimated to be given by this expression, which basically corresponds to the electron-ion collision frequency, taking into account that ions and electrons may have different densities, and that [ions] have Z times the electron charge.

Notes

Summary

5m 52s

Plasma resistivity



$\vec{E}$ drives a current in a plasma.

We focus on the electrons (higher mobility)

$$m_e n_e \frac{d\vec{u}_e}{dt} = -e n_e \vec{E} + \vec{R}_{ei}$$

Plasma

Okay, now we have all the elements to evaluate resistivity in a plasma. Let's imagine we have a plasma, we apply an electric field, charged particles will start to move, responding to this electric field. However, their motion will be slowed down because of the presence of collisions. And these overall dynamics can be represented as an effective resistivity of the plasma. So let's evaluate this resistivity. Our plasma will be made by ions, and electrons. Now, we apply an electric field, E, and particles will respond to this electric field. The ions will start to move in the direction of the electric field, along the electric field, while the electrons will be pushed against the electric field. This effectively will result in a current in a plasma. Electrons and ions will tend to respond in a different way. As a matter of fact, the ions are much heavier, and they will respond to the electric field less promptly than the electrons. Therefore, what we will do, we will focus on the electron dynamics which have a higher mobility. And the electrons will satisfy -will follow- an equation of motion, which states that they will be accelerated by the electric force, and then slowed down by collisions, collisions with the ions.

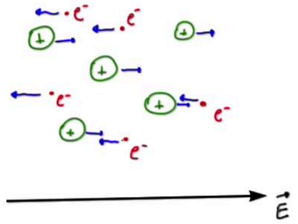
Notes

Summary

6m 59s

Plasma resistivity

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE



$\vec{E}$ drives a current in a plasma.

We focus on the electrons (higher mobility)

$m_e n_e \frac{d\vec{u}_e}{dt} = -e n_e \vec{E} + \vec{R}_{ei}$, being $\vec{R}_{ei} = -m_e n_e \langle \nu_{ei} \rangle (\vec{u}_e - \vec{u}_i)$ $\left(u_e \ll v_{th,e} \right)$

Observations

- Steady-state after a transient $\Rightarrow \frac{d}{dt} = 0$
- $\vec{j} = -n_e e (\vec{u}_e - \vec{u}_i)$

$\Rightarrow e^2 n_e \vec{E} = -m_e \langle \nu_{ei} \rangle e n_e (\vec{u}_e - \vec{u}_i) = m_e \langle \nu_{ei} \rangle \vec{j}$ $\Rightarrow$ $\vec{E} = \frac{m_e \langle \nu_{ei} \rangle}{e^2 n_e} \vec{j}$

Compare with Ohm's law $\vec{E} = \eta \vec{j}$ $\Rightarrow$ $\boxed{\eta = \frac{2^{\frac{1}{2}} m_e^{\frac{1}{2}} Z e^2 \ln\Lambda}{12 \pi^{\frac{3}{2}} \varepsilon_0^2 T_e^{3/2}}}$

Plasma

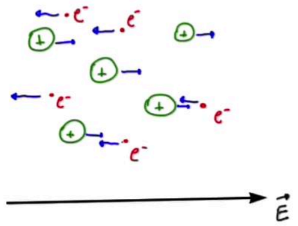
Where the collisions [term] with the ions -we have just evaluated- is given by the effective collision frequency for electrons against the ions that we have evaluated in the limit of the drift velocity, so the velocity responsible for the current, being much smaller than the thermal velocity. Now, in order to make some progress, let me make a couple of observations. The first thing is that after a transient, where particles are accelerated, and then slowed down by collision, there will be a steady state where the d/dt term will be equal to zero. The second thing we notice is that in general, j [current density] is written as - ne e (ue -ui) And if we multiply this equation now by e, we obtain this expression, where we have explicitly written the Rei term, and what we have here, actually, is the current. This will be actually given by, actually equal to me, collision frequency and current, which implies that E is equal to E = me <$\boldsymbol{\nu}_{ei}$>/(e^2 ne) j Now, we can compare this with Ohm's law that we know from medium, from all the other media, from metals, etc., It says that E = η j from which we obtain, by comparing these two expressions, and introducing the evaluation of the collision frequency that we have given previously, we obtain that the resistivity in a plasma is given by this expression, which represents the plasma resistivity due to electron-ion collisions.

Notes

Summary

9m 00s

Plasma resistivity



$\vec{E}$ drives a current in a plasma.

We focus on the electrons (higher mobility)

$$m_e n_e \frac{d\vec{u}_e}{dt} = -e n_e \vec{E} + \vec{R}_{ei} \quad , \text{ being } \quad \vec{R}_{ei} = -m_e n_e \langle \nu_{ei} \rangle (\vec{u}_e - \vec{u}_i) \quad (u_e \ll v_{th,e})$$

Observations

- Steady-state after a transient $\Rightarrow \frac{d}{dt} = 0$
- $\vec{j} = -n_e e (\vec{u}_e - \vec{u}_i)$

$$\Rightarrow e^2 n_e \vec{E} = -m_e \langle \nu_{ei} \rangle e n_e (\vec{u}_e - \vec{u}_i) = m_e \langle \nu_{ei} \rangle \vec{j} \quad \Rightarrow \quad \vec{E} = \frac{m_e \langle \nu_{ei} \rangle}{e^2 n_e} \vec{j}$$

Compare with Ohm's law $\vec{E} = \eta \vec{j}$ $\Rightarrow$

$$\boxed{\eta = \frac{2^{\frac{1}{2}} m_e^{1/2} Z e^2 \ln\Lambda}{12 \pi^{3/2} \varepsilon_0^2 T_e^{3/2}}}$$

Plasma resistivity
- decreases with T_e!
- independent of n!

Plasma

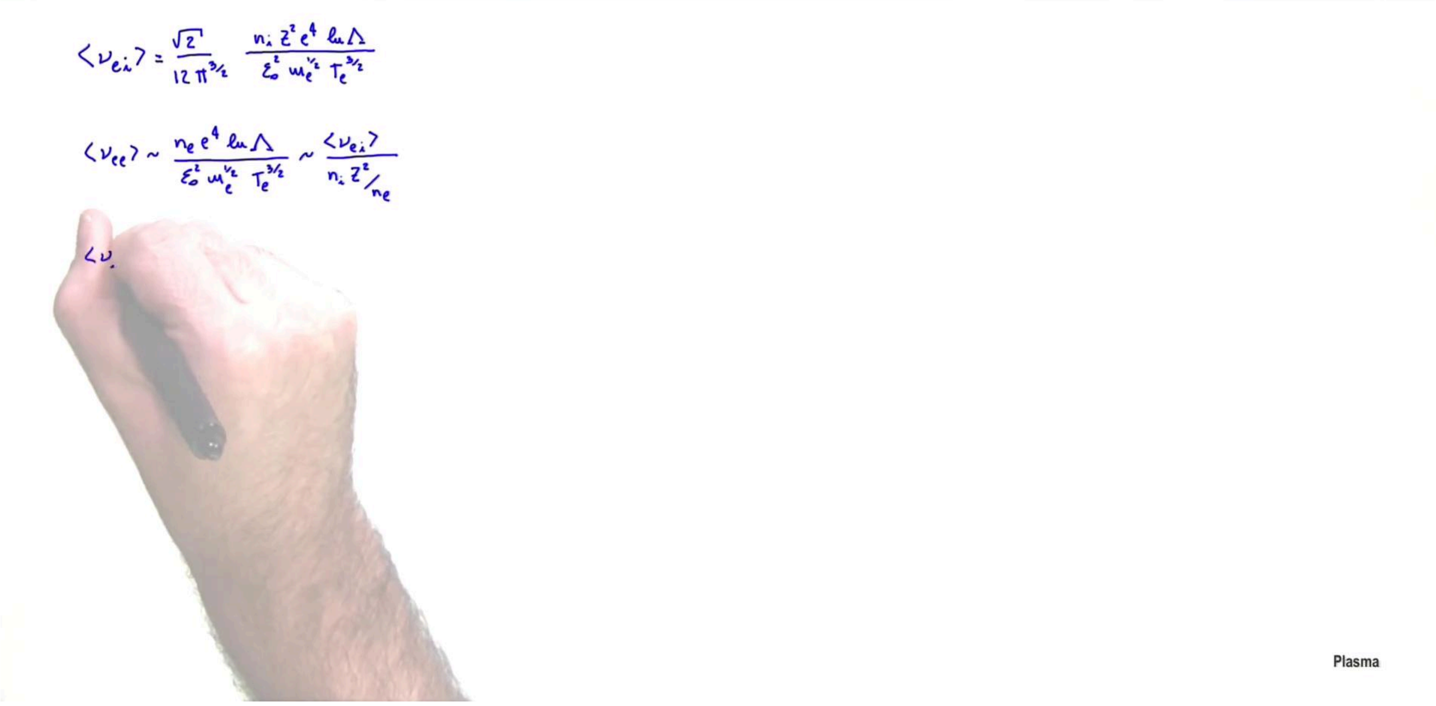
Two observations here. The first one is related to the dependence of the resistivity of the temperature, and what we observe is that the resistivity decreases with the temperature. This is fairly surprising, if you want, if you think about metals, the resistivity definitely increases with the temperature. And second observation is that this resistivity is independent of n, the density. Nowhere is there density, the plasma density. Why is that? Well, basically the reason is that while the number of carriers, of current carriers, increases with the density, also the collisions increase with the density, and the net effect will be that the resistivity is independent of density.

Notes

Summary

12m 05s

Overview of plasma collision frequency



We have evaluated the effective collision frequency related to the slowing down of an electron beam entering a plasma. We've also mentioned that the same beam can be slowed down by interaction with the other electrons. There are many collision frequencies that we can think of, ions impacting against other ions, or ions against electrons, and also we may be interested, for example, not only in slowing down and therefore the exchange of momentum, but rather on the exchange of energy. There are a number of collision frequencies, so what I'll try to do now is to give another view of all of them. The first collision frequency that we have just introduced was the collision frequency of the electrons against the ions, and this is one that we have evaluated rigorously, and we have found that it's equal to this expression here. We had briefly mentioned the electron-electron collisions. We did not go into the details of the derivation, we've just shown that the order of magnitude [of $\boldsymbol{\nu}_{ee}$] should be similar to the one for the electron-ion. And then, based on the electron-electron collision frequency, we can imagine what is the collision frequency for ions against the ions: this will basically be made by replacing the electron mass with the ion mass and the electron temperature, with the ion temperature.

Notes

Summary

13m 01s

Overview of plasma collision frequency

$$\langle \nu_{ei} \rangle = \frac{\sqrt{2}}{12\pi^{3/2}} \frac{n_i Z^2 e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}$$

$$\langle \nu_{ee} \rangle \sim \frac{n_e e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}} \sim \frac{\langle \nu_{ei} \rangle}{n_i Z^2 / n_e}$$

$$\langle \nu_{ii} \rangle \sim \frac{n_i Z^4 e^4 \ln\Lambda}{\varepsilon_0^2 m_i^{1/2} T_i^{3/2}} \sim Z^2 \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_e}{T_i}\right)^{3/2} \langle \nu_{ei} \rangle$$

Collision frequencies for momentum exchange

Energy gained by the ions when an electron beam enters a plasma?

$$m_e \Delta \vec{v}_e = m_i \Delta \vec{v}_i \Rightarrow \frac{1}{2} m_i |\Delta \vec{v}_i|^2 = \frac{m_e^2}{2 m_i} |\Delta \vec{v}_e|^2 = \frac{m_e^2}{2 m_i} |\Delta \vec{v}_{\perp e}|^2 \quad (\text{as } v_e \Delta v_{\parallel e} \sim \Delta v_{\perp e}^2)$$

$$\text{Since } \frac{d \langle \Delta v_{\perp e}^2 \rangle}{dt} = \frac{n_i Z^2 e^4 \ln\Lambda}{2\pi \varepsilon_0^2 m_e^2 v_e} \Rightarrow \langle \nu_E \rangle = \frac{n_i Z^2 e^4}{\quad}$$

Plasma

Besides, of course, the density and the charge. Now, these three collision frequencies are collision frequencies for momentum exchange. They represent the change of velocity, therefore the change of momentum of impacting particles. However, particles also exchange energy, for example, there will be some amount of energy that is gained by the ions when an electron beam enters in a plasma. What is this energy ? Because of momentum conservation, we will have that me Δve has to be equal to mi Δvi which implies, for the variation of energy for ions that 1/2 mi $(\Delta vi)^2$, this, by using this expression for Δvi will be equal to $me^2/(2\ mi)\ (\Delta ve)^2$ And now, as Δv⊥e is much larger than Δv∥e we will have that this is equal to $me^2/(2\ mi)\ (\Delta v\perp e)^2$ Why is that? Why is Δv⊥e much larger than Δv∥e? It's because, as we've derived in the previous module, we have that ve Δv∥e is comparable to $(\Delta v\perp e)^2$, and as 1/2 me ve^2 is much larger than the variation of the energy, we will have that Δv∥e can be neglected here in the evaluation of Δve. Now, as we have evaluated the variation in time of Δv⊥e, that is... we will have that the frequency at which energy is exchanged is equal to...

Notes

Summary

14m 47s

Overview of plasma collision frequency

$$\langle \nu_{ei} \rangle = \frac{\sqrt{2}}{12\pi^{3/2}} \frac{n_i Z^2 e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}$$

$$\langle \nu_{ee} \rangle \sim \frac{n_e e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}} \sim \frac{\langle \nu_{ei} \rangle}{n_i Z^2 / n_e}$$

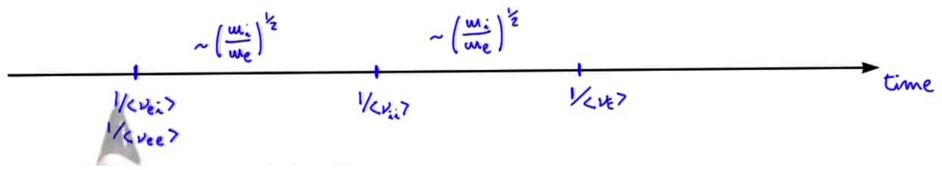
$$\langle \nu_{ii} \rangle \sim \frac{n_i Z^4 e^4 \ln\Lambda}{\varepsilon_0^2 m_i^{1/2} T_i^{3/2}} \sim Z^2 \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_e}{T_i}\right)^{3/2} \langle \nu_{ei} \rangle$$

Collision frequencies for momentum exchange

Energy gained by the ions when an electron beam enters a plasma?

$$m_e \Delta \vec{v}_e = m_i \Delta \vec{v}_i \Rightarrow \frac{1}{2} m_i |\Delta \vec{v}_i|^2 = \frac{m_e^2}{2 m_i} |\Delta \vec{v}_e|^2 = \frac{m_e^2}{2 m_i} |\Delta \vec{v}_{\perp e}|^2 \quad (\text{as } v_e \Delta v_{\parallel e} \sim \Delta v_{\perp e}^2)$$

Since $\frac{d \langle \Delta v_{\perp e}^2 \rangle}{dt} = \frac{n_i Z^2 e^4 \ln\Lambda}{2\pi \varepsilon_0^2 m_e^2 v_e} \Rightarrow \langle \nu_E \rangle = \frac{n_i Z^2 e^4 m_e^{1/2} \ln\Lambda}{3\pi (2\pi)^{1/2} \varepsilon_0^2 m_i T_e^{3/2}} \sim 2 \left(\frac{m_e}{m_i}\right) \langle \nu_{ei} \rangle$



Plasma

which can be estimated, looking at what is the expression, for the $\langle \nu_{ei} \rangle$ collision frequency for momentum exchange, as $\langle \nu_E \rangle = 2(m_e/m_i) \langle \nu_{ei} \rangle$. This is, as a matter of fact, a much smaller quantity than the collision frequency for momentum exchange of electrons with respect to ions. So here we are ready to summarize the results that we've found throughout these collision frequencies, and if we represent time, we will see that basically there are three families of collisions. The first one, which is the fastest one, which corresponds to a very fast time scale, is the one for electron-ion and electron-electron collisions. And then, after a time scale that is of the order of square root of mi/me, that in the case of hydrogen is about 40, then we will have the ion collisions time scale, and then again, after a time scale a factor of square root of (me/mi), we will have the exchange of energy. So what will happen in a plasma? Well, in a relatively short time scale, electrons will collide with ions and with the other electrons. This will lead to isotropization of the electron distribution function, which will therefore tend to a Maxwellian.

Notes

Summary

18m 06s

Overview of plasma collision frequency

$$\langle \nu_{ei} \rangle = \frac{\sqrt{2}}{12\pi^{3/2}} \frac{n_i Z^2 e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}$$

$$\langle \nu_{ee} \rangle \sim \frac{n_e e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}} \sim \frac{\langle \nu_{ei} \rangle}{n_i Z^2 / n_e}$$

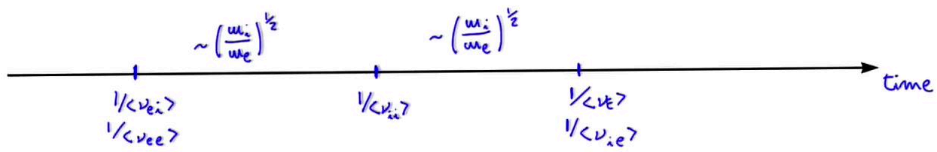
$$\langle \nu_{ii} \rangle \sim \frac{n_i Z^4 e^4 \ln\Lambda}{\varepsilon_0^2 m_i^{1/2} T_i^{3/2}} \sim Z^2 \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_e}{T_i}\right)^{3/2} \langle \nu_{ei} \rangle$$

Collision frequencies for momentum exchange

Energy gained by the ions when an electron beam enters a plasma?

$$m_e \Delta \vec{v}_e = m_i \Delta \vec{v}_i \Rightarrow \frac{1}{2} m_i |\Delta \vec{v}_i|^2 = \frac{m_e^2}{2 m_i} |\Delta \vec{v}_e|^2 = \frac{m_e^2}{2 m_i} |\Delta \vec{v}_{\perp e}|^2 \quad (\text{as } v_e \Delta v_{\parallel e} \sim \Delta v_{\perp e}^2)$$

Since $$\frac{d(\langle \Delta v_{\perp e}^2 \rangle)}{dt} = \frac{n_i Z^2 e^4 \ln\Lambda}{2\pi \varepsilon_0^2 m_e^2 v_e} \Rightarrow \langle \nu_E \rangle = \frac{n_i Z^2 e^4 m_e^{1/2} \ln\Lambda}{3\pi (2\pi)^{1/2} \varepsilon_0^2 m_i T_e^{3/2}} \sim 2 \left(\frac{m_e}{m_i}\right) \langle \nu_{ei} \rangle$$



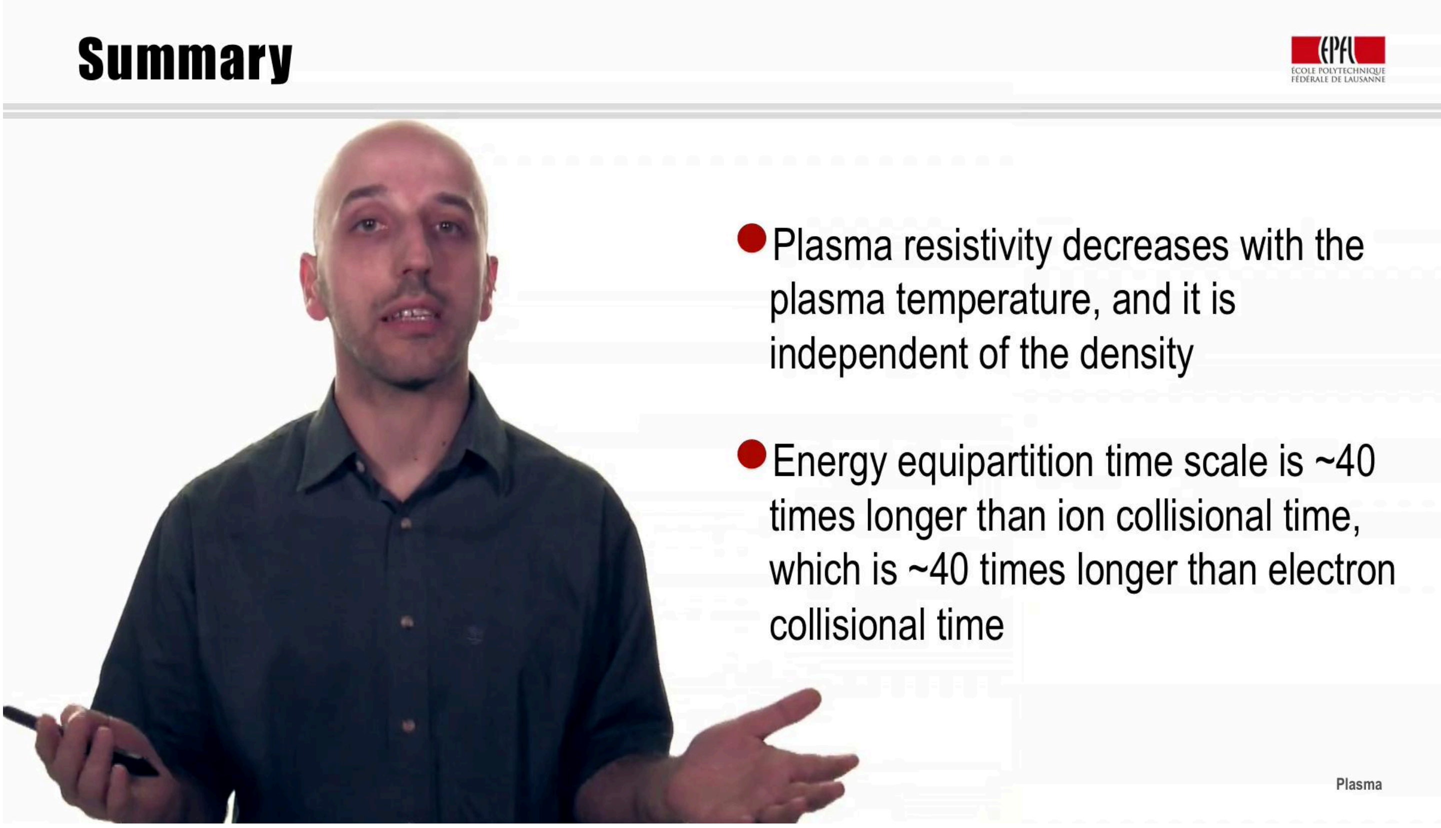
Plasma

On a time scale that is longer by a factor of 40, the ions will collide with the other ions, basically the ion distribution function will tend to thermalize, and after a factor of 40 to time scale, so on a much longer time scale, by an order of 40, then the electrons and the ions that have thermalized within each other will exchange energy, and they will tend to reach the same temperature. There is still one time scale that I would like to add, which we did not derive rigorously, however one can find that it's similar to the energy exchange collision frequency, and it's the one of the ions colliding against the electrons. So the slowing down of an ion beam entering a plasma because of collisions with the electrons. This may be important in some processes that occur in fusion plasmas.

Notes

Summary

20m 06s



We have considered a number of collisional processes that take place in a plasma. Well, let me just summarize the most important results. The first one concerns the plasma resistivity we have found, and this is quite surprising, that the plasma resistivity decreases with the temperature, and is independent of the density. We also looked at the collisional time scales present in a plasma, and we have seen that the energy equipartition time scale is about 40 times longer than the ion collisional time scale, which in turn is about 40 times longer than the electron collisional time scale. With this, with the present module, we conclude the detailed analysis of collisional processes in a plasma. In the next module, we will go back to the Boltzmann equation, and we will simplify it, assuming that we are looking at cases where collisions in plasmas can be neglected, and therefore deriving the equation that describes the dynamics of a collisionless plasma.

Notes

Summary

21m 14s